Rational Numbers

CHAPTER 1

1.1 Introduction

In Mathematics, we frequently come across simple equations to be solved. For example, the equation $x + 2 = 13$ (1)

is solved when $x = 11$, because this value of x satisfies the given equation. The solution 11 is a **natural number**. On the other hand, for the equation

$$x + 5 = 5 \quad (2)$$

the solution gives the **whole number** 0 (zero). If we consider only natural numbers, equation (2) cannot be solved. To solve equations like (2), we added the number zero to the collection of natural numbers and obtained the whole numbers. Even whole numbers will not be sufficient to solve equations of type

$$x + 18 = 5 \quad (3)$$

Do you see 'why'? We require the number –13 which is not a whole number. This led us to think of **integers, (positive and negative)**. Note that the positive integers correspond to natural numbers. One may think that we have enough numbers to solve all simple equations with the available list of integers. Consider the equations

$$2x = 3 \quad (4)$$

$$5x + 7 = 0 \quad (5)$$

for which we cannot find a solution from the integers. (Check this)

We need the numbers $\frac{3}{2}$ to solve equation (4) and $\frac{-7}{5}$ to solve equation (5). This leads us to the collection of **rational numbers**.

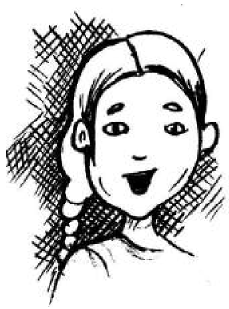
We have already seen basic operations on rational numbers. We now try to explore some properties of operations on the different types of numbers seen so far.

1.2 Properties of Rational Numbers

1.2.1 Closure

(i) Whole numbers

Let us revisit the closure property for all the operations on whole numbers in brief.



Operation	Numbers	Remarks
Addition	0 + 5 = 5, a whole number 4 + 7 = ... Is it a whole number? In general, $a + b$ is a whole number for any two whole numbers a and b.	Whole numbers are closed under addition.
Subtraction	5 – 7 = – 2, which is not a whole number.	Whole numbers are **not** closed under subtraction.
Multiplication	0 × 3 = 0, a whole number 3 × 7 = ... Is it a whole number? In general, if a and b are any two whole numbers, their product ab is a whole number.	Whole numbers are closed under multiplication.
Division	$5 \div 8 = \frac{5}{8}$, which is not a whole number.	Whole numbers are **not** closed under division.

Check for closure property under all the four operations for natural numbers.

(ii) Integers

Let us now recall the operations under which integers are closed.

Operation	Numbers	Remarks
Addition	– 6 + 5 = – 1, an integer Is – 7 + (–5) an integer? Is 8 + 5 an integer? In general, $a + b$ is an integer for any two integers a and b.	Integers are closed under addition.
Subtraction	7 – 5 = 2, an integer Is 5 – 7 an integer? – 6 – 8 = – 14, an integer	Integers are closed under subtraction.

	$-6-(-8)=2$, an integer Is $8-(-6)$ an integer? In general, for any two integers a and b, $a-b$ is again an integer. Check if $b-a$ is also an integer.	
Multiplication	$5\times 8=40$, an integer Is -5×8 an integer? $-5\times(-8)=40$, an integer In general, for any two integers a and b, $a\times b$ is also an integer.	Integers are closed under multiplication.
Division	$5\div 8=\frac{5}{8}$, which is not an integer.	Integers are **not** closed under division.

You have seen that whole numbers are closed under addition and multiplication but not under subtraction and division. However, integers are closed under addition, subtraction and multiplication but not under division.

(iii) Rational numbers

Recall that a number which can be written in the form $\frac{p}{q}$, where p and q are integers and $q\neq 0$ is called a **rational number**. For example, $-\frac{2}{3}, \frac{6}{7}$ are all rational numbers. Since the numbers 0, –2, 4 can be written in the form $\frac{p}{q}$, they are also rational numbers. (Check it!)

(a) You know how to add two rational numbers. Let us add a few pairs.

$$\frac{3}{8}+\frac{(-5)}{7}=\frac{21+(-40)}{56}=\frac{-19}{56} \quad \text{(a rational number)}$$

$$\frac{-3}{8}+\frac{(-4)}{5}=\frac{-15+(-32)}{40}=\ldots \quad \text{Is it a rational number?}$$

$$\frac{4}{7}+\frac{6}{11}=\ldots \quad \text{Is it a rational number?}$$

We find that sum of two rational numbers is again a rational number. Check it for a few more pairs of rational numbers.

We say that *rational numbers are closed under addition. That is, for any two rational numbers a and b, a + b is also a rational number*.

(b) Will the difference of two rational numbers be again a rational number?

We have,

$$\frac{-5}{7}-\frac{2}{3}=\frac{-5\times 3-2\times 7}{21}=\frac{-29}{21} \quad \text{(a rational number)}$$

$$\frac{5}{8}-\frac{4}{5}=\frac{25-32}{40}=\ldots$$ Is it a rational number?

$$\frac{3}{7}-\left(\frac{-8}{5}\right)=\ldots$$ Is it a rational number?

Try this for some more pairs of rational numbers. We find that *rational numbers are closed under subtraction. That is, for any two rational numbers a and b, a – b is also a rational number*.

(c) Let us now see the product of two rational numbers.

$$\frac{-2}{3}\times\frac{4}{5}=\frac{-8}{15};\ \frac{3}{7}\times\frac{2}{5}=\frac{6}{35}$$ (both the products are rational numbers)

$$-\frac{4}{5}\times\frac{-6}{11}=\ldots$$ Is it a rational number?

Take some more pairs of rational numbers and check that their product is again a rational number.

We say that *rational numbers are closed under multiplication. That is, for any two rational numbers a and b, a × b is also a rational number*.

(d) We note that $\frac{-5}{3}\div\frac{2}{5}=\frac{-25}{6}$ (a rational number)

$\frac{2}{7}\div\frac{5}{3}=\ldots$. Is it a rational number? $\frac{-3}{8}\div\frac{-2}{9}=\ldots$. Is it a rational number?

Can you say that rational numbers are closed under division?

We find that for any rational number a, $a \div 0$ is **not defined**.

So rational numbers are **not closed** under division.

However, if we exclude zero then the collection of, all other rational numbers is closed under division.

TRY THESE

Fill in the blanks in the following table.

Numbers	Closed under			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	Yes	Yes	...	No
Integers	...	Yes	...	No
Whole numbers	...	...	Yes	...
Natural numbers	...	No	...	...

1.2.2 Commutativity

(i) Whole numbers

Recall the commutativity of different operations for whole numbers by filling the following table.

Operation	Numbers	Remarks
Addition	$0 + 7 = 7 + 0 = 7$ $2 + 3 = ... + ... = ...$ For any two whole numbers a and b, $a + b = b + a$	Addition is commutative.
Subtraction	....	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

Check whether the commutativity of the operations hold for natural numbers also.

(ii) Integers

Fill in the following table and check the commutativity of different operations for integers:

Operation	Numbers	Remarks
Addition	....	Addition is commutative.
Subtraction	Is $5 - (-3) = -3 - 5$?	Subtraction is not commutative.
Multiplication	....	Multiplication is commutative.
Division	....	Division is not commutative.

(iii) Rational numbers

(a) Addition

You know how to add two rational numbers. Let us add a few pairs here.

$$\frac{-2}{3}+\frac{5}{7}=\frac{1}{21} \text{ and } \frac{5}{7}+\left(\frac{-2}{3}\right)=\frac{1}{21}$$

So, $$\frac{-2}{3}+\frac{5}{7}=\frac{5}{7}+\left(\frac{-2}{3}\right)$$

Also, $$\frac{-6}{5}+\left(\frac{-8}{3}\right) = ... \text{ and } \frac{-8}{3}+\left(\frac{-6}{5}\right)= ...$$

Is $$\frac{-6}{5}+\left(\frac{-8}{3}\right)=\left(\frac{-8}{3}\right)+\left(\frac{-6}{5}\right)?$$

Is $\frac{-3}{8}+\frac{1}{7}=\frac{1}{7}+\left(\frac{-3}{8}\right)$?

You find that two *rational numbers can be added in any order. We say that addition is commutative for rational numbers. That is, for any two rational numbers a and b, a + b = b + a.*

(b) Subtraction

Is $\frac{2}{3}-\frac{5}{4}=\frac{5}{4}-\frac{2}{3}$?

Is $\frac{1}{2}-\frac{3}{5}=\frac{3}{5}-\frac{1}{2}$?

You will find that subtraction is not commutative for rational numbers.

(c) Multiplication

We have, $\frac{-7}{3}\times\frac{6}{5}=\frac{-42}{15}=\frac{6}{5}\times\left(\frac{-7}{3}\right)$

Is $\frac{-8}{9}\times\left(\frac{-4}{7}\right)=\frac{-4}{7}\times\left(\frac{-8}{9}\right)$?

Check for some more such products.

You will find that *multiplication is commutative for rational numbers*.

In general, a × b = b × a for any two rational numbers a and b.

(d) Division

Is $\frac{-5}{4}\div\frac{3}{7}=\frac{3}{7}\div\left(\frac{-5}{4}\right)$?

You will find that expressions on both sides are not equal.

So division is **not commutative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Commutative for			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	Yes	...	...	...
Integers	...	No	...	...
Whole numbers	...	...	Yes	...
Natural numbers	...	...	...	No

1.2.3 Associativity

(i) Whole numbers

Recall the associativity of the four operations for whole numbers through this table:

Operation	Numbers	Remarks
Addition	....	Addition is associative
Subtraction	....	Subtraction is **not** associative
Multiplication	Is $7 \times (2 \times 5) = (7 \times 2) \times 5$? Is $4 \times (6 \times 0) = (4 \times 6) \times 0$? For any three whole numbers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	....	Division is **not** associative

Fill in this table and verify the remarks given in the last column.
Check for yourself the associativity of different operations for natural numbers.

(ii) Integers

Associativity of the four operations for integers can be seen from this table

Operation	Numbers	Remarks
Addition	Is $(-2) + [3 + (-4)]$ $= [(-2) + 3)] + (-4)$? Is $(-6) + [(-4) + (-5)]$ $= [(-6) + (-4)] + (-5)$? For any three integers a, b and c $a + (b + c) = (a + b) + c$	Addition is associative
Subtraction	Is $5 - (7 - 3) = (5 - 7) - 3$?	Subtraction is **not** associative
Multiplication	Is $5 \times [(-7) \times (-8)$ $= [5 \times (-7)] \times (-8)$? Is $(-4) \times [(-8) \times (-5)]$ $= [(-4) \times (-8)] \times (-5)$? For any three integers a, b and c $a \times (b \times c) = (a \times b) \times c$	Multiplication is associative
Division	Is $[(-10) \div 2] \div (-5)$ $= (-10) \div [2 \div (-5)]$?	Division is **not** associative

(iii) Rational numbers

(a) Addition

We have

$$\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\frac{-2}{3}+\left(\frac{-7}{30}\right)=\frac{-27}{30}=\frac{-9}{10}$$

$$\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)=\frac{-1}{15}+\left(\frac{-5}{6}\right)=\frac{-27}{30}=\frac{-9}{10}$$

So, $\frac{-2}{3}+\left[\frac{3}{5}+\left(\frac{-5}{6}\right)\right]=\left[\frac{-2}{3}+\frac{3}{5}\right]+\left(\frac{-5}{6}\right)$

Find $\frac{-1}{2}+\left[\frac{3}{7}+\left(\frac{-4}{3}\right)\right]$ and $\left[\frac{-1}{2}+\frac{3}{7}\right]+\left(\frac{-4}{3}\right)$. Are the two sums equal?

Take some more rational numbers, add them as above and see if the two sums are equal. We find that *addition is associative for rational numbers. That is, for any three rational numbers* a, b and c, $a + (b + c) = (a + b) + c$.

(b) Subtraction

Is $\frac{-2}{3}-\left[\frac{-4}{5}-\frac{1}{2}\right]=\left[\frac{2}{3}-\left(\frac{-4}{5}\right)\right]-\frac{1}{2}$?

Check for yourself.

Subtraction is **not associative** for rational numbers.

(c) Multiplication

Let us check the associativity for multiplication.

$$\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\frac{-7}{3}\times\frac{10}{36}=\frac{-70}{108}=\frac{-35}{54}$$

$$\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}=\ldots$$

We find that $\frac{-7}{3}\times\left(\frac{5}{4}\times\frac{2}{9}\right)=\left(\frac{-7}{3}\times\frac{5}{4}\right)\times\frac{2}{9}$

Is $\frac{2}{3}\times\left(\frac{-6}{7}\times\frac{4}{5}\right)=\left(\frac{2}{3}\times\frac{-6}{7}\right)\times\frac{4}{5}$?

Take some more rational numbers and check for yourself.

We observe that *multiplication is associative for rational numbers. That is for any three rational numbers* a, b and c, $a \times (b \times c) = (a \times b) \times c$.

(d) Division

Let us see if $\frac{1}{2} \div \left[\frac{-1}{3} \div \frac{2}{5}\right] = \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$

We have, LHS $= \frac{1}{2} \div \left(\frac{-1}{3} \div \frac{2}{5}\right) = \frac{1}{2} \div \left(\frac{-1}{3} \times \frac{5}{2}\right)$ (reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$)

$= \frac{1}{2} \div \left(-\frac{5}{6}\right) = \ldots$

RHS $= \left[\frac{1}{2} \div \left(\frac{-1}{3}\right)\right] \div \frac{2}{5}$

$= \left(\frac{1}{2} \times \frac{-3}{1}\right) \div \frac{2}{5} = \frac{-3}{2} \div \frac{2}{5} = \ldots$

Is LHS = RHS? Check for yourself. You will find that division is **not associative** for rational numbers.

TRY THESE

Complete the following table:

Numbers	Associative for			
	addition	**subtraction**	**multiplication**	**division**
Rational numbers	...	...	...	No
Integers	...	...	Yes	...
Whole numbers	Yes	...	...	...
Natural numbers	...	No	...	...

Example 1: Find $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

Solution: $\frac{3}{7} + \left(\frac{-6}{11}\right) + \left(\frac{-8}{21}\right) + \left(\frac{5}{22}\right)$

$= \frac{198}{462} + \left(\frac{-252}{462}\right) + \left(\frac{-176}{462}\right) + \left(\frac{105}{462}\right)$ (Note that 462 is the LCM of 7, 11, 21 and 22)

$= \frac{198 - 252 - 176 + 105}{462} = \frac{-125}{462}$

We can also solve it as.

$$\frac{3}{7}+\left(\frac{-6}{11}\right)+\left(\frac{-8}{21}\right)+\frac{5}{22}$$

$$=\left[\frac{3}{7}+\left(\frac{-8}{21}\right)\right]+\left[\frac{-6}{11}+\frac{5}{22}\right]$$ (by using commutativity and associativity)

$$=\left[\frac{9+(-8)}{21}\right]+\left[\frac{-12+5}{22}\right]$$ (LCM of 7 and 21 is 21; LCM of 11 and 22 is 22)

$$=\frac{1}{21}+\left(\frac{-7}{22}\right)=\frac{22-147}{462}=\frac{-125}{462}$$

Do you think the properties of commutativity and associativity made the calculations easier?

Example 2: Find $\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$

Solution: We have

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(-\frac{4\times 3}{5\times 7}\right)\times\left(\frac{15\times(-14)}{16\times 9}\right)$$

$$=\frac{-12}{35}\times\left(\frac{-35}{24}\right)=\frac{-12\times(-35)}{35\times 24}=\frac{1}{2}$$

We can also do it as.

$$\frac{-4}{5}\times\frac{3}{7}\times\frac{15}{16}\times\left(\frac{-14}{9}\right)$$

$$=\left(\frac{-4}{5}\times\frac{15}{16}\right)\times\left[\frac{3}{7}\times\left(\frac{-14}{9}\right)\right]$$ (Using commutativity and associativity)

$$=\frac{-3}{4}\times\left(\frac{-2}{3}\right)=\frac{1}{2}$$

1.2.4 The role of zero (0)

Look at the following.

$2 + 0 = 0 + 2 = 2$ (Addition of 0 to a whole number)

$-5 + 0 = \ldots + \ldots = -5$ (Addition of 0 to an integer)

$\frac{-2}{7} + \ldots = 0 + \left(\frac{-2}{7}\right) = \frac{-2}{7}$ (Addition of 0 to a rational number)

You have done such additions earlier also. Do a few more such additions.

What do you observe? You will find that when you add 0 to a whole number, the sum is again that whole number. This happens for integers and rational numbers also.

In general, $a + 0 = 0 + a = a$, where a is a whole number

$b + 0 = 0 + b = b$, where b is an integer

$c + 0 = 0 + c = c$, where c is a rational number

Zero is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers as well.

1.2.5 The role of 1

We have,

$$5 \times 1 = 5 = 1 \times 5 \quad \text{(Multiplication of 1 with a whole number)}$$

$$\frac{-2}{7} \times 1 = \ldots \times \ldots = \frac{-2}{7}$$

$$\frac{3}{8} \times \ldots = 1 \times \frac{3}{8} = \frac{3}{8}$$

What do you find?

You will find that when you multiply any rational number with 1, you get back that rational number as the product. Check this for a few more rational numbers. You will find that, $a \times 1 = 1 \times a = a$ for any rational number a.

We say *that* 1 *is the multiplicative identity for rational numbers.*

Is 1 the multiplicative identity for integers? For whole numbers?

THINK, DISCUSS AND WRITE

If a property holds for rational numbers, will it also hold for integers? For whole numbers? Which will? Which will not?

1.2.6 Negative of a number

While studying integers you have come across negatives of integers. What is the negative of 1? It is – 1 because $1 + (-1) = (-1) + 1 = 0$

So, what will be the negative of (–1)? It will be 1.

Also, $2 + (-2) = (-2) + 2 = 0$, so we say 2 is the **negative or additive inverse** of –2 and vice-versa. In general, for an integer a, we have, $a + (-a) = (-a) + a = 0$; so, a is the negative of $-a$ and $-a$ is the negative of a.

For the rational number $\frac{2}{3}$, we have,

$$\frac{2}{3} + \left(-\frac{2}{3}\right) = \frac{2 + (-2)}{3} = 0$$

Also, $\left(-\frac{2}{3}\right)+\frac{2}{3}=0$ (How?)

Similarly, $\frac{-8}{9}+\ldots=\ldots+\left(\frac{-8}{9}\right)=0$

$$\ldots+\left(\frac{-11}{7}\right)=\left(\frac{-11}{7}\right)+\ldots=0$$

In general, for a rational number $\frac{a}{b}$, we have, $\frac{a}{b}+\left(-\frac{a}{b}\right)=\left(-\frac{a}{b}\right)+\frac{a}{b}=0$. We say that $-\frac{a}{b}$ *is the additive inverse of* $\frac{a}{b}$ *and* $\frac{a}{b}$ *is the additive inverse of* $\left(-\frac{a}{b}\right)$.

1.2.7 Reciprocal

By which rational number would you multiply $\frac{8}{21}$, to get the product 1? Obviously by $\frac{21}{8}$, since $\frac{8}{21}\times\frac{21}{8}=1$.

Similarly, $\frac{-5}{7}$ must be multiplied by $\frac{7}{-5}$ so as to get the product 1.

We say that $\frac{21}{8}$ is the reciprocal of $\frac{8}{21}$ and $\frac{7}{-5}$ is the reciprocal of $\frac{-5}{7}$.

Can you say what is the reciprocal of 0 (zero)?

Is there a rational number which when multiplied by 0 gives 1? Thus, zero has no reciprocal.

We say that a rational number $\frac{c}{d}$ is called the **reciprocal** or **multiplicative inverse** of another rational number $\frac{a}{b}$ if $\frac{a}{b}\times\frac{c}{d}=1$.

1.2.8 Distributivity of multiplication over addition for rational numbers

To understand this, consider the rational numbers $\frac{-3}{4}$, $\frac{2}{3}$ and $\frac{-5}{6}$.

$$\frac{-3}{4}\times\left\{\frac{2}{3}+\left(\frac{-5}{6}\right)\right\}=\frac{-3}{4}\times\left\{\frac{(4)+(-5)}{6}\right\}$$

$$=\frac{-3}{4}\times\left(\frac{-1}{6}\right)=\frac{3}{24}=\frac{1}{8}$$

Also $\frac{-3}{4}\times\frac{2}{3}=\frac{-3\times 2}{4\times 3}=\frac{-6}{12}=\frac{-1}{2}$

And $$\frac{-3}{4} \times \frac{-5}{6} = \frac{5}{8}$$

Therefore $$\left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right) = \frac{-1}{2} + \frac{5}{8} = \frac{1}{8}$$

Thus, $$\frac{-3}{4} \times \left\{\frac{2}{3} + \frac{-5}{6}\right\} = \left(\frac{-3}{4} \times \frac{2}{3}\right) + \left(\frac{-3}{4} \times \frac{-5}{6}\right)$$

> **Distributivity of Multiplication over Addition and Subtraction.**
> For all rational numbers a, b and c,
> $a\ (b + c) = ab + ac$
> $a\ (b - c) = ab - ac$

TRY THESE

Find using distributivity. (i) $\left\{\frac{7}{5} \times \left(\frac{-3}{12}\right)\right\} + \left\{\frac{7}{5} \times \frac{5}{12}\right\}$ (ii) $\left\{\frac{9}{16} \times \frac{4}{12}\right\} + \left\{\frac{9}{16} \times \frac{-3}{9}\right\}$

Example 3: Write the additive inverse of the following:

(i) $\frac{-7}{19}$ (ii) $\frac{21}{112}$

> When you use distributivity, you split a product as a sum or difference of two products.

Solution:

(i) $\frac{7}{19}$ is the additive inverse of $\frac{-7}{19}$ because $\frac{-7}{19} + \frac{7}{19} = \frac{-7+7}{19} = \frac{0}{19} = 0$

(ii) The additive inverse of $\frac{21}{112}$ is $\frac{-21}{112}$ (Check!)

Example 4: Verify that $-(-x)$ is the same as x for

(i) $x = \frac{13}{17}$ (ii) $x = \frac{-21}{31}$

Solution: (i) We have, $x = \frac{13}{17}$

The additive inverse of $x = \frac{13}{17}$ is $-x = \frac{-13}{17}$ since $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$.

The same equality $\frac{13}{17} + \left(\frac{-13}{17}\right) = 0$, shows that the additive inverse of $\frac{-13}{17}$ is $\frac{13}{17}$

or $-\left(\frac{-13}{17}\right) = \frac{13}{17}$, i.e., $-(-x) = x$.

(ii) Additive inverse of $x = \frac{-21}{31}$ is $-x = \frac{21}{31}$ since $\frac{-21}{31} + \frac{21}{31} = 0$.

The same equality $\frac{-21}{31} + \frac{21}{31} = 0$, shows that the additive inverse of $\frac{21}{31}$ is $\frac{-21}{31}$, i.e., $-(-x) = x$.

Example 5: Find $\frac{2}{5} \times \frac{-3}{7} - \frac{1}{14} - \frac{3}{7} \times \frac{3}{5}$

Solution: $\frac{2}{5} \times \frac{-3}{7} - \frac{1}{14} - \frac{3}{7} \times \frac{3}{5} = \frac{2}{5} \times \frac{-3}{7} - \frac{3}{7} \times \frac{3}{5} - \frac{1}{14}$ (by commutativity)

$$= \frac{2}{5} \times \frac{-3}{7} + \left(\frac{-3}{7}\right) \times \frac{3}{5} - \frac{1}{14}$$

$$= \frac{-3}{7}\left(\frac{2}{5} + \frac{3}{5}\right) - \frac{1}{14} \quad \text{(by distributivity)}$$

$$= \frac{-3}{7} \times 1 - \frac{1}{14} = \frac{-6-1}{14} = \frac{-1}{2}$$

EXERCISE 1.1

1. Using appropriate properties find.

   (i) $-\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6}$ (ii) $\frac{2}{5} \times \left(-\frac{3}{7}\right) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$

2. Write the additive inverse of each of the following.

   (i) $\frac{2}{8}$ (ii) $\frac{-5}{9}$ (iii) $\frac{-6}{-5}$ (iv) $\frac{2}{-9}$ (v) $\frac{19}{-6}$

3. Verify that $-(-x) = x$ for.

   (i) $x = \frac{11}{15}$ (ii) $x = -\frac{13}{17}$

4. Find the multiplicative inverse of the following.

   (i) -13 (ii) $\frac{-13}{19}$ (iii) $\frac{1}{5}$ (iv) $\frac{-5}{8} \times \frac{-3}{7}$

   (v) $-1 \times \frac{-2}{5}$ (vi) -1

5. Name the property under multiplication used in each of the following.

   (i) $\frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = -\frac{4}{5}$ (ii) $-\frac{13}{17} \times \frac{-2}{7} = \frac{-2}{7} \times \frac{-13}{17}$

   (iii) $\frac{-19}{29} \times \frac{29}{-19} = 1$

6. Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$.

7. Tell what property allows you to compute $\frac{1}{3} \times \left(6 \times \frac{4}{3}\right)$ as $\left(\frac{1}{3} \times 6\right) \times \frac{4}{3}$.

8. Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

9. Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not?

10. Write.

(i) The rational number that does not have a reciprocal.

(ii) The rational numbers that are equal to their reciprocals.

(iii) The rational number that is equal to its negative.

11. Fill in the blanks.

(i) Zero has ________ reciprocal.

(ii) The numbers ________ and ________ are their own reciprocals

(iii) The reciprocal of – 5 is ________.

(iv) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is ________.

(v) The product of two rational numbers is always a _______.

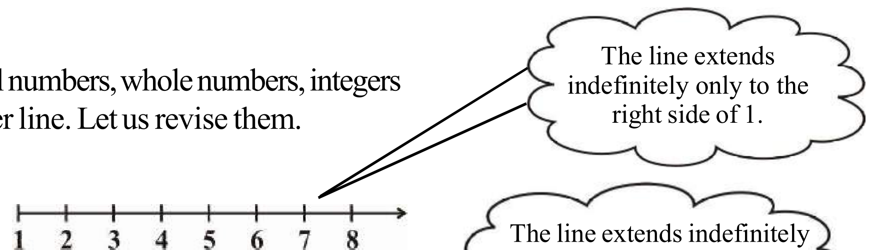
(vi) The reciprocal of a positive rational number is ________.

1.3 Representation of Rational Numbers on the Number Line

You have learnt to represent natural numbers, whole numbers, integers and rational numbers on a number line. Let us revise them.

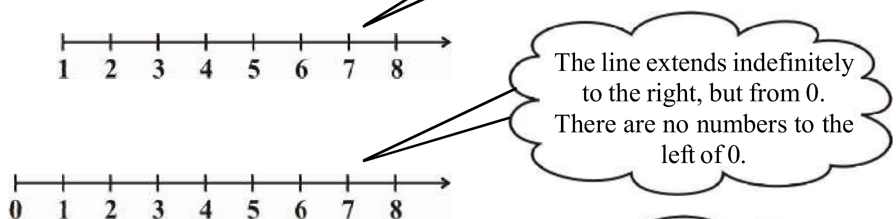
Natural numbers

(i)



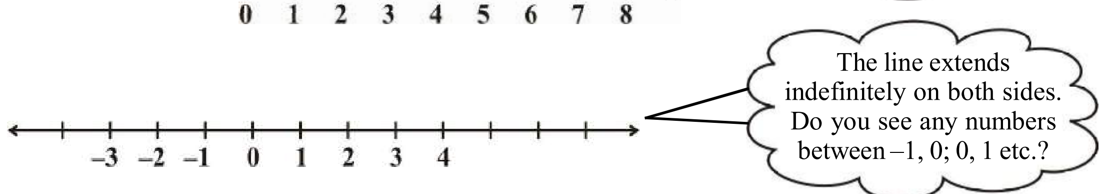
Whole numbers

(ii)



Integers

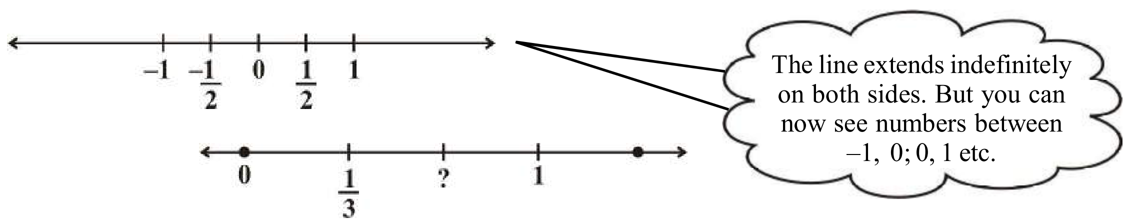
(iii)



Rational numbers

(iv)

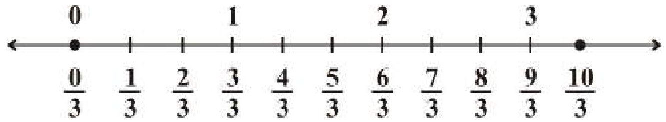
(v)

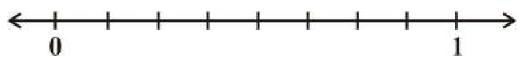


The point on the number line (iv) which is half way between 0 and 1 has been labelled $\frac{1}{2}$. Also, the first of the equally spaced points that divides the distance between 0 and 1 into three equal parts can be labelled $\frac{1}{3}$, as on number line (v). How would you label the second of these division points on number line (v)?

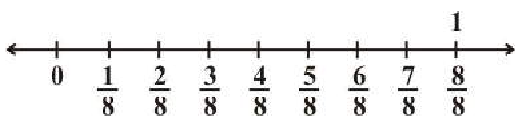
The point to be labelled is twice as far from and to the right of 0 as the point labelled $\frac{1}{3}$. So it is two times $\frac{1}{3}$, i.e., $\frac{2}{3}$. You can continue to label equally-spaced points on the number line in the same way. The next marking is 1. You can see that 1 is the same as $\frac{3}{3}$.

Then comes $\frac{4}{3}, \frac{5}{3}, \frac{6}{3}$ (or 2), $\frac{7}{3}$ and so on as shown on the number line (vi)

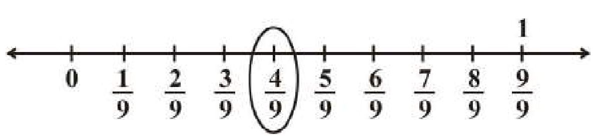
(vi) 

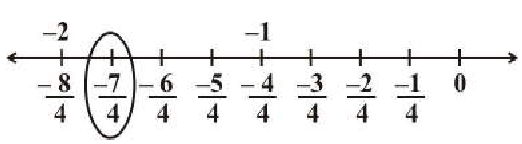
Similarly, to represent $\frac{1}{8}$, the number line may be divided into eight equal parts as shown: 

We use the number $\frac{1}{8}$ to name the first point of this division. The second point of division will be labelled $\frac{2}{8}$, the third point $\frac{3}{8}$, and so on as shown on number line (vii)

(vii) 

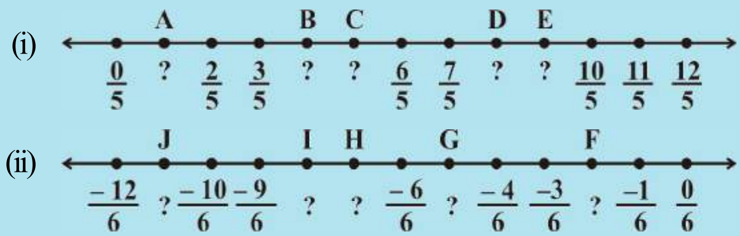
Any rational number can be represented on the number line in this way. In a rational number, the numeral below the bar, i.e., the denominator, tells the number of equal parts into which the first unit has been divided. The numeral above the bar i.e., the numerator, tells 'how many' of these parts are considered. So, a rational number such as $\frac{4}{9}$ means four of nine equal parts on the right of 0 (number line viii) and for $\frac{-7}{4}$, we make 7 markings of distance $\frac{1}{4}$ each on the *left* of zero and starting from 0. The seventh marking is $\frac{-7}{4}$ [number line (ix)].

(viii) 

(ix) 

TRY THESE

Write the rational number for each point labelled with a letter.



1.4 Rational Numbers between Two Rational Numbers

Can you tell the natural numbers between 1 and 5? They are 2, 3 and 4.
How many natural numbers are there between 7 and 9? There is one and it is 8.
How many natural numbers are there between 10 and 11? Obviously none.
List the integers that lie between –5 and 4. They are – 4, – 3, –2, –1, 0, 1, 2, 3.
How many integers are there between –1 and 1?
How many integers are there between –9 and –10?

You will find a definite number of natural numbers (integers) between two natural numbers (integers).

How many rational numbers are there between $\frac{3}{10}$ and $\frac{7}{10}$?

You may have thought that they are only $\frac{4}{10}, \frac{5}{10}$ and $\frac{6}{10}$.

But you can also write $\frac{3}{10}$ as $\frac{30}{100}$ and $\frac{7}{10}$ as $\frac{70}{100}$. Now the numbers, $\frac{31}{100}, \frac{32}{100}, \frac{33}{100}$, ... $\frac{68}{100}, \frac{69}{100}$, are all between $\frac{3}{10}$ and $\frac{7}{10}$. The number of these rational numbers is 39.

Also $\frac{3}{10}$ can be expressed as $\frac{3000}{10000}$ and $\frac{7}{10}$ as $\frac{7000}{10000}$. Now, we see that the rational numbers $\frac{3001}{10000}, \frac{3002}{10000}, ..., \frac{6998}{10000}, \frac{6999}{10000}$ are between $\frac{3}{10}$ and $\frac{7}{10}$. These are 3999 numbers in all.

In this way, we can go on inserting more and more rational numbers between $\frac{3}{10}$ and $\frac{7}{10}$. So unlike natural numbers and integers, the number of rational numbers between two rational numbers is not definite. Here is one more example.

How many rational numbers are there between $\frac{-1}{10}$ and $\frac{3}{10}$?

Obviously $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}$ are rational numbers between the given numbers.

If we write $\frac{-1}{10}$ as $\frac{-10000}{100000}$ and $\frac{3}{10}$ as $\frac{30000}{100000}$, we get the rational numbers $\frac{-9999}{100000}, \frac{-9998}{100000}, ..., \frac{-29998}{100000}, \frac{29999}{100000}$, between $\frac{-1}{10}$ and $\frac{3}{10}$.

You will find that *you get countless rational numbers between any two given rational numbers*.

Example 6: Write any 3 rational numbers between –2 and 0.

Solution: –2 can be written as $\frac{-20}{10}$ and 0 as $\frac{0}{10}$.

Thus we have $\frac{-19}{10}, \frac{-18}{10}, \frac{-17}{10}, \frac{-16}{10}, \frac{-15}{10}, \ldots, \frac{-1}{10}$ between –2 and 0.

You can take any three of these.

Example 7: Find any ten rational numbers between $\frac{-5}{6}$ and $\frac{5}{8}$.

Solution: We first convert $\frac{-5}{6}$ and $\frac{5}{8}$ to rational numbers with the same denominators.

$$\frac{-5 \times 4}{6 \times 4} = \frac{-20}{24} \quad \text{and} \quad \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

Thus we have $\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, ..., \frac{14}{24}$ as the rational numbers between $\frac{-20}{24}$ and $\frac{15}{24}$.

You can take any ten of these.

Another Method

Let us find rational numbers between 1 and 2. One of them is 1.5 or $1\frac{1}{2}$ or $\frac{3}{2}$. This is the **mean** of 1 and 2. You have studied mean in Class VII.

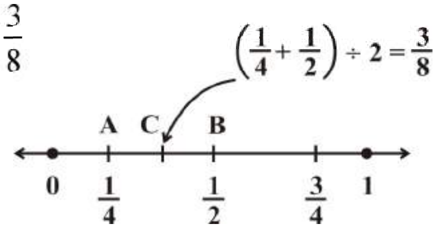
We find that *between any two given numbers, we need not necessarily get an integer but there will always lie a rational number*.

We can use the idea of mean also to find rational numbers between any two given rational numbers.

Example 8: Find a rational number between $\frac{1}{4}$ and $\frac{1}{2}$.

Solution: We find the mean of the given rational numbers.

$$\left(\frac{1}{4} + \frac{1}{2}\right) \div 2 = \left(\frac{1+2}{4}\right) \div 2 = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$



$\frac{3}{8}$ lies between $\frac{1}{4}$ and $\frac{1}{2}$.

This can be seen on the number line also.

We find the mid point of AB which is C, represented by $\left(\frac{1}{4}+\frac{1}{2}\right) \div 2 = \frac{3}{8}$.

We find that $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.

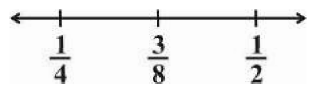
If a and b are two rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

This again shows that there are countless number of rational numbers between any two given rational numbers.

Example 9: Find three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

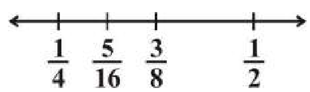
Solution: We find the mean of the given rational numbers.

As given in the above example, the mean is $\frac{3}{8}$ and $\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$.



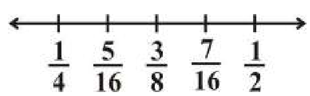
We now find another rational number between $\frac{1}{4}$ and $\frac{3}{8}$. For this, we again find the mean of $\frac{1}{4}$ and $\frac{3}{8}$. That is, $\left(\frac{1}{4}+\frac{3}{8}\right) \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$

$$\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{1}{2}$$



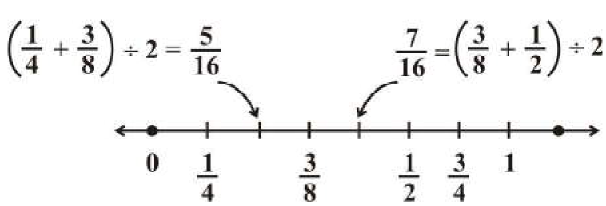
Now find the mean of $\frac{3}{8}$ and $\frac{1}{2}$. We have, $\left(\frac{3}{8}+\frac{1}{2}\right) \div 2 = \frac{7}{8} \times \frac{1}{2} = \frac{7}{16}$

Thus we get $\frac{1}{4} < \frac{5}{16} < \frac{3}{8} < \frac{7}{16} < \frac{1}{2}$.



Thus, $\frac{5}{16}, \frac{3}{8}, \frac{7}{16}$ are the three rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

This can clearly be shown on the number line as follows:



In the same way we can obtain as many rational numbers as we want between two given rational numbers . You have noticed that there are countless rational numbers between any two given rational numbers.

EXERCISE 1.2

1. Represent these numbers on the number line. (i) $\frac{7}{4}$ (ii) $\frac{-5}{6}$
2. Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$ on the number line.
3. Write five rational numbers which are smaller than 2.
4. Find ten rational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$.
5. Find five rational numbers between.

   (i) $\frac{2}{3}$ and $\frac{4}{5}$ (ii) $\frac{-3}{2}$ and $\frac{5}{3}$ (iii) $\frac{1}{4}$ and $\frac{1}{2}$
6. Write five rational numbers greater than –2.
7. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$.

WHAT HAVE WE DISCUSSED?

1. Rational numbers are **closed** under the operations of addition, subtraction and multiplication.
2. The operations addition and multiplication are
   (i) **commutative** for rational numbers.
   (ii) **associative** for rational numbers.
3. The rational number 0 is the **additive identity** for rational numbers.
4. The rational number 1 is the **multiplicative identity** for rational numbers.
5. The **additive inverse** of the rational number $\frac{a}{b}$ is $-\frac{a}{b}$ and vice-versa.
6. The **reciprocal** or **multiplicative inverse** of the rational number $\frac{a}{b}$ is $\frac{c}{d}$ if $\frac{a}{b} \times \frac{c}{d} = 1$.
7. **Distributivity** of rational numbers: For all rational numbers a, b and c,
   $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
8. Rational numbers can be represented on a number line.
9. Between any two given rational numbers there are countless rational numbers. The idea of **mean** helps us to find rational numbers between two rational numbers.

CHAPTER

2

Linear Equations in One Variable

2.1 Introduction

In the earlier classes, you have come across several **algebraic expressions** and **equations**. Some examples of expressions we have so far worked with are:

$$5x,\ 2x - 3,\ 3x + y,\ 2xy + 5,\ xyz + x + y + z,\ x^2 + 1,\ y + y^2$$

Some examples of equations are: $5x = 25$, $2x - 3 = 9$, $2y + \frac{5}{2} = \frac{37}{2}$, $6z + 10 = -2$

You would remember that equations use the *equality* (=) sign; it is missing in expressions.

Of these given expressions, many have more than one variable. For example, $2xy + 5$ has two variables. We however, restrict to expressions with only one variable when we form equations. Moreover, the expressions we use to form equations are linear. This means that the highest power of the variable appearing in the expression is 1.

These are linear expressions:

$$2x,\ 2x + 1,\ 3y - 7,\ 12 - 5z,\ \frac{5}{4}(x - 4) + 10$$

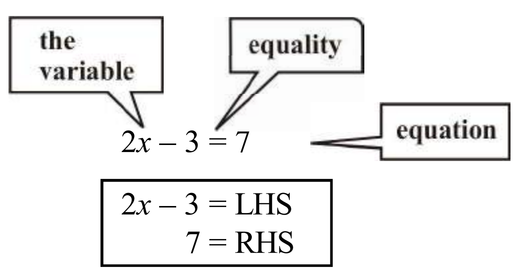
These are **not** linear expressions:

$x^2 + 1,\ y + y^2,\ 1 + z + z^2 + z^3$ (since highest power of variable > 1)

Here we will deal with equations with linear expressions in one variable only. Such equations are known as **linear equations in one variable**. The simple equations which you studied in the earlier classes were all of this type.

Let us briefly revise what we know:

(a) *An algebraic equation is an equality involving variables*. It has an *equality sign*. The expression on the left of the equality sign is the *Left Hand Side* (LHS). The expression on the right of the equality sign is the *Right Hand Side* (RHS).



(b) In an equation the *values of the expressions on the LHS and RHS are equal*. This happens to be *true* only for certain values of the variable. These values are the **solutions** of the equation.

> $x = 5$ is the solution of the equation $2x - 3 = 7$. For $x = 5$,
> LHS $= 2 \times 5 - 3 = 7 =$ RHS
> On the other hand $x = 10$ is not a solution of the equation. For $x = 10$, LHS $= 2 \times 10 - 3 = 17$. This is not equal to the RHS

(c) *How to find the solution of an equation?*

We assume that the two sides of the equation are balanced. We perform the same mathematical operations on both sides of the equation, so that the balance is not disturbed. A few such steps give the solution.

2.2 Solving Equations which have Linear Expressions on one Side and Numbers on the other Side

Let us recall the technique of solving equations with some examples. Observe the solutions; they can be any rational number.

Example 1: Find the solution of $2x - 3 = 7$

Solution:

Step 1 Add 3 to both sides.

$$2x - 3 + 3 = 7 + 3 \quad \text{(The balance is not disturbed)}$$

or $$2x = 10$$

Step 2 Next divide both sides by 2.

$$\frac{2x}{2} = \frac{10}{2}$$

or $$x = 5 \quad \text{(required solution)}$$

Example 2: Solve $2y + 9 = 4$

Solution: Transposing 9 to RHS

$$2y = 4 - 9$$

or $$2y = -5$$

Dividing both sides by 2, $$y = \frac{-5}{2} \quad \text{(solution)}$$

To check the answer: LHS $= 2\left(\frac{-5}{2}\right) + 9 = -5 + 9 = 4 =$ RHS (as required)

Do you notice that the solution $\left(\frac{-5}{2}\right)$ is a rational number? In Class VII, the equations we solved did not have such solutions.

Example 3: Solve $\frac{x}{3}+\frac{5}{2}=-\frac{3}{2}$

Solution: Transposing $\frac{5}{2}$ to the RHS, we get $\frac{x}{3}=\frac{-3}{2}-\frac{5}{2}=-\frac{8}{2}$

or $\frac{x}{3}=-4$

Multiply both sides by 3, $x=-4\times 3$

or $x=-12$ (solution)

***Check*:** LHS $=-\frac{12}{3}+\frac{5}{2}=-4+\frac{5}{2}=\frac{-8+5}{2}=\frac{-3}{2}=$ RHS (as required)

Do you now see that the coefficient of a variable in an equation need not be an integer?

Example 4: Solve $\frac{15}{4}-7x=9$

Solution: We have $\frac{15}{4}-7x=9$

or $-7x=9-\frac{15}{4}$ (transposing $\frac{15}{4}$ to R H S)

or $-7x=\frac{21}{4}$

or $x=\frac{21}{4\times(-7)}$ (dividing both sides by -7)

or $x=-\frac{3\times 7}{4\times 7}$

or $x=-\frac{3}{4}$ (solution)

***Check*:** LHS $=\frac{15}{4}-7\left(\frac{-3}{4}\right)=\frac{15}{4}+\frac{21}{4}=\frac{36}{4}=9=$ RHS (as required)

EXERCISE 2.1

Solve the following equations.

1. $x-2=7$
2. $y+3=10$
3. $6=z+2$
4. $\frac{3}{7}+x=\frac{17}{7}$
5. $6x=12$
6. $\frac{t}{5}=10$
7. $\frac{2x}{3}=18$
8. $1.6=\frac{y}{1.5}$
9. $7x-9=16$

10. $14y - 8 = 13$ **11.** $17 + 6p = 9$ **12.** $\frac{x}{3} + 1 = \frac{7}{15}$

2.3 Some Applications

We begin with a simple example.

Sum of two numbers is 74. One of the numbers is 10 more than the other. What are the numbers?

We have a puzzle here. We do not know either of the two numbers, and we have to find them. We are given two conditions.

(i) One of the numbers is 10 more than the other.

(ii) Their sum is 74.

We already know from Class VII how to proceed. If the smaller number is taken to be x, the larger number is 10 more than x, i.e., $x + 10$. The other condition says that the sum of these two numbers x and $x + 10$ is 74.

This means that $x + (x + 10) = 74$.

or $2x + 10 = 74$

Transposing 10 to RHS, $2x = 74 - 10$

or $2x = 64$

Dividing both sides by 2, $x = 32$. This is one number.

The other number is $x + 10 = 32 + 10 = 42$

The desired numbers are 32 and 42. (Their sum is indeed 74 as given and also one number is 10 more than the other.)

We shall now consider several examples to show how useful this method is.

Example 5: What should be added to twice the rational number $\frac{-7}{3}$ to get $\frac{3}{7}$?

Solution: Twice the rational number $\frac{-7}{3}$ is $2 \times \left(\frac{-7}{3}\right) = \frac{-14}{3}$. Suppose x added to this number gives $\frac{3}{7}$; i.e.,

$$x + \left(\frac{-14}{3}\right) = \frac{3}{7}$$

or $$x - \frac{14}{3} = \frac{3}{7}$$

or $$x = \frac{3}{7} + \frac{14}{3} \qquad \left(\text{transposing } \frac{14}{3} \text{ to RHS}\right)$$

$$= \frac{(3 \times 3) + (14 \times 7)}{21} = \frac{9 + 98}{21} = \frac{107}{21}.$$

Thus $\frac{107}{21}$ should be added to $2 \times \left(\frac{-7}{3}\right)$ to give $\frac{3}{7}$.

Example 6: The perimeter of a rectangle is 13 cm and its width is $2\frac{3}{4}$ cm. Find its length.

Solution: Assume the length of the rectangle to be x cm.

The perimeter of the rectangle = 2 × (length + width)

$$= 2 \times (x + 2\frac{3}{4})$$

$$= 2\left(x + \frac{11}{4}\right)$$

The perimeter is given to be 13 cm. Therefore,

$$2\left(x + \frac{11}{4}\right) = 13$$

or $\quad x + \frac{11}{4} = \frac{13}{2} \quad$ (dividing both sides by 2)

or $\quad x = \frac{13}{2} - \frac{11}{4}$

$$= \frac{26}{4} - \frac{11}{4} = \frac{15}{4} = 3\frac{3}{4}$$

The length of the rectangle is $3\frac{3}{4}$ cm.

Example 7: The present age of Sahil's mother is three times the present age of Sahil. After 5 years their ages will add to 66 years. Find their present ages.

Solution: Let Sahil's present age be x years.

We could also choose Sahil's age 5 years later to be x and proceed. Why don't you try it that way?

	Sahil	Mother	Sum
Present age	x	$3x$	
Age 5 years later	$x + 5$	$3x + 5$	$4x + 10$

It is given that this sum is 66 years.

Therefore, $\quad 4x + 10 = 66$

This equation determines Sahil's present age which is x years. To solve the equation, we transpose 10 to RHS,

$$4x = 66 - 10$$

or $$4x = 56$$

or $$x = \frac{56}{4} = 14 \qquad \text{(solution)}$$

Thus, Sahil's present age is 14 years and his mother's age is 42 years. (You may easily check that 5 years from now the sum of their ages will be 66 years.)

Example 8: Bansi has 3 times as many two-rupee coins as he has five-rupee coins. If he has in all a sum of ₹ 77, how many coins of each denomination does he have?

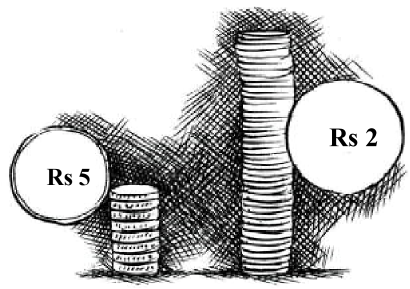
Solution: Let the number of five-rupee coins that Bansi has be x. Then the number of two-rupee coins he has is 3 times x or $3x$.

The amount Bansi has:

(i) from 5 rupee coins, ₹ $5 \times x$ = ₹ $5x$

(ii) from 2 rupee coins, ₹ $2 \times 3x$ = ₹ $6x$

Hence the total money he has = ₹ $11x$



But this is given to be ₹ 77; therefore,

$$11x = 77$$

or $$x = \frac{77}{11} = 7$$

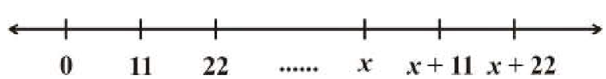
Thus, number of five-rupee coins = $x = 7$

and number of two-rupee coins = $3x = 21$ (solution)

(You can check that the total money with Bansi is ₹ 77.)

Example 9: The sum of three consecutive multiples of 11 is 363. Find these multiples.

Solution: If x is a multiple of 11, the next multiple is $x + 11$. The next to this is $x + 11 + 11$ or $x + 22$. So we can take three consecutive multiples of 11 as x, $x + 11$ and $x + 22$.



It is given that the sum of these consecutive multiples of 11 is 363. This will give the following equation:

$$x + (x + 11) + (x + 22) = 363$$

or $$x + x + 11 + x + 22 = 363$$

or $$3x + 33 = 363$$

or $$3x = 363 - 33$$

or $$3x = 330$$

> Alternatively, we may think of the multiple of 11 immediately before x. This is $(x - 11)$. Therefore, we may take three consecutive multiples of 11 as $x - 11$, x, $x + 11$.
>
> In this case we arrive at the equation
>
> $$(x - 11) + x + (x + 11) = 363$$
>
> or $$3x = 363$$

or $x = \frac{330}{3}$

$= 110$

Hence, the three consecutive multiples are 110, 121, 132 (answer).

or $x = \frac{363}{3} = 121$. Therefore,
$x = 121$, $x - 11 = 110$, $x + 11 = 132$
Hence, the three consecutive multiples are 110, 121, 132.

We can see that we can adopt different ways to find a solution for the problem.

Example 10: The difference between two whole numbers is 66. The ratio of the two numbers is 2 : 5. What are the two numbers?

Solution: Since the ratio of the two numbers is 2 : 5, we may take one number to be $2x$ and the other to be $5x$. (Note that $2x : 5x$ is same as 2 : 5.)

The difference between the two numbers is $(5x - 2x)$. It is given that the difference is 66. Therefore,

$$5x - 2x = 66$$

or $3x = 66$

or $x = 22$

Since the numbers are $2x$ and $5x$, they are 2×22 or 44 and 5×22 or 110, respectively.

The difference between the two numbers is $110 - 44 = 66$ as desired.

Example 11: Deveshi has a total of ₹ 590 as currency notes in the denominations of ₹ 50, ₹ 20 and ₹ 10. The ratio of the number of ₹ 50 notes and ₹ 20 notes is 3:5. If she has a total of 25 notes, how many notes of each denomination she has?

Solution: Let the number of ₹ 50 notes and ₹ 20 notes be $3x$ and $5x$, respectively.

But she has 25 notes in total.

Therefore, the number of ₹ 10 notes = $25 - (3x + 5x) = 25 - 8x$

The amount she has

from ₹ 50 notes : $3x \times 50$ = ₹ $150x$

from ₹ 20 notes : $5x \times 20$ = ₹ $100x$

from ₹ 10 notes : $(25 - 8x) \times 10$ = ₹ $(250 - 80x)$

Hence the total money she has $= 150x + 100x + (250 - 80x)$ = ₹ $(170x + 250)$

But she has ₹ 590. Therefore, $170x + 250 = 590$

or $170x = 590 - 250 = 340$

or $x = \frac{340}{170} = 2$

The number of ₹ 50 notes she has $= 3x$

$= 3 \times 2 = 6$

The number of ₹ 20 notes she has $= 5x = 5 \times 2 = 10$

The number of ₹ 10 notes she has $= 25 - 8x$

$= 25 - (8 \times 2) = 25 - 16 = 9$

EXERCISE 2.2

1. If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$. What is the number?
2. The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?
3. The base of an isosceles triangle is $\frac{4}{3}$ cm. The perimeter of the triangle is $4\frac{2}{15}$ cm. What is the length of either of the remaining equal sides?
4. Sum of two numbers is 95. If one exceeds the other by 15, find the numbers.
5. Two numbers are in the ratio 5:3. If they differ by 18, what are the numbers?
6. Three consecutive integers add up to 51. What are these integers?
7. The sum of three consecutive multiples of 8 is 888. Find the multiples.
8. Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3 and 4 respectively, they add up to 74. Find these numbers.
9. The ages of Rahul and Haroon are in the ratio 5:7. Four years later the sum of their ages will be 56 years. What are their present ages?
10. The number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. What is the total class strength?
11. Baichung's father is 26 years younger than Baichung's grandfather and 29 years older than Baichung. The sum of the ages of all the three is 135 years. What is the age of each one of them?
12. Fifteen years from now Ravi's age will be four times his present age. What is Ravi's present age?
13. A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

14. Lakshmi is a cashier in a bank. She has currency notes of denominations ₹ 100, ₹ 50 and ₹ 10, respectively. The ratio of the number of these notes is 2:3:5. The total cash with Lakshmi is ₹ 4,00,000. How many notes of each denomination does she have?
15. I have a total of ₹ 300 in coins of denomination ₹ 1, ₹ 2 and ₹ 5. The number of ₹ 2 coins is 3 times the number of ₹ 5 coins. The total number of coins is 160. How many coins of each denomination are with me?
16. The organisers of an essay competition decide that a winner in the competition gets a prize of ₹ 100 and a participant who does not win gets a prize of ₹ 25. The total prize money distributed is ₹ 3,000. Find the number of winners, if the total number of participants is 63.

2.4 Solving Equations having the Variable on both Sides

An equation is the equality of the values of two expressions. In the equation $2x - 3 = 7$, the two expressions are $2x - 3$ and 7. In most examples that we have come across so far, the RHS is just a number. But this need not always be so; both sides could have expressions with variables. For example, the equation $2x - 3 = x + 2$ has expressions with a variable on both sides; the expression on the LHS is $(2x - 3)$ and the expression on the RHS is $(x + 2)$.

- We now discuss how to solve such equations which have expressions with the variable on both sides.

Example 12: Solve $2x - 3 = x + 2$

Solution: We have

$$2x = x + 2 + 3$$

or $$2x = x + 5$$

or $$2x - x = x + 5 - x \quad \text{(subtracting } x \text{ from both sides)}$$

or $$x = 5 \quad \text{(solution)}$$

Here we subtracted from both sides of the equation, not a number (constant), but a term involving the variable. We can do this as variables are also numbers. Also, note that subtracting x from both sides amounts to transposing x to LHS.

Example 13: Solve $5x + \frac{7}{2} = \frac{3}{2}x - 14$

Solution: Multiply both sides of the equation by 2. We get

$$2 \times \left(5x + \frac{7}{2}\right) = 2 \times \left(\frac{3}{2}x - 14\right)$$

$$(2 \times 5x) + \left(2 \times \frac{7}{2}\right) = \left(2 \times \frac{3}{2}x\right) - (2 \times 14)$$

or $$10x + 7 = 3x - 28$$

or $$10x - 3x + 7 = -28 \quad \text{(transposing } 3x \text{ to LHS)}$$

or $$7x + 7 = -28$$

or $$7x = -28 - 7$$

or $$7x = -35$$

or $x = \frac{-35}{7}$ or $x = -5$ (solution)

EXERCISE 2.3

Solve the following equations and check your results.

1. $3x = 2x + 18$ **2.** $5t - 3 = 3t - 5$ **3.** $5x + 9 = 5 + 3x$

4. $4z + 3 = 6 + 2z$ **5.** $2x - 1 = 14 - x$ **6.** $8x + 4 = 3(x - 1) + 7$

7. $x = \frac{4}{5}(x + 10)$ **8.** $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$ **9.** $2y + \frac{5}{3} = \frac{26}{3} - y$

10. $3m = 5m - \frac{8}{5}$

2.5 Some More Applications

Example 14: The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number?

Solution: Take, for example, a two-digit number, say, 56. It can be written as $56 = (10 \times 5) + 6$.

If the digits in 56 are interchanged, we get 65, which can be written as $(10 \times 6) + 5$.

Let us take the two digit number such that the digit in the units place is b. The digit in the tens place differs from b by 3. Let us take it as $b + 3$. So the two-digit number is $10(b + 3) + b = 10b + 30 + b = 11b + 30$.

With interchange of digits, the resulting two-digit number will be

$$10b + (b + 3) = 11b + 3$$

> Could we take the tens place digit to be $(b - 3)$? Try it and see what solution you get.

If we add these two two-digit numbers, their sum is

$$(11b + 30) + (11b + 3) = 11b + 11b + 30 + 3 = 22b + 33$$

It is given that the sum is 143. Therefore, $22b + 33 = 143$

or $22b = 143 - 33$

or $22b = 110$

or $b = \frac{110}{22}$

or $b = 5$

> Remember, this is the solution when we choose the tens digits to be 3 more than the unit's digits. What happens if we take the tens digit to be $(b - 3)$?

The units digit is 5 and therefore the tens digit is $5 + 3$ which is 8. The number is 85.

Check: On interchange of digits the number we get is 58. The sum of 85 and 58 is 143 as given.

> The statement of the example is valid for both 58 and 85 and both are correct answers.

Example 15: Arjun is twice as old as Shriya. Five years ago his age was three times Shriya's age. Find their present ages.

Solution: Let us take Shriya's present age to be x years.

Then Arjun's present age would be $2x$ years.

Shriya's age five years ago was $(x - 5)$ years.

Arjun's age five years ago was $(2x - 5)$ years.

It is given that Arjun's age five years ago was three times Shriya's age.

Thus, $2x - 5 = 3(x - 5)$

or $2x - 5 = 3x - 15$

or $15 - 5 = 3x - 2x$

or $10 = x$

So, Shriya's present age $= x = 10$ years.

Therefore, Arjun's present age $= 2x = 2 \times 10 = 20$ years.

EXERCISE 2.4

1. Amina thinks of a number and subtracts $\frac{5}{2}$ from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?
2. A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other new number. What are the numbers?
3. Sum of the digits of a two-digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?
4. One of the two digits of a two digit number is three times the other digit. If you interchange the digits of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?
5. Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?
6. There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11:4. At the rate ₹100 per metre it will cost the village panchayat ₹ 75000 to fence the plot. What are the dimensions of the plot?
7. Hasan buys two kinds of cloth materials for school uniforms, shirt material that costs him ₹ 50 per metre and trouser material that costs him ₹ 90 per metre.

For every 3 meters of the shirt material he buys 2 metres of the trouser material. He sells the materials at 12% and 10% profit respectively. His total sale is ₹ 36,600. How much trouser material did he buy?

8. Half of a herd of deer are grazing in the field and three fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.
9. A grandfather is ten times older than his granddaughter. He is also 54 years older than her. Find their present ages.
10. Aman's age is three times his son's age. Ten years ago he was five times his son's age. Find their present ages.

2.6 Reducing Equations to Simpler Form

Example 16: Solve $\frac{6x+1}{3}+1=\frac{x-3}{6}$

Solution: Multiplying both sides of the equation by 6,

> Why 6? Because it is the smallest multiple (or LCM) of the given denominators.

$$\frac{6\,(6x+1)}{3}+6\times 1=\frac{6(x-3)}{6}$$

or $2\,(6x + 1) + 6 = x - 3$

or $12x + 2 + 6 = x - 3$ (opening the brackets)

or $12x + 8 = x - 3$

or $12x - x + 8 = -3$

or $11x + 8 = -3$

or $11x = -3 - 8$

or $11x = -11$

or $x = -1$ (required solution)

Check: LHS $=\frac{6(-1)+1}{3}+1=\frac{-6+1}{3}+1=\frac{-5}{3}+\frac{3}{3}=\frac{-5+3}{3}=\frac{-2}{3}$

RHS $=\frac{(-1)-3}{6}=\frac{-4}{6}=\frac{-2}{3}$

LHS = RHS. (as required)

Example 17: Solve $5x - 2\,(2x - 7) = 2\,(3x - 1) + \frac{7}{2}$

Solution: Let us open the brackets,

LHS $= 5x - 4x + 14 = x + 14$

$$\text{RHS} = 6x - 2 + \frac{7}{2} = 6x - \frac{4}{2} + \frac{7}{2} = 6x + \frac{3}{2}$$

The equation is $x + 14 = 6x + \frac{3}{2}$

or $\quad 14 = 6x - x + \frac{3}{2}$

or $\quad 14 = 5x + \frac{3}{2}$

or $\quad 14 - \frac{3}{2} = 5x \qquad$ (transposing $\frac{3}{2}$)

or $\quad \frac{28-3}{2} = 5x$

or $\quad \frac{25}{2} = 5x$

or $\quad x = \frac{25}{2} \times \frac{1}{5} = \frac{5 \times 5}{2 \times 5} = \frac{5}{2}$

Did you observe how we simplified the form of the given equation? Here, we had to multiply both sides of the equation by the LCM of the denominators of the terms in the expressions of the equation.

Therefore, required solution is $x = \frac{5}{2}$.

***Check*:** LHS $= 5 \times \frac{5}{2} - 2\left(\frac{5}{2} \times 2 - 7\right)$

$$= \frac{25}{2} - 2(5-7) = \frac{25}{2} - 2(-2) = \frac{25}{2} + 4 = \frac{25+8}{2} = \frac{33}{2}$$

$$\text{RHS} = 2\left(\frac{5}{2} \times 3 - 1\right) + \frac{7}{2} = 2\left(\frac{15}{2} - \frac{2}{2}\right) + \frac{7}{2} = \frac{2 \times 13}{2} + \frac{7}{2}$$

$$= \frac{26+7}{2} = \frac{33}{2} = \text{LHS. (as required)}$$

Note, in this example we brought the equation to a simpler form by opening brackets and combining like terms on both sides of the equation.

EXERCISE 2.5

Solve the following linear equations.

1. $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$ **2.** $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$ **3.** $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{2}$

4. $\dfrac{x-5}{3}=\dfrac{x-3}{5}$ **5.** $\dfrac{3t-2}{4}-\dfrac{2t+3}{3}=\dfrac{2}{3}-t$ **6.** $m-\dfrac{m-1}{2}=1-\dfrac{m-2}{3}$

Simplify and solve the following linear equations.

7. $3(t-3)=5(2t+1)$ **8.** $15(y-4)-2(y-9)+5(y+6)=0$

9. $3(5z-7)-2(9z-11)=4(8z-13)-17$

10. $0.25(4f-3)=0.05(10f-9)$

2.7 Equations Reducible to the Linear Form

Example 18: Solve $\dfrac{x+1}{2x+3}=\dfrac{3}{8}$

Solution: Observe that the equation is not a linear equation, since the expression on its LHS is not linear. But we can put it into the form of a linear equation. We multiply both sides of the equation by $(2x+3)$,

$$\left(\frac{x+1}{2x+3}\right)\times(2x+3)=\frac{3}{8}\times(2x+3)$$

Note that $2x+3\neq 0$ (Why?)

Notice that $(2x+3)$ gets cancelled on the LHS We have then,

$$x+1=\frac{3\,(2x+3)}{8}$$

We have now a linear equation which we know how to solve.
Multiplying both sides by 8

$$8\,(x+1)=3\,(2x+3)$$

This step can be directly obtained by 'cross-multiplication'

$$\frac{x+1}{2x+3}\nearrow\!\!\!\!\!\nwarrow\frac{3}{8}$$

or $8x+8=6x+9$

or $8x=6x+9-8$

or $8x=6x+1$

or $8x-6x=1$

or $2x=1$

or $x=\dfrac{1}{2}$

The solution is $x=\dfrac{1}{2}$.

Check : Numerator of LHS $=\dfrac{1}{2}+1=\dfrac{1+2}{2}=\dfrac{3}{2}$

Denominator of LHS $=2x+3=2\times\dfrac{1}{2}+3=1+3=4$

LHS = numerator ÷ denominator = $\frac{3}{2} \div 4 = \frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$

LHS = RHS.

Example 19: Present ages of Anu and Raj are in the ratio 4:5. Eight years from now the ratio of their ages will be 5:6. Find their present ages.

Solution: Let the present ages of Anu and Raj be $4x$ years and $5x$ years respectively.

After eight years. Anu's age = $(4x + 8)$ years;

After eight years, Raj's age = $(5x + 8)$ years.

Therefore, the ratio of their ages after eight years = $\frac{4x+8}{5x+8}$

This is given to be 5 : 6

Therefore, $\frac{4x+8}{5x+8} = \frac{5}{6}$

Cross-multiplication gives $6(4x + 8) = 5(5x + 8)$

or $24x + 48 = 25x + 40$

or $24x + 48 - 40 = 25x$

or $24x + 8 = 25x$

or $8 = 25x - 24x$

or $8 = x$

Therefore, Anu's present age = $4x = 4 \times 8 = 32$ years

Raj's present age = $5x = 5 \times 8 = 40$ years

EXERCISE 2.6

Solve the following equations.

1. $\frac{8x-3}{3x} = 2$ **2.** $\frac{9x}{7-6x} = 15$ **3.** $\frac{z}{z+15} = \frac{4}{9}$

4. $\frac{3y+4}{2-6y} = \frac{-2}{5}$ **5.** $\frac{7y+4}{y+2} = \frac{-4}{3}$

6. The ages of Hari and Harry are in the ratio 5:7. Four years from now the ratio of their ages will be 3:4. Find their present ages.

7. The denominator of a rational number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

WHAT HAVE WE DISCUSSED?

1. An algebraic equation is an equality involving variables. It says that the value of the expression on one side of the equality sign is equal to the value of the expression on the other side.
2. The equations we study in Classes VI, VII and VIII are linear equations in one variable. In such equations, the expressions which form the equation contain only one variable. Further, the equations are linear, i.e., the highest power of the variable appearing in the equation is 1.
3. A linear equation may have for its solution any rational number.
4. An equation may have linear expressions on both sides. Equations that we studied in Classes VI and VII had just a number on one side of the equation.
5. Just as numbers, variables can, also, be transposed from one side of the equation to the other.
6. Occasionally, the expressions forming equations have to be simplified before we can solve them by usual methods. Some equations may not even be linear to begin with, but they can be brought to a linear form by multiplying both sides of the equation by a suitable expression.
7. The utility of linear equations is in their diverse applications; different problems on numbers, ages, perimeters, combination of currency notes, and so on can be solved using linear equations.

CHAPTER 3

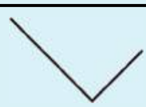
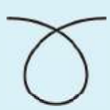
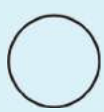
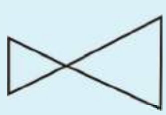
Understanding Quadrilaterals

3.1 Introduction

You know that the paper is a model for a **plane surface**. When you join a number of points without lifting a pencil from the paper (and without retracing any portion of the drawing other than single points), you get a **plane curve**.

Try to recall different varieties of curves you have seen in the earlier classes.

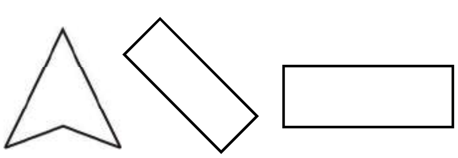
Match the following: (Caution! A figure may match to more than one type).

Figure		Type	
(1)		(a)	Simple closed curve
(2)		(b)	A closed curve that is not simple
(3)		(c)	Simple curve that is not closed
(4)		(d)	Not a simple curve

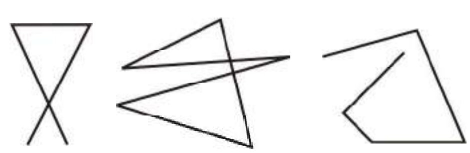
Compare your matchings with those of your friends. Do they agree?

3.2 Polygons

A simple closed curve made up of only line segments is called a **polygon**.



Curves that are polygons

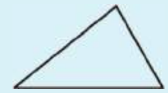
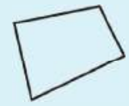
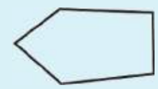
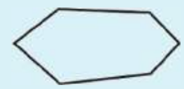
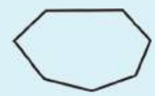
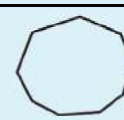


Curves that are not polygons

Try to give a few more examples and non-examples for a polygon.
Draw a rough figure of a polygon and identify its sides and vertices.

3.2.1 Classification of polygons

We classify polygons according to the number of sides (or vertices) they have.

Number of sides or vertices	Classification	Sample figure
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octagon	
9	Nonagon	
10	Decagon	
⋮	⋮	⋮
n	n-gon	

3.2.2 Diagonals

A diagonal is a line segment connecting two non-consecutive vertices of a polygon (Fig 3.1).

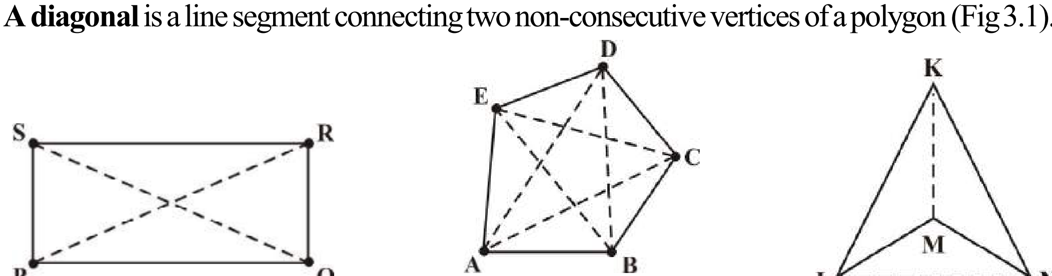
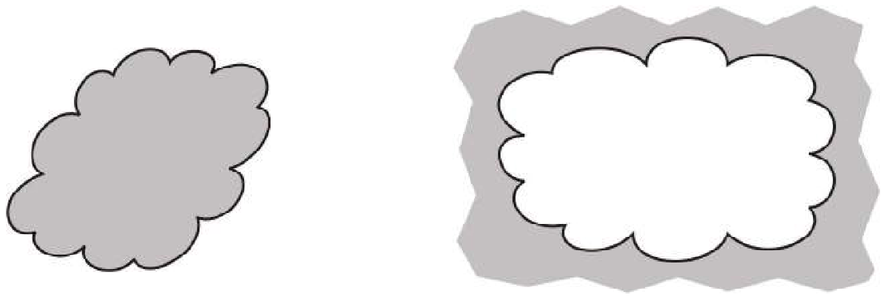


Fig 3.1

Can you name the diagonals in each of the above figures? (Fig 3.1)

Is $\overline{PQ}$ a diagonal? What about $\overline{LN}$?

You already know what we mean by **interior** and **exterior** of a closed curve (Fig 3.2).

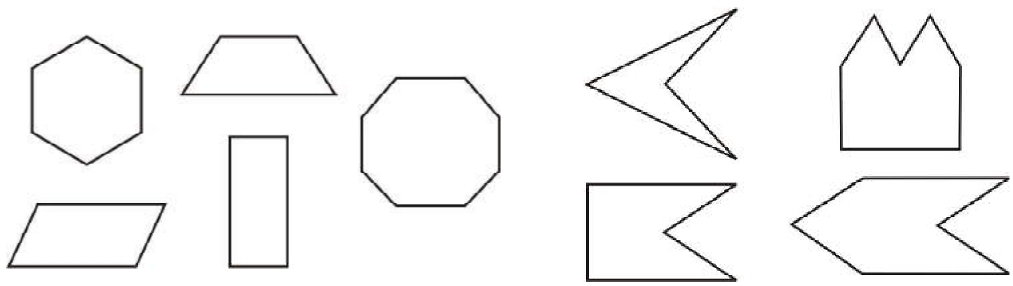


Interior **Fig 3.2** **Exterior**

The interior has a boundary. Does the exterior have a boundary? Discuss with your friends.

3.2.3 Convex and concave polygons

Here are some convex polygons and some concave polygons. (Fig 3.3)



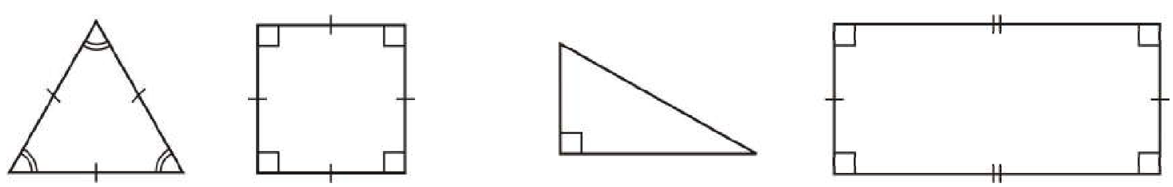
Convex polygons **Fig 3.3** **Concave polygons**

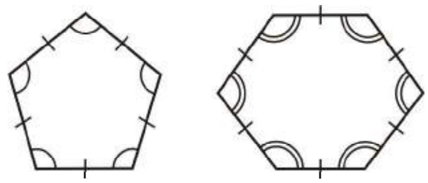
Can you find how these types of polygons differ from one another? Polygons that are convex have no portions of their diagonals in their exteriors. Is this true with concave polygons? Study the figures given. Then try to describe in your own words what we mean by a convex polygon and what we mean by a concave polygon. Give two rough sketches of each kind.

In our work in this class, we will be dealing with convex polygons only.

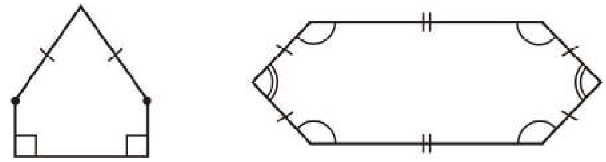
3.2.4 Regular and irregular polygons

A regular polygon is both 'equiangular' and 'equilateral'. For example, a square has sides of equal length and angles of equal measure. Hence it is a regular polygon. A rectangle is equiangular but not equilateral. Is a rectangle a regular polygon? Is an equilateral triangle a regular polygon? Why?





Regular polygons



Polygons that are not regular

[**Note:** Use of ⩘ or ⩘ indicates segments of equal length].

In the previous classes, have you come across any quadrilateral that is equilateral but not equiangular? Recall the quadrilateral shapes you saw in earlier classes – Rectangle, Square, Rhombus etc.

Is there a triangle that is equilateral but not equiangular?

3.2.5 Angle sum property

Do you remember the angle-sum property of a triangle? The sum of the measures of the three angles of a triangle is 180°. Recall the methods by which we tried to visualise this fact. We now extend these ideas to a quadrilateral.

DO THIS

1. Take any quadrilateral, say ABCD (Fig 3.4). Divide it into two triangles, by drawing a diagonal. You get six angles 1, 2, 3, 4, 5 and 6.

   Use the angle-sum property of a triangle and argue how the sum of the measures of ∠A, ∠B, ∠C and ∠D amounts to 180° + 180° = 360°.

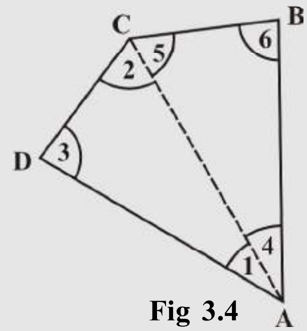


Fig 3.4

2. Take four congruent card-board copies of any quadrilateral ABCD, with angles as shown [Fig 3.5 (i)]. Arrange the copies as shown in the figure, where angles ∠1, ∠2, ∠3, ∠4 meet at a point [Fig 3.5 (ii)].

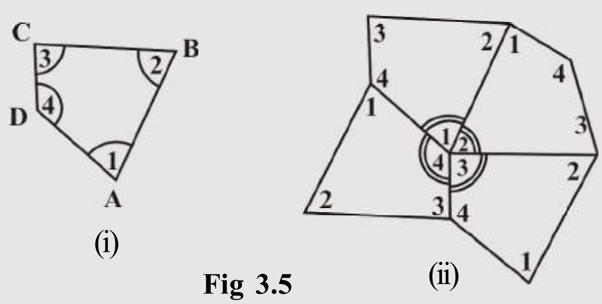


Fig 3.5

For doing this you may have to turn and match appropriate corners so that they fit.

What can you say about the sum of the angles ∠1, ∠2, ∠3 and ∠4?

[**Note:** We denote the angles by ∠1, ∠2, ∠3, etc., and their respective measures by *m*∠1, *m*∠2, *m*∠3, etc.]

The sum of the measures of the four angles of a quadrilateral is____________.

You may arrive at this result in several other ways also.

3. As before consider quadrilateral ABCD (Fig 3.6). Let P be any point in its interior. Join P to vertices A, B, C and D. In the figure, consider ΔPAB. From this we see $x = 180° - m\angle 2 - m\angle 3$; similarly from ΔPBC, $y = 180° - m\angle 4 - m\angle 5$, from ΔPCD, $z = 180° - m\angle 6 - m\angle 7$ and from ΔPDA, $w = 180° - m\angle 8 - m\angle 1$. Use this to find the total measure $m\angle 1 + m\angle 2 + ... + m\angle 8$, does it help you to arrive at the result? Remember $\angle x + \angle y + \angle z + \angle w = 360°$.

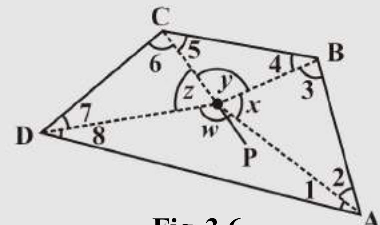


Fig 3.6

4. These quadrilaterals were convex. What would happen if the quadrilateral is not convex? Consider quadrilateral ABCD. Split it into two triangles and find the sum of the interior angles (Fig 3.7).

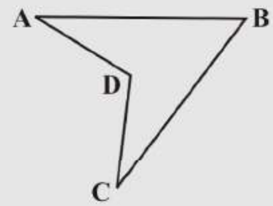
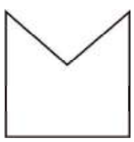


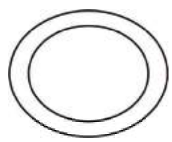
Fig 3.7

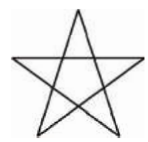
EXERCISE 3.1

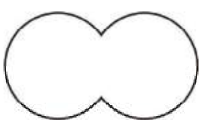
1. Given here are some figures.


(1)

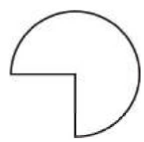

(2)


(3)


(4)


(5)


(6)


(7)


(8)

Classify each of them on the basis of the following.

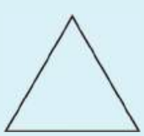
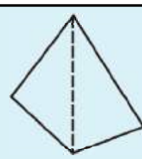
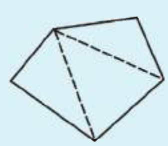
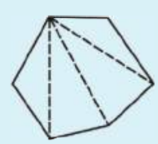
(a) Simple curve (b) Simple closed curve (c) Polygon
(d) Convex polygon (e) Concave polygon

2. How many diagonals does each of the following have?

(a) A convex quadrilateral (b) A regular hexagon (c) A triangle

3. What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and try!)

4. Examine the table. (Each figure is divided into triangles and the sum of the angles deduced from that.)

Figure				
Side	3	4	5	6
Angle sum	180º	$2 \times 180°$ $= (4 - 2) \times 180°$	$3 \times 180°$ $= (5 - 2) \times 180°$	$4 \times 180°$ $= (6 - 2) \times 180°$

What can you say about the angle sum of a convex polygon with number of sides?

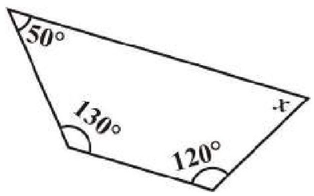
(a) 7 (b) 8 (c) 10 (d) n

5. What is a regular polygon?

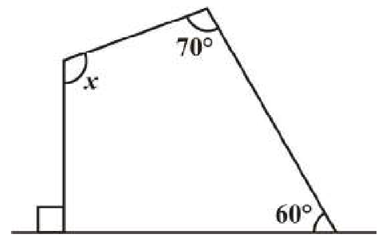
State the name of a regular polygon of

(i) 3 sides (ii) 4 sides (iii) 6 sides

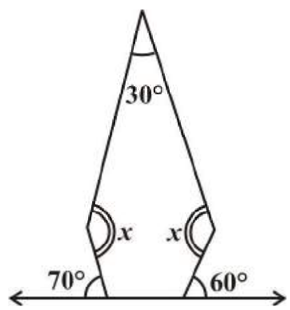
6. Find the angle measure x in the following figures.



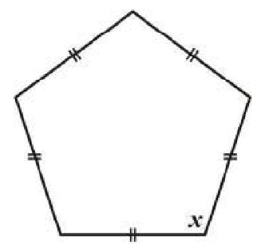
(a)



(b)

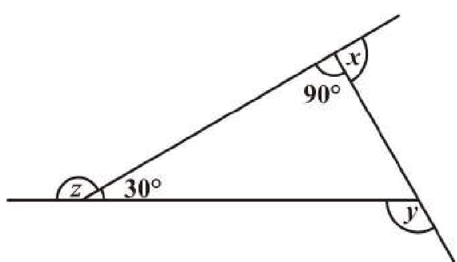


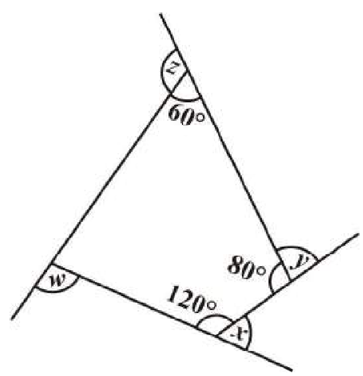
(c)



(d)

7.





(a) Find $x + y + z$ (b) Find $x + y + z + w$

3.3 Sum of the Measures of the Exterior Angles of a Polygon

On many occasions a knowledge of exterior angles may throw light on the nature of interior angles and sides.

DO THIS

Draw a polygon on the floor, using a piece of chalk. (In the figure, a pentagon ABCDE is shown) (Fig 3.8).

We want to know the total measure of angles, i.e, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5$. Start at A. Walk along $\overline{AB}$. On reaching B, you need to turn through an angle of $m\angle 1$, to walk along $\overline{BC}$. When you reach at C, you need to turn through an angle of $m\angle 2$ to walk along $\overline{CD}$. You continue to move in this manner, until you return to side AB. You would have in fact made one complete turn.

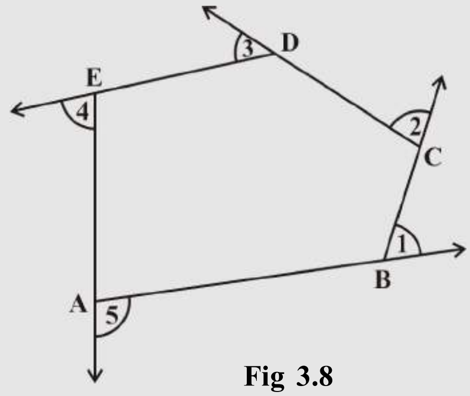


Fig 3.8

Therefore, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$

This is true whatever be the number of sides of the polygon.

Therefore, *the sum of the measures of the external angles of any polygon is 360°.*

Example 1: Find measure x in Fig 3.9.

Solution:

$$x + 90° + 50° + 110° = 360° \quad \text{(Why?)}$$
$$x + 250° = 360°$$
$$x = 110°$$

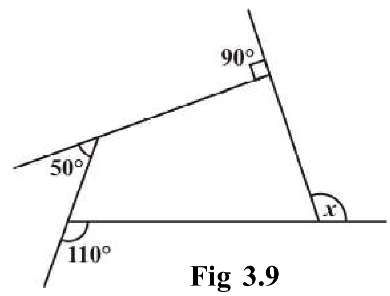


Fig 3.9

TRY THESE

Take a regular hexagon Fig 3.10.

1. What is the sum of the measures of its exterior angles x, y, z, p, q, r?
2. Is $x = y = z = p = q = r$? Why?
3. What is the measure of each?
   (i) exterior angle (ii) interior angle
4. Repeat this activity for the cases of
   (i) a regular octagon (ii) a regular 20-gon

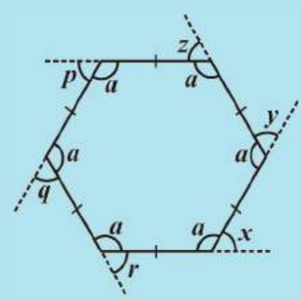


Fig 3.10

Example 2: Find the number of sides of a regular polygon whose each exterior angle has a measure of 45°.

Solution: Total measure of all exterior angles = 360°

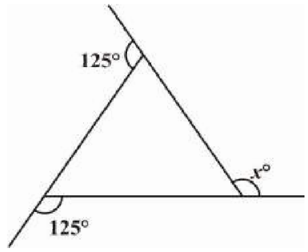
Measure of each exterior angle = 45°

Therefore, the number of exterior angles $= \frac{360}{45} = 8$

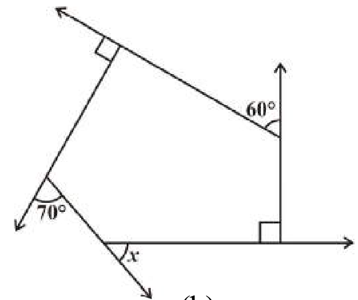
The polygon has 8 sides.

EXERCISE 3.2

1. Find x in the following figures.



(a)



(b)

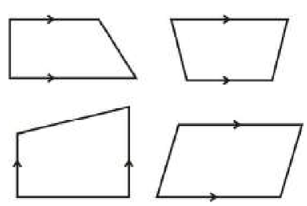
2. Find the measure of each exterior angle of a regular polygon of
   (i) 9 sides   (ii) 15 sides
3. How many sides does a regular polygon have if the measure of an exterior angle is 24°?
4. How many sides does a regular polygon have if each of its interior angles is 165°?
5. (a) Is it possible to have a regular polygon with measure of each exterior angle as 22°?
   (b) Can it be an interior angle of a regular polygon? Why?
6. (a) What is the minimum interior angle possible for a regular polygon? Why?
   (b) What is the maximum exterior angle possible for a regular polygon?

3.4 Kinds of Quadrilaterals

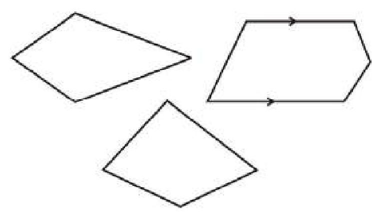
Based on the nature of the sides or angles of a quadrilateral, it gets special names.

3.4.1 Trapezium

Trapezium is a quadrilateral with a pair of parallel sides.



These are trapeziums



These are not trapeziums

Study the above figures and discuss with your friends why some of them are trapeziums while some are not. (**Note:** *The arrow marks indicate parallel lines*).

DO THIS

1. Take identical cut-outs of congruent triangles of sides 3 cm, 4 cm, 5 cm. Arrange them as shown (Fig 3.11).

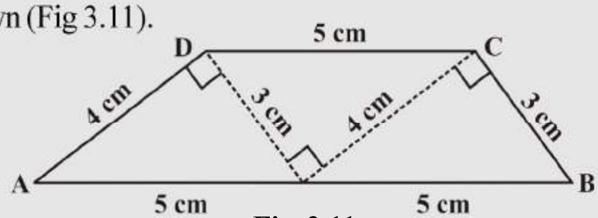


Fig 3.11

You get a trapezium. (Check it!) Which are the parallel sides here? Should the non-parallel sides be equal?

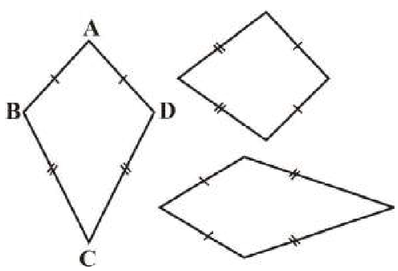
You can get two more trapeziums using the same set of triangles. Find them out and discuss their shapes.

2. Take four set-squares from your and your friend's instrument boxes. Use different numbers of them to place side-by-side and obtain different trapeziums.

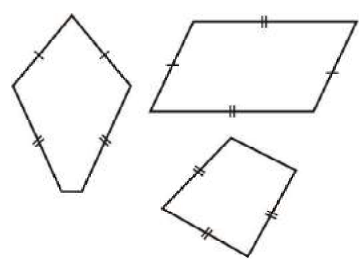
If the non-parallel sides of a trapezium are of equal length, we call it an *isosceles trapezium*. Did you get an isoceles trapezium in any of your investigations given above?

3.4.2 Kite

Kite is a special type of a quadrilateral. The sides with the same markings in each figure are equal. For example AB = AD and BC = CD.



These are kites



These are not kites

Study these figures and try to describe what a kite is. Observe that

(i) A kite has 4 sides (It is a quadrilateral).

(ii) There are exactly two **distinct consecutive pairs** of sides of equal length.

DO THIS

Take a thick white sheet.

Fold the paper once.

Draw two line segments of different lengths as shown in Fig 3.12.

Cut along the line segments and open up.

You have the shape of a kite (Fig 3.13).

Has the kite any line symmetry?

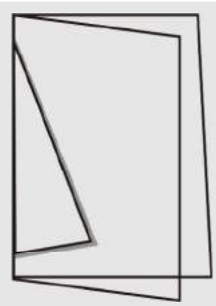


Fig 3.12

Show that ΔABC and ΔADC are congruent. What do we infer from this?

Fold both the diagonals of the kite. Use the set-square to check if they cut at right angles. Are the diagonals equal in length?

Verify (by paper-folding or measurement) if the diagonals bisect each other.

By folding an angle of the kite on its opposite, check for angles of equal measure.

Observe the diagonal folds; do they indicate any diagonal being an angle bisector?

Share your findings with others and list them. A summary of these results are given elsewhere in the chapter for your reference.

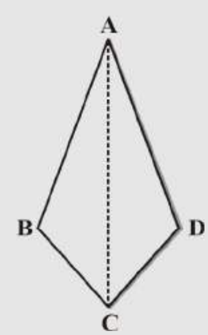
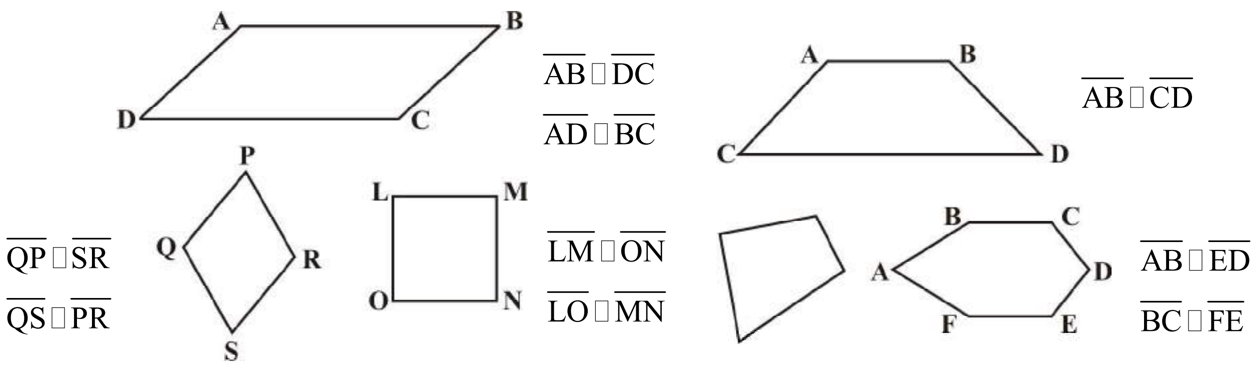


Fig 3.13

3.4.3 Parallelogram

A parallelogram is a quadrilateral. As the name suggests, it has something to do with parallel lines.



These are parallelograms **These are not parallelograms**

Study these figures and try to describe in your own words what we mean by a parallelogram. Share your observations with your friends.

DO THIS

Take two different rectangular cardboard strips of different widths (Fig 3.14).

Strip 1 **Fig 3.14** **Strip 2**

Place one strip horizontally and draw lines along its edge as drawn in the figure (Fig 3.15).

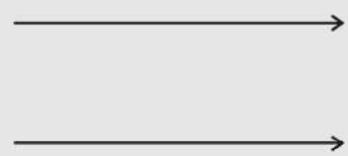


Fig 3.15

Now place the other strip in a slant position over the lines drawn and use this to draw two more lines as shown (Fig 3.16).

These four lines enclose a quadrilateral. This is made up of two pairs of parallel lines (Fig 3.17).

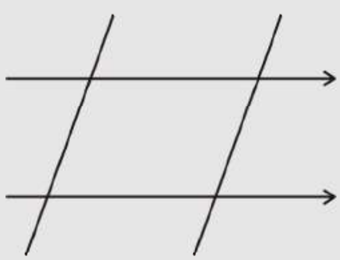


Fig 3.16

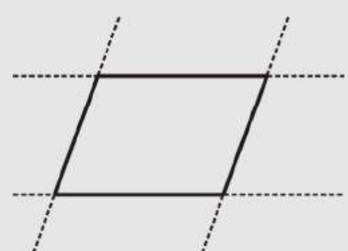


Fig 3.17

It is a parallelogram.

A parallelogram is a quadrilateral whose opposite sides are parallel.

3.4.4 Elements of a parallelogram

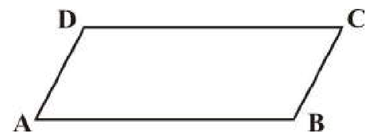


Fig 3.18

There are four sides and four angles in a parallelogram. Some of these are equal. There are some terms associated with these elements that you need to remember.

Given a parallelogram ABCD (Fig 3.18).

$\overline{AB}$ and $\overline{DC}$, are **opposite sides**. $\overline{AD}$ and $\overline{BC}$ form another pair of opposite sides.

$\angle$A and $\angle$C are a pair of **opposite angles**; another pair of opposite angles would be $\angle$B and $\angle$D.

$\overline{AB}$ and $\overline{BC}$ are **adjacent sides**. This means, one of the sides starts where the other ends. Are $\overline{BC}$ and $\overline{CD}$ adjacent sides too? Try to find two more pairs of adjacent sides.

$\angle$A and $\angle$B are **adjacent angles**. They are at the ends of the same side. $\angle$B and $\angle$C are also adjacent. Identify other pairs of adjacent angles of the parallelogram.

DO THIS

Take cut-outs of two identical parallelograms, say ABCD and A′B′C′D′ (Fig 3.19).

Fig 3.19

Here $\overline{AB}$ is same as $\overline{A'B'}$ except for the name. Similarly the other corresponding sides are equal too.

Place $\overline{A'B'}$ over $\overline{DC}$. Do they coincide? What can you now say about the lengths $\overline{AB}$ and $\overline{DC}$?

Similarly examine the lengths $\overline{AD}$ and $\overline{BC}$. What do you find?

You may also arrive at this result by measuring $\overline{AB}$ and $\overline{DC}$.

Property: *The opposite sides of a parallelogram are of equal length.*

TRY THESE

Take two identical set squares with angles 30° – 60° – 90° and place them adjacently to form a parallelogram as shown in Fig 3.20. Does this help you to verify the above property?

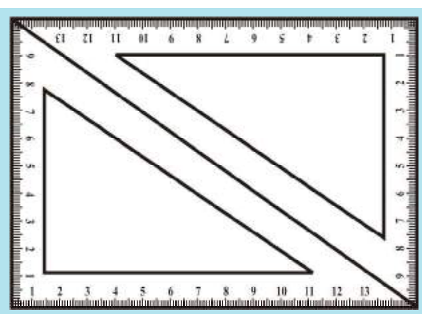


Fig 3.20

You can further strengthen this idea through a logical argument also.

Consider a parallelogram ABCD (Fig 3.21). Draw any one diagonal, say $\overline{AC}$.

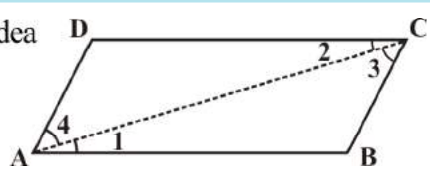


Fig 3.21

Looking at the angles,

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4 \quad \text{(Why?)}$$

Since in triangles ABC and ADC, $\angle 1 = \angle 2$, $\angle 3 = \angle 4$

and $\overline{AC}$ is common, so, by ASA congruency condition,

$$\Delta\, ABC \cong \Delta\, CDA \qquad \text{(How is ASA used here?)}$$

This gives $AB = DC$ and $BC = AD$.

Example 3: Find the perimeter of the parallelogram PQRS (Fig 3.22).

Solution: In a parallelogram, the opposite sides have same length.

Therefore, PQ = SR = 12 cm and QR = PS = 7 cm

So, Perimeter = PQ + QR + RS + SP

= 12 cm + 7 cm + 12 cm + 7 cm = 38 cm

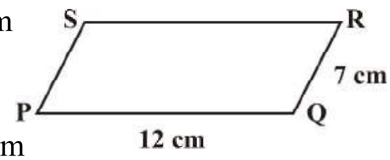


Fig 3.22

3.4.5 Angles of a parallelogram

We studied a property of parallelograms concerning the (opposite) sides. What can we say about the angles?

DO THIS

Let ABCD be a parallelogram (Fig 3.23). Copy it on a tracing sheet. Name this copy as A′B′C′D′. Place A′B′C′D′ on ABCD. Pin them together at the point where the diagonals meet. Rotate the transparent sheet by 180°. The parallelograms still concide; but you now find A′ lying exactly on C and vice-versa; similarly B′ lies on D and vice-versa.

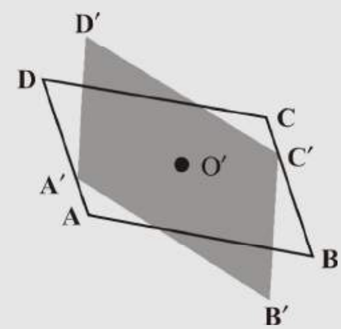


Fig 3.23

Does this tell you anything about the measures of the angles A and C? Examine the same for angles B and D. State your findings.

Property: *The opposite angles of a parallelogram are of equal measure.*

TRY THESE

Take two identical 30° – 60° – 90° set-squares and form a parallelogram as before. Does the figure obtained help you to confirm the above property?

You can further justify this idea through logical arguments.

If $\overline{AC}$ and $\overline{BD}$ are the diagonals of the parallelogram, (Fig 3.24) you find that

$$\angle 1 = \angle 2 \quad \text{and} \quad \angle 3 = \angle 4 \quad \text{(Why?)}$$

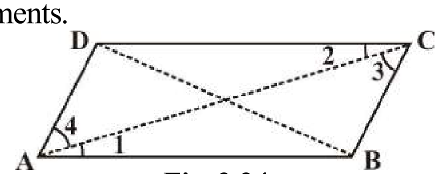


Fig 3.24

Studying ΔABC and ΔADC (Fig 3.25) separately, will help you to see that by ASA congruency condition,

$$\Delta\,ABC \cong \Delta\,CDA \qquad \text{(How?)}$$

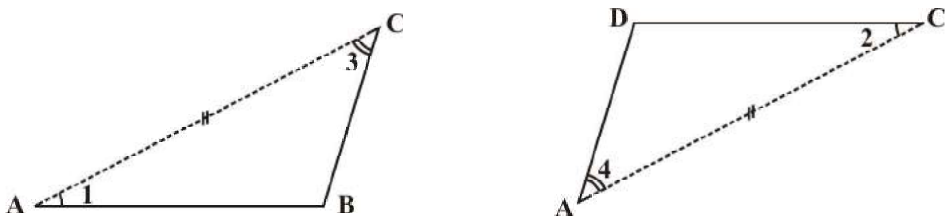


Fig 3.25

This shows that $\angle$B and $\angle$D have same measure. In the same way you can get $m\angle A = m\angle C$.

Example 4: In Fig 3.26, BEST is a parallelogram. Find the values *x*, *y* and *z*.

Solution: S is opposite to B.

So, $x = 100°$ (opposite angles property)

$y = 100°$ (measure of angle corresponding to $\angle x$)

$z = 80°$ (since $\angle y$, $\angle z$ is a linear pair)

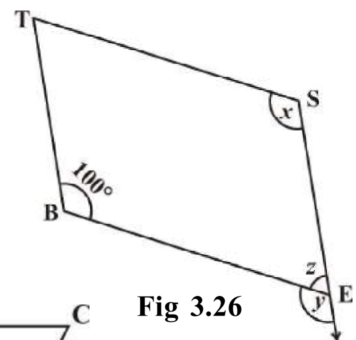


Fig 3.26

We now turn our attention to adjacent angles of a parallelogram.

In parallelogram ABCD, (Fig 3.27).

$\angle$A and $\angle$D are supplementary since $\overline{DC} \parallel \overline{AB}$ and with transversal $\overline{DA}$, these two angles are interior opposite.

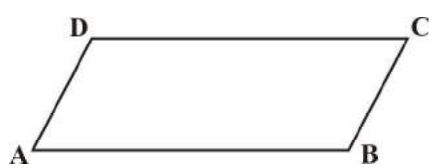


Fig 3.27

$\angle$A and $\angle$B are also supplementary. Can you say 'why'?

$\overline{AD} \parallel \overline{BC}$ and $\overline{BA}$ is a transversal, making $\angle$A and $\angle$B interior opposite.

Identify two more pairs of supplementary angles from the figure.

Property: *The adjacent angles in a parallelogram are supplementary.*

Example 5: In a parallelogram RING, (Fig 3.28) if $m\angle R = 70°$, find all the other angles.

Solution: Given $m\angle R = 70°$

Then $m\angle N = 70°$

because $\angle$R and $\angle$N are opposite angles of a parallelogram.

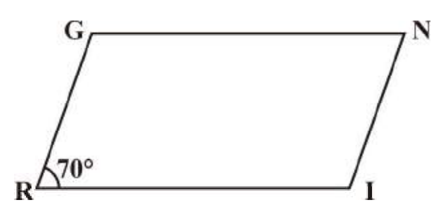


Fig 3.28

Since $\angle$R and $\angle$I are supplementary,

$$m\angle I = 180° - 70° = 110°$$

Also, $m\angle G = 110°$ since $\angle$G is opposite to $\angle$I

Thus, $m\angle R = m\angle N = 70°$ and $m\angle I = m\angle G = 110°$