# 1. Rational and Irrational number

- The numbers, which can be written in the form $\frac{p}{q}$, where $p$ and $q$ are integers and $q \neq 0$, are called rational numbers. Rational numbers can be positive as well as negative. Rational numbers include all integers and fractions.

  For example

  $-\frac{2}{7}, \frac{41}{366}, 2 = \frac{2}{1}$, etc.

- **Rational numbers on number line**

  Rational numbers can be represented on number line in the similar manner like fractions and integers.

  Negative rational numbers are marked to the left of 0 while positive rational numbers are marked to the right of 0.

  **Example:**

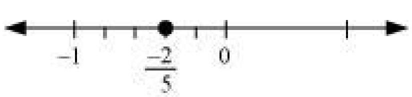
  Represent $-\frac{2}{5}$ on number line.

  **Solution:**

  The given rational number is negative. Therefore, it will lie to the left of 0.

  The space between –1 and 0 is divided into 5 equal parts. Therefore, each part represents $-\frac{1}{5}$.

  Marking $-\frac{2}{5}$ at 2 units to the left of 0, we obtain the number line as shown below.




- Two positive rational numbers can be compared as in fractions.
- Two negative rational numbers can be compared by ignoring their negative signs and then reversing their order.

  **Example**: Compare $-\frac{2}{3}$ and $-\frac{1}{5}$

  **Solution**: To compare $-\frac{2}{3}$ and $-\frac{1}{5}$, we first compare $\frac{2}{3}$ and $\frac{1}{5}$.

HCF (3, 5) = 15

$\therefore \frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15} \text{and} \frac{1}{5} = \frac{1\times3}{5\times3} = \frac{3}{15}$

Now, $\frac{10}{15} > \frac{3}{15}$

$\Rightarrow \frac{2}{3} > \frac{1}{5}$

$\Rightarrow -\frac{2}{3} < -\frac{1}{5}$

- Rational numbers between two rational numbers can be found by first converting them to rational numbers with same denominator.

  **Example:** Find 4 rational numbers between $-\frac{2}{3} \text{and} -\frac{1}{5}$.

  **Solution:**

  $-\frac{2}{3} \times \frac{5}{5} = -\frac{10}{15}$

  $= \frac{1}{5} \times \frac{3}{3} = -\frac{3}{15}$

  $\therefore \frac{10}{15} < -\frac{9}{15} < -\frac{8}{15} < -\frac{7}{15} < -\frac{6}{15} < -\frac{3}{15}$

  Thus, four rational numbers between $-\frac{3}{2} \text{and} -\frac{1}{5} \text{are} -\frac{9}{15}, -\frac{8}{15}, -\frac{7}{15} \text{and} -\frac{6}{15}$

- There are unlimited rational numbers between two rational numbers.

- Decimal expansion of a rational number can be of two types:

  (i) Terminating

  (ii) Non-terminating and repetitive

  In order to find decimal expansion of rational numbers we use long division method.

  For example, to find the decimal expansion of $\frac{1237}{25}$.

  We perform the long division of 1237 by 25.

```
       49.48
25 ) 1237.00
     100
     ----
      237
      225
     ----
       120
       100
     -----
        200
        200
     -----
          0
```

Hence, the decimal expansion of $\frac{1237}{25}$ is 49.48. Since the remainder is obtained as zero, the decimal number is terminating.

- Every number of the form $\sqrt{p}$, where $p$ is a prime number is called an irrational number. For example, $\sqrt{3}$, $\sqrt{11}$, $\sqrt{12}$ etc.

**Theorem:** If a prime number $p$ divides $a^2$, then $p$ divides $a$, where $a$ is a positive integer.

**Example:**
Prove that $\sqrt{7}$ is an irrational number.

**Solution:**
If possible, suppose $\sqrt{7}$ is a rational number.
Then, $\sqrt{7}=\frac{p}{q}$, where $p$, $q$ are integers, $q \neq 0$.
If HCF $(p, q) \neq 1$, then by dividing $p$ and $q$ by HCF$(p, q)$, $\sqrt{7}$ can be reduced as
$\sqrt{7}=\frac{a}{b}$ where HCF $(a, b) = 1$ … (1)
$\Rightarrow \sqrt{7}b = a$
$\Rightarrow 7b^2 = a^2$
$\Rightarrow a^2$ is divisible by 7
$\Rightarrow a$ is divisible by 7 … (2)
$\Rightarrow a = 7c$, where $c$ is an integer
$\therefore \sqrt{7}c = b$
$\Rightarrow 7b^2 = 49c^2$
$\Rightarrow b^2 = 7c^2$
$\Rightarrow b^2$ is divisible by 7
$\Rightarrow b$ is divisible by 7 … (3)
From (2) and (3), 7 is a common factor of $a$ and $b$. which contradicts (1)

$\therefore \sqrt{7}$ is an irrational number.

**Example:**
Show that $\sqrt{12}-6$ is an irrational number.

**Solution:**
If possible, suppose $\sqrt{12}-6$ is a rational number.
Then $\sqrt{12}-6=\frac{p}{q}$ for some integers $p$, $q$ ($q$ ¹ 0)
Now,

$\sqrt{12}-6=\frac{p}{q}$

$\Rightarrow 2\sqrt{3}=\frac{p}{q}+6$

$\Rightarrow \sqrt{3}=\frac{1}{2}\left(\frac{p}{q}+6\right)$

As $p$, $q$, 6 and 2 are integers, $\frac{1}{2}\left(\frac{p}{q}+6\right)$ is rational number, so is $\sqrt{3}$.
This conclusion contradicts the fact that $\sqrt{3}$ is irrational.
Thus, $\sqrt{12}-6$ is an irrational number.

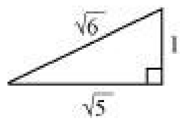
- **Represent irrational numbers on the number line:**

We can represent irrational numbers of the form $\sqrt{n}$ on the number line by first plotting $\sqrt{n} - 1$, where *n* is any positive integer.

**Example:** Locate $\sqrt{6}$ on the number line.

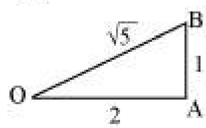
**Solution:**

As $\sqrt{6} = \sqrt{\left(\sqrt{5}\right)^2 + 1^2}$




To locate $\sqrt{6}$ on the number line, we first need to construct a length of $\sqrt{5}$.

$\sqrt{5} = \sqrt{2^2 + 1}$




By Pythagoras theorem, $OB^2 = OA^2 + AB^2 = 2^2 + 1^2 = 5$

$\Rightarrow OB = \sqrt{5}$

**Steps:**

Mark O at 0 and A at 2 on the number line, and then draw AB of unit length perpendicular to OA.

Then, by Pythagoras Theorem, $OB = \sqrt{5}$.

Construct BD of unit length perpendicular to OB. Thus, by Pythagoras theorem,

$OD = \sqrt{\left(\sqrt{5}\right)^2 + 1^2} = \sqrt{6}$

Using a compass with centre O and radius OD, draw an arc intersecting the number line at point P.

Thus, P corresponds to the number $\sqrt{6}$.