

Class X
(CBSE 2019)
Mathematics
Abroad (Set-3)

General Instructions:

- (i) **All** questions are compulsory.
 - (ii) The question paper consists of **30** questions divided into four sections – **A, B, C** and **D**.
 - (iii) Section **A** comprises **6** questions of **1** mark each. Section **B** contains **6** questions of **2** marks each. Section **C** contains **10** questions of **3** marks each. Section **D** contains **8** questions of **4** marks each.
 - (iv) There is no overall choice. However, an internal choice has been provided in **two** questions of **1** mark, **two** questions of **2** marks, **four** questions of **3** marks each and **three** questions of **4** marks each. You have to attempt only **one** of the alternative in all such questions.
 - (v) Use of calculators is **not** permitted.
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Question 1

Which term of the A.P. $-4, -1, 2, \dots$ is 101?

SOLUTION:

We have been given an arithmetic progression where

$$a = -4, d = -1 - (-4) = 3 \text{ and } a_n = 101$$

We need to find which term of the given AP is 101 so, we need to find n .

$$\text{Using } a_n = a + (n - 1)d$$

Substituting the values in the formula we get

$$101 = -4 + (n - 1)3$$

$$101 + 4 = 3n$$

$$3n = 105$$

$$n = 35$$

Therefore, 35th term of given A.P is 101.

Question 2

Evaluate:

$$\frac{\tan 65^\circ}{\cot 25^\circ}$$

OR

Express $(\sin 67^\circ + \cos 75^\circ)$ in terms of trigonometric ratios of the angle between 0° and 45° .

SOLUTION:

$$\begin{aligned} & \frac{\tan 65^\circ}{\cot 25^\circ} \\ &= \frac{\tan(90^\circ - 25^\circ)}{\cot 25^\circ} \quad (\because \tan(90^\circ - \theta) = \cot \theta) \\ &= \frac{\cot 25^\circ}{\cot 25^\circ} \\ &= 1 \end{aligned}$$

OR

$$(\sin 67^\circ + \cos 75^\circ)$$

$$= (\sin(90^\circ - 23^\circ) + \cos(90^\circ - 25^\circ)) \quad (\because \sin(90^\circ - \theta) = \cos \theta \text{ and } \cos(90^\circ - \theta) = \sin \theta) \\ = (\cos 23^\circ + \sin 25^\circ)$$

Question 3

Find the value of k for which the quadratic equation $kx(x - 2) + 6 = 0$ has two equal roots.

SOLUTION:

Given quadratic equation is:

$$kx(x - 2) + 6 = 0$$

$$\Rightarrow kx^2 - 2kx + 6 = 0$$

For a quadratic equation to have equal roots,

$$b^2 - 4ac = 0$$

Comparing the given equation with general equation $ax^2 + bx + c = 0$

We get $a = k$, $b = -2k$ and $c = 6$

$$(-2k)^2 - 4(k)(6) = 0$$

$$\Rightarrow 4k^2 - 24k = 0$$

$$\Rightarrow 4k(k - 6) = 0$$

Therefore, $k = 0$ and $k = 6$

Question 4

Find a rational number between $\sqrt{2}$ and $\sqrt{7}$.

OR

Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$.

SOLUTION:

We know

$$\sqrt{2} = 1.414$$

$$\sqrt{7} = 1.732$$

So, rational number between $\sqrt{2}$ and $\sqrt{7}$ will be $1.5 = \frac{3}{2}$.

OR

Given prime factorisation is $2^2 \times 5^3 \times 3^2 \times 17$.

A number will have zero at the end when we have 2×5 .

In $2^2 \times 5^3 \times 3^2 \times 17$ we will have 2 zeroes as $(2^2 \times 5^2) \times 5 \times 3^2 \times 17$.

Question 5

Find the distance between the points (a, b) and $(-a, -b)$.

SOLUTION:

Using distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = a$, $y_1 = b$, $x_2 = -a$ and $y_2 = -b$

On substituting the values in the formula we get

$$\sqrt{(-a - a)^2 + (-b - b)^2}$$

$$= \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2}$$

$$= 2\sqrt{a^2 + b^2}$$

Therefore, the distance between (a, b) and $(-a, -b)$ is $2\sqrt{(a)^2 + (b)^2}$

Question 6

Let $\triangle ABC \sim \triangle DEF$ and their areas be respectively, 64 cm^2 and 121 cm^2 . If $EF = 15.4 \text{ cm}$, find BC .

SOLUTION:

Given: $\triangle ABC \sim \triangle DEF$

We know ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{\text{ar}\triangle ABC}{\text{ar}\triangle DEF} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \frac{64}{121} = \left(\frac{BC}{15.4}\right)^2$$

$$\Rightarrow \left(\frac{8}{11}\right)^2 = \left(\frac{BC}{15.4}\right)^2$$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow BC = \frac{8 \times 15.4}{11} = 11.2 \text{ cm}$$

Thus, $BC = 11.2 \text{ cm}$.

Question 7

Find the solution of the pair of equation :

$$\frac{3}{x} + \frac{8}{y} = -1; \frac{1}{x} - \frac{2}{y} = 2, x, y \neq 0$$

OR

Find the value(s) of k for which the pair of equations $\begin{cases} kx + 2y = 3 \\ 3x + 6y = 10 \end{cases}$ has a unique solution.

SOLUTION:

The given equations are

$$\frac{3}{x} + \frac{8}{y} = -1 \quad \dots\dots (1)$$

$$\frac{1}{x} - \frac{2}{y} = 2 \quad \dots\dots (2)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

(1) and (2) will become

$$3u + 8v = -1 \quad \dots\dots (3)$$

$$u - 2v = 2 \quad \dots\dots (4)$$

Multiply (4) with 4

$$4u - 8v = 8 \quad \dots\dots (5)$$

Adding (3) and (5) we get

$$7u = 7$$

$$\Rightarrow u = 1$$

Putting this value in (4)

$$1 - 2v = 2$$

$$\Rightarrow v = \frac{-1}{2}$$

Now

$$\frac{1}{x} = u$$

$$\Rightarrow \frac{1}{x} = 1$$

$$\Rightarrow x = 1$$

And

$$\frac{1}{y} = v$$

$$\Rightarrow \frac{1}{y} = \frac{-1}{2}$$

$$\Rightarrow y = -2$$

OR

The given equations are

$$kx + 2y = 3$$

$$3x + 6y = 10$$

For a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

where $a_1 = k$, $a_2 = 3$, $b_1 = 2$, $b_2 = 6$

$$\frac{k}{3} \neq \frac{2}{6}$$

$$\Rightarrow k \neq 1$$

For all values of k except 1, the given linear equations will have unique solution.

Question 8

Use Euclid's division algorithm to find the HCF of 255 and 867.

SOLUTION:

The given numbers are 255 and 867.

Now $867 > 255$. So, on applying Euclid's algorithm we get

$$867 = 255 \times 3 + 102$$

Now the remainder is not 0 so, we repeat the process again on 255 and 102

$$255 = 102 \times 2 + 51$$

The algorithm is applied again but this time on the numbers 102 and 51

$$102 = 51 \times 2 + 0$$

Thus, the HCF obtained is 51.

Question 9

The point R divides the line segment AB , where $A (-4, 0)$ and $B (0, 6)$ such that $AR = \frac{3}{4} AB$. Find the coordinates of R .

SOLUTION:

We have given that R divides the line segment AB

$$AR + RB = AB$$

$$\frac{3}{4}AB + RB = AB$$

$$\Rightarrow RB = \frac{AB}{4}$$

$$\Rightarrow AR : RB = 3 : 1$$

Using section formula:

$$x = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2} \right), y = \left(\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right)$$

$$m_1 = 3, m_2 = 1$$

$$x_1 = -4, y_1 = 0$$

$$x_2 = 0, y_2 = 6$$

Plugging values in the formula we get

$$x = \frac{3 \times 0 + 1 \times (-4)}{3 + 1}, y = \frac{3 \times 6 + 1 \times 0}{3 + 1}$$

$$x = \frac{-4}{4}, y = \frac{18}{4}$$

$$\Rightarrow x = -1, y = \frac{9}{2}$$

Therefore, the coordinates of R are $\left(-1, \frac{9}{2}\right)$.

Question 10

How many multiples of 4 lie between 10 and 205 ?

OR

Determine the A.P. whose third term is 16 and 7th term exceeds the 5th by 12.

SOLUTION:

We need to find the number of multiples of 4 between 10 and 205.

So, multiples of 4 gives the sequence 12, 16, ..., 204

$$a = 12, d = 4 \text{ and } a_n = 204$$

$$\text{Using the formula } a_n = a + (n - 1)d$$

Plugging values in the formula we get

$$204 = 12 + (n - 1)4$$

$$204 = 12 + 4n - 4$$

$$4n = 196$$

$$n = 49$$

Thus, there are 49 multiples of 4 between 10 and 205.

OR

Given: 3rd term of the AP is 16.

$$a_3 = 16$$

$$a + (3 - 1)d = 16$$

$$a + 2d = 16 \quad \dots(1)$$

Also, 7th term exceeds the 5th term by 12.

$$a_7 - a_5 = 12$$

$$[a + (7 - 1)d] - [a + (5 - 1)d] = 12$$

$$(a + 6d) - (a + 4d) = 12$$

$$2d = 12$$

$$d = 6$$

From equation (1), we obtain

$$a + 2(6) = 16$$

$$a + 12 = 16$$

$$a = 4$$

Therefore, A.P. will be 4, 10, 16, 22, ...

Question 11

Three different coins are tossed simultaneously. Find the probability of getting exactly one head.

SOLUTION:

Total possible outcomes of tossing three coins simultaneously are {TTT, TTH, THT, THH, HTT, HTH, HHT, HHH}

that is 8

We have to find the probability of getting exactly one head.

Cases of exactly one head are {TTH,THT,HTT}

that is 3

Probability of getting exactly on head is $\frac{3}{8}$.

Question 12

A die is thrown once. Find the probability of getting.

- (a) a prime number.
- (b) an odd number

SOLUTION:

Total outcomes of throwing a dice once are 1, 2, 3, 4, 5 and 6

(1)Probability of getting a prime number

Prime numbers are 2, 3 and 5 in throwing a die once

Probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$

(2)Probability of getting an odd number

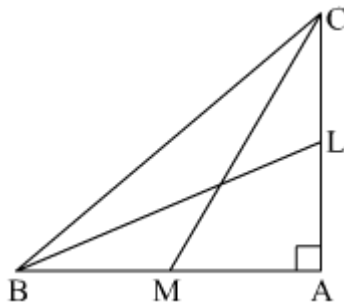
odd numbers are those that are not divisible by 2.

So, there three odd numbers in throwing a dice once which is 1, 3 and 5.

Probability of getting an odd number = $\frac{3}{6} = \frac{1}{2}$

Question 13

In Figure 1, BL and CM are medians of a $\triangle ABC$ right-angled at A. Prove that $4(BL^2 + CM^2) = 5 BC^2$.



OR

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

SOLUTION:

To prove: $4 (BL^2 + CM^2) = 5 BC^2$

Proof: In $\triangle CAB$,

Applying Pythagoras theorem,

$$AB^2 + AC^2 = BC^2 \quad \dots(1)$$

In $\triangle ABL$,

$$AL^2 + AB^2 = BL^2$$

$$\Rightarrow \left(\frac{AC}{2}\right)^2 + AB^2 = BL^2$$

$$\Rightarrow AC^2 + 4 AB^2 = 4 BL^2 \quad \dots\dots(2)$$

In $\triangle CAM$,

$$CA^2 + MA^2 = CM^2$$

$$\Rightarrow \left(\frac{BA}{2}\right)^2 + CA^2 = CM^2$$

$$\Rightarrow BA^2 + 4 CA^2 = 4 CM^2 \quad \dots\dots(3)$$

Adding (2) and (3)

$$AC^2 + 4 AB^2 + BA^2 + 4 CA^2 = 4 BL^2 + 4 CM^2$$

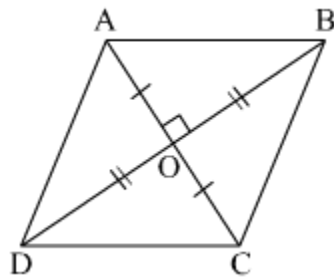
$$\Rightarrow 5 AC^2 + 5 AB^2 = 4 (BL^2 + CM^2)$$

$$\Rightarrow 5 (AC^2 + AB^2) = 4 (BL^2 + CM^2)$$

$$\Rightarrow 5 (BC^2) = 4 (BL^2 + CM^2) \quad (\text{From (1)})$$

Hence Proved.

OR



In $\triangle AOB$, $\triangle BOC$, $\triangle COD$, $\triangle AOD$,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \quad \dots (1)$$

$$BC^2 = BO^2 + OC^2 \quad \dots (2)$$

$$CD^2 = CO^2 + OD^2 \quad \dots (3)$$

$$AD^2 = AO^2 + OD^2 \quad \dots (4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2(AO^2 + OB^2 + OC^2 + OD^2)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

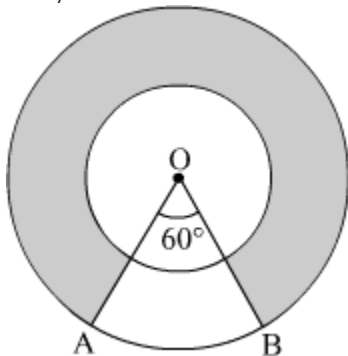
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

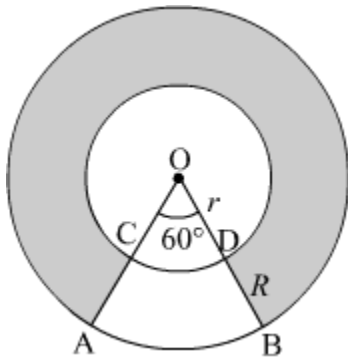
$$= (AC)^2 + (BD)^2$$

Question 14

In Figure 2, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60^\circ$, find the area of the shaded region.



SOLUTION:



Radius of inner circle, $OC = 21$ cm

Radius of outer circle, $OA = 42$ cm

Area of circle with radius $R = \pi R^2 = \pi(42)^2$

Area of circle with radius $r = \pi r^2 = \pi(21)^2$

Area of sector AOB = $\frac{\theta}{360} \times \pi R^2 = \frac{60}{360} \times \pi(42)^2 = \frac{\pi(42)^2}{6}$

Area of sector COD = $\frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \pi(21)^2 = \frac{\pi(21)^2}{6}$

Area of shaded portion = Area of circle with radius R - Area of circle with radius r - [Area of sector AOB - Area of sector COD]

$$= \pi(42)^2 - \pi(21)^2 - \left[\frac{\pi(42)^2}{6} - \frac{\pi(21)^2}{6} \right]$$

$$= \pi \left[(42)^2 - (21)^2 - \frac{1}{6} \left[(42)^2 - (21)^2 \right] \right]$$

$$= \pi \left[\left((42)^2 - (21)^2 \right) \left(1 - \frac{1}{6} \right) \right]$$

$$= \pi \left[(42 - 21)(42 + 21) \frac{5}{6} \right]$$

$$= \frac{22}{7} \times \frac{5}{6} \times 21 \times 63$$

$$= 3465 \text{ cm}^2$$

Question 15

A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere and hence find the surface area of this sphere.

OR

A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, how much time will the tank be filled ?

SOLUTION:

A cone has been reshaped in sphere

Height of cone is 24 cm and radius of base is 6 cm

Volume of sphere = volume of cone

$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

Plugging the values in the formula we get

$$\text{volume of cone} = \frac{1}{3} \pi (6)^2 24$$

$$= 288\pi \text{ cm}^3$$

Let the radius of sphere be r

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3$$

Since, volume of cone = volume of sphere

Volume of sphere = $288\pi \text{ cm}^3$

So,

$$288\pi = \frac{4}{3}\pi r^3$$

$$\Rightarrow 288 = \frac{4}{3}r^3$$

$$\Rightarrow r^3 = 216$$

$$\Rightarrow r = 6 \text{ cm}$$

Hence, radius of reshaped sphere is 6 cm

Now, surface area of sphere = $4\pi r^2$

$$= 4\pi(6)^2$$

$$= 144 \times \frac{22}{7}$$

$$= 452.5 \text{ cm}^2$$

Therefore, surface area of sphere is 452.57 cm^2 .

OR



Consider an area of cross-section of pipe as shown in the figure.

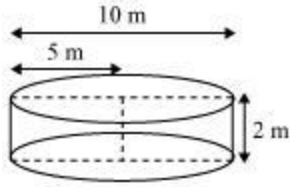
$$\text{Radius } (r_1) \text{ of circular end of pipe} = \frac{20}{200} = 0.1 \text{ m}$$

$$\text{Area of cross-section} = \pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi \text{ m}^2$$

$$\text{Speed of water} = 3 \text{ km/h} = \frac{3000}{60} = 50 \text{ metre/min}$$

$$\text{Volume of water that flows in 1 minute from pipe} = 50 \times 0.01\pi = 0.5\pi \text{ m}^3$$

$$\text{Volume of water that flows in } t \text{ minutes from pipe} = t \times 0.5\pi \text{ m}^3$$



Radius (r_2) of circular end of cylindrical tank $= \frac{10}{2} = 5\text{m}$

Depth (h_2) of cylindrical tank $= 2\text{ m}$

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe $=$ Volume of water in tank

$$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$$

$$t \times 0.5 = 5^2 \times 2$$

$$t = 100$$

Therefore, the cylindrical tank will be filled in 100 minutes.

Question 16

Calculate the mode of the following distribution :

Class :	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency :	4	7	20	8	1

SOLUTION:

Modal class is the class with highest frequency

modal class is 20 - 25

lower limit of modal class i.e $l = 20$

class size i.e $h = 5$

frequency of modal class $f_1 = 20$

frequency of preceding class $f_0 = 7$

frequency of succeeding class $f_2 = 8$

Using the formula

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Plugging the values in the formula we get

$$\text{mode} = 20 + \left(\frac{20-7}{2 \times 20 - 7 - 8} \right) \times 5$$

$$\text{mode} = 20 + \left(\frac{13}{25} \right) \times 5$$

$$\text{mode} = 20 + \frac{13}{5}$$

$$\text{mode} = \frac{113}{5} = 22.6$$

Question 17

Show that $\frac{2+3\sqrt{2}}{7}$ is not a rational number, given that $\sqrt{2}$ is an irrational number.

SOLUTION:

To prove: $\frac{2+3\sqrt{2}}{7}$ is irrational, let us assume that $\frac{2+3\sqrt{2}}{7}$ is rational.

$$\frac{2+3\sqrt{2}}{7} = \frac{a}{b}; b \neq 0 \text{ and } a \text{ and } b \text{ are integers.}$$

$$\Rightarrow 2b + 3\sqrt{2}b = 7a$$

$$\Rightarrow 3\sqrt{2}b = 7a - 2b$$

$$\Rightarrow \sqrt{2} = \frac{7a-2b}{3b}$$

Since a and b are integers so, $7a - 2b$ will also be an integer.

So, $\frac{7a-2b}{3b}$ will be rational which means $\sqrt{2}$ is also rational.

But we know $\sqrt{2}$ is irrational(given).

Thus, a contradiction has risen because of incorrect assumption.

Thus, $\frac{2+3\sqrt{2}}{7}$ is irrational.

Question 18

Obtain all the zeroes of the polynomial $2x^4 - 5x^3 - 11x^2 + 20x + 12$ when 2 and -2 are two zeroes of the above polynomial

SOLUTION:

We know that if $x = \alpha$ is a zero of a polynomial, and then $x - \alpha$ is a factor of $f(x)$.

Since 2 and -2 are zeros of $f(x)$.

Therefore

$$(x - 2)(x + 2) = x^2 - 4$$

$(x^2 - 4)$ is a factor of $f(x)$. Now, we divide $2x^4 - 5x^3 - 11x^2 + 20x + 12$ by

$g(x) = (x^2 - 4)$ to find the zero of $f(x)$.

$$\begin{array}{r}
 2x^2 - 5x - 3 \\
 x^2 - 4 \overline{) 2x^4 - 5x^3 - 11x^2 + 20x + 12} \\
 \underline{2x^4 + 0x^3 - 8x^2} \\
 -5x^3 - 3x^2 + 20x + 12 \\
 \underline{-5x^3 + 0x^2 + 20x} \\
 + -3x^2 + 12 \\
 \underline{-3x^2 + 12} \\
 + - \\
 \hline
 0
 \end{array}$$

By using division algorithm we have $f(x) = g(x) \times q(x) - r(x)$

$$\begin{aligned}
 2x^4 - 5x^3 - 11x^2 + 20x + 12 &= (x^2 - 4)(2x^2 - 5x - 3) \\
 &= (x - 2)(x + 2)[2x(x - 3) + (x - 3)] \\
 &= (x - 2)(x + 2)(x - 3)(2x + 1)
 \end{aligned}$$

Hence, the zeros of the given polynomial are 2, -2, 3, $-\frac{1}{2}$.

Question 19

A motorboat whose speed is 18 km/hr in still water takes an hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

SOLUTION:

Speed of the motorboat is 18 km/h

Let us assume the speed of the stream be x km/h

speed of motorboat upstream is $(18 - x)$ km/h

speed of motorboat downstream is $(18 + x)$ km/h

Time taken to go downstream is $\left(\frac{24}{18+x}\right)$ hr

Time taken to go upstream is $\left(\frac{24}{18-x}\right)$ hr

Equation becomes:

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$

Solving the above equation

$$\frac{24(18+x) - 24(18-x)}{(18-x)(18+x)} = 1$$

$$\frac{432 + 24x - 432 + 24x}{324 - x^2} = 1$$

$$\frac{48x}{324 - x^2} = 1$$

$$48x = 324 - x^2$$

$$x^2 + 48x - 324 = 0$$

Solving for the value of x

$$\text{using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-48 \pm \sqrt{(48)^2 - 4(1)(-324)}}{2}$$

$$x = \frac{-48 \pm \sqrt{2304 + 1296}}{2}$$

$$x = \frac{-48 \pm \sqrt{3600}}{2}$$

$$x = \frac{-48 \pm 60}{2}$$

$$x = \frac{12}{2}, \frac{-108}{2}$$

$$x = 6, -54$$

Since, speed can not be negative

So, the speed of the stream is 6 km/h.

Question 20

Prove that:

$$(\sin \theta + 1 + \cos \theta)(\sin \theta - 1 + \cos \theta) \cdot \sec \theta \operatorname{cosec} \theta = 2$$

OR

Prove that :

$$\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta$$

SOLUTION:

$$\text{LHS} = (\sin \theta + 1 + \cos \theta)(\sin \theta - 1 + \cos \theta) \cdot \sec \theta \operatorname{cosec} \theta$$

$$= [\sin^2 \theta - \sin \theta + \sin \theta \cos \theta + \sin \theta - 1 + \cos \theta + \sin \theta \cos \theta - \cos \theta + \cos^2 \theta] \frac{1}{\cos \theta} \frac{1}{\sin \theta} \quad \left(\because \sec \theta = \frac{1}{\cos \theta} \text{ and } \operatorname{cosec} \theta = \frac{1}{\sin \theta} \right)$$

$$= [1 + 2 \sin \theta \cos \theta - 1] \frac{1}{\cos \theta} \frac{1}{\sin \theta}$$

$$= [2 \sin \theta \cos \theta] \frac{1}{\cos \theta} \frac{1}{\sin \theta}$$

$$= 2 = \text{RHS}$$

Hence proved

OR

$$\begin{aligned}\sqrt{\frac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} &= \frac{\sqrt{\sec \theta - 1}}{\sqrt{\sec \theta + 1}} + \frac{\sqrt{\sec \theta + 1}}{\sqrt{\sec \theta - 1}} \\&= \frac{\sqrt{\sec \theta - 1}\sqrt{\sec \theta - 1} + \sqrt{\sec \theta + 1}\sqrt{\sec \theta + 1}}{\sqrt{\sec \theta + 1}\sqrt{\sec \theta - 1}} \\&= \frac{(\sqrt{\sec \theta - 1})^2 + (\sqrt{\sec \theta + 1})^2}{\sqrt{(\sec \theta - 1)(\sec \theta + 1)}} \\&= \frac{\sec \theta - 1 + \sec \theta + 1}{\sqrt{\sec^2 \theta - 1}} \\&= \frac{2 \sec \theta}{\sqrt{\tan^2 \theta}} \\&= \frac{2 \sec \theta}{\tan \theta} \\&= 2 \frac{1}{\frac{\cos \theta}{\sin \theta}} \\&= 2 \frac{1}{\cos \theta} \\&= 2 \frac{1}{\sin \theta} \\&= 2 \operatorname{cosec} \theta\end{aligned}$$

Question 21

In what ratio does the point P(-4, y) divide the line segment joining the points A(-6, 10) and B(3, -8) ? Hence find the value of y.

OR

Find the value of p for which the points (-5, 1), (1, p) and (4, -2) are collinear.

SOLUTION:

Let P divides the line segment AB in the ratio $k : 1$

Using section formula

$$x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

A(-6, 10) and B(3, -8)

$$m_1 : m_2 = k : 1$$

plugging values in the formula we get

$$-4 = \frac{k \times 3 + 1 \times (-6)}{k+1}, y = \frac{k \times (-8) + 1 \times 10}{k+1}$$

$$-4 = \frac{3k-6}{k+1}, y = \frac{-8k+10}{k+1}$$

Considering only x coordinate to find the value of k

$$-4k - 4 = 3k - 6$$

$$-7k = -2$$

$$k = \frac{2}{7}$$

$$k : 1 = 2 : 7$$

Now, we have to find the value of y

so, we will use section formula only in y coordinate to find the value of y

$$y = \frac{2 \times (-8) + 7 \times 10}{2+7}$$

$$y = \frac{-16+70}{9}$$

$$y = 6$$

Therefore, P divides the line segment AB in 2 : 7 ratio

And value of y is 6.

OR

Points are collinear means the area of triangle formed by the collinear points is 0.

Using

$$\text{area of triangle} = \frac{1}{2} [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

$$= \frac{1}{2} [-5 (p - (-2)) + 1 (-2 - 1) + 4 (1 - p)]$$

$$= \frac{1}{2} [-5 (p + 2) + 1 (-3) + 4 (1 - p)]$$

$$= \frac{1}{2} [-5p - 10 - 3 + 4 - 4p]$$

$$= \frac{1}{2} [-5p - 9 - 4p]$$

Area of triangle will be zero points being collinear

$$\frac{1}{2} [-5p - 4p - 9] = 0$$

$$\frac{1}{2} [-9p - 9] = 0$$

$$9p + 9 = 0$$

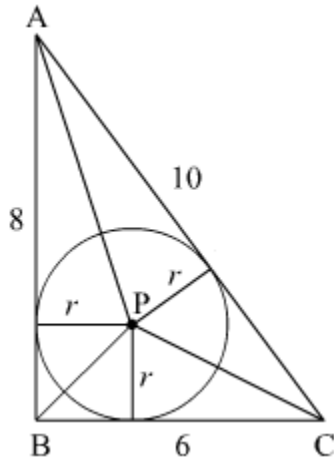
$$p = -1$$

Therefore, the value of $p = -1$.

Question 22

ABC is a right triangle in which $\angle B = 90^\circ$. If $AB = 8$ cm and $BC = 6$ cm, find the diameter of the circle inscribed in the triangle.

SOLUTION:



We have given that a circle is inscribed in a triangle

Using pythagoras theorem

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = (8)^2 + (6)^2$$

$$(AC)^2 = 64 + 36$$

$$(AC)^2 = 100$$

$$\Rightarrow AC = 10$$

Area of $\triangle ABC$ = area of $\triangle APB$ + area of $\triangle BPC$ + area of $\triangle APC$

$$\frac{1}{2} \times b \times h = \frac{1}{2} \times b_1 \times h_1 + \frac{1}{2} \times b_2 \times h_2 + \frac{1}{2} \times b_3 \times h_3$$

$$\frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r$$

$$24 = 4r + 3r + 5r$$

$$24 = 12r$$

$$\Rightarrow r = 2$$

$$\therefore d = 2r$$

$$\Rightarrow d = 2 \times 2$$

$$\Rightarrow d = 4 \text{ cm}$$

Question 23

In an A.P., the first term is -4 , the last term is 29 and the sum of all its terms is 150 . Find its common difference

SOLUTION:

$$a = -4, l = 29 \text{ and } S_n = 150$$

Using

$$S_n = \frac{n}{2} [a + l]$$

$$150 = \frac{n}{2} [-4 + 29]$$

$$300 = 25n$$

$$n = 12$$

Now, we will find d

$$\text{using } a_n = a + (n - 1)d$$

Plugging the values in the formula we get

$$29 = -4 + (12 - 1)d$$

$$33 = 11d$$

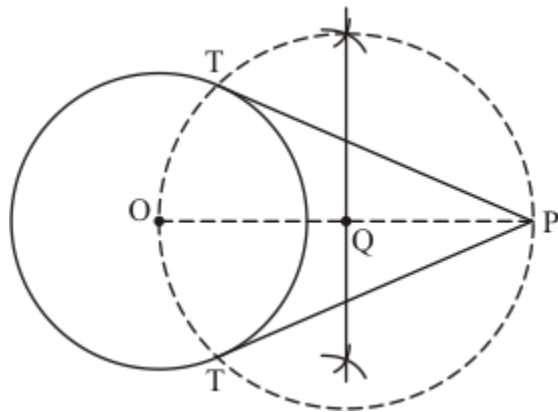
$$d = 3$$

Therefore, the common difference is 3 .

Question 24

Draw a circle of radius 4 cm. From a point 6 cm away from its centre, construct a pair of tangents to the circle and measure their lengths

SOLUTION:



Step of construction

Step: I- First of all we draw a circle of radius $AB = 4$ cm.

Step: II- Mark a point P from the centre at a distance of 6 cm from the point O .

Step: III - Draw a perpendicular bisector of OP , intersecting OP at Q .

Step: IV- Taking Q as centre and radius OQ = PQ, draw a circle to intersect the given circle at T and T'.

Step: V- Join PT and PT' to obtain the required tangents.

Thus, PT and PT' are the required tangents.

The length of PT = PT' $\approx$ 4.5 cm

Question 26

Solve for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$; $x \neq 0$, $x \neq \frac{-2a-b}{2}$, $a, b \neq 0$

OR

The sum of the areas of two squares is 640 m². If the difference of their perimeters is 64 m, find the sides of the square.

SOLUTION:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

$$\frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$$

$$\frac{2x-2a-b-2x}{4ax+2bx+4x^2} = \frac{b+2a}{2ab}$$

$$(-2a-b)(2ab) = (b+2a)(4ax+2bx+4x^2)$$

$$\frac{-(b+2a)(2ab)}{(b+2a)} = (4ax+2bx+4x^2)$$

$$-(2ab) = (4ax+2bx+4x^2)$$

$$4x^2 + 2bx + 4ax + 2ab = 0$$

$$2x(2x+b) + 2a(2x+b) = 0$$

$$(2x+2a)(2x+b) = 0$$

$$\Rightarrow (2x+2a) = 0$$

$$\Rightarrow x = -a$$

or

$$(2x+b) = 0$$

$$\Rightarrow x = \frac{-b}{2}$$

Therefore, values of x are $-a$ and $\frac{-b}{2}$

OR

Let the side of one square be x
And the side of other square be y
Sum of area of two square is 640
Equation becomes

$$x^2 + y^2 = 640 \quad (\because \text{area of square is side}^2) \quad \dots(1)$$

Now, difference of their perimeters is 64
Equation becomes

$$4x - 4y = 64 \quad (\because \text{perimeter of square is } 4 \times \text{side})$$
$$x - y = 16$$

$$\Rightarrow x = y + 16 \quad \dots\dots(2)$$

Solving the two equation by substitution method
Substitute (2) in (1)

$$(16 + y)^2 + y^2 = 640$$

$$256 + y^2 + 32y + y^2 = 640$$

$$2y^2 + 32y - 384 = 0$$

$$y^2 + 16y - 192 = 0$$

$$\text{Using } y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Plugging the values in the formula we get

$$y = \frac{-16 \pm \sqrt{256 - 4(-192)}}{2}$$

$$y = \frac{-16 \pm \sqrt{1024}}{2}$$

$$y = \frac{-16 \pm 32}{2}$$

$$y = \frac{-48}{2}, \frac{16}{2}$$

$$y = -24, 8$$

Since, sides can not be negative

Therefore, $y = 8$

Put $y = 8$ in (2)

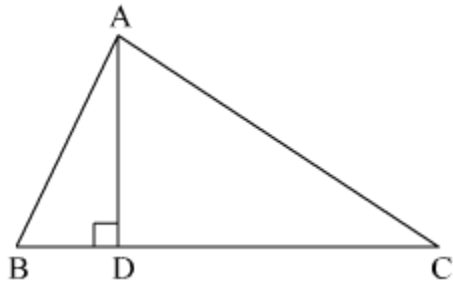
$$x = 8 + 16$$

$$x = 24$$

Therefore, the sides of square are 24 m and 8 m.

Question 27

In $\triangle ABC$ (Figure 3), $AD \perp BC$. Prove that
 $AC^2 = AB^2 + BC^2 - 2BC \times BD$



SOLUTION:

Applying Pythagoras theorem in $\triangle ADB$, we obtain

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2 \quad \dots(1)$$

Applying Pythagoras theorem in $\triangle ADC$, we obtain

$$AD^2 + DC^2 = AC^2$$

$$AB^2 - BD^2 + DC^2 = AC^2 \text{ [Using equation (1)]}$$

$$AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$$

$$AC^2 = AB^2 + BC^2 - 2BC \times BD$$

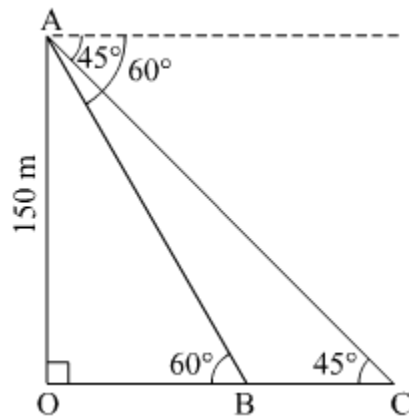
Question 28

A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/min.

OR

There are two poles, one each on either bank of a river just opposite to each other. One pole is 60 m high. From the top of this pole, the angle of depression of the top and foot of the other pole are 30° and 60° respectively. Find the width of the river and height of the other pole.

SOLUTION:



Let AO be the cliff of height 150 m.

Let the speed of boat be x metres per minute.

And BC be the distance which man travelled.

So, $BC = 2x$ [$\because$ Distance = Speed $\times$ Time]

$$\tan(60^\circ) = \frac{AO}{OB}$$

$$\sqrt{3} = \frac{150}{OB}$$

$$\Rightarrow OB = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$$

$$\tan(45^\circ) = \frac{AO}{OC}$$

$$\Rightarrow 1 = \frac{150}{OC}$$

$$\Rightarrow OC = 150$$

$$\text{Now } OC = OB + BC$$

$$\Rightarrow 150 = 50\sqrt{3} + 2x$$

$$\Rightarrow x = \frac{150 - 50\sqrt{3}}{2}$$

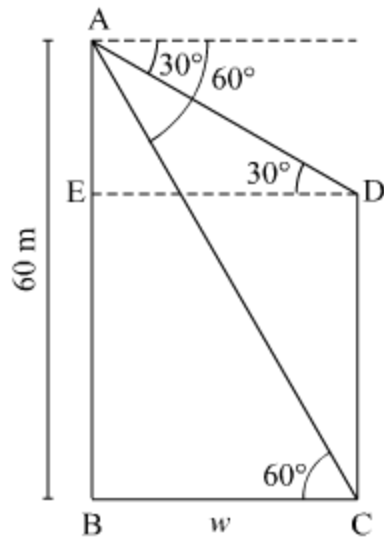
$$\Rightarrow x = 75 - 25\sqrt{3}$$

$$\text{Using } \sqrt{3} = 1.73$$

$$x = 75 - 25 \times 1.732 \approx 32 \text{ m/min}$$

Hence, the speed of the boat is 32 metres per minute.

OR



Let the width of the river be w .

In $\triangle ABC$,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60}{w}$$

$$\Rightarrow w = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

In $\triangle AED$,

$$\tan 30^\circ = \frac{AE}{ED}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{w}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AE}{20\sqrt{3}}$$

$$\Rightarrow AE = 20$$

Height of pole CD = AB - AE

$$= 60 - 20 = 40 \text{ m}$$

Thus, width of river is $20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m}$

Height of pole = 40 m

Question 29

Calculate the mean of the following frequency distribution :

Class :	10-30	30-50	50-70	70-90	90-110	110-130
Frequency :	5	8	12	20	3	2

OR

The following table gives production yield in kg per hectare of wheat of 100 farms of a village :

Production yield (kg/hectare) :	40-45	45-50	50-55	55-60	60-65	65-70
Number of farms	4	6	16	20	30	24

Change the distribution to a 'more than type' distribution, and draw its ogive.

SOLUTION:

Class	frequency (f_i)	Class mark (x_i)	$f_i x_i$
10-30	5	$\frac{10+30}{2} = 20$	100
30-50	8	$\frac{30+50}{2} = 40$	320
50-70	12	$\frac{50+70}{2} = 60$	720
70-90	20	$\frac{70+90}{2} = 80$	1600
90-110	3	$\frac{90+110}{2} = 100$	300
110-130	2	$\frac{110+130}{2} = 120$	240
	$\sum f_i = 50$		$\sum f_i x_i = 3280$

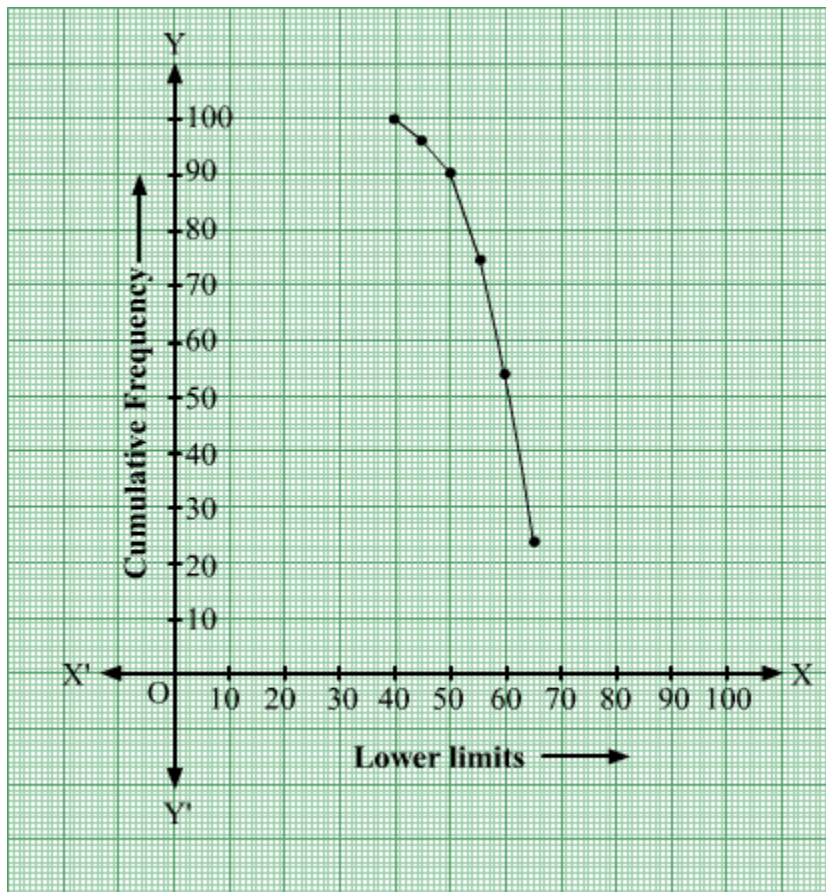
Using: $\text{mean} = \frac{\sum f_i x_i}{\sum f_i}$

substituting the values in the formula

$\text{mean} = \frac{3280}{50} = 65.6$

OR

Production yield	Cumulative frequency
more than 40	100
more than 45	96
more than 50	90
more than 55	74
more than 60	54
more than 65	24



Question 30

A container opened at the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of ₹ 50 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 10 per 100 cm². (Take $\pi = 3.14$)

SOLUTION:

We have to find the cost of milk which can completely fill the container

Volume of container = Volume of frustum

$$= \frac{1}{3}\pi h (r_1^2 + r_2^2 + r_1 r_2)$$

Here,

height = 16 cm

radius of upper end = 20 cm

And radius of lower end = 8 cm

Plugging the values in the formula we get

$$\begin{aligned}
 \text{Volume of container} &= \frac{1}{3} \times 3.14 \times 16 \left((20)^2 + (8)^2 + 20 \times 8 \right) \\
 &= \frac{1}{3} \times 50.24 (400 + 64 + 160) \\
 &= \frac{1}{3} \times 50.24 (624) \\
 &= 10449.92 \text{ cm}^3 \\
 &= 10.449 \text{ litre} \quad (\because 1 \text{ litre} = 1000 \text{ cm}^3)
 \end{aligned}$$

Cost of 1 litre milk is Rs 50

Cost of 10.449 litre milk = $50 \times 10.449 = \text{Rs } 522.45$

We will find the cost of metal sheet to make the container

Firstly, we will find the area of container

Area of container = Curved surface area of the frustum + area of bottom circle ($\because$ container is closed from bottom)

$$\text{Area of container} = \pi (r_1 + r_2)l + \pi r^2$$

Now, we will find l

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$l = \sqrt{(16)^2 + (20 - 8)^2}$$

$$l = \sqrt{(16)^2 + (12)^2}$$

$$l = \sqrt{256 + 144}$$

$$l = \sqrt{400}$$

$$l = 20 \text{ cm}$$

$$\begin{aligned}
 \text{Area of frustum} &= 3.14 \times 20 (20 + 8) \\
 &= 1758.4 \text{ cm}^2
 \end{aligned}$$

$$\text{Area of bottom circle} = 3.14 \times 8^2 = 200.96 \text{ cm}^2$$

$$\begin{aligned}
 \text{Area of container} &= 1758.4 + 200.96 \\
 &= 1959.36 \text{ cm}^2
 \end{aligned}$$

$$\text{Cost of making } 100 \text{ cm}^2 = \text{Rs } 10$$

$$\text{Cost of making } 1 \text{ cm}^2 = \frac{10}{100} = \text{Rs } \frac{1}{10}$$

$$\text{Cost of making } 1959.36 \text{ cm}^2 = \frac{1}{10} \times 1959.36 = 195.936$$

Hence, cost of milk is Rs 522.45

And cost of metal sheet is Rs 195.936