Fluid Mechanics

Exercise Solutions

Solution 1:

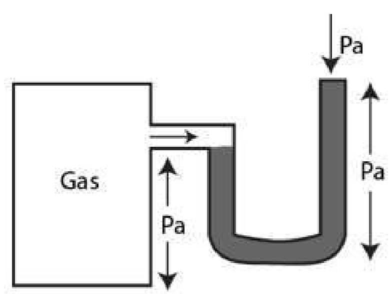
Pressure = P = ρgh

Here ρ = 1000 kg/m^3, g = 10 m s^{-2} and h = 4 m

=> P = 40000 N/m^2

Yes, it necessary to state that the tap is closed. If tap is open the level of water will decrease and hence the pressure will decrease, the pressure at the tap is atmospheric.

Solution 2:



(a) Pressure at the bottom of the tubes should be same when considered for both limbs.

From figure,

$P_a + \rho_{Hg} h_1 g = P_g + \rho_{Hg} h_2 g$

=> $P_g = P_a + \rho_{Hg} g(h_1 - h_2)$

(b) Pressure of mercury at the bottom of tube

$P = P_a + \rho_{Hg} h_1 g$

Solution 3:

Pressure = Force/Area

F = mg = 45 x 9.8 = 441 N

and A = 900 cm^2 or 0.09 m^2

Therefore, Pressure = 441/ 0.09 = 4900 N ...(1)

Also, pressure can be represented as, P = ρg Δh
Where, Δh is the height difference and ρ = density

Density of water = ρ = 1000 kg/m^3

=> P = 1000 x 9.8 x Δh ...(2)

from (1) and (2)

4900 = 1000 x 9.8 x Δh

or Δh = 50 cm

Solution 4:

(a) Pressure at bottom of the container = pressure due to water + atmospheric pressure.

$P = \rho g\, h + P_{atm}$

= (1000 x 10 x 0.2) + 1 x 10^5

= 1.02 x 10^5 N/m^2

Now, Force exerted on the bottom of the container = F = Px Area

= (1.02 x 10^5) x 0.002

F = 204 N

(b) Vertical forces (F') includes the weight of the water and the force due to atmospheric pressure.

F' = mg + [P_atm x area]

= (0.5 x 9.8) + (0.002 x 1 x 10^5)

F' = 205 N

Therefore, total upward force = F' - F = 1 N

Solution 5:

If the glass is covered by a jar and the air inside the jar is completely pumped out.

Pressure at the bottom of the glass = P = ρ gh = 1000 x 10 x 0.2 = 2000 N/m^2

The downward force is F = P x area = 2000 x 0.002 = 4N

(b) Vertical force is equal to the weight of the water = mg = 0.5 x 10 = 5N
The horizontal forces again cancel out. So the total upward force = 1N

It glass of different shape is used provided the volume, height and area remain same, no change in answer will occur.

Solution 6:

Standard atmospheric pressure is always pressure exerted by 76 cm Hg column.

$P_{atm}|Hg = \rho_{Hg}\, gh = 13.6 \times 10^3 \times g \times 0.76$ ...(1)

For water, $P_{atm}|water = \rho_{water}\, gh = 1000 \times g \times 0.76$ ...(2)

Atmospheric pressure: P_atm|Hg = P_atm|water

equating both the equations, we get

h = 1033.6 cm, is the required height of the water.

Solution 7:

Pressure = P = ρgh = 1000 x 10 x 500 = 5 x 10^6 N/m^2

and force = F = Pressure x Area = (5 x 10^6) x 2 = 10^7 N

Solution 8:

Dimensions of rectangular tank : 3 m × 2 m × 1 m.

(a) Pressure at the bottom of the tank = P = ρgh

= 1000 x 10 x 1 = 10^4 N/m^2

Area of the bottom of the tank = A = lb = 3x2 = 6m^2

Now, force = F = Pressure x Area = 10^4 x 6 = 60000 N

(b) P = ρgh = ρgx

Area of strip = A = 2 dx

So, the force on the strip = F = PA = ρgx x 2 dx

= (20000xdx) N

(c) Let us first find the perpendicular distance(d) of the strip from the bottom edge.

d = h-x = (1-x) m

Now, Torque = Force x perpendicular distance = Fd

= (20000x(1-x)dx) N-m

(d) Total force

$$F = 20000\int_0^1 x\,dx = 20000\left|\frac{x^2}{2}\right|_0^1 = 10000N$$

(e) Total Torque

$$\tau = \int_0^1 20000x(1-x)dx = \frac{10000}{3} N - m$$

Solution 9: The density of copper and gold are 8.9 and 19.3 respectively.

V = x/8.9 + (36-x)/19.3 ...(1)

When ornament placed in water, it displaces water equal to its weight. The density of water = ρ_{water} = 1 g/cm^3

Weight of the water displaced = w = mg = v ρ_{water} g = vg ....(2)

ornament weighs 34 g in water.(Given)

The buoyant force = F_b = mg = (36-34) = 2g ...(3)

Buoyant force is equal to weight of the water displaced.

vg = 2g

=> [x/8.9 + (36-x)/19.3] g = 2g

or x = 23.14/10.4 = 2.22 grams (approx)

The weight of copper in the ornament is 2.22 grams.

Solution 10:

Total volume of the Ornament(y)= Volume of gold + volume of cavity

Total volume (y)= 36/19.3 +v

Using, Principle of Flotation,

The volume of the water displaced is also y cm^3

The mass of the water displaced =y grams

The weight of the water displaced = yg

yg = 2g

or y = 2

Therefore, 36/19.3 +v = 2

or v = 0.13 cm^3

Solution 11:

Volume of metal immersed, = V = m/ρ = 0.16/8000 = 0.00002 m^3

Weight of the water displaced = F_b = V x ρ_{water} x g

= 0.00002 x 1000 x 10

= 0.2

Find the normal force exerted by the bottom of the glass on the metal piece:
Vertical forces consist of the normal reaction(N) from the surface (upwards), the weight of the metal(downwards) and the buoyant force(upwards) to maintain the balance of vertical forces.

N + F_b = mg

=> N = (0.16x10)-0.2 = 1.4 N

The normal reaction is 1/4 N in total.

Solution 12:

(a) Buoyant force should be equal to the weight of the boat.

v_w g = mg

v x 1000 = 50

or v = 1/20 m^3

(b) Let V_b be the volume of boat filled with water before water starts coming in form the sides. here V_b 1 m^3

Therefore, buoyant force = F_b = V_b ρ_w g = 1000 gN

The volume of water = V_w = m_w/ρ_w = 950/1000 = 0.95 m^3

[Here m_w + 50 = 1000 or m_w = 950 kg]

Now, fraction of water filled in the boat = 0.95 = 19/20

Solution 13:

Let the edge of the cube be "x cm".

Volume of the cube = $V = x^3/10^6\ m^3$ ...(1)

And buoyant force = $F_b = m_{mg} + V\ \rho_i\ g = 0.5 + 900\ v$ ...(2)

Also, buoyant force is equal to the weight of the water displaced

=> V x 1000 g = 0.5 + 900

=> 100V = 0.5

Using (1),

$x^3/10^6 = 0.5/100$

or x = 17 cm (approx)

Solution 14:

Density of water = ρ_w = 1000 kg/m^3

Density of ice = ρ_i = 0.9 x 1000 = 900 kg/m^3

Density of K.oil = ρ_o = 1000 kg/m^3

The weight of the cube of ice = w_i = volume x density x g = (a+b)900g ...(1)

Where a = volume of cube immersed in water and b = volume immersed in K.oil

The total buoyant force = sum of forces exerted by the liquids.

=> $F_b = a\ \rho_w + b\rho_o$= a(1000) + b(800) ....(2)

From (1) and (2)

a(1000) + b(800) = (a+b)900

=> a/b = 1

The ratio of the volumes immersed is 1:1.

Solution 15:

Density of iron = ρ_i = 8000 kg/m^3 and density of water = 1000 kg m^{-3}.
Let "x cm" be the edge of the iron sheet.

Surface area = $6x^2$ cm^2

Volume of the iron sheets = v = $6x^2 \times 0.1 = (0.6/10^6)$ m^3

Therefore, mass of the iron box= $m_b = \rho_i \times v = 4.8 \times 10^{-3} x^2$ kg ...(1)

volume of the cube = $V' = x^3$ cm^3 = $(x^3/10^6)$ m^3

buoyant force = $F_b = V' \rho_w g$

$= (x^3/10^6) \times 1000 \times g$

$= (x^3/10^3)$ gN .....(2)

Here, buoyant force is equal to the weight of the box.

From (1) and (2)
$4.8 \times 10^{-3} x^2 = (x^3/10^3)g$

=> x = 4.8 cm

Solution 16:
Total mass of the system = mass of wood (m_w) + mass of lead(m_{Pb})

= 200 + m_{Pb} ...(1)

Density of wood = ρ_w = 0.8 g/cm^3

Volume of the wooden block = $v_w = m_w/\rho_w$ = 250 cm^3

Density of lead = ρ_{Pb} = 11.3 g/cm^3

Volume of lead piece = $v_{Pb} = m_{Pb}/\rho_{Pb} = (m_{Pb}/11.3)$ cm^3

Therefore, the volume of water displaced would be equal to the total volume.

$V = v_w + v_{Pb} = 250 + (m_{Pb}/11.3)$ cm^3

$F_b = v\, \rho_w\, g = (250 + m_{Pb}/11.3) \times 1 \times g$ ....(2)

Now, (1) = (2), we get

$200 + m_{Pb} = (250 + m_{Pb}/11.3) \times 1 \times g$

=> m_{Pb} = 54.8 g

Solution 17:
volume immersed is only due to the wooden block.

The buoyant force = F_b = 250g

Now,
Total mass = F_b

=> $200 + m_{Pb}$ = 250 g

or m_{Pb} = 50 g

The mass of the lead piece is 50 g.

Solution 18:

Volume immersed in mercury= $v_m = (12^3/5)\ cm^3$

Buoyant force on the block is equal to the weight of the mercury displaced = $\rho_m \times v_m$

$= 13.6 \times (12^3/5)\ g$ ....(1)

Volume of block in water = $v_w = 12^3\ h\ cm^3$

Volume of block in mercury = $v'_m = 12^2(12-h)\ cm^3$

Now, buoyant force = $F_b = [v_m \rho_w + v_m' \rho_m]g$

$= 12^2\ h + 12^2\ (12-h)13.6\ g$ .....(2)

The buoyant force is equal to the weight of the cube.

(1)=(2)

$\Rightarrow 13.6 \times (12^3/5)\ g = 12^2\ h + 12^2\ (12-h)13.6\ g$

Solving above equation, we have

h = 10.4 cm

Therefore, water needs to be poured to a height of 10.4 cm.

Solution 19:

A hollow spherical body of inner and outer radii 6 cm and 8 cm respectively floats half-submerged in water.

Find density of the material of the sphere:

Here, Mass of the water displaced = Mass of the material,

$\rho_w \times V_i = v_b \times \rho$

$\Rightarrow 1000 \times (2/3)\ \pi\ 8^3 = (4/3)\pi\ (8^3 - 6^3)\rho$

Solving above equation, we have

$\rho = 865\ kg/m^3$

Solution 20:
A solid sphere of radius 5 cm floats in water. If a maximum load of 0.1 kg can be put on it without wetting the load

Volume of sphere = $v = 4\pi r^3/3 = (5\pi/3) \times 10^{-4}\ m^3$

Weight of water displaced = $\rho_w vg = (5\pi/3)$ N

Now, Weight of sphere + Load = weight of displaced water
$\rho_s vg + 0.1 \times 10 = \rho_w vg$

or $\rho_s = [\rho_w vg - 1]/vg$

Therefore, specific gravity = $\rho_s/\rho_w = 1 - 1/[5\pi/3] = 0.81$

Which is the specific gravity of sphere.

Solution 21:
The weight of iron block = $w_i = m_i g - v_i \rho_a g = m_i [1 - (1/\rho_i) \rho_a]g$

and weight of wooden block = $w_w = m_w g - v_w \rho_a g = m_w [1 - (1/\rho_w) \rho_a]g$

Now, ratio of w_i and w_w:

$$\frac{W_i}{W_w} = \frac{m_i\left[1 - \frac{1}{\rho_i}\rho_a\right]g}{m_w\left[1 - \frac{1}{\rho_w}\rho_a\right]g}$$

$$\Rightarrow \frac{W_i}{W_w} = \frac{1-\frac{1.293}{7800}}{1-\frac{1.293}{800}} = 1.0015$$

Solution 22:
Buoyant force = $F_b = v \rho_w g = (\pi r^2 x)\rho_w g$

Where v = volume of the immersed object

and ma = $(\pi r^2 x)\rho_w g$

or a = $(\pi r^2 x \rho_w g)/2000$

Now, time period = T = 2π √(displacement/acceleration)

=> T = 0.5 sec (approx)

Solution 23:

(a)In the equilibrium condition, the weight of the cylinder, is supported by the spring and the buoyant force.

$kx + v \rho_w g = mg$

where v is volume = $\pi r^2(h/2)$

=> $500x + (\pi(0.05)^2 x0.1x10x1000) = \pi((0.05)^2 x\ 0.2x8000x10)$

Solving above equation, we get

x = 23.5 cm

(b) Driving force = F = $kx + v\rho_w g$

=> ma = $kx + \pi r^2 x \rho_w g$

=> a = 2π √(displacement/acceleration)/m

Time period, T = 2π √(displacement/acceleration)

= $2\pi \sqrt{0.5x/[500+(\pi(0.05)^2 x1000x10)x]}$

= 0.935 sec

Solution 24:

(a) The weight of the block is balanced by the spring and the buoyant force.

=> $mg = kx + v \rho_w g$

=> $0.5 \times 10 = 50x + (0.5/800) \times 1000 \times 10$

Solving for x,

$x = 2.5$ cm

(b) $a = kx/m$
$w^2x = kx/m$

Time period = $T = 2\pi \sqrt{(m/k)} = (\pi/5)$ sec

Solution 25:

The weight of the remaining ice will be balanced by the buoyant force provided from the melted water.

$mg = v \rho_w g$

The mass of the ice = $m = x^3 \times \rho_i$

=> $x^3 \times 0.9 = x^2 \times h \times 1$

=> $h = 0.9\ x$

Volume of water formed = $4^3 - x^3 = \pi r^2 h - x^2 h$

$x = 2.26$ cm

Solution 26:
Balance all the vertical forces on the tube.

P_a A + Alpha$_o$ = P_a + Ahρg

Where P_a is atmospheric pressure.

=> a_o I = hg

or h = a_0/g

Solution 27:
The sum of the volume flow from Alaknanda and Bhagirathi is equal to the volume flow in Ganga.
Let the depth of the rivers be "d"

v_A x 12 x d + v_B x 8 x d = v_G x 16 x d

=> v_G = 23 km/h

Solution 28:

(a)The total volume is equal to area times velocity.
v_A x a_v = v

Where v_A is the velocity in tube.

=> 4 x 10^{-2} x v_A = 1

=> V_A = 25 cm/s

(b) Let the velocity in tube B, v_B

For steady flow: v_A x a_A = v_B x a_B

=> 25 x 4 x 10^{-2} = v_B x 2 x 10^{-2}

=> V_B = 50 cm/s

(c) From Bernoulli equation,

$P_A + (1/2)\rho v_A^2 = P_B + (1/2)\rho V_B^2$

=> $P_A - P_B = (1/2)\rho [v_B^2 - v_A^2]$

= $(1/2) \times 1000 \times (0.5^2 - 0.25^2)$

= 94 N/m^2

Solution 29:

V_A = 25 cm/s and V_B = 50 cm/s [from previous problem solution]
As changing the orientation doesn't change the volume of water flowing from the tubes.

(c) From Bernoulli's equation

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

$$\Rightarrow P_A - P_B = \frac{1}{2} \times 1000 \times [0.5^2 - 0.25^2] - 1000 \times 10 \times \frac{15}{16 \times 10^2}$$

$$= 0$$

Therefore, pressure difference is 0 N/m^2

Solution 30:

(a)The total volume is equal to area times velocity.

$V = a_B \times v_B$

Where, $v = 1\ cm^3$, a_A = area of cross-section of tube A = 4 mm^2 and a_B = area of cross-section of tube B = 2 mm^2

=> $v_B = 50$ cm/s

(b) let v_b be the velocity in tube

$v_A \times a_A = v_B \times a_B$

=> $v_A = 25$ cm/s

(c) From Bernoulli equation

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$

$$\Rightarrow P_A - P_B = \frac{1}{2} \times 1000 \times [0.5^2 - 0.25^2] + 1000 \times 10 \times \frac{15}{16 \times 10^2}$$

$$= 187.5 N/m^2$$

$$\Rightarrow P_A - P_B = 188 N/m^2 \text{ (approx)}$$

Solution 31:

(a)From equation of continuity,

$v_A \times a_A = v_B \times av$

$0.1 \times 1 = 0.5 \times v_B$

=> v_B = 20cm/s

(b) From Bernoulli's equation

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g h_B$$
$$\Rightarrow P_B - P_A = \rho g[h_B - h_A] - \frac{1}{2}\rho[v_B^2 - v_A^2]$$
$$= 1000 \times 10 \times 0.05 - \frac{1}{2} \times 1000 \times [0.2^2 - 0.1^2]$$
$$= 485 N/m^2$$

Solution 32:

Equation of continuity: v_A x a_A = v_B x a_B

=> $v_B = 2\ v_A$

Where v_A and v_B speed of water at A and speed of water at B respectively.

a_A = Area of cross-section of tube A and a_B = Area of cross-section of tube B

Also, given h = difference in height in the two columns = 2 cm

Pressure difference between A and B: $P_A - P_B = \rho gh = 1000 \times 10 \times 0.02 = 200\ N/m^2$

By Bernoulli's equation,

$P_A + (1/2)\rho[v_B^2 - v_A^2]$

$200 = (1/2) \times 1000 \times [4v_A^2 - v_A^2]$

or $v_A^2 = 400/(3\times100)$

or $v_A = 36.51$ cm/s

Rate of flow = v_A x a_A = 36.51 x 4 = 146 cm^3/s (approx)

Solution 33:

Equation of continuity: $v_A \times a_A = v_B \times a_B$

Where v_A and v_B are velocities of flow at A and B

$v_A \times 5 = 500$

=> $v_A = 100$ cm/s or 1 m/s

Similarly, $v_B = 250$ cm/s or 2.5 m/s

Now, the pressure difference between the points,

$P_A - P_B = \rho_{Hg} gh = 13.6 \times 10 \times h$

From the Bernoulli's equation:

$$P_A + \frac{1}{2}\rho v_A^2 = P_B + \frac{1}{2}\rho v_B^2$$

$$P_A - P_B = \rho_{Hg} gh = \frac{1}{2}\rho_w[v_B^2 - v_A^2]$$

$$=> 13.6 \times 980 \times h = \frac{1}{2} \times 1 \times [250^2 - 100^2]$$

$$=> h = \frac{26250}{13.6 \times 980} = 1.969 \text{ cm}$$

Solution 34:

(a) The velocity of water exiting at "a"

$v_A = \sqrt{2gh}$

=> $v_A = \sqrt{(2 \times 10 \times 9.8)} = 4$ m/s

(b) $v_B = \sqrt{(2g.h/2)} = \sqrt{(2x10x0.4)} = \sqrt{8}$ m/s

(c) volume flow = V = Area x height = Ah

Therefore, dv/dt = A dh/dt ...(1)

We know, V = av dt

or V = a ($\sqrt{2gh}$) t

Differentiating above equation, we have

dV/dt = A x $\sqrt{2gh}$ ...(2)

From (1) and (2)

a ($\sqrt{2gh}$) = A dh/dt

=> 2 x 10^{-6} ($\sqrt{2gh}$) = 0.4 dh/dt

=> dh/($\sqrt{2gh}$) = 5x10^{-6} dt

(d) integrating above equation ,we have

$$5 \times 10^{-6} \int_0^t dt = \frac{1}{\sqrt{28}} \int_{0.8}^{0.4} \frac{dh}{\sqrt{h}}$$

$$\Rightarrow t = \frac{1}{\sqrt{20}} \times 2 \times [(0.4)^{\frac{1}{2}} - (0.8)^{\frac{1}{2}} \times \frac{1}{5 \times 10^{-6}}$$

solving for t,

t = 6.51 h

Solution 35: Water level is maintained in a cylindrical vessel up to a fixed height H.

Height of water above the hole = H-h

Velocity with which water = v = $\sqrt{[2g[H-h]}$ and time of flight be t

$t = \sqrt{(2h/g)}$

The horizontal distance travelled = x = vt = $\sqrt{[2g[H-h]} \times \sqrt{(2h/g)}$

=> $x = \sqrt{[4[Hh-h^2]}$

To maximize this function: dx/dh = 0

$d/dh \sqrt{[4[Hh-h^2]} = 0$

$h = H/2$