Chapter 5

Continuity and Differentiability

Exercise 5.6

Q. 1 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

$x = 2at^2$, $y = at^4$

Answer:

It is given that

$x = 2at^2$, $y = at^4$

So, now

$$\frac{dx}{dt} = \frac{d(2at^2)}{dt}$$

$$= 2a\,\frac{d(t^2)}{dt}$$

= 2a.2t

= 4at ………… (1)

And

$$\frac{dy}{dt} = \frac{d(at^4)}{dt}$$

$$= a\,\frac{d(t^4)}{dt}$$

$= a.4.t^3$

$= 4at^3$………… (2)

Therefore, form equation (1) and (2). we get

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4at^3}{4at} = t^2$$

Hence, the value of $\frac{dy}{dx}$ is t^2

Q. 2 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = a cos θ, y = b cos θ

Answer:

It is given that

x = a cos θ, y = b cos θ

Then, we have

$$\frac{dx}{d\theta} = \frac{d(a\cos\theta)}{d\theta}$$

= a(-sinθ)

= -a sin θ………… (1)

$$\frac{dy}{d\theta} = \frac{d(b\cos\theta)}{d\theta}$$

= b (-sin θ)

= -b sin θ …… (2)

From equation (1) and (2), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$$

Hence, the value of $\frac{dy}{dx}$ is $\frac{b}{a}$

Q. 3 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = sin t, y = cos 2t

Answer:

It is given that

x = sin t, y = cos 2t

Then, we have

$\frac{dx}{dt} = \frac{d(\sin t)}{dt}$

= cost ………… (1)

$\frac{dy}{dx} = \frac{d(\cos 2t)}{dt} = -\sin 2t\frac{d(2t)}{dt}$

= -2sin2t………… (2)

So, equation (1) and (2), we get

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{\cos t}$

= $\frac{-2.2\sin t\cos t}{\cos t}$, Since sin2t = 2sintcost

= -4sint

Hence, the value of $\frac{dy}{dx}$ is -4sint

Q. 4 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = 4t, y = 4/t

Answer:

It is given that

x = 4t, y = $\frac{4}{t}$

Then, we have

$\frac{dx}{dt} = \frac{d(4t)}{dt}$

= 4 ……… (1)

$\frac{dy}{dt} = \frac{d\left(\frac{4}{t}\right)}{dt} = 4\,\frac{-1}{t^2} = \frac{-4}{t^2}$ ………… (2)

Therefore, from equation (1) and (2), we get

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-4}{t^2}}{4} = \frac{-1}{t^2}$ = - 4sin t

Hence, the value of $\frac{dy}{dx}$ is $\frac{-1}{t^2}$

Q. 5 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = cos θ – cos 2θ, y = sin θ – sin 2θ

Answer:

It is given that

x = cos θ – cos 2θ, y = sin θ – sin 2θ

Then, we have

$\frac{dx}{d\theta} = \frac{d(\cos\theta - \cos 2\theta)}{d\theta}$

$= \frac{d(\cos\theta)}{d\theta} - \frac{d(\cos 2\theta)}{d\theta}$

= -sin θ – (-2sin2θ)

= 2sin2θ – sin θ …………… (1)

$\frac{dy}{d\theta} = \frac{d(\sin\theta - \sin\theta)}{d\theta}$

$= \frac{d(\sin\theta)}{d\theta} - \frac{d(\sin\theta)}{d\theta}$

= cos θ – 2cos2θ

= -b sin θ ………… (2)

From equation (1) and (2), we get,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$$

Hence, the value of $\frac{dy}{dx}$ is $\frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}$

Q. 6 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = a (θ – sin θ), y = a (1 + cos θ)

Answer:

It is given that

x = a (θ – sin θ), y = a (1 + cos θ)

Then, we have

$$\frac{dx}{d\theta} = a\left[\frac{d(\theta)}{d\theta} - \frac{d(\sin\theta)}{d\theta}\right]$$

= a(1-cosθ) …………………… (1)

$$\frac{dy}{d\theta} = a\left[\frac{d(1)}{d\theta} - \frac{d(\cos\theta)}{d\theta}\right]$$

= a [0 + (-sin θ)]

= -a sin θ …………………… (2)

From equation (1) and (2), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$$

$$= \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2sin^2\frac{\theta}{2}}$$

$= \frac{-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$ = - cot $\frac{\theta}{2}$

Hence, the value of $\frac{dy}{dx}$ is –cot $\frac{\theta}{2}$

Q. 8 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = a $\left(\cos t + \log\tan\frac{t}{2}\right)$ y = a sin t

Answer:

It is given that

x = a $\left(\cos t + \log\tan\frac{t}{2}\right)$ y = a sin t

Then, we have

$$\frac{dx}{dt} = a\left[\frac{d(\cos t)}{dt} + \frac{d\left(\log\tan\frac{t}{2}\right)}{dt}\right]$$

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\frac{d\left(\tan\frac{t}{2}\right)}{dt}\right]$$

$$= a\left[-\sin t + \cot\frac{t}{2}.sec^2\frac{t}{2}\frac{d\left(\frac{t}{2}\right)}{dt}\right]$$

$$= a\left[-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}} \times \frac{1}{cos^2\frac{t}{2}} \times \frac{1}{2}\right]$$

$$= a\left[-\sin t + \frac{2}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right]$$

$$= a\left[\frac{1-sin^2 t}{\sin t}\right]$$

$= a\frac{cos^2 t}{\sin t}$ …………………… (1)

$\frac{dy}{dt} = a\frac{d(\sin t)}{dt}$

= a cos t…………………… (2)

From equation (1) and (2), we get

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{\left(a\frac{cos^2 t}{\sin t}\right)}$

$= \frac{\sin t}{\cos t}$

= tan t

Hence, the value of $\frac{dy}{dx}$ is tan t

Q. 9 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = a sec θ, y = b tan θ

Answer:

It is given that

x

= a sec θ, y = b tan θ

Then, we have

$\frac{dx}{d\theta} = a\frac{d(\sec\theta)}{d\theta}$

= a sec θ tan θ…………………… (1)

$\frac{dy}{d\theta} = b\frac{d(\tan\theta)}{d\theta}$

= bsec2θ…………………… (2)

From equation (1) and (2), we get,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{bsec^2\theta}{\text{asec}\,\theta\tan\theta}$$

$$= \frac{b}{a}\sec\theta\cot\theta$$

$$= \frac{b\cos\theta}{a\cos\theta\sin\theta}$$

$$= \frac{b}{a}\times\frac{1}{\sin\theta}$$

$$= \frac{b}{a}\ cosec\ \theta$$

Hence, the value of $\frac{dy}{dx}$ is $cosec\ \theta$

Q. 10 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

x = a (cos θ + θ sin θ), y = a (sin θ – θ cos θ)

Answer:

It is given that

x = a (cos θ + θ sin θ), y = a (sin θ – θ cos θ)

Then, we have

$$\frac{dx}{d\theta} = a\left[\frac{d(\cos\theta)}{d\theta} + \frac{d(\theta\sin\theta)}{d\theta}\right]$$

$$= a\left[-\sin\theta + \frac{\theta d(\sin\theta)}{d\theta} + \sin\theta\frac{d(\theta)}{d\theta}\right]$$

= a [-sin θ + θ cos θ + sin θ]

= a θ cos θ…………………… (1)

$$\frac{dy}{d\theta} = a\left[\frac{d(\sin\theta)}{d\theta} - \frac{d(\theta\cos\theta)}{d\theta}\right]$$

$= a\left[\cos\theta - \left\{\frac{\theta d(\cos\theta)}{d\theta} + \cos\theta\frac{d(\theta)}{d\theta}\right\}\right]$

= a [cos θ + θ sin θ – cos θ]

= a θ sin θ…………………… (2)

From (1) and (2) we get,

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta\sin\theta}{a\theta\cos\theta}$

= tan θ

Hence, the value of $\frac{dy}{dx}$ is tan θ

Q. 11 If x and y are connected parametrically by the equations given in without eliminating the parameter, Find dy/dx.

If, x = $\sqrt{a^{sin^{-1}t}}, y = \sqrt{a^{cos^{-1}t}}$ show that $\frac{dy}{dx} = -\frac{y}{x}$

Answer:

It is given that

x = $\sqrt{a^{sin^{-1}t}}$, y = $\sqrt{a^{cos^{-1}t}}$

Now,

x = $\sqrt{a^{sin^{-1}t}}$ = $x = \left(a^{sin^{-1}t}\right)^{\frac{1}{2}} = x = a^{\frac{1}{2}sin^{-1}t}$

Similarly, y = $\sqrt{a^{cos^{-1}t}} = y\left(a^{cos^{-1}t}\right)^{\frac{1}{2}} = y = a^{\frac{1}{2}cos^{-1}t}$

Let us consider,

x = $a^{\frac{1}{2}sin^{-1}t}$

Taking Log on both sides, we get

log x = $\frac{1}{2}sin^{-1}t\log a$

Therefore, $\frac{1}{x}.\frac{dx}{dt} = \frac{1}{2}\log a.\frac{d(sin^{-1}t)}{dt}$

= $\frac{dx}{dt} = \frac{x}{2}\log a.\frac{1}{\sqrt{1-t^2}}$

= $\frac{dx}{dt} = \frac{x\log a}{2\sqrt{1-t^2}}$ … (1)

Now, Consider

y = $a^{\frac{1}{2}cos^{-1}t}$

Taking Log on both sides, we get

log y = $\frac{1}{2}cos^{-1}t \log a$

Therefore, $\frac{1}{y}.\frac{dy}{dt} = \frac{1}{2}\log a.\frac{d(cos^{-1}t)}{dt}$

= $\frac{dy}{dt} = \frac{y}{2}\log a.\frac{-1}{\sqrt{1-t^2}}$

= $\frac{dy}{dt} = \frac{-y\log a}{2\sqrt{1-t^2}}$ …………….. (2)

So, from equation (1) and (2), we get

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{-y\log a}{2\sqrt{1-t^2}}}{\frac{x\log a}{2\sqrt{1-t^2}}} = -\frac{y}{x}$$

Therefore, L.H.S. = R.H.S.

Hence Proved