Mathematics - Standard

Section - A

1. (B) 2.

2. (C) $\left(\frac{13}{7}, 0\right)$

3. (A) 12

4. (D) not-real

5. (A) 1650

6. (C) 8cm

7. (C) 1

8. (D) $\frac{1}{12}$.

9. (C) $\frac{3}{7}$

10. (B) $\frac{6}{7}$

11. (D) $\Delta ADP \sim \Delta CBP$

12. (A) increases by 2.

13. (C) $\frac{1}{10}$

14. (B) $5\sqrt{2}$ cm

15. (A) 5 units

16. (B) 16th

17. (D) 40°

18. (B) made

19. (A) Both Assertion (A) and Reason (R) are true. Reason (R) is the correct explanation of Assertion (A).

20. (D) Assertion (A) is not true but Reason (R) is true.

Section - B

21. If the number $(15)^n$, n being a natural number, were to end with the digit 0, then it would be divisible by 2 and 5.
This means it must have 2 among its prime factors.
However $(15)^n$ can only be factorised as $(3)^n \times (5)^n = (3 \times 5)^n$.
It does not have 2 among its factors.
The uniqueness of the 'Fundamental Theorem of Arithmetic' guarantees that the composition of 15 has only the prime nos 3 and 5.
Hence $(15)^n$ cannot end with the digit zero ever.

22. Given : A(1, 0) ; B(-5, 0) ; C(-2, 5)

To find : Type of triangle formed

Solution : Using the distance formula ie, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

we find the distance between each side.

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(1+5)^2+(0-0)^2}$$
$$= \sqrt{36}$$
$$= 6 \text{ units}$$

$$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-5+2)^2+(0-5)^2}$$
$$= \sqrt{(-3)^2+(-5)^2} \Rightarrow \sqrt{9+25}$$
$$= \sqrt{34} \text{ units}$$

$$CA = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(1+2)^2+(0-5)^2}$$
$$= \sqrt{(3)^2+(5)^2} \Rightarrow \sqrt{9+25}$$
$$= \sqrt{34} \text{ units}$$

Since ΔABC has 2 sides (CA and BC) of equal length (each $\sqrt{34}$ units).

we can conclude that ΔABC is isosceles.

$\therefore$ ΔABC is an isosceles triangle.

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23. (b) Given: $2(\sin A + B) = \sqrt{3}$ and $\cos(A-B) = 1$

To find: values of A and B

Solution: Consider $2\sin(A+B) = \sqrt{3}$

$\Rightarrow \sin(A+B) = \sqrt{3}/2$

$\Rightarrow \sin(A+B) = \sin(60°)$

$A + B = 60° \longrightarrow (1)$

Also it is given, $\cos(A-B) = 1$

$\Rightarrow \cos(A-B) = \cos(0°)$

$\Rightarrow A - B = 0 \longrightarrow (2)$

now adding (1) and (2), $A + \cancel{B} + A - \cancel{B} = 60° + 0°$

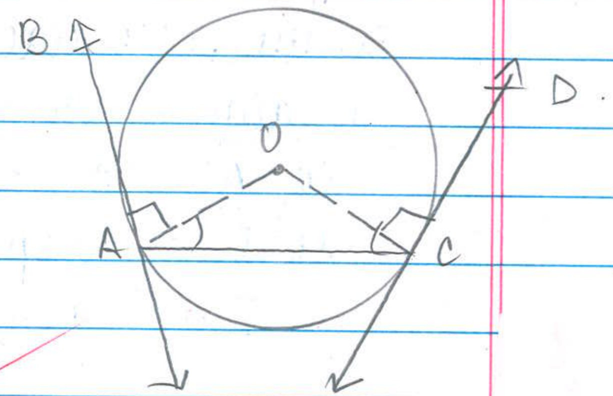
$2A = 60°$

$\therefore A = 30°$

substituting value of A in (1), we get $B = 30°$

Thus values of A and B are 30° and 30° respectively.

24. Given: AB and CD are tangents to circle with centre O and chord AC.



To find: if $\angle BAC = \angle DCA$

Construction: Join OA and OC

Solution: Consider $\triangle OAC$ where

$OA = OC$ (radii of same circles

so $\angle OAC = \angle OCA \rightarrow$ (1)

(since angles opposite to equal sides in a triangle are equal)

We also know that a tangent to a circle is perpendicular to radius at the point of contact. Since AB and CD are both tangents,

$\angle OAB = \angle OCD \rightarrow$ (2) (as each is equal to 90°)

adding (1) and (2), $\angle OAC + \angle OAB = \angle OCA + \angle OCD$

$\angle BAC = \angle DCA$

hence proved that $\angle BAC = \angle DCA$.

* Q25 solved at end of paper.

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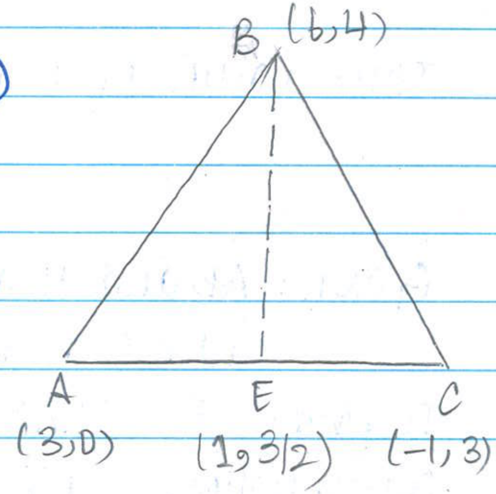
★ Q25 solved at the end of the paper ★

25. (b) Given: ΔABC with vertices $A(3,0)$; $B(6,4)$ and $C(-1,3)$, along with median BE

To find: Length of BE.



Solution: Consider ΔABC. We know that a median bisects the base to which it is drawn.

ie, AE = EC

so E = midpoint of AC

$$E = \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$$

$$E = \left[\frac{3-1}{2}, \frac{0+3}{2}\right]$$

$$E = \left[\frac{2}{2}, \frac{3}{2}\right]$$

∴ coordinates are $E(1, 3/2)$ or $E[1, 1.5]$

now length BE is given by distance formula →

$$\Rightarrow \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

substituting values, $\sqrt{(6-1)^2 + \left(4-\frac{3}{2}\right)^2}$

$$\Rightarrow \sqrt{(5)^2 + (5/2)^2}.$$

$$\Rightarrow \sqrt{25 + \frac{25}{4}}.$$

$$\Rightarrow \sqrt{\frac{100 + 25}{4}}$$

$$\Rightarrow \sqrt{\frac{125}{4}}.$$

$$\Rightarrow \frac{\sqrt{125}}{\sqrt{4}}.$$

$$\Rightarrow \frac{\sqrt{25 \times 5}}{\sqrt{4}}$$

$$\Rightarrow \frac{5\sqrt{5}}{4} \text{ units}$$

Thus length of median BE $= \frac{5\sqrt{5}}{4}$ units

Section-C

26. (a) Given : $S_m = S_n$

To prove : $S_{(m+n)} = 0$

Proof : We know that sum of first 'x' terms in AP is given by -

$$\boxed{S_x = \frac{x}{2}[2a+(x-1)d]}$$

so $S_m = \frac{m}{2}[2a+(m-1)d]$

and $S_n = \frac{n}{2}[2a+(n-1)d]$.

but $S_m = S_n$ (given)

$\Rightarrow \frac{m}{\cancel{2}}[2a+(m-1)d] = \frac{n}{\cancel{2}}[2a+(n-1)d]$

$\Rightarrow m[2a+md-d] = n[2a+nd-d]$.

$\Rightarrow 2am+m^2d-dm = 2an+n^2d-dn$

$\Rightarrow 2am-2an+m^2d-n^2d-dm+dn = 0$

$\Rightarrow 2a(m-n)+d(m^2-n^2)-d(m-n) = 0$

$\Rightarrow (m-n)[2a+d(m+n)-d] = 0$

$\Rightarrow (m-n)\,[2a + d(m+n-1)] = 0.$

$\Rightarrow [2a + d(m+n-1)] = 0 \quad -(1)$

now sum of first $(m+n)$ terms will be,

$S(m+n) = \frac{(m+n)}{2}[2a + (m+n-1)d].$

but using (1) we get,

$S_{(m+n)} = \frac{(m+n)}{2}[0]$

thus $S(m+n) = 0$

hence proved that sum of first $(m+n)$ terms of this AP will definitely be zero.

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27. Let us assume to the contrary that $\sqrt{5}$ is rational. Then it can be expressed in the form (a/b) where 'a' and 'b' are integers and co-primes. Also, $b \neq 0$.

So, $\sqrt{5} = \frac{a}{b}$

$\Rightarrow \quad b\sqrt{5} = a$

On squaring both sides we get $(b\sqrt{5})^2 = (a)^2$

$\Rightarrow \quad 5b^2 = a^2$

Since 5 divides a^2, then 5 divides a. (Since if 'p' a prime no. divides a^2, it surely divides a).

Now let $a = 5c$ (for any positive integer c).

substituting, $5b^2 = a^2$

$\Rightarrow \quad 5b^2 = (5c)^2$

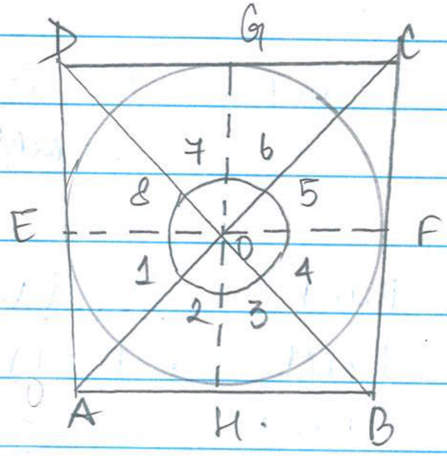
$\Rightarrow \quad 5b^2 = 25c^2$

$\Rightarrow \quad b^2 = 5c^2$

Here since 5 divides b^2, we know that 5 divides b.

However this means that 'a' and 'b' have a common factor 5, apart from 1. This contradicts the fact that they are co-primes. This contradiction arose due to incorrect assumption ie, that $\sqrt{5}$ is rational.

Hence, we conclude that $\sqrt{5}$ is irrational.



28. (b) Given : AB, BC, CD and DA are tangents to circle with centre O, forming quad. ABCD

To prove : $\angle AOB + \angle COD = 180°$

$\Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

Construction : Draw 2 diameters ie, EF and GH.

Join OG, OF, OH and OE.

Proof : We know that tangents drawn from an external point to a circle are equally inclined to the centre of the circle ie,

$\angle 1 = \angle 2$; $\angle 3 = \angle 4$; $\angle 5 = \angle 6$; $\angle 7 = \angle 8$ $\rightarrow$ (1)

(since AB, BC, CD and DA are tangents to same circles.

Also, sum of all angles around a point is 360°. So,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

from (1), we get $2(\angle 2) + 2(\angle 3) + 2(\angle 6) + 2(\angle 7) = 360°$

$2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360°$

$\underbrace{\angle 2 + \angle 3}_{} + \underbrace{\angle 6 + \angle 7}_{} = 180°$

$\angle AOB + \angle COD = 180°$

Hence proved that $\angle AOB + \angle COD = 180°$.

29. To prove : $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$

Proof : Consider LHS.

$\Rightarrow \frac{(\sec\theta-\tan\theta+1)}{(\tan\theta+\sec\theta+1)}$

$\Rightarrow \frac{\sec\theta-\tan\theta+(\sec^2\theta-\tan^2\theta)}{(\tan\theta+\sec\theta+1)}$ $[\because 1+\tan^2\theta = \sec^2\theta]$

$\Rightarrow \frac{(\sec\theta-\tan\theta)[1+(\sec\theta+\tan\theta)]}{(\tan\theta+\sec\theta+1)}$ $[\because (a^2-b^2) = (a-b)(a+b)]$

$\Rightarrow \frac{(\sec\theta-\tan\theta)\cancel{(1+\sec\theta+\tan\theta)}}{\cancel{(\tan\theta+\sec\theta+1)}}$

$\Rightarrow \sec\theta-\tan\theta$.

$\Rightarrow \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$ ie, RHS.

thus LHS = RHS. Hence proved.

30. Let assumed mean (A) for the data = 25

class interval (marks)	x_i $\left[\frac{\text{upper limit + lower limit}}{2}\right]$	f_i (frequency)	d_i $(x_i - A)$	$f_i d_i$
0 - 10	5	12	$(5-25) \Rightarrow -20$	-240
10 - 20	15	23	$(15-25) \Rightarrow -10$	-230
20 - 30	A $\boxed{25}$	34	$(25-25) \Rightarrow 0$	0
30 - 40	35	25	$(35-25) \Rightarrow 10$	250
40 - 50	45	6	$(45-25) \Rightarrow 20$	120
Total		100		(-100)

Now using the 'Assumed mean method', mean $= \left(A + \frac{\Sigma f_i d_i}{\Sigma f_i}\right)$

substituting the values, $25 + \left[\frac{-100}{100}\right]$

$\Rightarrow 25 - 1$

$\Rightarrow \underline{24}$

Thus the mean marks obtained by the students is 24.

31. Let the digit be in the form $(10x+y)$. So, digit in units place $= y$ and digit in tens place $= x$.

Now, we know $y = (x-5) \rightarrow$ ① [given]

also product of digits $= 36$.

$$xy = 36$$

$$x(x-5) = 36 \quad \text{[Substituting value of } y \text{ from ①]}$$

$$x^2 - 5x = 36$$

$$x^2 - 5x - 36 = 0$$

by splitting the middle term we get,

$$x^2 - 9x + 4x - 36 = 0$$

$$x(x-9) + 4(x-9) = 0$$

$$(x-9)(x+4)$$

$\therefore x = 9$ or $x = -4$.

We reject $(x = -4)$ as this number cannot be negative.

So, $x = 9$. from ①, we get, value of $y \Rightarrow (9-5) = 4$.

Thus the number is $10x + y \Rightarrow 94$.

Hence the required number is 94.

Section - D

32. (a) Consider the given equations.

(i) $3x + y + 4 = 0$

Required values of x and y are :

x	1	0	(−3)
y	(−7)	(−4)	5

(ii) $3x - y + 2 = 0$

Required values of x and y are:

x	1	0	(−2)
y	5	2	(−4)

[Lines are made on graph on page 23]

From page 23,

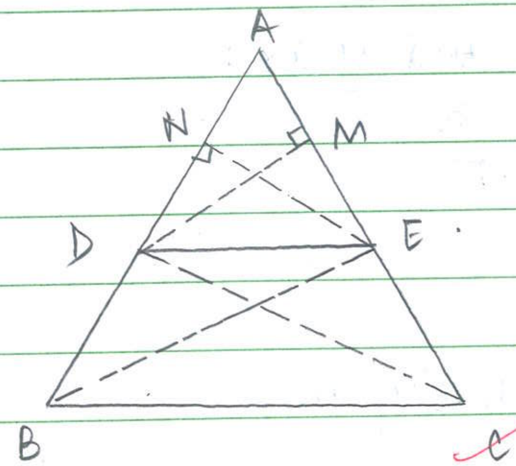
Point of intersection is $G(-1, -1)$.

Hence value of $x = -1$ and $y = -1$.

We observe that this value satisfies each equation.

Hence solved.

33. (a) **Statement:** If a line is drawn parallel to one side of a triangle to intersect the other 2 sides at two distinct points, the other two sides are divided in the same ratio.



Given: ΔABC where $DE \parallel BC$ and DE intersects AB at D and intersects AC at E.

To prove: $\frac{AB}{BD} = \frac{AE}{EC}$

Construction: Draw $DM \perp AC$ and $EN \perp AB$. Join BE and AC.

Proof : consider ΔADE and ΔBDE.

area $\Delta ADE = 1/2 \times base \times height \Rightarrow 1/2 \times AD \times EN$

area $\Delta BDE = 1/2 \times base \times height \rightarrow 1/2 \times BD \times EN$

$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\cancel{1/2} \times AD \times \cancel{EN}}{\cancel{1/2} \times BD \times \cancel{EN}} = \boxed{\frac{AD}{BD}} \quad \text{— (1)}$$

Similarly, consider ΔADE and ΔCDE.

area $\Delta ADE = 1/2 \times base \times height \Rightarrow 1/2 \times AE \times DM$

area $\Delta CDE = 1/2 \times base \times height \Rightarrow 1/2 \times EC \times DM$

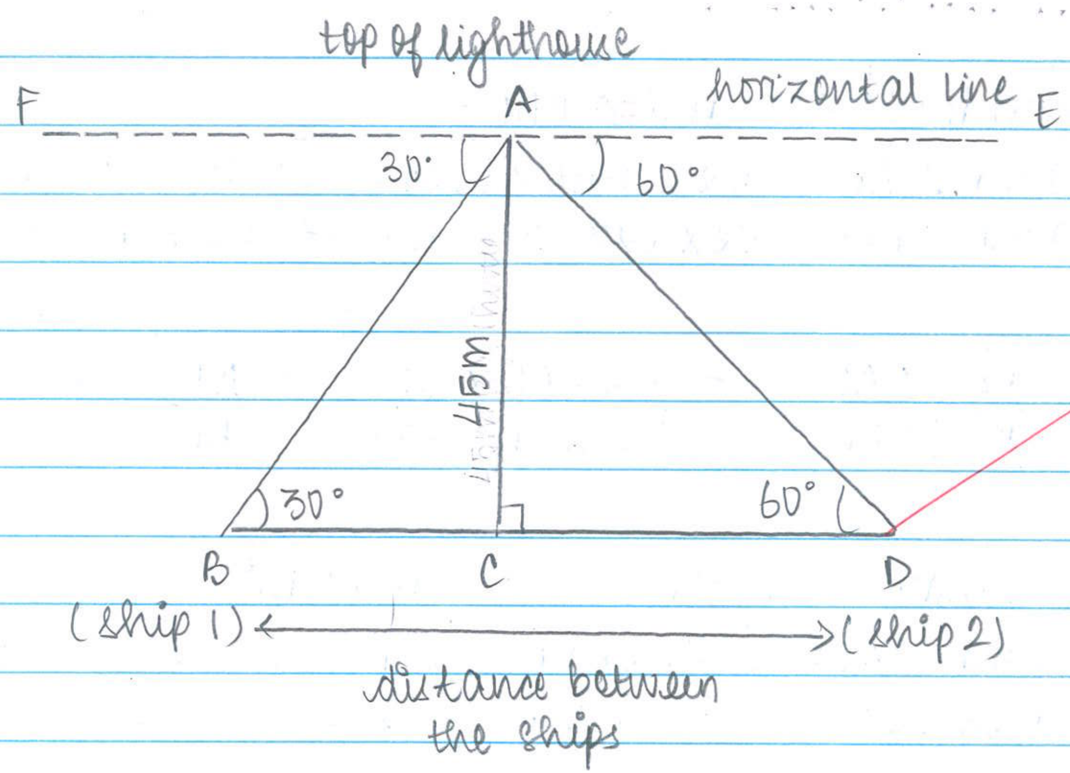
$$\Rightarrow \frac{ar(\Delta ADE)}{ar(\Delta CDE)} = \frac{\cancel{1/2} \times AE \times \cancel{DM}}{\cancel{1/2} \times EC \times \cancel{DM}} = \boxed{\frac{AE}{EC}} \quad \text{— (2)}$$

Since ΔBDE and ΔCDE lie on the same base DE and between the same parallel lines, ~~there are~~ (BC and DE), there area is equal. Hence (1) and (2) are equal.

from (1) and (2), $\boxed{\frac{AD}{DB} = \frac{AE}{EC}}$

Hence proved.

34. **Given :**



To find : Distance between ships ie, BD.

Solution : Consider $\triangle ABC$ where $\angle B = 30°$ and $\angle C = 90°$

$$\tan A = \frac{AC}{BC}$$

$$\Rightarrow \tan 30° = \frac{45m}{BC}.$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{45}{BC}$$

$\therefore$ Length of BC $= 45\sqrt{3}$m — 1

$$\begin{array}{r} 4\ \ 1\ \ \\ 1\,7.3 \\ \times 6 \\ \hline 10\,3.8 \end{array}$$

now consider ΔACD where $\angle D = 60°$ and $\angle C = 90°$.

$$\tan 60° = \frac{AC}{CD}.$$

$$\sqrt{3} = \frac{45}{CD}.$$

$$CD = \frac{45}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{45\sqrt{3}}{3}$$

$$= 15\sqrt{3}m$$

so length of CD $= 15\sqrt{3}$m — 2

now length BD $=$ BC $+$ CD

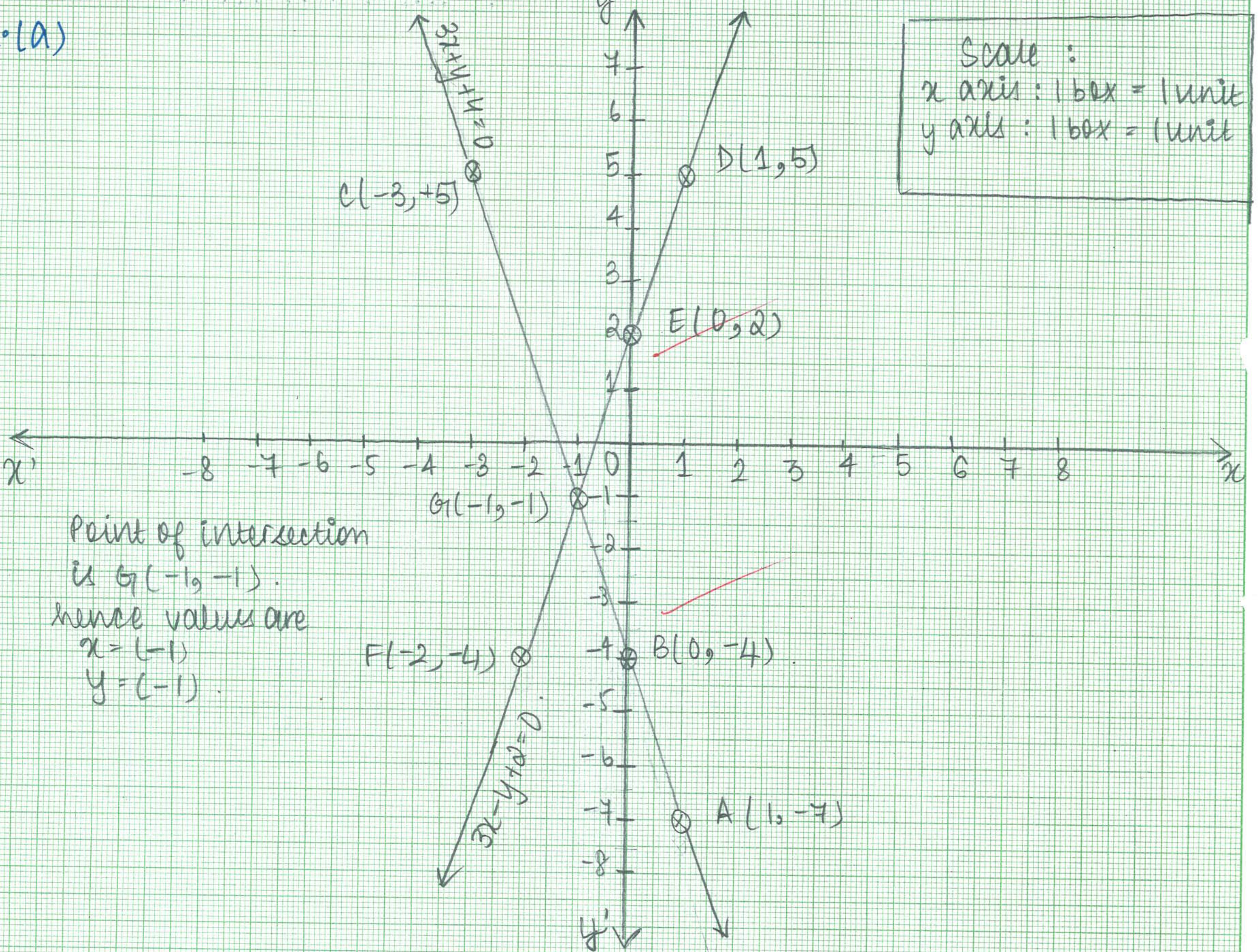
$= 45\sqrt{3} + 15\sqrt{3}$m (from 1 and 2)

$= \sqrt{3}\,[45+15]$

$= (60)(1.73) = 103.8$m

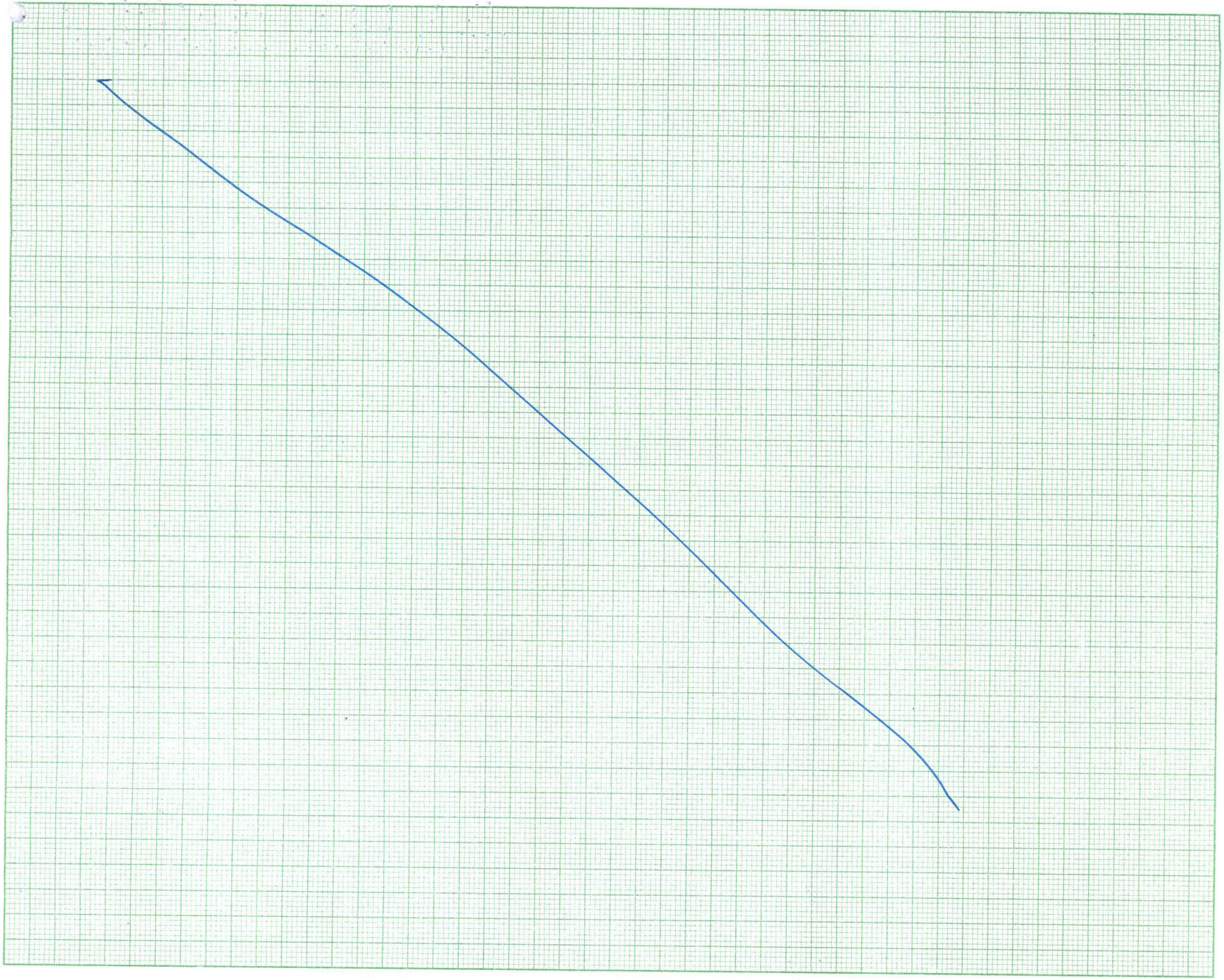
Thus distance between 2 ship is 103.8m or $\sqrt{3}(60)$m .

32·(a)



Scale :
x axis : 1 box = 1 unit
y axis : 1 box = 1 unit

Point of intersection
is G(-1, -1).
hence values are
$x = (-1)$
$y = (-1)$.

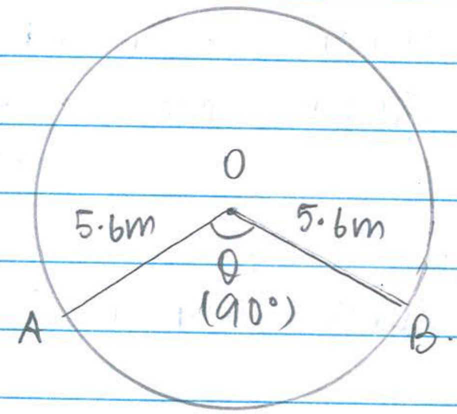


35. Given : circle with centre O and radius 5.6m.

Perimeter of sector OAB is 20m

To find : Area of sector OAB.

Solution : Consider sector OAB.



Perimeter of sector = 20m

2 radius + arc = 20m

2(5.6) + arc = 20

length of arc = $\underline{8.8m}$

length of arc = $\frac{\theta}{360°} \times 2\pi r$.

$\Rightarrow \frac{\theta}{360°} \times 2 \times \frac{22}{7} \times 5.6 = 8.8$

$$\theta = \frac{\overset{4}{\cancel{8.8}} \times \overset{18}{\cancel{360}} \times 7}{\cancel{2} \times \cancel{22} \times 5.6}$$

$$= \frac{4 \times 18}{0.8} = \underline{\underline{90°}}$$

Hence the required angle $(\theta) = 90°$

Area of sector $= \frac{\theta}{360°} \times \pi r^2$

Substituting values, $\frac{\cancel{90}}{4\ \cancel{360}} \times \frac{22}{\cancel{7}} \times 5.6 \times \cancel{5.6}\ 0.8$

$\Rightarrow \frac{22 \times 5.6 \times \cancel{8}}{\cancel{40}\ 5}$

$\Rightarrow \frac{123.2}{5}$

$\Rightarrow 24.64 m^2$

Thus the required area of the sector is $\underline{\underline{24.64 m^2}}$

Section - E

36. (i) Zeroes of the polynomial are given by the x-coordinate at the point of intersection with the coordinate axes.
Hence, zeroes of polynomial are 5 and 0.

(ii) The maximum height was achieved by the ball after 5/2 seconds.

$h = 25t - 5t^2.$

$= \left(25 \times \frac{5}{2}\right) - \left(5 \times \frac{25}{4}\right).$

$\Rightarrow \frac{125}{2} - \frac{125}{4}.$

$\Rightarrow \frac{250 - 125}{4}.$

$\Rightarrow \frac{125}{4}$

$\Rightarrow$ ~~36.25m~~ . 31.25m

hence maximum height was ~~36.25m~~ 31.25m

$$\begin{array}{r} 31.2 \\ 4\overline{)125} \\ -12 \\ \hline 05 \\ -4 \\ \hline \end{array}$$

(iii) (b) $h = 25t - 5t^2$

$20 = 25t - 5t^2$.

$0 = 25t - 5t^2 - 20$.

$5t^2 - 25t + 20 = 0$

~~$5t^2 - 5t - 20t + 20 = 0$~~ .

$t^2 - 5t + 4 = 0$

$t^2 - t - 4t + 4 = 0$

$t(t-1) - 4(t-1) = 0$

$(t-1)(t-4)$

Thus the two different values of t are '1' and '4'.

37. (i) $l = \sqrt{h^2 + r^2}$

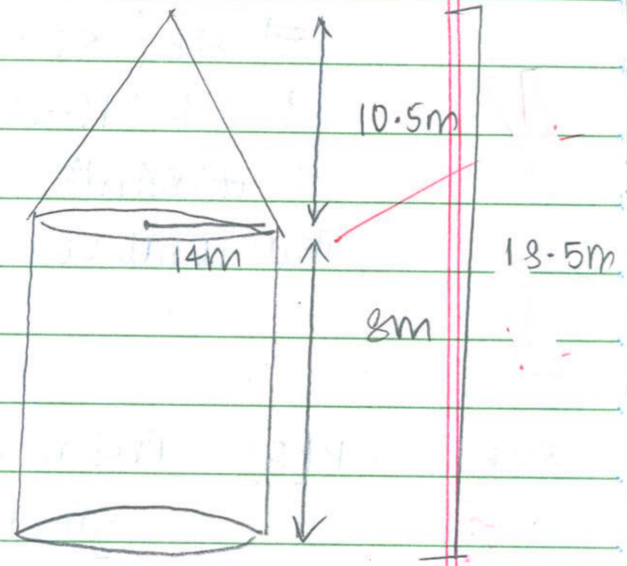
$= \sqrt{(10.5)^2 + (14)^2}$

$= \sqrt{110.25 + 196}$

$= \sqrt{306.25}$

$= 17.5m$

hence the slant height is $\underline{\underline{17.5m}}$



(ii) Floor area $= \pi r^2$

$= \pi (14)^2$

$= \frac{22}{\cancel{7}} \times 14 \times \cancel{14}\,2$

$= 44 \times 14$

$= 616m^2$

Thus floor area is $\underline{\underline{616m^2}}$

$$3 \overline{)\,10.5\,} = 3.15$$
$$-9$$
$$15$$

(iii) (b) Volume = volume of cylinder + volume of cone

$= \pi r^2 h' + 1/3 \pi r^2 h$

$= \pi r^2 (h' + 1/3 h)$

$= \left[\frac{22}{7} \times 14 \times 14\right]\left[8 + \left(\frac{1}{3} \times 10.5\right)\right]$

$\Rightarrow 616(8+3\cdot5)$

$\Rightarrow 616(11\cdot5)$

$\Rightarrow$ 7084 m^3

Thus total volume is $\underline{\underline{7084 m^3}}$

38. (i) $P(E)$ = Probability of bus or ship

$$= \frac{\text{no. of outcomes favourable to E}}{\text{total possible outcomes of E}}$$

$$= \boxed{\frac{23}{120}}$$

since no. of people who travel by bus or ship = $\frac{69}{360} \times 120 \Rightarrow \underline{\underline{23}}$

Required probability is $\boxed{\frac{23}{120}}$.

(ii) 'Car' is the favourite mode of transport.

No. of people who travel by it $= \frac{177}{360} \times 120$

$= 59$ people

Out of the 120 surveyed people, 59 use cars.

(iii) (b) No. of people who use planes $= \frac{7}{60} \times 120 \Rightarrow$ 14 people.

Thus, total revenue = ₹(5000)(14)

= ₹70,000

Thus total money made is ₹70,000.

☆ Q25 is behind ☆.

25. (a) Let the points be A(3,-5) and B(-1, 6) divided by the point C(x, y) such that x = y.

section formula gives us the coordinates for points divided =

$$(m : n) = \left[\frac{(m)(x_2) + (n)(x_1)}{m+n}, \frac{(m)(y_2)(n)(y_1)}{m+n} \right]$$

$$\Rightarrow (k:1) = \left[\frac{(k)(-1) + (1)(3)}{k+1}, \frac{(k)(+6) + (1)(-5)}{k+1} \right]$$

$$\Rightarrow (x,y) = \left[\frac{-k+3}{k+1}, \frac{+6k-5}{k+1} \right]$$

thus x coordinate $= \left(\frac{-k+3}{k+1} \right)$ and y coordinate $= \left(\frac{+6k-5}{k+1} \right)$

But we know that x = y

$$so, \frac{6k+3}{k+1} = \frac{-k-5}{k+1}$$

but we know that $x = y$.

$$\frac{-k+3}{\cancel{k+1}} = \frac{+6k-5}{\cancel{k+1}}$$

$$3 + 5 = 6k + k.$$

$$8 = 7k.$$

$$k = \frac{8}{7}.$$

hence the ratio is $k:1$ ie, $\underline{8:7}$

Required ratio is $8:7$.

end of paper