5

Introduction to Plane Geometry, Line and Angle

5.01 Historical Introduction

Harappa and Mohan-jo-dro (now in Pakistan), Kalibanga (Rajasthan) and Lothal (Gujrat) testify clearly that a rich civilization flourished in a large track of land during the period extending from 2500 B.C. to 1750 B.C. in ancient India. The relics of this civilization prove that the people of this period had special knowledge of Geometry and Geometrical formations. On the basis of this knowledge they constructed buildings, roads, circles, arcs where mensuration is of great importance. Babylonian has also fomulated formulae for finding out the area of various linear shapes which are available in Rhind Papyrus (1650 B.C.)

5.02 History of Indian Geometry

India has been the birth place of Geometry too. The foundation of Geometry had been laid in the shulav age or in the age of Vedang and Astronomy. During this period it was known by various names, such as Shulav mathematics, Shulav Science, Rajju Mathematics, Rajju-Figures. Rajju being synonym of shulav it began to be called Rajju Mathematics which later on changed to Geometry.

Similarly such terms as Rajju Mathematics, Kshetra Samas, Kshetra behaviour, Mensuration, Geometry and Boomiti were used for the work of measuring fields. Since ancient times altras for the performance of yagya used to be built. Geometry was the basis of their construction.

It is said in astronomy that :-

"Veda hi yajanartham bhi pravaritah"

That is , the Vedas also have been used in the work of performing yagya. According to the need of various yajanans (yagya). Different types of altars were required to be built. For this, various shape such as rectangular arcs, rectangular, triangular etc. were developed while constructing an altar care was taken that the areas of all they altars must be equal to the area of standard altar. Hence for this knowledge of geometrical formation such as forming a square on straight line, converting it into circle equal to the area of the square drawing a circle around the square, drawing a circle within the square and doubling the area of the circle etc. was very essential.

If we think a bit seriously, we come to know that two words are very important in Geometry. They are Rajju (rope) and measurement. Hence the science or mathematics which was developed with the help of Shulav, began to be called Shulav science and Shulav mathematics. Indian mathematicians contributed a lot in this field. They formulated formulae for forming various shapes which were known by their names such as Baudhayan Shulav Formula. Apastamb Shulav formula, Katyayan Shulav formula, Manav Shulav formula, Mestrayan Shulav formula, Varah Shulav formula, Baudhav Shulv formula etc.

Among the main achievements of Shulav period "Sumkon Tribhuz ka Prameya" (theorem of right angle tringle), that is a square formed on hypotenuse is equel to the sum of the squares formed on remaining two arms. This theorem was widely used in India many centuries before Pythagoras (580 B.C.).

Baudhayan Therem (800 B.C.)

"Deergh chatur srisya khasnya Rajjuh Parshavmani
Tiryankmani yatprithambhute kurutastadubhayam karoti"

The sum of the areas of squares formed on the perpendicular line and the base line of a rectangle is equal to the area of the square formed on the diagonal. It is worth noting that Pythagoras (580 B.C.) established this theorem about 300 years after 'Baudhayan'. Hence it is proper to call this theorem as Baudhayan theorem.

Among Indian Geometricians are Bhraham Gupta (598 A.D.) who found out the area of cyclic quadrilateral in terms of its perimeter and rams, Arya Bhat (476 A.D.) who found out the area of equilateral tringle, volume of pyramid and value of π and Bhaskar-II (1114 A.D.) who proved the Baudhayan theorem, by split method.

Later on the Greek mathematicians (300 B.C.) systematized this knowledge by providing its facts through inductive reasoning and published it in the book titled 'Elements". These days we study Geometry in this way.

5.03 Basic Concepts

Geometry is studied by taking some basic concepts as basis. These basic concepts are understood by examples and experiences. For these, no proofs are given. There are three basic concepts in the study of Geometry which are very important. These are (1) Point (2) Line and (3) Surface. Now we shall study these with the help of some examples.

(1) Point : A minute sign made by a fine pointed pencil, the corner of the black board are the examples of a point. If the point of the pencil is fine, it will make a fine point. Generally the points are shown by the capital letter of English alphabet that is A, B, C, D etc.

(2) Line : If we fold a piece 3 of paper, a line is formed on the surface of the piece of paper. Line is also regarded as a concepts in the same way as a point is regarded. In the figure 5.01, the line AB is shown by AB. The line can also be shown by small letters of English alphabet such as *l, m, n*.

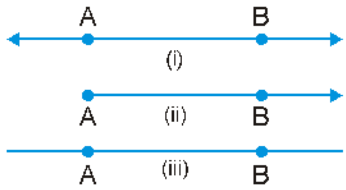


Fig. 5.01

(3) Plane : Floor, roof, wall, table are similar examples of plane. Although there are some dimensions of these examples, yet the geometrical planes extend endlessly in all the directions. A plane is extended in length and width. is has no thickness.

(4) Postulates : In Geometry there are some concepts which are regrarded as true without any proof and on the basis of which other geometrical facts are proved. such facts are called 'Postulates'. Some of main postulates are-

(i) There are infinite points on a line.

(ii) A line can be extended as much as we desire.

(iii) Infinite number of lines can be drawn from a point.

(iv) One and only one straight line can be drawn through two points.

(v) One and only one line can be drawn from a point paralled to a line.

(vi) All the right angles are equal.

(vii) Like supplementary and complementary angles are equal to each other.

(viii) A line segment can be bisected at one point only.

(ix) An angle can be bisected by one line only.

5.04 Inductive and Deductive Reasoning

Various rules which are established by various examples. Empirical findings are called inductive reasoning. Such findings are always true.

A special method of reasoning wherein the rules are proved with the help of evidences is known as deductive reasoning.

Theorem (Prameya) and Construction (Nirmeya) :

(1) Theorem : The rules which are verified by the inductive reasoning are called theorem. In geometry following steps are adopted in proving a theorem.

Corollary : On proving the theorem some results are drawn, which are understood easily. Such results are called “Corollary”.

Constructions : The geometrical forms formed by using geometric rules are called 'Construction'.

5.05 Geometric Symbols

The terms used in geometry are written in he form of symbols. Symbols of some words are given in the following table :

S.No.	Words	Symbol	S.No.	Words	Symbol
1.	Because	∵	8.	Right angle	∟
2.	Therefore	∴	9.	Perpendicular	⊥
3.	Greaterthan	>	10.	Triangle	△
4.	Lessthan	<	11.	Parallel	||
5.	Congruent	≅	12.	Circle	○
6.	Similar	~	13.	Arc	⌒
7.	Angle	∠	14.	Not equal to	≠

5.06 Angle and its Measurements

Angle : Any two rays whose starting point is same make an angle. In fig 5.02, starting from O and two rays $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are starting. In this figure angle at point O is known as angle. According to figure, one ray is at point A and second ray is at point B, then this angle can be expressed as $\angle AOB$ or $\angle BOA$,OA and OB are sides of or Common point O is known as vertex of angle. Sometimes for convenience an angle is denoted by numbers or words as given is fig. 5.03 and 5.04.

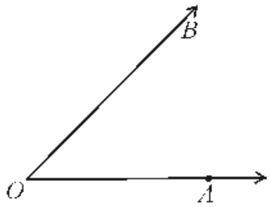


Fig. 5.02

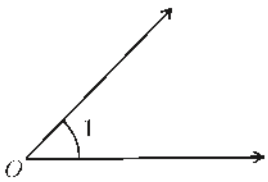


Fig. 5.03

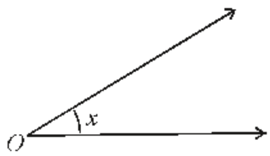


Fig. 5.04

Measurement of Angle : Suppose a revolving line with respect to point O is in OA position reaches in OB position as shown in fig. 5.05, then this revolving position is known as $\angle AOB$ and quantity of this angle is known as its measurement.

If a line OA by completing a round around O and comes in its original position, then measurement of such angle is distrubuted is 360 equal parts and it is denoted by 360^0 (degree) so similiarly 1 Part =1 degree=1^0 and 360 part = 360^0.

In rotation form 1 degree; 1 minute and 1 seconds represents 1^0, 1 and 1" respectively.

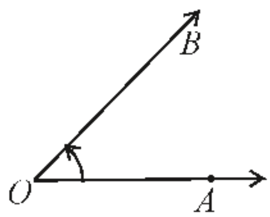


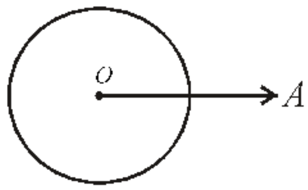
Fig. 5.05

If every degree is divided into 60 equal parts, then such each part is known as one kala (1 minute), if one minute is divided into 60 equal parts, then each part is known as vikala (second).

$1^0 = 60$ (minutes) = 60 kala

1 kala (minuts) = 60 vikala

In symbolic form of one degree, one minute and one second can be expressed 1^0, 1' and 1" For measurement of angle use of a protractor, which have 0^0 to 180^0 scale.



(1 round = 360°)

Fig. 5.06

Vertically Opposite Angles

Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays. In Fig. 5.07 lines AB and CD intersects each other at point O and makes angle $\angle AOC$ and $\angle BOD$, $\angle AOD$, $\angle BOC$ which are vertically opposite angle.

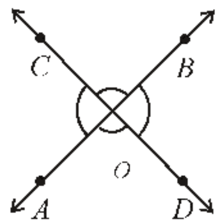


Fig. 5.07

Angle around a point

The sides of vertically angles are opposite rays.

Angle on around of a point : if various rays are starting from a point and angles obtained in this manner are know as angles around a point. As we studied that in this measurement an angle made around a point by revolving ray is $360°$.

Here, sum of angle around point O is $360°$.

or, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360°$

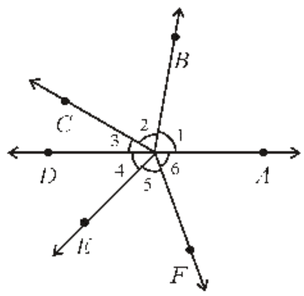


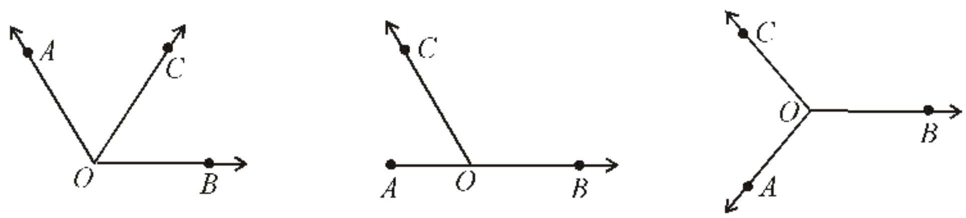
Fig. 5.08

5.07 A linear Pair of Angles

In Fig. 5.09, carefully observe the pair of angles.

Every pair of angles ($\angle AOC$ and $\angle BOC$) are adjacent angles.

In Fig. 5.09 (ii), a pair of angle is such that their total is 180^0 such angle is known as linear angle combination. It is clear that linear angle combination of adjacent angles are supplementary.



(i) (ii) (iii)

Fig. 5.09

Defination :

If the non-common arms of two adjacent angles form a line, then these angles are called linear pair of angles.

Linear Pair Axiom

Theorem 5.1 : If two lines intersect each other, then the vertically opposite angles are equal to each other.

Given : Lines AB and CD intersect each other at point O.

To Prove : Vertically opposite angles

$\angle AOC = \angle DOB$ and $\angle AOD = \angle BOC$

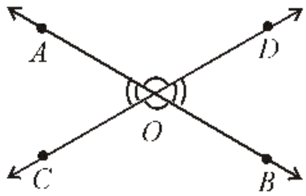


Fig. 5.10

Proof:

$\because$ Ray OD stands on line AB.

$\therefore \quad \angle AOD + \angle DOB = 180°$ (By linear pair Axiom) ....(i)

Similiary, $\quad \angle AOC + \angle AOD = 180°$ ......(ii)

From equation (i) and (ii), we get

$$\angle AOD + \angle DOB = \angle AOC + \angle AOD$$

$$\Rightarrow \quad \angle AOC = \angle DOB$$

Similarly, we can prove that :

$$\angle AOD = \angle BOC$$

Hence proved.

Cordallary 1 : If two or more than two lines intersect each other on a point, then the sum of the angles on intersecting point is $360°$.

Cordallary 2 : Bisector of vertically opposite angles are in a line.

Illustrative Examples

Example 1. In Fig. 5.11, $\angle 1$ and $\angle 2$ are linear pair $\angle 2 - \angle 1 = 18°$. If then find out the value of $\angle 1$ and $\angle 2$.

Sol. : $\angle 2 + \angle 1 = 180°$ (Linear pair axiom) ....(1)

Given, $\quad \angle 2 - \angle 1 = 18°$ ....(2)

On adding equation (1) and (2), we get

$$\angle 2 = \frac{198}{2} = 99° \quad ....(3)$$

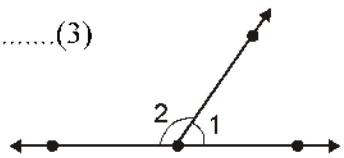


Fig. 5.11

Substitution $\angle 2 = 99°$ in equation (1), we get

$$99° + \angle 1 = 180°$$

$$\angle 1 = 180° - 99° = 81°$$

Thus, $\angle 1 = 81^\circ$ and $\angle 2 = 99^\circ$

Example 2. In Fig. 5.12, find the value of $\angle AOB, \angle BOC, \angle COD$ and $\angle DOE$ where $\angle AOE = 100^\circ$.

Sol : We know that the sum of the angles at a point is 360⁰

$\therefore \quad y + 2y + 4y + 6y + 100^\circ = 360^\circ$

$\Rightarrow \quad 13y = 260^\circ$

$\Rightarrow \quad y = \dfrac{260}{13} = 20^\circ$

$\angle AOB = y = 20^\circ,$

$\angle BOC = 2y = 40^\circ,$

$\angle COD = 4y = 80^\circ$

$\angle DOE = 6y = 120^\circ$

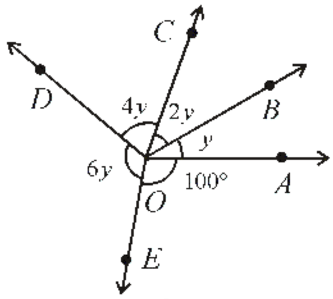


Fig. 5.12

Example 3. In Fig. 5.13, find the value of $\angle x, \angle y$ and $\angle z$.

Sol : Here, $\angle y = 140^\circ$ (Vertically opposite angles)

and $\angle x + 140^\circ = 180^\circ$ (Linear pair)

$\Rightarrow \quad \angle x = 40^\circ$

Also $\quad \angle x = \angle z$ (Verticalliy opposite angle)

$\Rightarrow \quad \angle z = 40^\circ$

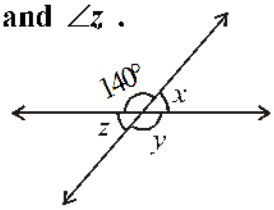


Fig. 5.13

Example 3. In Fig. 5.14, line AB and line OP meet at point O. line OD and OE are the bisectors of angle $\angle BOP$ and $\angle POA$, then find the neeasure of $\angle EOD$.

Sol : Let $\angle BOP = x$, and $\angle POA = y$

From figure,

$\angle x + \angle y = 180^\circ$ (Linear pair) .....(i)

From figure

$\angle x = 2\angle 1, \angle y = 2\angle 2$ ....(ii)

$\therefore$ From equation (i) and (ii), we get

$2\angle 1 + 2\angle 2 = 180^\circ$ (Linear pair)

$\Rightarrow \quad 2(\angle 1 + \angle 2) = 180^\circ$

$\Rightarrow \quad \angle 1 + \angle 2 = 90^\circ$

Thus $\quad \angle EOD = 90^\circ$

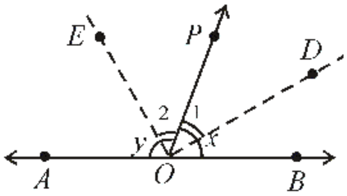


Fig. 5.14

Example 5. An angle is half of its supplementary angle, then find out the value of each angle.

Sol : Let one of the angle is x, then the value of its supplement is angle = $\dfrac{(180 - x)}{2}$

We know that sum of supplimentry angles is 180^0.

So $\quad x = \dfrac{180 - x}{2}$

$2x = 180 - x$

$3x = 180^0$

$\therefore \quad x = 60^0$

Hence, the angles are 120° and 60° .

Exercise 5.1

1. If angles $(2x+4)$ and $(x-1)$ form a linear pair, then find out the measure of the angles.
2. In Fig. 5.15 :

   (i) Find the measure of $\angle BOD$

   (ii) Find the measure of $\angle AOD$

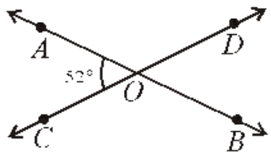


Fig. 5.15

   (iii) Which pair of angles have vertically opposite angles?

   (iv) Find the adjacent supplementary angles of $\angle AOC$.

3. In the given figure 5.16. if $\angle PQR = \angle PRQ$, then prove $\angle PQS = \angle PRT$.

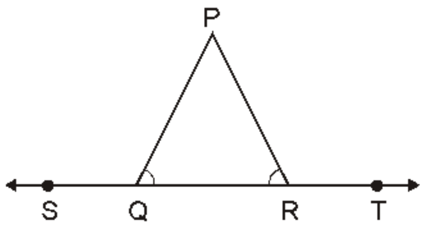


Fig. 5.16

4. In fig. 5.17, OP, OQ OR and OS are four rays, then prove that :

   $\angle POQ + \angle QOR + \angle SOR + \angle POS = 360^\circ$

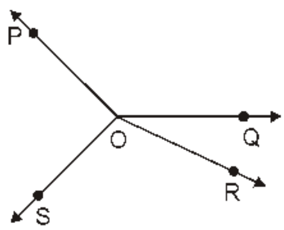


Fig. 5.17

5. In Fig. 5.18 $\angle x + \angle y = \angle p + \angle q$, then prove that AOB is a straight line.

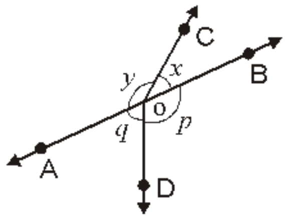


Fig. 5.18

5.08 Intersecting lines and Parallel Lines

If you are asked to draw pairs of two straight lines, then lines will be definately shown as in Fig. 5.19.

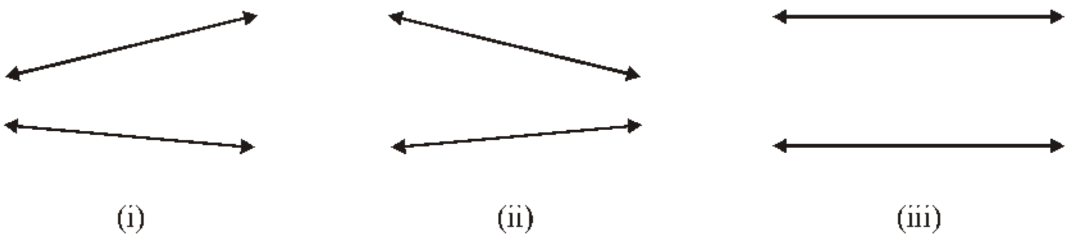


Fig. 5.19

Now, measure the distance between the lines in two different places. What do you observe?

No doubt you will see that in Fig. 5.19 (i) and (ii), the distance between the pair of lines is not equal at every point, i.e. if we extend these lines in forward or backward direction, then these lines will interest each other. Hence, these lines are intersecting lines. In Fig. 5.19 (iii), the difference between the lines is equal at every point, if we extend the lines in forward or backward direction up to infinite, then they will not intersect each other, Hence, there are parallel lines.

Transversal : If the group of two or more lines are insected by a line at different points, then it is known as transversal as shown in Fig. 5.20, line l, ntersect lines m and n at points P and Q respectively . So line l is a transversal for lines m and n .Do you observe that four angles at points P and Q. Yes, at point P, four angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ and at point Q four angles $\angle 5$, $\angle 6$, $\angle 7$, $\angle 8$ are formed.

Here $\angle 1$, $\angle 4$, $\angle 6$ and $\angle 7$ are exterior angles and $\angle 2$, $\angle 3$, $\angle 5$ and $\angle 8$ are interior angles.

Remember you have studied the numbering of angles made by a transversal. Let us remind them again.

(a) Corresponding angle :

(i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$

(iii) $\angle 3$ and $\angle 7$ (iv) $\angle 4$ and $\angle 8$

(b) Alternate interior angles :

(i) $\angle 2$ and $\angle 8$ (ii) $\angle 3$ and $\angle 5$

(c) Alternate exterior angles:

(i) $\angle 1$ and $\angle 7$ (ii) $\angle 4$ and $\angle 6$

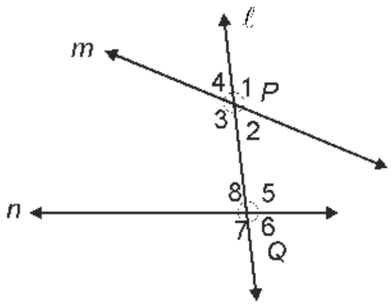


Fig. 5.20

(d) Interior angles on the same side of the transversal:

(i) $\angle 2$ and $\angle 5$ (ii) $\angle 3$ and $\angle 8$

When a transversal line intersect two or more parallel lines, then :

(i) Corresponding angles are equal

(ii) Alternate angles are equal

(iii) Interior angles are on the same side of a transversal line are supplementary and Opposite all of the statements are also true. If a transversal line cuts two lines and corresponding angles are same then the lines are perallel.

Theorem 5.2. ***If a transversal intersect two or more parallel lines, then alternate interior angles are equal to each other.***

Given : *l* and *m* are two parallel lines and a transversal intersect them at points *G* and *H*. $(\angle 2, \angle 1)$ and $(\angle 3, \angle 4)$ are pair of alternate angles.

To Prove : $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$

Proof

$\angle 2 = \angle 6$ (Vertically opposite angles) ... (i)

$\angle 1 = \angle 6$ (Corresponding angles) ... (ii)

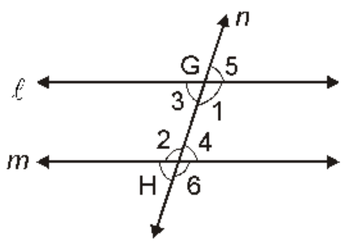


Fig. 5.21

From equation (1) and (2), we get

$\Rightarrow$ $\angle 1 = \angle 2$

Similarly $\angle 4 = \angle 5$ (Corresponding angles) ... (iii)

and $\angle 3 = \angle 5$ (Vertically opposite angles) ... (iv)

From equation (iii) and (iv) , we get

$\Rightarrow$ $\angle 3 = \angle 4$ **Hence proved**

Throrem 5.3 (Converse of Theorm 5.2)

If a transversal intersects two lines and alternate interior angles are equal, then the lines are paralles to each other.

Given : *l* and *m* are two lines which is interected by a transversal *n* at points *G* and *H*, then alternate angles are $\angle 2 = \angle 8$ and $\angle 3 = \angle 5$.

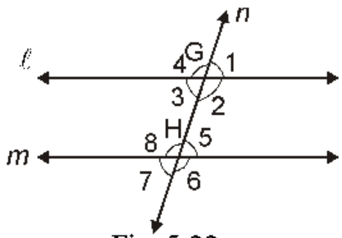


Fig. 5.22

To Prove : $\ell \parallel m$

Proof :

$\angle 1 = \angle 3$ (Alternate interior angles) ....(1)

$\angle 3 = \angle 5$ (Given) ....(2)

From equation (1) and (2), we get $\angle 1 = \angle 5$ (But these are corresponding angles)

So, $\ell \parallel m$ **Hence Proved.**

Theorem 5.4. ***If a transversal intersect two parallel lines, then the sum interior angles is equal to two right angles or 180°.***

Given : ℓ and *m* two parallel lines which are intersected by a transversal *n* at point *G* and *H* and angles at *G* and *H* are $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5, \angle 6, \angle 7, \angle 8$ are formed.

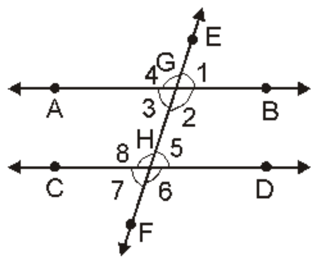


Fig. 5.23

To Prove : $\angle 2 + \angle 5 = 180°$ and $\angle 3 + \angle 8 = 180°$

Proof $\angle 1 + \angle 2 = 180°$ (Linear pair) ...(i)

$\angle 1 = \angle 5$ (Corresponding angles) ... (ii)

From equation (i) and (ii), we get

$\angle 2 + \angle 5 = 180°$

Similarly, $\angle 3 + \angle 4 = 180°$ (linear pair) ... (iii)

and $\angle 4 = \angle 8$ (Corresponding angles) ... (iv)

From equations (iii) and (iv), we get

$\angle 3 + \angle 8 = 180°$ **Hence proved**

Theorem 5.5 (Converse of Theorem 5.4)

If a transversal intersect two lines, and the sum of angles of the same side of the transversal is two right angle, then lines are parallel.

Given : ℓ and *m* are two lines which is intersected by a transversal *n* at points *G* and *H*, and formed angles $\angle 1, \angle 2, \angle 3, \angle 4$ and $\angle 5, \angle 6, \angle 7, \angle 8$ then alternate angles are such that : $\angle 2 + \angle 5 = 180°$ and $\angle 3 + \angle 8 = 180°$

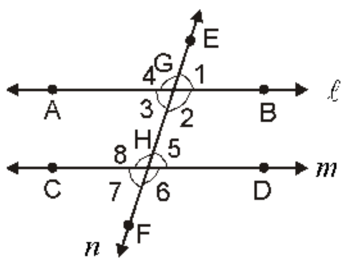


Fig. 5.24

To prove : $\ell \parallel m$

Proof :

$\angle 1 + \angle 2 = 180°$ (Linear pair axiom) ...(1)

$\angle 2 + \angle 5 = 180°$ (Given) ...(2)

From equation (1) and (2), we get

$\angle 1 = \angle 5$

But these are corresponding angles.

Thus, $\ell \parallel m$

Theorem 5.6 : ***If two lines are parallel to third line, then all three lines will be parallel to each other,i.e., if*** $\ell \parallel n$ ***and*** $m \parallel n$***, then*** $\ell \parallel m$

To prove : $\ell \parallel m$

Proof : Since, $l \parallel n$

$\Rightarrow$ $\angle 1 = \angle 9$

(Corresponding angles) . . . (1)

m || n and PQ is transversal

$\angle 5 = \angle 9$

(Corresponding angles) . . . (2)

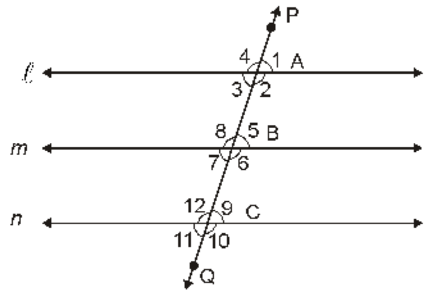


Fig. 5.25

From equations (1) and (2)

$\angle 1 = \angle 5$ . . . (3)

Since, $\angle 1$ and $\angle 5$ are corresponding angles, therefore by corresponding angles property $\ell \parallel m$.

Hence Proved

Example 6. In Fig. 5.26 lines $\ell \parallel m$ and a transverasal n is intersects them. If $\angle 1 = 55°$, then find out the remaining angles.

Sol : Since, $\angle 3 = \angle 1$ (Vertically opposite angles)

$\angle 1 = 55°$ (Given)

Therefore, $\angle 3 = 55°$

$\angle 1 + \angle 4 = 180°$ (Linear Pair axiom)

$\Rightarrow$ $55° + \angle 4 = 180° \Rightarrow \angle 4 = 125°$

Since, $\angle 2 = \angle 4$ (Vertically opposite angles)

$\Rightarrow$ $\angle 2 = 125°$

Since, $\angle 1 = \angle 6$ (Corresponding angles)

$\angle 1 = 55°$

Since, $\angle 4 = \angle 8$ (Corresponding angles)

$\angle 4 = 125°$

Since, $\angle 7 = \angle 6$ (Vertically opposite angles)

$\Rightarrow$ $\angle 6 = \angle 7 = 55°$

Since, $\angle 5 = \angle 8$ (Corresponding angles)

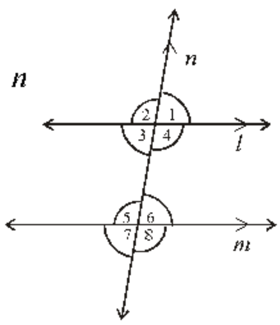


Fig. 5.26

$\Rightarrow \qquad \angle 6 = 55^\circ$

$\angle 5 = \angle 8$

and $\quad \angle 8 = 125^\circ \qquad \therefore \qquad \angle 5 = 125^\circ$

Example 7. **In Fig. 5.27, $\ell \parallel m$, then find out the equal angles and also write the reason.**

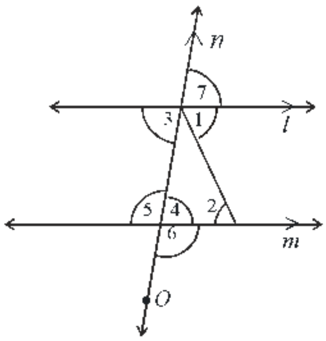


Fig.5.27

Sol : Here $\angle 1 = \angle 2$ (Alternate angle)

$\angle 3 = \angle 4$ (Alternate angle)

$\angle 5 = \angle 6$ (Vertically opposite angle)

$\angle 3 = \angle 7$ (Vertically opposite angle)

$\angle 4 = \angle 7$ (Corresponding angle)

Example 8. **In Fig. 5.28, *m* and *n* two plane mirror which are perpendicular to each other. Show that incident ray *CA* is parallel to reflected ray *BD*.**

Sol : Let us consider that perpendicular of *A* and *B* meet at point *P* because both the glasses are perpendicular.

$\therefore \qquad BP \parallel OA$ and $AP \parallel OB$

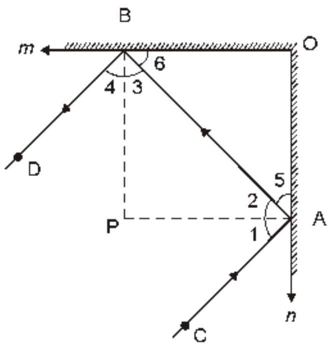


Fig.5.28

$\because \quad BP \parallel OA$ and BA is a transveral.

$\therefore \qquad \angle 3 = \angle 5$ (Alternate angles) ... (1)

and $PA \perp OA \Rightarrow \angle PAO = 90^\circ$

$\angle PAO = \angle 2 + \angle 5 = 90^\circ$ ... (2)

From equation (1) and (2), we get

$\angle 2 + \angle 3 = 90^\circ$ ... (3)

$\because \quad \angle 1 = \angle 2$ and $\angle 4 = \angle 3$ (angle of Incidence = angle Reflection) ... (4)

From equation (3) and (4), we get

$\angle 1 + \angle 4 = 90^\circ$ ... (5)

Adding equations (3) and (5), we get

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$$

$\Rightarrow \angle CAB + \angle DBA = 180^\circ$ (Interior angles on the same side of the transversal)

Thus, $CA \parallel BD$

Example 9. **In Fig. 5.29, BA || ED and BC || EF. Show that :** $\angle ABC = \angle DEF$

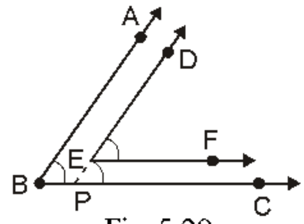


Fig. 5.29

Sol : Extend DE in such away that it will meet BC at point P.

$\because$ $BC \parallel EF$ (Given)

If we consider DP as transversal, then

$\angle DEF = \angle DPC$ (Corresponding angles) . . . (1)

$AB \parallel DE$ (Given)

then $AB \parallel DP$ (DE is extended uo to P) and consider BC as a transversal.

$\angle ABC = \angle DPC$ (Corresponding angles) . . . (2)

From equation (1) and (2), we get

$$\angle ABC = \angle DEF$$

Exercise 5.2

1. In Fig. 5.30, lines AB, CD and EF are parallel to each other. Find the values of $\angle x, \angle y, \angle z$ and $\angle p$

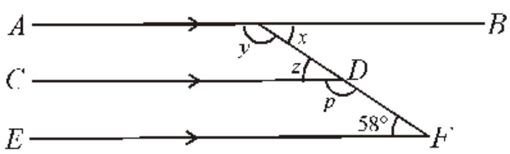


Fig. 5.30

2. In Fig. 5.31, $AB \parallel EF$ find the values of $\angle x$ and $\angle y$.

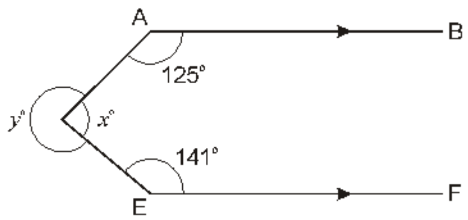


Fig. 5.31

3. In Fig. 5.32, if $\ell \parallel m$, then find the angles equivalent to $\angle 1$.

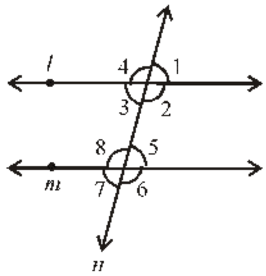


Fig. 5.32

4. In Fig. 5.33, $\angle 1 = 60^\circ$ and $\angle 6 = 120^\circ$, then show that m and n are parallel.

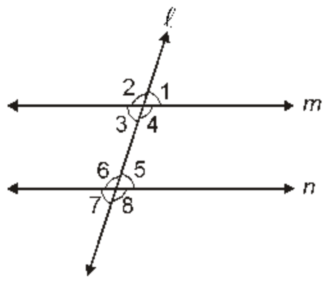


Fig. 5.33

5. AP and BQ are two bisectors of alternate angles of two parallel lines ℓ and m and its transversal n. Show that $AP \parallel BQ$.

6. In Fig. 5.34, $BA \parallel ED$, then show : $\angle ABC + \angle DEF = 180^\circ$.

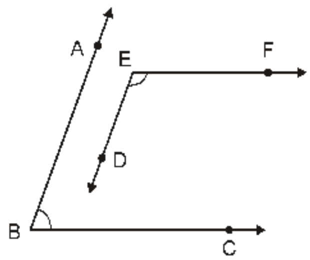


Fig. 5.34

7. In Fig. 5.35, $DE \parallel QR$ and AP and BP are bisectors of $\angle EAB$ and $\angle RBA$, then

find out the value of $\angle APB$.

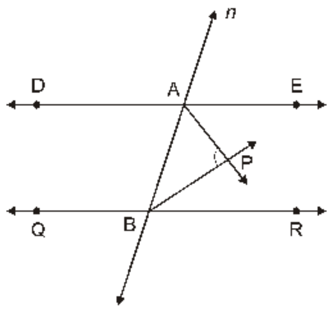


Fig. 5.35

8. If two straight lines are perpendicular to two parallel lines, then show that these straight lines are parallel to each other.

9. In Fig. 5.36, $AB \parallel CD, CD \parallel EF$ and $y : z = 3 : 7$, then find the value of x.

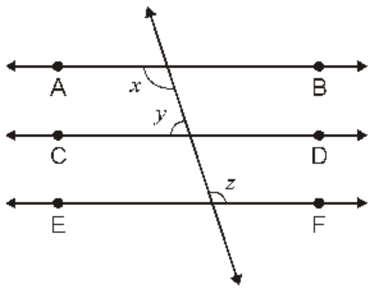


Fig. 5.36

10. In Fig. 5.37, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strickes the mirror PQ to B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that $AB \parallel CD$.

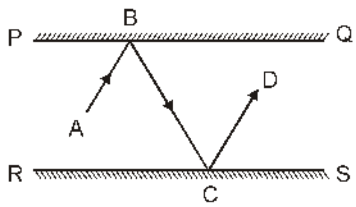


Fig. 5.37

11. In Fig. 5.38, if $PQ \parallel RS$, $\angle MXQ = 135°$ and $\angle MYR = 40°$, then find $\angle XMY$.

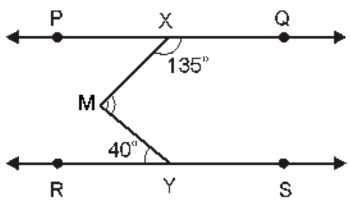


Fig. 5.38

5.9. Basic Constructions

While proving the theorem or solving the questions related to theorem which figures,shapes are made they have very less accuracy but geometric construction with the help of geometric tools like rural, protector, set squares and compasses can be made more accurate. During drawing following points should be remember.

1. Whatever geometric is to be made, first we have to draw the rough diagram with the dark pencil.
2. Steps of drawing should be explained in words.

Few Simple Construction :

You have studied some geometric construction in previous classes such as to draw a line of desired length and to draw an angle of desired measure. You have also studied various geometric facts in pervious chapter in this book. On the basis of previous chapter's knowledge and facts we can draw few geometric construction.

Geometric construction 1

Bisection of a given line segment i.e., bisection of line segment AB.

Construction : Draw a line segment AB of desired length. Taking A and B as a centre and take arc of more than half radius on the both side of line segment AB which intersect each other at point P and Q. Now, join PQ where it will intersect the line AB marked that point PQ where it will intersect the line AB maked that point O. Point O is the point of bisection of line AB as shown in Fig. 5.39.

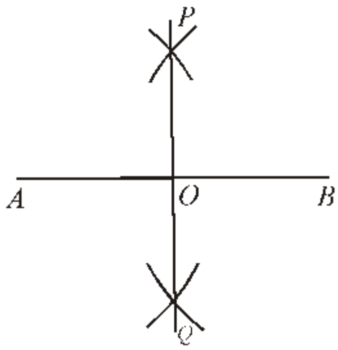


Fig. 5.39

Base of Construction : The point which have equal distance from the two given points is perpendicular bisector of the line joining them. PQ is the perpendicular bisector of line segment AB.

Geometric Construction 2

Bisection of any given angle say $\angle BAC$

Construction : Taking point A as centre of given angle $\angle BAC$ and with any radius draw an arc, which interest the side AB and AC at point P and Q. Now, taking P and Q as a centre and radius more than half of PQ, draw two arcs which intersect each other at point O. Now, join AO which bisect $\angle BAC$.

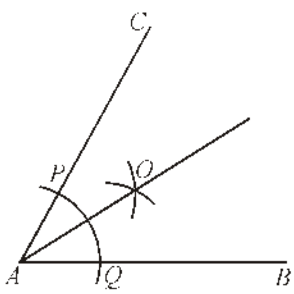


Fig. 5.40

Base of Construction : The points which have equal distance from lines AC and AB is bisecter of $\angle CAB$. Now join AO. The line between the angles is known as bisector of $\angle BAC$.

Geometric Construction 3

Draw an angle of 60° with ruler and compass.

Draw an angle of $60°$ on line segment AB at point O.

Construction : Taking O as centre draw an arc of any radius on line AB which intersect AB at point P. Now taking P as centre draw an arc with same radius which will a intersect the arc at point Q, join OQ. Thus, $\angle BOQ = 60°$ is the required angle as shown in Fig. 5.41.

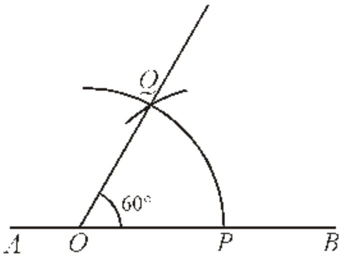


Fig. 5.41

Base of Construction : Equilateral triangle ΔPOQ which have every angle $60°$.

Geometric Construction 4

Draw an angle of 120° with the help of ruler and compass.

Draw an angle of $120°$ on line AB at point O .

Construction : Taking O as a centre draw an arc of any radius which will intersect AB at point P. Now, taking P as centre draw an arc of same radius as two times (twice) which will intersect the arc on points Q and R respectively. Now, join OR. So, $\angle BOR = 120°$, as shown in Fig. 5.42.

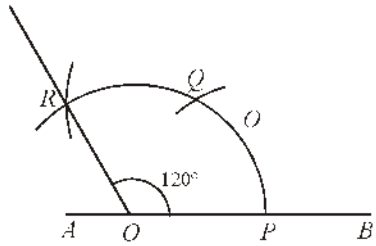


Fig. 5.42

Geometric Construction 5

Construction of various angles

(A) *Construction of angle* 30° :

$$\left[\because 30° = \frac{60°}{2}\right]$$

Construction : With the help of ruler and compass draw an angle BOQ of 60° according to compass and geometric construction no. 2. Draw the bisector of $\angle BOQ$.

Hence, $\angle BOR = 30°$ (Fig. 5.43).

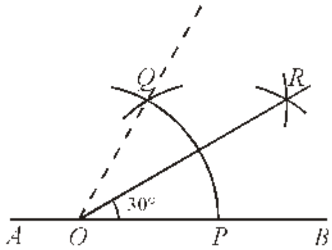


Fig. 5.43

(A) *Construction of* 90° *angle :*

First Method :

$$\left[\because 90° = 60° + \frac{60°}{2}\right]$$

Construction : With the help of ruler and compass

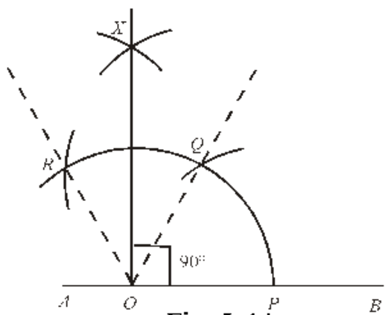


Fig. 5.44

draw $60°$ and $120°$ take two arcs and mark then Q and R points. On the basis of compass and geometric construction 2. Draw the bisector $\angle QOR$ and draw OX similar $\angle BOX = 90°$ (fig. 5.44)

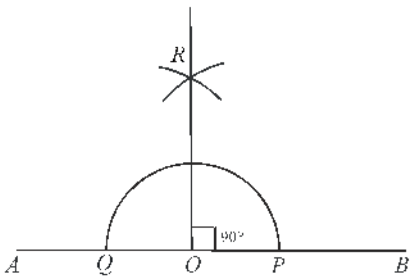


Fig. 5.45

Second Method :

$$\left[\because 90° = \frac{180°}{2}\right]$$

Construction : Angle on straight line is $BOA = 180°$ according to compass and geometric construction and bisector OR.

$\therefore \quad \angle BOR = 90°$ (Fig. 5.45)

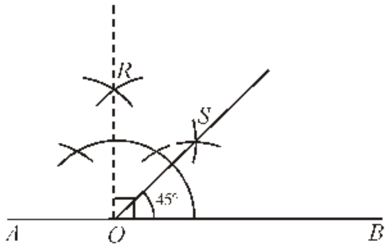


Fig. 5.46

(C) Construction of 45°

$$\left[\because 45° = \frac{90°}{2}\right]$$

Construction : Draw a $90°$ angle with the help of ruler and compass $\angle BOR = 90°$. According to compass and geometric construction 2, bisector of $\angle BOR$ by OS (Fig. 5.46).

So, $\quad \angle BOS = 45°$

(D) *Construction of* 135° :

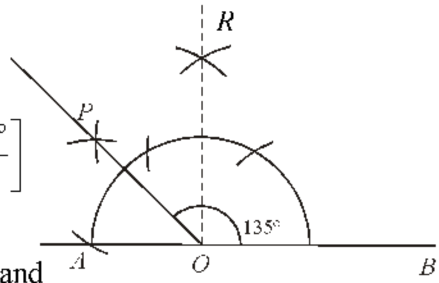


Fig. 5.47

$$\left[\because 135° = 90° + \frac{90°}{2} \quad \text{or} \quad 180° - \frac{90°}{2}\right]$$

Construction : Draw a $90°$ angle with the help of ruler and compass $\angle BOR$ according to compass and geometric construction 2. Draw the bisector of $\angle ROA$ and its bisector by OP.

So, $\quad \angle BOP = 135°$. (Fig. 5.47)

Similarly, you can draw the following angles

1. $15° = \frac{30°}{2}$
2. $75° = 60° + 15°$
3. $105° = 90° + 15°$
4. $150° = 120° + 30°$ or $150° = 180° - \frac{60°}{2}$

   $= 180° - 30°$

Geometric Construction 6

Draw an equivelant angle to an angle drawn on a point of a line.

Construction : Draw an equivalent angle equal to $\angle D$ of line segment AB at a point O. Taking D as centre and taking appropriate radius. Draw an arc which will intersect the sides at P and Q. Now, taking O as centre, draw the same arc which will intersect the line segment AB at R.

Now, taking R as a centre, draw an arc of radius PQ which will intersect the pervious arc at S. Now, join OS then $\angle BOS$ is desired angle.

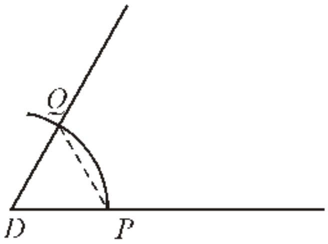


Fig. 5.48

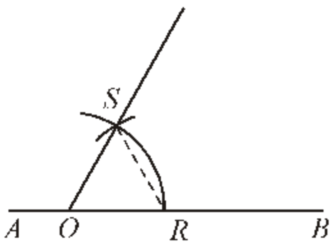


Fig. 5.49

Geometric Construction 7.

In pervious classes we have learnt to draw the various angles with the help of angle measuring instrument. Let us draw the various angles without angle mesurement instrument with the help of scale and protractor.

Step 1 : Draw a line segment AB of 6 cm as shown in Fig. 5.50.

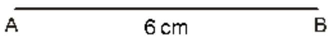


Fig. 5.50

Step 2 : Open the compass upto 6 cm and taking A as centre, draw a circle such that it passes through B. Now, taking 6 cm as radius draw equal arcs on the circle and mark the cuts as C, D, E, F and G.

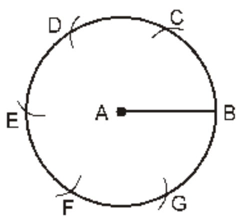


Fig. 5.51

Step 3 : Now, join AB and BC. Join in such away that it will start from where O is started and marked them 1-1 cm. Similarly , CD, DE, EF, FG and GB.

Here marked after one cm distance makes a 10° angle form each outer from point A.

if it divided into mm, then it will make 1° degree angle at distance of 1 mm form point A.

Suppose, we have to draw an angle of 40°, then we have to move up to 4th point where 40 is written. Now, join A to that point $\angle BAH = 40^\circ$.

Suppose, we have to draw 130° angle then we have to move upto point D and after that 130°, then $\angle BAK = 130^\circ$.

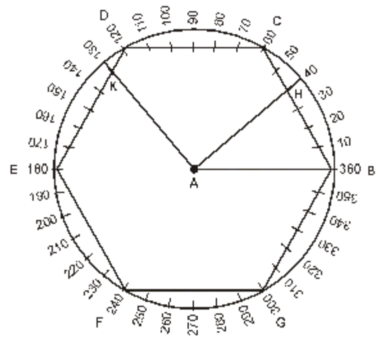


Fig. 5.52

Note : In this method, by using millimeter sacle is used then we can drawn angle 1° to 360°.

Remark : In the above method 6 cm straight line and another end where angle to be drawn. A 6 cm circle to be drawn and divide circlein 6 equal parts. (Hence, every part But not joining every point we have to join the desired line.

Geometric Construction 8

On any given line to draw a perpendicular form a point which is outside the line.

Method 1 : AB is straight line there is a point P outside the line. Draw a perpendicular.

Composition : Taking P as a centre and radius more than the distance from P to AB draw two arc which will intersect the line AB at points C and D. Taking C and D as a centre and radius more than half of CD draw two arcs below AB which will intersect each other at Q. Now join PQ. It will meet AB at point O. Thus, PO is the desired perpendicular.

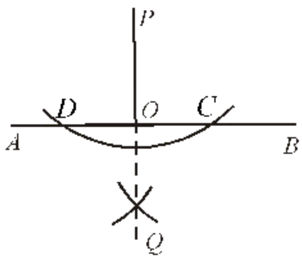


Fig. 5.53

Base of Construction : PQ is the perpendicular bisector of C and D which have equal distance.

Method 2 : Keep the one side of set square on line AB. Keep second set square in such a way so that it can move on sacle and it should be stable. Move set square so that it should move up to P. Draw line PQ to AB form point P. Hence, PQ is the required perpendicular (Fig. 5.54).

Geometric Construction 9

Draw a perpendicular at any point O on line AB.

Construction : Let a is any point on ; line AB Taking O as centre of given line AB draw an arc which will intersect the line AB at points C and

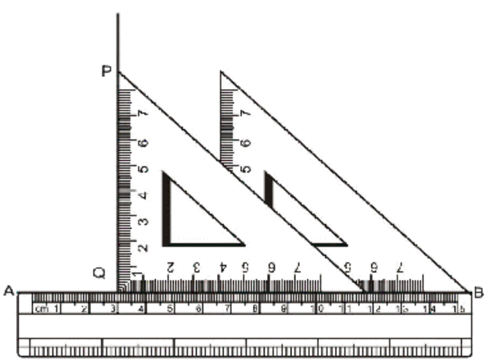


Fig. 5.54

D . Now taking C and D as centre taking more than half distance in compass draw an arc which will intetrsect each other at point P. Now join PO. Hence, PO is the required perpendicualr Fig. (5.55).

Base of Construction : If we join PC and PD, then ΔPOC and ΔPOD are congurent. Result is $\angle POC = \angle POD = 90^\circ$.

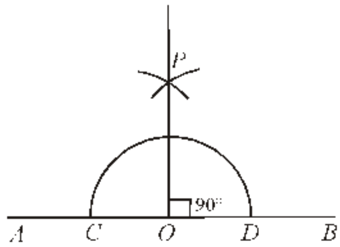


Fig. 5.55

5.10. Construction of Parallel Lines

Geometric Construction 10

From any outer point P draw a line parallel to a given line.

When two lines are intersected by a transversal, corresponding and alerante angles are equal then lines are parallel to each other.

(A) Corresponding Angle Method :

Taking a point Q on line AB and join with P. Extend QPto R. Now, at point P on QR draw $\angle DPR$ which is equal to $\angle BQR$. Hence, CD is the required line parallel to AB as shown in (Fig. 5.56).

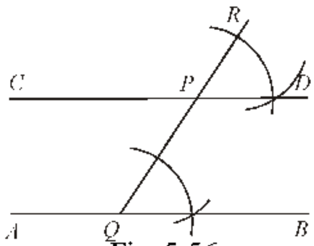


Fig. 5.56

(B) Alternate Angle Method :

Take a point Q on AB and join this with P of QP draw alternate angles $\angle CPQ = \angle BQP$. Hence, CD is the required line parallel to AB as shown in Fig. 5.57.

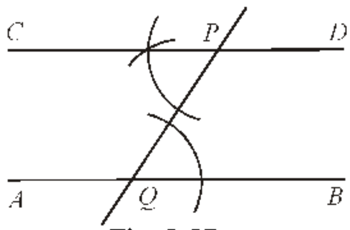


Fig. 5.57

(C) Construction of Parallel Lines with the help of Set Square :

Let AB be a straight line to which a parallel line has to be drawn from point P. Except the diagonal of set square keep other side on AB. Its another side set in such a way that its one side should be fix. Move the set square in such a way that it should be moved on P. Now, draw line PC. Hence, PC is the required parallel line (Fig. 5.58).

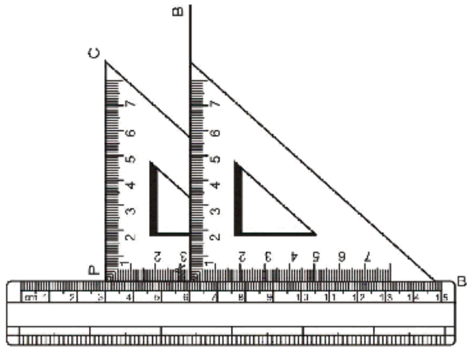


Fig. 5.58

Exercise 5.3

1. Draw a line segment $AB = 10$ cm. Draw its bisector and verify your answer.
2. Draw an angle of $120°$. Draw bisector of this angle. Measure both the angles and verify your answer.
3. Draw an angle of $40°$ with the protractor also draw an angle equal to this angle equal to this angle with ruler and compass.
4. Draw a line of 6 cm. Draw a perpendicular to it from an outer point P.
5. Draw $\angle ABC = 120°$. Draw a line parallel to BC from A.
6. Draw a line segment of 9 cm. Divide this in three equal parts with ruler and compass.
7. Draw a line segment of 10 cm. With the help of ruler and compass divide it into 3 : 2. Measure their length.
8. Draw a line segment of 6 cm. Divide it into 1 : 2 : 3 with the help of ruler and compass.
9. Draw the following angles with ruler and compass $45°$, $75°$, $105°$, $150°$.
10. Without using protractor draw the following angles :

    (i) $12°$ (ii) $20°$ (iii) $80°$ (iv) $100°$

    (v) $155°$ (vi) $218°$ (vii) $307°$ (viii) $127°$

    Verify these angles with protractor.

❑

Important Points

1. When two rays are originated form a point, then it makes a shape of angle. Common point of rays is known as vertex and rays are known as sides of angle.
2. If one line is standing on another line, then they make a right angle.
3. If a revolving ray takes a complete round, then it makes angles equal to four right angles, *i.e.*, $360°$.
4. An acute angle has a magnitude between $0°$ and $90°$.
5. An obtuse angle has a magnitude between $90°$ and $180°$.
6. Measurement of straight angle on a plane is equal to two right angles.
7. A reflex angle has a magnitude of more than two right angles but less than four right angles.
8 .If the sum of two angles is equal to $90°$, then these angles are known as complementary angles.
9. If the sum of two angles is equal to $180°$, then these angles are known as supplementary angles.
10. If in two vertex and one side is common and other side is opposite to common side then angles are known as adjacent angle.
11. If the measurment of two adjacent angles is $180°$, then these angles are known as linear pair.
12. Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays and they are equal.
13. It two parallel lines are intersected by a trasnversal, then :
    (i) Corresponding angles are equal,
    (ii) Alternate angles are equal,
    (iii) Sum of interior angles of one side is equal to $180°$.
14. If two lines are intersectd by a transversal and the angles made by them are such that
    (i) Corresponding angles are equal, or
    (ii) Alternate angles are equal, or
    (iii) Sum of interior angle on one side is equal to $180°$, then the lines are parallel to each other.

Miscellaneous Exercise-5

1. In fig. 5.59 If $AB \mid\ CD \,||\, EF,\ PQ \quad RS,\ \angle RQD = 25^o$ and $\angle CQP = 60^o$, then the value of $\angle QRS$ is equal to :

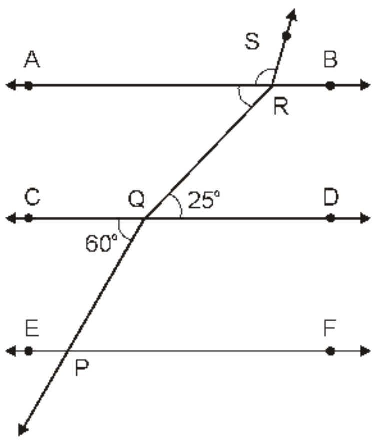


Fig. 5.59

(A) 85° (B) 135° (C) 145° (C) 110°

2. In Fig. 5.60., POQ is a straight line. The value of x is :

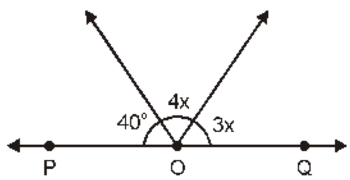


Fig. 5.60

(A) 20° (B) 25° (C) 30° (C) 35°

3. Fig. 5.61, $OP \quad RS,\ \angle OPQ = 110^o$ and $\angle QRS = 130^o$, then $\angle PQR$ is equal to :

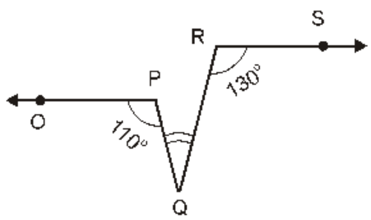


Fig. 5.61

(A) 40° (B) 50° (C) 60° (C) 70°

4. In Fig. 5.62, reflex angle $\angle AOB$ is equal to :

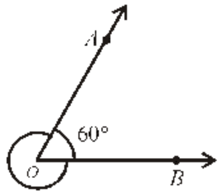


Fig. 5.62

(A) 60° (B) 120° (C) 300° (C) 360°

5. In Fig. 5.63, two straight lines AB and CD are intersecting each other at point O . Angles so formed are marked Fig. 5.63.

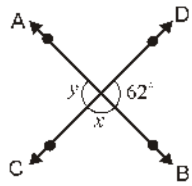


Fig. 5.63

Here, value of $\angle x - \angle y$ is :

(A) 56° (B) 118° (C) 62° (D) 180°

6. In Fig. 5.64., which pair of angles is not a pair of corresponding angles :

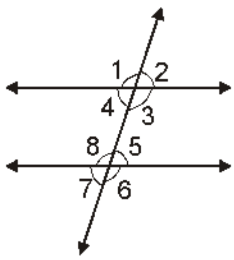


Fig. 5.64

(A) $\angle 1, \angle 5$ (B) $\angle 2, \angle 6$ (C) $\angle 3, \angle 7$ (D) $\angle 3, \angle 5$

7. In fig. 5.65, if l and m are two parallel lines which are intersected by transversal line at points G and H and the angles so formed are shown.

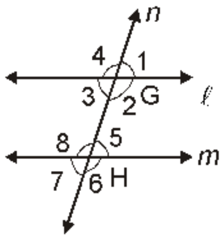


Fig. 5.65

If $\angle 1$ is an acute angle, then which statement is false :

(A) $\angle 1+\angle 2=180°$ (B) $\angle 2+\angle 5=180°$

(C) $\angle 3+\angle 8=180°$ (D) $\angle 2+\angle 6=180°$

8. Find the value of x in Fig. 5.66.

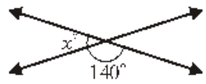


Fig. 5.66

9. In Fig. 5.67, $AB \parallel CD$. Find the value of $\angle x$ and $\angle y$.

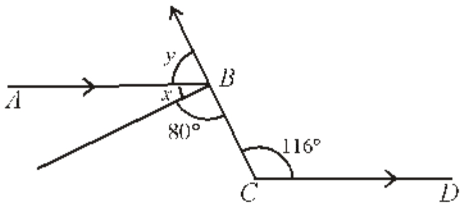


Fig. 5.67

10. In Fig. 5.68, lines *l* and *m* are parallel. Find the value of $\angle x$.

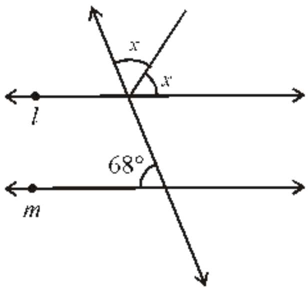


Fig. 5.68

11. In Fig. 5.69, which lines are parallel and why ?

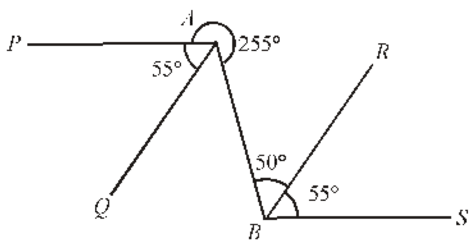


Fig. 5.69

12. In Fig. 5.70, $AC \parallel PQ$ and $AB \parallel RS$ then find the value of $\angle y$. Also, write the statements you have used.

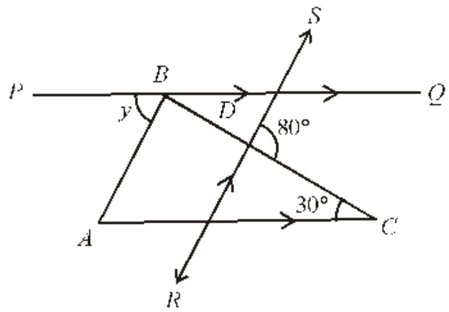


Fig. 5.70

13. In Fig. 5.71, $AB \parallel CD$ and $PQ \parallel EF$, then find the value of $\angle x$.

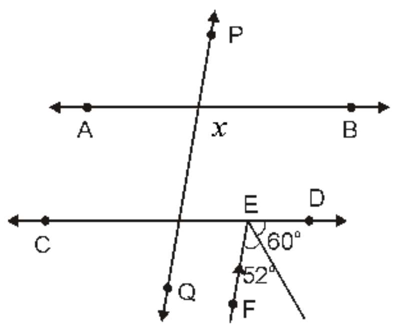


Fig. 5.71

14. In Fig. 5.72, out of lines *l, m, n, p, q* and *r* which lines are parallel and why.

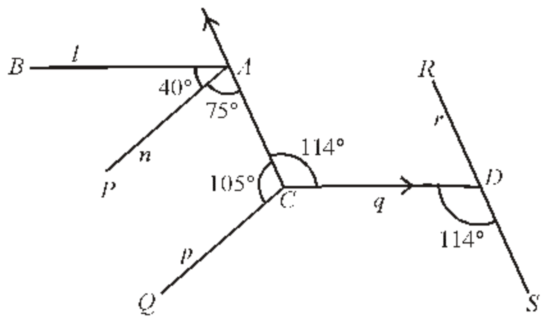


Fig. 5.72

15. In Fig. 5.73, two straight lines are intersecting each other. If $\angle 1 + \angle 2 + \angle 3 = 230^\circ$ then find the values of $\angle 1$ and $\angle 4$.

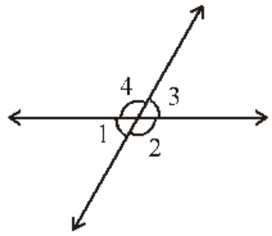


Fig. 5.73

16. In Fig. 5.74, PQ and QR are two mirrors which are joined at Q at an angle of 30° to each other. If incident ray AB is parallel to mirror RC, then find the values of $\angle BCQ$, $\angle CBQ$ and $\angle BDC$.

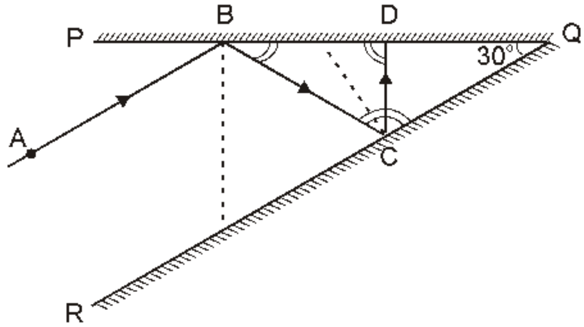


Fig. 5.74

Answer

Exercise 5.1

1. 122° and 58°

2. (i) 52° (ii) 128° (iii) $\angle AOC, \angle BOD$ and $\angle AOD, \angle BOC$

(iv) $\angle AOD$ and $\angle BOC$

Exercise 5.2

1. $\angle x = 58^\circ,\ \angle y = 122^\circ,\ \angle z = 58^\circ,\ \angle p = 122^\circ$

2. $\angle x = 94^\circ,\ \angle y = 266^\circ$

3. $\angle 3,\ \angle 5,\ \angle 7$

7. 90°

9. 126°

11. 85°

Miscellaneous Exercise - 5

1. (C) **2.** (A) **3.** (C) **4.** (C)

5. (A) **6.** (D) **7.** (B)

8. 40°

9. $x = 36,\ y = 64$

10. $x = 56^\circ$

11. $AQ \parallel BR,\ AP \parallel BS$

12. $y = 110^\circ$

13. 112°

14. $m \parallel r,\ n \parallel p$

15. $\angle 1 = 50,\ \angle 4 = 113^\circ$

16. $\angle BCQ = 120^\circ,\ \angle CBQ = 30^\circ,\ \angle BDC = 90^\circ$