Probability

INTRODUCTION

The word *probability* or *chance* is very frequently used in day-to-day life. For example, we generally say, 'He may come today' or 'probably it may rain tomorrow' or 'most probably he will get through the examination'. All these phrases involve an element of uncertainty and probability is a concept which measures these uncertainties. The probability when defined in simplest way is the chance of occurring of a certain event when expressed quantitatively, i.e., probability is a quantitative measure of the certainty.

The probability has its origin in the problems dealing with games of chance such as gambling, coin tossing, die throwing and playing cards. In all these cases, the outcome of a trial is uncertain. These days probability is widely used in business and economics in the field of predictions for future.

The following remarks may be important for learning this chapter on probability.

1. **Die:** A die is a small cube used in games of chance. On its six faces, dots are marked as

   . .. ... :: :.: :::

   Plural of die is dice. The outcome of throwing (or tossing) a die is the number of dots on its uppermost face. An ace on a die means one dot.

2. **Cards:** A pack (or deck) of playing cards has 52 cards, divided into four suits:

   (i) Spades हुकम (♠) (ii) Clubs चिड़ी (♣)
   (iii) Hearts पान (♥) (iv) Diamonds ईंट (♦)

   Each suit has 13 cards, nine cards numbered 2 to 10, an Ace (इक्का), a King (बादशाह), Queen (बेगम) and a Jack or Knave (गुलाम). Spades and Clubs are black-faced cards while Hearts and Diamonds are red-faced cards. The Aces, Kings, Queens and Jacks are called *face cards* and other cards are called *number cards*. The Kings, Queens and Jacks are called *court cards*.

3. The number of combinations of n objects taken r at a time $(r \le n)$ is denoted by $C(n, r)$ or ${}^nC_r$ and is defined as

$$ {}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2)\ldots \text{to } r \text{ factors}}{1\times 2\times 3 \ldots r}. $$

Illustration 1 ${}^5C_3 = \dfrac{5\times4\times3}{1\times2\times3} = 10$, ${}^nC_0 = 1$ and ${}^nC_n = 1$

If $r > \dfrac{n}{2}$, then it is better to simplify ${}^nC_r$ as ${}^nC_{n-r}$

Illustration 2
$$ {}^{52}C_{50} = {}^{52}C_{52-50} = {}^{52}C_2 = \frac{52\times51}{2\times1} = 26.51 = 1326 $$

When, $r > n, {}^nC_r = 0$

Some Important Terms and Concepts

Random Experiment or Trial: The performance of an experiment is called a *trial*. An experiment is characterized by the property that its observations under a given set of circumstances do not always lead to the same observed outcome but rather to the different outcomes. If in an experiment, all the possible outcomes are known in advance and none of the outcomes can be predicted with certainty, then such an experiment is called a *random experiment*.

For example, tossing a coin or throwing a die are random experiments.

Event: The possible outcomes of a trial are called *events*. Events are generally denoted by capital letters A, B, C and so on.

Illustration 3 (i) When a coin is tossed the outcome of getting a head or a tail is an event

(ii) When a die is thrown the outcome of getting 1 or 2 or 3 or 4 or 5 or 6 is an event

Sample Space: The set of all possible outcomes of an experiment is called a *sample space*. We generally denote it by S.

Illustration 4 (i) When a coin is tossed, $S = \{H, T\}$ where H = head, T = tail

(ii) When a die is thrown, $S = \{1, 2, 3, 4, 5, 6\}$

(iii) When two coins are tossed simultaneously,

$$S = \{HH, HT, TH, TT\}$$

Equally Likely Events: Events are said to be *equally likely* if there is no reason to expect any one in preference to other. Thus, equally likely events mean outcome is as likely to occur as any other outcome.

Illustration 5 In throwing a die, all the six faces (1, 2, 3, 4, 5, 6) are equally likely to occur

Simple and Compound Events

In the case of *simple events*, we consider the probability of happening or non-happening of single events.

Illustration 6 We might be interested in finding out the probability of drawing an ace from a pack of cards.

In the case of *compound events*, we consider the joint occurrence of two or more events.

Illustration 7 If from a bag, containing 8 red and 5 green balls, two successive draws of 2 balls are made, we will be finding out the probability of getting 2 red balls in the first draw and 2 green balls in the second draw. We are thus dealing with a compound event.

Exhaustive Events: It is the total number of all possible outcomes of any trial.

Illustration 8 (i) When a coin is tossed, either head or tail may turn up and therefore, there are two exhaustive cases.

(ii) There are six exhaustive cases or events in throwing a die.

(iii) If two dice are thrown simultaneously, the possible outcomes are

(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Thus, in this case, there are $36 (=6^2)$ ordered pairs. Hence, the number of exhaustive cases in the simultaneous throw of two dice is 36

(iv) Three dice are thrown, the number of exhaustive cases is 6^3, i.e., 216

Algebra of Events

If A and B are two events associated with sample space S, then

(i) $A \cup B$ is the event that either A or B or both occur.

(ii) $A \cap B$ is the event that A and B both occur simultaneously.

(iii) $\overline{A}$ is the event that A does not occur.

(iv) $\overline{A} \cap \overline{B}$ is an event of non-occurrence of both A and B, i.e., none of the events A and B occurs.

Illustration 9 In a single throw of a die, let A be the event of getting an even number and B be the event of getting a number greater than 2. Then,

$$A = \{1, 3, 5\}, B = \{3, 4, 5, 6\}$$

$$\therefore \quad A \cup B = \{1, 3, 4, 5, 6\}$$

$A \cup B$ is the event of getting an odd number or a number greater than 2

$$A \cap B = \{3, 5\}$$

$A \cap B$ is the event of getting an odd number greater than 2.

$\overline{A} = \{2, 4, 6\}$ [Those elements of S which are not in A.]

$\overline{A}$ is the event of not getting an odd number, i.e., getting an even number.

$$\overline{B} = \{1, 2\}$$

$\overline{B}$ is the event of not getting a number greater that 2, i.e., getting a number less than or equal to 2.

$$\overline{A} \cap \overline{B} = \{2\}$$

$\overline{A} \cap \overline{B}$ is the event of neither getting an odd number nor a number greater than 2

Mutually Exclusive Events

In an experiment, if the occurrence of an event precludes or rules out the happening of all the other events in the same experiment.

Illustration 10 (i) When a coin is tossed either head or tail will appear. Head and tail cannot appear simultaneously. Therefore, occurrence of a head or a tail are two mutually exclusive events.

(ii) In throwing a die, all the 6 faces numbered 1 to 6 are mutually exclusive since if any one of these faces comes, the possibility of others in the same trial, is ruled out.

Note:

A and B are mutually exclusive events $\Leftrightarrow A \cap B = \phi$, i.e., A and B are disjoint sets.

Illustration 11 (i) If the random experiment is '*a* die is thrown' and A, B are the events, A:the number is less than 3; B:the number is more than 4, then $A = \{1, 2\}$, $B = \{5, 6\}$

$A \cap B = \phi$, thus A and B are mutually exclusive events.

(ii) If the random experiment is '*a* card is drawn from a well-shuffled pack of cards' and A, B are the events A: the card is Black; B: the card is an ace.

Since a black card can be an ace, $A \cap B \neq \phi$, thus A and B are not mutually exclusive events.

Mutually Exclusive and Exhaustive Events

Events $E_1, E_2, ..., E_n$ are called mutually exclusive and exhaustive if $E_1 \cup E_2 \cup ... \cup E_n = S$, i.e., $\bigcup_{i=1}^{n} E_i = S$ and $E_i \cap E_j = \phi$ for all $i \neq j$.

For example, in a single throw of a die, let A be the event of getting an even number and B be event of getting odd numbers, then

$$A = \{2, 4, 6\}, B = \{1, 3, 5\}$$
$$A \cap B = \phi, A \cup B = \{1, 2, 3, 4, 5, 6\} = S$$

$\therefore$ A and B are mutually exclusive and exhaustive events.

Illustration 12 Two dices are thrown and the sum of the numbers which come up on the dice noted. Let us consider the following events:

A : 'the sum is even'

B : 'the sum is a multiple of 3'

C : 'the sum is less than 4'

D : 'the sum is greater than 11'

Which pairs of these events are mutually exclusive?

Solution: There are $6 \times 6 = 36$ elements in the sample space (Refer to Example 2).

A is the event "the sum is even". It means we have to consider those ordered pairs (x, y) in which $x + y$ is even. Thus,

$A = [(1, 1), (2, 2) (1, 3), (1, 5), (2, 4), (2, 6), (3, 1),$
$(3, 3), (3, 5), (4, 2), (4, 4), (4, 6)$
$(5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)].$

Similarly,

$B = [(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2),$
$(3, 6), (6, 3), (4, 5), (5, 4), (6, 6)]$

$C = [(1, 1), (2, 1), (1, 2)]$ $D = [(6, 6)].$

We find that $A \cap B = [(1, 5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 6)] \neq \phi$

Thus, A and B are not mutually exclusive.

Similarly, $A \cap C \neq \phi$, $A \cap D \neq \phi$, $B \cap C \neq \phi$, $B \cap D \neq \phi$, $C \cap D = \phi$. Thus, C and D are mutually exclusive.

PROBABILITY OF AN EVENT

The probability of an event is defined in the following two ways:

(i) Mathematical (or *a priori*) definition

(ii) Statistical (or empirical) definition

Mathematical Definition of Probability: Probability of an event A, denoted as $P(A)$, is defined as

$$P(A) = \frac{\text{Number of cases favourable to } A}{\text{Number of possible outcomes}}$$

Thus, if an event A can happen in m ways and fails (does not happen) in n ways and each of $m + n$ ways is equally likely to occur then the probability of happening of the event A (also called success of A) is given by

$$P(A) = \frac{m}{m+n}$$

and that the probability of non-occurrence of the A (also called its failure) is given by

$$P\text{ (not } A) \text{ or } P(\bar{A}) = \frac{n}{m+n}$$

If the probability of the happening of a certain event is denoted by p and that of not happening by q, then

$$p + q = \frac{m}{m+n} + \frac{n}{m+n} = 1$$

Here, p, q are non-negative and cannot exceed unity, i.e., $0 \leq p \leq 1$ and $0 \leq q \leq 1$

When, $p = 1$, then the event is certain to occur.

When, $p = 0$, then the event is impossible. For example, the probability of throwing eight with a single die is zero.

Probability as defined above is sometimes called **Priori Probability,** i.e., it is determined before hand, that is, before the actual trials are made.

Illustration 13 A coin is tossed once. What are all possible outcomes? What is the probability of the coin coming up 'tails'?

Solution: The coin can come up either "heads" (H) or "tails" (T). Thus, the set S of all possible outcomes is $S = \{H, T\}$

$$\therefore \qquad P(T) = \frac{1}{2}$$

Illustration 14 What is the probability of getting an even number in a single throw of a die?

Solution: Clearly, a die can fall with any of its faces upper most. The number on each of the faces is, therefore, a possible outcome. Thus, there are total 6 outcomes. Since there are 3 even numbers on the die, namely, 2, 4 and 6,

$$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$$

Illustration 15 What is the probability of drawing a 'king' from a well-shuffled deck of 52 cards?

Solution: Well-shuffled ensures equally likely outcomes. There are 4 kings in a deck. Thus,

$$P(\text{a king}) = \frac{4}{52} - \frac{1}{13}$$

Odds of an Event

Suppose, there are m outcomes favourable to a certain event and n outcomes unfavourable to the event in a sample space, then

odds in favour of the event

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}} = \frac{m}{n}$$

and odds against the event

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcome}} = \frac{n}{m}.$$

If odds in favour of an event A are $a:b$, then the probability of happening of event $A = P(A) = \frac{a}{a+b}$ and probability of not happening of event $A = P(\bar{A}) = \frac{b}{a+b}$.

If odds against happening of an event A are $a:b$, then probability of happening of event $A = P(A) = \frac{b}{a+b}$ and probability of not happening of event

$$A = P(\bar{A}) = \frac{a}{a+b}.$$

Illustration 16 What are the odds in favour of getting a '3' in a throw of a die? What are the odds against getting a '3'?

Solution: There is only one outcome favourable to the event "getting" a 3, the other five outcomes, namely, 1, 2, 4, 5, 6 are unfavourable. Thus,

Odds in favour of getting a '3'

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of unfavourable outcomes}}$$

$$= \frac{1}{5} \text{ or, 1 to 5}$$

Odd against getting a '3'

$$= \frac{\text{Number of unfavourable outcomes}}{\text{Number of favourable outcomes}}$$

$$= \frac{5}{1} \text{ or, 5 to 1.}$$

Illustration 17 If the odds in favour of an event are 4 to 5, find the probability that it will occur.

Solution: The odds in favour of the event are $\frac{4}{5}$. Thus,

$$\frac{P(A)}{1-P(A)} = \frac{4}{5}, \text{ i.e., } 4[1-P(A)] = 5P(A),$$

i.e., $$P(A) = \frac{4}{9}$$

The probability that it will occur $= \frac{4}{9}$.

FUNDAMENTAL THEOREMS ON PROBABILITY

Theorem 1 In a random experiment, if S is the sample space and E is an event, then

(i) $P(E) \geq 0$ (ii) $P(\phi) = 0$ (iii) $P(S) = 1$.

Remarks: It follows from above results that

(i) probability of occurrence of an event is always non-negative;

(ii) probability of occurrence of an impossible event is 0;

(iii) probability of occurrence of a sure event is 1.

Theorem 2 If E and F are mutually exclusive events, then

(i) $P(E \cap F) = 0$ and

(ii) $P(E \cup F) = P(E) + P(F)$.

Notes:

1. For mutually exclusive events E and F, we have

$$P(E \text{ or } F) = P(E \cup F) = P(E) + P(F).$$

2. If $E_1, E_2, \ldots, E_k$ are mutually exclusive events, then

$$P(E_1 \cup E_1 \cup, \ldots, \cup E_k) = P(E_1) + P(E_2) + \ldots + P(E_k).$$

Theorem 3 If E and F are two mutually exclusive and exhaustive events, then $P(E) + P(F) = 1$.

Theorem 4 Let E be any event and be its complementary event, then $= 1 - P(E)$.

Theorem 5 For any two events E and F,

$$P(E - F) = P(E) - P(E \cap F).$$

Theorem 6 **(Addition Theorem).** For any two events E and F,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Notes:

1. We may express the above results as
$$P\,(E \text{ or } F) = P\,(E) + (F) - P\,(E \text{ and } F)$$
2. If E and F are mutually exclusive, then
$P(E \cap F) = 0$ and so $P(E \cup F) = P(E) + P(F)$.

Theorem 7 If E_1 and E_2 be two events such that $E_1 \subseteq E_2$, then prove that $P(E_1) \le P(E_2)$.

Theorem 8 If E is an event associated with a random experiment, then $0 \le P\,(E) \le 1$.

Theorem 9. For any three events E, F, G

$$P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(E \cap F) - P(F \cap G) - P(E \cap G) + P(E \cap F \cap G)$$

Illustration 18 *A* card is drawn at random from a well-shuffled pack of 52 cards. Find the probability of getting

(i) a jack or a queen or a king.

(ii) a two of heart or diamond.

Solution: (i) In a pack of 52 cards, we have

4 jacks, 4 queens and 4 kings.

Now, clearly a jack and a queen and a king are mutually exclusive events.

Also, $P(\text{a jack}) = \dfrac{{}^4C_1}{{}^{52}C_1} = \dfrac{4}{52} = \dfrac{1}{13}$

$$\text{P(a queen)} = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

$$\text{P(a king)} = \frac{{}^4C_1}{{}^{52}C_1} = \frac{4}{52} = \frac{1}{13}$$

$\therefore$ By the addition theorem of Probability,

$P(\text{a jack or a queen or a king})$

$$= P(\text{a jack}) + P(\text{a queen}) + P(\text{a king})$$

$$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13} = \frac{3}{13}$$

(ii) $P(\text{two of heart or two of diamond})$

$$= P(\text{two of heart}) + P(\text{two of diamond})$$

$$= \frac{1}{52} + \frac{1}{52} = \frac{2}{52} = \frac{1}{26}$$

Illustration 19 Find the probability of getting a sum of 7 or 11 in a simultaneous throw of two dice.

Solution: When two dice are thrown we have observed that there are 36 possible outcomes. Now, we can have a sum of 7 as

$$1 + 6 = 7,\ 2 + 5 = 7,\ 3 + 4 = 7,\ 4 + 3 = 7,\ 5 + 2 = 7,\ 6 + 1 = 7$$

Thus, the six favourable cases are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

$$\therefore \quad P(\text{a sum of 7}) = \frac{6}{36} = \frac{1}{6}$$

Again, the favourable cases of getting a sum of 11 are (5, 6), (6, 5)

$$\therefore \quad P(\text{a sum of 11}) = \frac{2}{36} = \frac{1}{18}$$

Since the events of getting 'a sum of 7' or 'a sum of 11' are mutually exclusive:

$\therefore P(\text{a sum of 7 or 11})$

$$= P(\text{a sum of 7}) + P(\text{a sum of 11})$$

$$= \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9}.$$

Illustration 20 From a well-shuffled pack of 52 cards, a card is drawn at random, find the probability that it is either a heart or a queen.

Solution: A:Getting a heart card $\quad B$:Getting a queen card

$$P(A) = \frac{13}{52},\ P(B) = \frac{4}{52},\ P(A \cap B) = \frac{1}{52}$$

Required probability $= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

INDEPENDENT EVENTS

Two event A and B are said to be independent if the occurrence (or non-occurrence of one does not affect the probability of the occurrence (and hence non-occurrence) of the other.

Illustration 21 In the simultaneous throw of two coins, 'getting a head' on first coin and 'getting a tail on the second coin are independent events.

Illustration 22 When a card is drawn from a pack of well-shuffled cards and replaced before the second card is drawn, the result of second draw is independent of first draw. We now state, without proof, the theorem which gives the probabilities of simultaneous occurrence of the independent events.

Theorem 10 If A and B are two independent events, then

$$P(A \text{ and } B) = P(A).\ P(B)$$

Illustration 23 Two dice are thrown. Find the probability of getting an odd number on the one die and a multiple of three on the other.

Solution: Since the events of 'getting an odd number' on one die and the event of getting a multiple of three on the other are independent events,

$$P(A \text{ and } B) = P(A) \times P(B) \qquad (1)$$

Now, $P(A) = P(\text{an odd number}) = \frac{3}{6} = \frac{1}{2}$ [There are three odd numbers 1, 3, 5] and $P(B) = P(\text{a multiple of } 3) = \frac{2}{6} = \frac{1}{3}$ [Multiples of 3 are 3 and 6]

$\therefore$ From (1), required probability $= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Illustration 24 Arun and Tarun appear for an interview for two vacancies. The probability of Arun's selection is one-third and that of Tarun's selection is one-fifth. Find the probability that

(i) only one of them will be selected.

(ii) none of them be selected.

Solution: Let A:Arun is s selected B:Tarun is selected.

Then, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$

Clearly, 'A' and 'not B' are independent also 'not A' and 'not B' are independent, 'B' and 'not A' are independent.

(i) P (only one of them will be selected)

$$= P(A \text{ and not } B \text{ or, } B \text{ and not } A)$$

$$= P(A)\,P(\text{not } B) + P(B)\,P(\text{not } A)$$

$$= \frac{1}{3}\left(1-\frac{1}{5}\right)+\frac{1}{5}\left(1-\frac{1}{3}\right)$$

$$= \frac{1}{3}\times\frac{4}{5}\times\frac{1}{5}\times\frac{2}{3}=\frac{4}{15}+\frac{2}{12}$$

$$= \frac{6}{15} = \frac{2}{5}$$

(ii) P (only one of them be selected)

$$= P(\text{not } A \text{ and not } B)$$

$$= P(\text{not } A) \times P(\text{not } B)$$

$$= \left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{5}\right)$$

$$= \frac{2}{3}\times\frac{4}{5} = \frac{8}{15}.$$

Practice Exercises

Difficulty Level-1 (Based on Memory)

1. Suppose six coins are flipped. Then the probability of getting at least one tail is:

(a) $\frac{71}{72}$ (b) $\frac{53}{54}$

(c) $\frac{63}{64}$ (d) $\frac{1}{12}$

[Based on MAT, 2002]

2. If events A and B are independent and $P(A) = 0.15$, $P(A \cup B) = 0.45$, then, $P(B) =$

(a) $\frac{6}{13}$ (b) $\frac{6}{17}$

(c) $\frac{6}{19}$ (d) $\frac{6}{23}$

[Based on MAT, 2005]

3. One hundred identical coins each with probability p of showing up heads are tossed. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads on 51 coins; then the value of p is:

(a) $\frac{1}{2}$ (b) $\frac{49}{101}$

(c) $\frac{50}{101}$ (d) $\frac{51}{101}$

[Based on MAT, 1999]

4. The probability that a marksman will hit a target is given as one-fifth. Then, his probability of atleast one hit in 10 shots is:

(a) $\frac{1}{6^{10}}$ (b) $1-\left(\frac{4}{5}\right)^{10}$

(c) $1-\frac{1}{5^{10}}$ (d) $1-\frac{1}{5^{19}}$

[Based on MAT, 2005]

5. Two dice are tossed. The probability that the total score is a prime number is:

(*a*) $\frac{1}{6}$ (*b*) $\frac{5}{12}$

(*c*) $\frac{1}{2}$ (*d*) $\frac{7}{9}$

[Based on MAT, 2000]

6. Four different objects 1, 2, 3, 4 are distributed at random in four places marked 1, 2, 3, 4. What is the probability that none of the objects occupy the place corresponding to its number?

(*a*) 17/24 (*b*) 3/8

(*c*) 1/2 (*d*) 5/8

[Based on MAT, 2001]

7. Three students try to solve a problem independently with a probability of solving it as 1/3, 2/5, 5/12, respectively. What is the probability that the problem is solved?

(*a*) 1/18 (*b*) 12/30

(*c*) 23/30 (*d*) 1/2

[Based on IIT Joint Man. Ent. Test, 2004]

8. If the probability of rain on any given day in Pune city is 50% then what is the probability that it rains on exactly 3 days in a five-day period?

(*a*) 8/125 (*b*) 5/16

(*c*) 8/25 (*d*) 2/25

[Based on SCMHRD Ent. Exam., 2003]

9. A Chartered Accountant applies for a job in two firms *X* and *Y*. The probability of his being selected in firm *X* is 0.7, and being rejected at *Y* is 0.5 and the probability of his application being rejected is 0.6. What is the probability that he will be selected in one of the firm?

(*a*) 0.8 (*b*) 0.2

(*c*) 0.4 (*d*) 0.7

[Based on MAT, 2008]

10. There are two boxes *A* and *B*. Box *A* has 5 oranges and 6 apples in it and box *B* contains 3 apples and 4 oranges in it. A fruit is taken from *A* and placed in *B*, after which a fruit is then transfered from *B* to *A*. What is the probability that the configuration of boxes does not change due to the transfers?

(*a*) 49/88 (*b*) 45/88

(*c*) 25/44 (*d*) None of these

[Based on MAT (Feb), 2011]

11. Eight chits are numbered 1 to 8. Three are drawn one by one with replacement. The probability that the least number on any selected chit is 6, is:

(*a*) $1-\left(\frac{3}{4}\right)^3$ (*b*) $\left(\frac{3}{4}\right)^3$

(*c*) $\left(\frac{3}{8}\right)^3$ (*d*) $1-\left(\frac{3}{8}\right)^3$

[Based on MAT (Feb), 2011]

12. Two events *A* and *B* have probabilities 0.25 and 0.50 respectively. The probability that both *A* and *B* occur simultaneously is 0.12. Then, the probability that neither *A* nor *B* occurs is:

(*a*) 0.38 (*b*) 0.13

(*c*) 0.63 (*d*) 0.37

[Based on MAT (Feb), 2011]

13. *A* has 3 shares in a lottery containing 3 prizes and 9 blanks. *B* has two shares in a lottery containing 2 prizes and 6 blanks. Compare their chances of success.

(*a*) 145:362 (*b*) 952:715

(*c*) 123:213 (*d*) 145:716

[Based on MAT (Feb), 2011]

14. A classroom has 3 electric lamps. From a collection of 10 electric bulbs of which 6 are good, 3 are selected at random and put in the lamps. Find the probability that all lamps are burning.

(*a*) 1/8 (*b*) 1/4

(*c*) 1/2 (*d*) 1/6

[Based on MAT (Dec), 2010]

15. The odds in favour of *A* winning a game of badminton against *B* are 5:2 If three games are to be played, what are the odds in favour of *A*'s winning aleast one game?

(*a*) 425:5 (*b*) 365:1

(*c*) 335:8 (*d*) None of these

[Based on MAT (Dec), 2010]

16. *A* and *B* play a game where each is asked to select a number from 1 to 16. If the two numbers match, both of them win a prize. Find the probability that they will not win a prize in a single trial.

(*a*) 15/16 (*b*) 12/17

(*c*) 14/15 (*d*) 12/13

[Based on MAT (Dec), 2010]

17. The probability that a computer company will get a computer hardware contract is 2/3 and the probability that it will not get a software contract is 5/9. If the probability of getting atleast one contract is 4/5, what is the probability that it will get both the contracts?

(*a*) 14/45 (*b*) 17/45

(*c*) 16/45 (*d*) 11/45

[Based on MAT (Dec), 2010]

18. *A* speaks truth in 75% of cases and *B* in 80% of cases. *A* and *B* agree in a statement .What is the probability that the statement is true?

(*a*) 12/13 (*b*) 11/15

(*c*) 14/17 (*d*) 17/29

[Based on MAT (Sept), 2010]

19. There are 10 persons, including *A* and *B* who stand in the form of a circle. If the arrangement of the persons is at random, then the probability that there are exactly 3 persons between A and B is:

(*a*) 1/9 (*b*) 7/9

(*c*) 2/9 (*d*) 1/3

[Based on MAT (May), 2010]

20. The odds in favour of A winning a game against B is 4:3. If three games are to be played to decide the overall winner, the odds in favour of A winning atleast once is:

(*a*) 343:27 (*b*) 316:27

(*c*) 343:316 (*d*) None of these

[Based on MAT (May), 2010]

21. Cards each marked with the numbers 1, 2, 3, ..., 10 are placed in a box and mixed throughly. One card is drawn at random from the box. Find the probability of getting a prime number.

(*a*) 2/5 (*b*) 4/9

(*c*) 5/9 (*d*) 1/6

[Based on MAT (May), 2010]

22. Namrata want to visit four cities A, B, C and D on an official trip. The probability that she visits A just before B is:

(*a*) 1/2 (*b*) 1/12

(*c*) 1/6 (*d*) 1/4

[Based on MAT (Feb), 2010]

23. A lot of 12 bulbs contains 4 defective bulbs. Three bulbs are drawn at random from the lot, one after the other. The probability that all three are non defective is:

(*a*) 14/55 (*b*) 8/12

(*c*) 1/27 (*d*) None of these

[Based on MAT (Feb), 2010]

24. Three riflemen take one shot each at the same target. The probability of the first rifleman hitting the target is 0.4, the probability of the second rifleman hitting the target is 0.5 and the probability of the third rifleman hitting the target is 0.8. The probability that exactly two of them hit the target, is:

(*a*) 0.54 (*b*) 0.44

(*c*) 0.32 (*d*) 0.52

[Based on MAT (Dec), 2009]

25. 100 students appeared for two examinations. 60 passed the first, 50 passed the second and 30 passed both. The probability that a student selected at random has failed in both examinations is:

(*a*) 0.3 (*b*) 0.2

(*c*) 0.4 (*d*) 0.1

[Based on MAT (Dec), 2009]

26. A and B stand in a ring with 11 other persons. If the arrangement of the 13 persons is at random, then the probability that there are exactly 3 persons between A and B is:

(*a*) 3/12 (*b*) 2/11

(*c*) 1/6 (*d*) 4/9

[Based on MAT (Dec), 2009]

27. There are 100 students in a college class of which 36 are boys studying Statistics and 13 girls are not studying Statistics. If there are 55 girls in all, the probability that a boy picked up at random is not studying Statistics is:

(*a*) 1/5 (*b*) 2/5

(*c*) 3/5 (*d*) 4/5

[Based on MAT (Dec), 2009]

28. A can hit a target 4 times in 5 shots, B hits 3 times in 4 shots and C hits thrice in 3 shots, they fire together. Find the probability that atleast two shots hit the target.

(*a*) 13/30 (*b*) 5/6

(*c*) 11/40 (*d*) None of these

[Based on MAT (Sept), 2009]

29. The odds that A speaks the truth are 3:2 and the odds that B speaks the truth are 5:3. In what per cent of cases are they likely to agree each other on an identical point?

(*a*) 47.5% (*b*) 37.5%

(*c*) 63.5% (*d*) None of these

[Based on MAT (Sept), 2009]

30. In a class of 25 students with Roll No. 1 to 25. A student is picked up at random to answer a question. Find the probability that the roll number of the selected student is either multiple of 5 or 7.

(*a*) 6/25 (*b*) 4/25

(*c*) 8/25 (*d*) 7/25

[Based on MAT (Sept), 2009]

31. A bag contains 5 white and 3 black balls and 4 are successively drawn out and not replaced; what is the probability they are alternately of different colours?

(*a*) $\frac{1}{7}$ (*b*) $\frac{1}{14}$

(*c*) $\frac{3}{14}$ (*d*) $\frac{2}{7}$

[Based on MAT (May), 2009]

32. The probabilities that three man hit a target are $\frac{1}{6}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. If only one hits the target, what is the probability that it was the first man?

(*a*) $\frac{11}{57}$ (*b*) $\frac{21}{57}$

(*c*) $\frac{12}{67}$ (*d*) $\frac{2}{9}$

[Based on MAT (May), 2009]

33. A bag contains 100 tickets numbered 1, 2, 3, ..., 100. If a ticket is drawn out of it at random, what is the probability that the ticket drawn has the digit 2 appearing on it?

(*a*) 21/100 (*b*) 19/100
(*c*) 32/100 (*d*) 23/100

[Based on MAT (Feb), 2009]

34. A box contains 5 brown and 4 white socks. A man takes out two socks. The probability that they are of the same colour is:

(*a*) 1/6 (*b*) 5/108
(*c*) 5/18 (*d*) 4/9

[Based on MAT (Feb), 2009]

35. A fair coin is tossed repeatedly. If head appears on the first four tosses, then the probability of appearance of tail on the fifth toss is:

(*a*) 1/2 (*b*) 1/7
(*c*) 3/7 (*d*) 2/3

[Based on MAT (Feb), 2009]

36. A car is parked by an owner amongst 25 cars in a row, not at either end. On his return, he finds that exactly 15 places are still occupied. The probability that both the neighbouring parking places are empty is:

(*a*) 91/276 (*b*) 15/164
(*c*) 15/92 (*d*) 91/164

[Based on MAT (May), 2008]

37. A committee consists of 9 experts taken from three institutions *A*, *B* and *C* of which 2 are from *A*, 3 from *B* and 4 from *C*. If three experts resign, then the probability that they belong to different institutions is:

(*a*) 1/729 (*b*) 1/24
(*c*) 1/21 (*d*) 2/7

[Based on MAT (May), 2008]

38. A Chartered Accountant applied for a job in two firms *X* and *Y*. The ability of his being selected in firm *X* is 0.7 and being rejected at *Y* is 0.5 and the probability of atleast one of his applications being rejected is 0.6. What is the probability that he will be selected atleast one of the firms?

(*a*) 0.8 (*b*) 0.2
(*c*) 0.4 (*d*) 0.7

[Based on MAT (May), 2008]

39. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting atleast one contract is 4/5, what is the probability that he will get both?

(*a*) 31/45 (*b*) 8/45
(*c*) 14/45 (*d*) None of these

[Based on MAT (May), 2008]

40. A fair coin is tossed a fixed number of times. If the probability of getting 4 heads equals the probability of getting 7 heads, then the probability of getting 2 heads is:

(*a*) 1/1024 (*b*) 55/2048
(*c*) 3/4096 (*d*) None of these

[Based on MAT (Feb), 2008]

41. An ordinary cube has four blank faces, one face marked 2 and another marked 3, then the probability of obtained 12 in 5 throws is:

(*a*) 5/1944 (*b*) 5/1296
(*c*) 5/2592 (*d*) None of these

[Based on MAT (Feb), 2008]

42. There are three events *A*, *B* and *C*, one of which must and only one can happen. The odds are 8 to 3 against *A*, 5 to 2 against *B*. Find the odds against *C*.

(*a*) 43:34 (*b*) 43:77
(*c*) 34:43 (*d*) 77:43

[Based on MAT (Dec), 2007]

43. A card is drawn at random from a well-shuffled pack of 52 cards. What is the probability of getting a two of hearts or a two of diamonds?

(*a*) $\frac{3}{26}$ (*b*) $\frac{2}{17}$
(*c*) $\frac{1}{26}$ (*d*) $\frac{4}{13}$

[Based on MAT (Dec), 2007]

44. An instrument manufactured by a company consists of two parts *A* and *B* in manufacturing part *A*, 9 out of 100 are likely to be defective and in manufacturing part *B*, 5 out of 100 are likely to be defective. Calculate the probability that both the instruments will not be defective.

(*a*) 0.91 (*b*) 0.86
(*c*) 0.95 (*d*) 0.83

[Based on MAT (Dec), 2007]

45. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting points 0, 1, 2 are 0.45, 0.05 and 0.50 respectively. Assuming that outcomes are independent the probability of India getting atleast 7 points is:

(*a*) 0.8750 (*b*) 0.06875
(*c*) 0.0875 (*d*) 0.0250

[Based on MAT (Dec), 2007]

46. Three persons work independently on a problem. If the respective probabilities that they will solve it are one-third, one-fourth and one-fifth, then the probability that none can solve it is:

(*a*) 1/5 (*b*) 1/3
(*c*) 2/5 (*d*) None of these

[Based on MAT (Dec), 2007]

47. A class consists of 100 students; 25 of them are girls and 75 boys; 20 of them are rich and the remaining poor; 40 of them are fair-complexioned. The probability of selecting a fair complexioned rich girl is:

(*a*) 0.05 (*b*) 0.04

(*c*) 0.02 (*d*) 0.08

[Based on MAT (May), 2007]

48. Four boys and three girls stand in queue for an interview. The probability that they will stand in alternate positions is:

(*a*) 1/34 (*b*) 1/35

(*c*) 1/17 (*d*) 1/68

[Based on MAT (Dec), 2006]

49. *A* and *B* play a game where each is asked to select a number from 1 to 5. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is:

(*a*) 1/25 (*b*) 24/25

(*c*) 2/25 (*d*) None of these

[Based on MAT (Dec), 2006]

50. Three groups *A*, *B* and *C* are contesting for a position in the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3 and 0.2 respectively. If the group *A* wins, then the probability of introducing a new product is 0.7 and the corresponding probabilities for group *B* and *C* are 0.6 and 0.5 respectively. The probability that the new product will be introduced is:

(*a*) 0.52 (*b*) 0.74

(*c*) 0.63 (*d*) None of these

[Based on MAT (May), 2006]

51. An article manufactured by a company consists of two parts *A* and *B*. In the process of manufacture of part *A*, 9 out of 100 are likely to be defective.

Similarly, 5 out of 100 are likely to be defective in the process of manufacture of part *B*. The probability that the assembled part will not be defective is:

(*a*) 0.8645 (*b*) 0.9645

(*c*) 0.6243 (*d*) None of these

[Based on MAT (May), 2006]

52. A programmer noted the results of attempting to run 20 programs. The results showed that 2 programs ran correctly in the first attempt, 7 ran correctly in the second attempt, 5 ran correctly in the third attempt, 4 ran correctly in the fourth attempt and 2 ran correctly in the fifth attempt. What is the probability that his next program will run correctly on the third run?

(*a*) 1/4 (*b*) 1/3

(*c*) 1/6 (*d*) 1/5

[Based on MAT, 1997]

53. With the same data as for the previous question, what is the probability that the next program will run correctly after the third run but not earlier?

(*a*) 9/10 (*b*) 3/10

(*c*) 7/20 (*d*) 1/10

[Based on MAT, 1997]

54. Course materials are sent to students by a distance teaching institution. The probability that they will send a wrong programme's study material is 1/5. There is a probability of 3/4 that the package is damaged in transit, and there is a probability of 1/3 that there is a short shipment. What is the probability that the complete material for the course arrives without any damage in transit?

(*a*) 4/5 (*b*) 8/60

(*c*) 8/15 (*d*) 4/20

[Based on MAT, 1997]

55. With the same data as for the previous question, what is the probability that the same student on two successive occasions gets the wrong study material?

(*a*) 1/25 (*b*) 1/5

(*c*) 4/25 (*d*) 3/25

[Based on MAT, 1997]

56. What is the probability of getting a sum as 6 when two dice are thrown simultaneously?

(*a*) $\frac{5}{36}$ (*b*) $\frac{35}{66}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{3}{8}$

[Based on MAT, 1998]

57. Eight coins are tossed simultaneously. The probability of getting at least 6 heads is:

(*a*) $\frac{1}{13}$ (*b*) $\frac{37}{256}$

(*c*) $\frac{25}{57}$ (*d*) None of these

[Based on MAT, 1998]

58. Two cards are drawn together from a pack of 52 playing cards at random. What is the probability that both are kings?

(*a*) $\frac{1}{13}$ (*b*) $\frac{25}{57}$

(*c*) $\frac{37}{257}$ (*d*) None of these

[Based on MAT, 1998]

59. Three light bulbs are selected at random from 20 bulbs of which 5 are defective. The probability that none of the bulbs that is picked up is defective is:

(a) $\frac{4}{7}$ (b) $\frac{140}{228}$

(c) $\frac{137}{228}$ (d) $\frac{91}{228}$

[Based on MAT, 1998]

60. Two cards are drawn together from a pack of 52 cards (*a* set of traditional playing cards) at random. The probability that one is a spade and the other is a heart is:

(*a*) $\frac{13}{102}$ (*b*) $\frac{3}{20}$

(*c*) $\frac{47}{100}$ (*d*) $\frac{29}{34}$

[Based on MAT, 1998]

61. A bag has 4 red and 5 black balls. A second bag has 3 red and 7 black balls. One ball is drawn from the first bag and two from the second. The probability that there are two black balls and a red ball is:

(*a*) $\frac{14}{45}$ (*b*) $\frac{11}{45}$

(*c*) $\frac{7}{15}$ (*d*) $\frac{9}{54}$

[Based on MAT, 1998]

62. Three boxes contain 6 red, 4 black; 4 red, 6 black and 5 red, 5 black balls respectively. One of these boxes is selected at random and a ball is drawn from it. If the ball drawn is red, then the probability that it is drawn from the first box is:

(*a*) $\frac{3}{4}$ (*b*) $\frac{27}{83}$

(*c*) $\frac{15}{59}$ (*d*) $\frac{2}{5}$

[Based on MAT, 1998]

63. A dice is thrown 6 times. If 'getting an odd number' is 'success'. The probability of 5 successes is:

(*a*) $\frac{1}{10}$ (*b*) $\frac{3}{32}$

(*c*) $\frac{5}{6}$ (*d*) $\frac{25}{36}$

[Based on MAT, 1998]

64. In the West Indies, there is a 3-match one-day International tournament between West Indies and India. At the end of every match, either a team wins or loses. There is no draw. Find the probability that India wins the series by winning at least 2 consecutive matches.

(*a*) 1 (*b*) $\frac{5}{8}$

(*c*) $\frac{3}{8}$ (*d*) $\frac{1}{2}$

65. A coin is tossed 5 times. What is the probability that head appears an odd number of times?

(*a*) 2/5 (*b*) 1/5

(*c*) 1/2 (*d*) 4/25

[Based on MAT, 1998]

66. A bag contains 5 white, 7 red and 8 black balls. If 4 balls are drawn one-by-one with replacement, what is the probability that all are white?

(*a*) 1/256 (*b*) 1/16

(*c*) 4/20 (*d*) 4/8

[Based on MAT, 1998]

67. Atal can hit a target 3 times in 6 shots, Bhola can hit the target 2 times in 6 shots and Chandra can hit the target 4 times in 4 shots. What is the probability that at least 2 shots hit the target?

(*a*) 1/2 (*b*) 2/3

(*c*) 1/3 (*d*) 11/18

[Based on MAT, 1998]

68. A bag contains 3 white balls and 2 black balls. Another bag contains 2 white balls and 4 black balls. A bag and a ball are picked at random. The probability that the ball will be white is:

(*a*) $\frac{7}{11}$ (*b*) $\frac{7}{30}$ (*c*) $\frac{5}{11}$ (*d*) $\frac{7}{15}$

[Based on MAT, 1999]

69. There are 6 positive and 8 negative numbers. Four numbers are chosen at random and multiplied. The probability that the product is a positive number is:

(*a*) $\frac{500}{1001}$ (*b*) $\frac{503}{1001}$

(*c*) $\frac{505}{1001}$ (*d*) $\frac{101}{1001}$

[Based on MAT, 1999]

70. One hundred identical coins each with probability p of showing up heads are tossed. If $0 < p < 1$ and the probability of heads on 50 coins is equal to that of heads on 51 coins, then the value of p is:

(*a*) $\frac{1}{2}$ (*b*) $\frac{49}{101}$

(*c*) $\frac{50}{101}$ (*d*) $\frac{51}{101}$

[Based on MAT, 1999]

71. The odds against certain event are 5:2 and the odds favour of another independent event are 6:5. The probability that at least one of the event will happen is:

(*a*) $\frac{12}{77}$ (*b*) $\frac{25}{77}$

(*c*) $\frac{52}{77}$ (*d*) $\frac{65}{77}$

[Based on IIFT, 2005]

72. Three letters are drawn from the alphabet of 26 letters without replacement. The probability that they appear in alphabetical order is:

(a) ${}^{3}C_{1}/{}^{26}C_{3}$ (b) $1/{}^{26}C_{3}$

(c) 1/3 (d) 1/6

[Based on IIFT, 2005]

73. A and B are independent events such that $p(A) = p(A \cup B) = 0.4$. Then, $p(B)$ equals:

(a) $\frac{1}{7}$ (b) 0.7

(c) 0.12 (d) $\frac{1}{8}$

[Based on IIFT, 2005]

74. A letter is taken out at random from ASSISTANT and another taken out from STATISTICS. The probability that they are the same letter is:

(a) 1/45 (b) 13/90

(c) 19/90 (d) None of the above

75. If the chances that the electricity goes off for a particular day is 50% then what is the probability that in a week it will go off exactly for three days?

(a) $\frac{8}{15}$ (b) $\frac{5}{16}$

(c) $\frac{35}{128}$ (d) None of these

76. If the chance that a vessel arrives safely at a port is 9/10 then what is the chance that out of 5 vesels expected at least 4 will arrive safely?

(a) $\frac{(14\times 9^{4})}{10^{5}}$ (b) $\frac{(15\times 9^{5})}{10^{4}}$

(c) $\frac{(14\times 9^{3})}{10^{4}}$ (d) $\frac{(14\times 9^{6})}{10^{5}}$

[Based on JMET, 2011]

77. There are four hotels in a town. If 3 men check into the hotels in a day then what is the probability that each checks into a different hotel?

(a) $\frac{6}{7}$ (b) $\frac{1}{8}$

(c) $\frac{3}{8}$ (d) $\frac{5}{9}$

[Based on JMET, 2011]

78. If 5% of the electric bulbs manufactured by a company are defective then what is the probability that in a sample of 100 bulbs none is defective?

(a) e^{-3} (b) e^{-5}

(c) e^{5} (d) e

[Based on JMET, 2011]

79. The letters B, G, I, N and R are rearranged to form the word 'Bring'. Find its probability.

(a) $\frac{1}{120}$ (b) $\frac{1}{5^{4}}$

(c) $\frac{1}{24}$ (d) $\frac{5}{5}\times 4^{2}$

80. There are three events A, B and C, one of which must and only can happen. If the odds are 8:3 against A, 5:2 against B, the odds against C must be:

(a) 13:7 (b) 3:2

(c) 43:34 (d) 43:77

[Based on ATMA, 2008]

81. A bag contains 4 five rupee coins, 3 two rupee coins and 3 one rupee coins. If 6 coins are drawn from the bag at random. What are the odds in favour of the draw yielding maximum amount?

(a) 1:70 (b) 1:69

(c) 69:70 (d) 70:1

[Based on MAT, 2012]

82. A point is selected at random from the interior of a circle. The probability that the point is closer to the center then the boundary of the circle is:

(a) 3/4 (b) 1/2

(c) 1/4 (d) None of these

[Based on MAT, 2013]

83. In a compitition A, B and C are participating. The probability that A wins is twice that of B, the probability that B wins is twice that of C. The probability that A loses is:

(a) 1/7 (b) 2/7

(c) 4/7 (d) 3/7

[Based on MAT, 2013]

84. It has been found that if A and B play a game 12 times A wins 6 times, B wins 4 times and they draw twice. A and B take part in a series of 3 games. The probability that they will win alternately is:

(a) 2/12 (b) 5/36

(c) 1/6 (d) 3/4

[Based on MAT, 2013]

85. Ten cars are parked in a row, what is the probability that there are exactly five cars between the particular two cars:

(a) $\frac{8\times 5}{10!}$ (b) $\frac{4\times 5!\times 2!}{10!}$

(c) $\frac{8\times 8!}{10}$ (d) $\frac{16\times 8!}{10!}$

[Based on MAT, 2013]

86. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A dice is cast. If the face 1 or 3 turns up, a ball is taken from the first bag and if any other face turns up a ball is chosen from the second bag. The probability of choosing a black ball is:

(*a*) $\frac{11}{21}$ (*b*) $\frac{10}{21}$

(*c*) $\frac{12}{21}$ (*d*) $\frac{9}{21}$

[Based on MAT, 2013]

87. Out of 20 consecutive positive integers, two are chosen at random. The probability that their sum is odd is:

(*a*) $\frac{10}{19}$ (*b*) $\frac{1}{20}$

(*c*) $\frac{19}{20}$ (*d*) $\frac{9}{19}$

[Based on MAT, 2013]

88. A fair coin is tossed repeatedly. If head appears on the first four tosses, then the probability of appearance of tail on the fifth toss is:

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{7}$

(*c*) $\frac{3}{7}$ (*d*) $\frac{2}{3}$

[Based on MAT, 2013]

89. Two fair dices are thrown. Given that, the sum of the dice is less than or equal to 4, find the probability that only one dice shows two.

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{2}$

(*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

[Based on MAT, 2014]

90. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing it until he draws a heart. What is the probability that he has to make 3 trials?

(*a*) $\frac{9}{64}$ (*b*) $\frac{3}{64}$

(*c*) $\frac{5}{64}$ (*d*) $\frac{1}{64}$

[Based on MAT, 2014]

91. The probability that a contractor will get a plumbing contract is 2/3 and the probability that he will not get an electric contract is 5/9. If the probability of getting at least one contract is 4/5. What is the probability that he will get both?

(*a*) 31/45 (*b*) 8/45

(*c*) 14/45 (*d*) None of these

[Based on MAT, 2012]

92. A candidate is selected for interview for three posts. For the first post, there are 5 candidates, for the second there are 8 and for third there are 7. What are the chances for him getting at least one post?

(*a*) $\frac{1}{5}$ (*b*) $\frac{3}{5}$

(*c*) $\frac{2}{5}$ (*d*) $\frac{4}{5}$

[Based on ATMA, 2008]

Difficulty Level-2 (Based on Memory)

1. A subscriber was dialing a telephone number. When he forgot the last 3 digits and remembering that they were different, he dialled at random. What is the probability that he dialled the right number?

(*a*) $\frac{3}{10}$ (*b*) $\frac{3}{720}$

(*c*) $\frac{1}{720}$ (*d*) Cannot be determined

2. To open a lock, a key is taken out of a collection of n keys at random. If the lock is not opened with this key, it is put back into the collection and another key is tried. The process is repeated again and again. It is given that with only one key in the collection, the lock can be opened. The probability that the lock will open in n trials is:

(*a*) $\left(\frac{1}{n}\right)^n$ (*b*) $\left(\frac{n-1}{n}\right)^n$

(*c*) $1-\left(\frac{n-1}{n}\right)^n$ (*d*) $1-\left(\frac{1}{n}\right)^n$

3. Two buckets contain black and white balls. The first has five black and three white and the second one has 4 white and 4 black balls. A ball is picked from the first bucket and put into the second bucket. Next a ball from the second bucket is picked and put into the first bucket. Find the probability of the event that after these actions the composition of first bucket has become that of the second one and the composition of the second bucket has become that of first bucket.

(a) 12/19 (b) 5/18
(c) 4/9 (d) 7/18

4. If n positive integers are taken at random and multiplied together, then the probability that the last digit of the product be 2, 4, 6 or 8 is:

(a) $\frac{4^n + 2^n}{5^n}$ (b) $\frac{4^n \times 2^n}{5^n}$
(c) $\frac{4^n - 2^n}{5^n}$ (d) $\frac{4^n \times 5^n}{2^n}$

5. India plays two matches each with the West Indies and Australia. In any match the probability of India getting points 0, 1 and 2 are 0.45, 0.05 and 0.50, respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is:

(a) 0.8750 (b) 0.0875
(c) 0.0625 (d) 0.0250

6. What is the probability of drawing 2 black balls randomly and in succession from a bag containing 5 black balls and 3 white balls if the first ball drawn is replaced before the second draw is made?

(a) $\frac{25}{64}$ (b) $\frac{23}{64}$
(c) $\frac{25}{56}$ (d) $\frac{23}{56}$

7. The odds against a certain event are 5:2, and the odds in favour of another event independent of the former are 6:5. The chance that at least one of the events will happen is:

(a) $\frac{25}{77}$ (b) $\frac{52}{77}$
(c) 1 (d) None of the above

8. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trials of the experiment are formed. The probability that the event A happens at least once is:

(a) 0.934 (b) 0.784
(c) 0.548 (d) 0.343

[Based on FMS (Delhi), 2002]

9. Thirty days are in September, April, June and November. Some months are of thirty one days. A month is chosen at random. Then its probability of having exactly three days less than maximum of 31 is:

(a) 15/16 (b) 1
(c) 3/48 (d) None of these

[Based on SNAP, 2007]

10. Ram has 3 shares in a lottery in which there are 3 prizes and 6 blanks. Mohan has 1 share in lottery in which there is 1 prize and 2 blanks. What is the ratio of Ram's chance of success to Mohan's chance of success?

(a) 9:14 (b) 16:7
(c) 10:7 (d) 12:11

11. A binary number is made up of 8 digits. Suppose that the probability of an incorrect digit appearing is p and that the errors in different digits are independent of each other. Then the probability of forming an incorrect number is:

(a) p^8 (b) $p/8$
(c) $(1 - p^8)$ (d) $1 - (1 - p)^8$

12. A special lottery is to be held to select a student who will live in the only deluxe room in a hostel. There are 100 years-III, 150 year-II, and 500 years-I students who applied. Each year-III's name is place in the lottery 3 times; each year-II's name, 2 times; and each year-I's name, 1 time. What is the probability that a year-III's name will be chosen?

(a) 1/8 (b) 2/9
(c) 2/7 (d) 3/8

[Based on SNAP, 2007]

13. A man who is firing at a distant target has 10% chance of hitting the target in one shot. The number of times he must fire at the target to have about 50% chance of hitting the target is (given log2 = 0.3010 and log3 = 0.4771):

(a) 11 (b) 9
(c) 7 (d) 5

14. In the above question, if in the 3 matches, one match ends in a draw, then what is the probability that India wins the series by winning 2 consecutive matches?

(a) 1 (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{6}$

15. In a right triangle, the probability of one of the angles to be 60° is. Then what is the probability of atleast one of the angles to be of 30° degrees?

(a) $\frac{1}{4}$ (b) 1
(c) $\frac{1}{2}$ (d) Can't be said

16. A solid cube of side 4 cm is painted on all sides. Then the cube is cut to form, cubes of side 1 cm. If a cube is selected at random, what is the probability that none of its sides are painted?

(a) $\frac{1}{8}$ (b) $\frac{3}{4}$
(c) $\frac{1}{2}$ (d) $\frac{7}{8}$

17. In a factory, each day the expected number of accidents is related to the number of overtime hour by a linear equation. Suppose that on one day there were 1000 overtime hours logged and 8 accidents reported and on another day there were 400 overtime hours logged and 5 accidents. What is the expected number of accidents when no overtime hours are logged?

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

[Based on SNAP, 2007]

18. Thirty days are in September, April, June and November. Some months are of thirty one days. A month is chosen at random. Then its probability of having exactly three days less than maximum of 31 is:

(*a*) 15/16 (*b*) 1

(*c*) 3/48 (*d*) None of these

[Based on SNAP, 2007]

19. If all the angles of a triangle are integers. What is the probability that an isosceles triangle is equilateral?

(*a*) $\frac{1}{59}$ (*b*) $\frac{1}{60}$

(*c*) $\frac{1}{89}$ (*d*) $\frac{1}{90}$

20. A dice is rolled three times and sum of three numbers appearing on the uppermost face is 15. The chance that the first roll was four is:

(*a*) 2/5 (*b*) 1/5

(*c*) 1/6 (*d*) None of these

[Based on SNAP, 2009]

21. A five-digit number is formed by using the digits 1, 2, 3, 4 and 5 without repetitions. What is the probability that the number is divisible by 4?

(*a*) 1/5 (*b*) 5/6

(*c*) 4/5 (*d*) None of these

[Based on SNAP, 2009]

22. A dice is rolled three times and sum of three numbers appearing on the uppermost face is 15. The chance that the first roll was a four is:

(*a*) $\frac{2}{5}$ (*b*) $\frac{1}{5}$

(*c*) $\frac{1}{6}$ (*d*) None of these

[Based on SNAP, 2010]

23. A five-digit number is formed by using the digits 1, 2, 3, 4 and 5 without repetitions. What is the probability that the number is divisible by 4?

(*a*) $\frac{1}{5}$ (*b*) $\frac{5}{6}$

(*c*) $\frac{4}{5}$ (*d*) None of these

[Based on SNAP, 2010]

24. While investigating the case of recent blasts in Delhi, the Delhi Police submitted two evidences E_1 and E_2 suggesting the involvement of a suspect in the crime to a local court. The court wants to decide whether the suspect is guilty (G) on the basis of pieces of evidence E_1 and E_2. Suppose for both the evidences E_1 and E_2 the court determines the probability of guilt $P(G \mid E_1)$ and $P(G \mid E_2)$ to be 0.60 and 0.70, respectively. What is the probability of guilt on the basis of both the evidences E_1 and E_2, i.e., $P(G \mid E_1, E_2)$?

(*a*) 0.42 (*b*) 0.60

(*c*) 0.65 (*d*) 0.78

[Based on FMS, 2009]

25. Three of the men participating in a race are A, B and C. The probability of A winning the race is $\frac{1}{6}$, the probability for B winning the race is $\frac{2}{5}$ and for C is $\frac{4}{13}$ if dead heat among them is possible, what is the probability that only one of the three winning the race?

(*a*) $\frac{177}{380}$ (*b*) $\frac{59}{130}$

(*c*) $\frac{39}{157}$ (*d*) $\frac{134}{390}$

26. In a pizza stall, Ajay and Mohan, being the lucky customers, were given the option of drawing tickets from a pot containing x number of tickets for the knife throwing show and y number of tickets for the talking doll show. Both Ajay and Mohan being excited about the knife throwing show, start drawing tickets from the pot until they get one for the show, replacing any drawn ticket for the talking dolls show in the pot. Ajay draws the ticket first, followed by Mohan. Given this, mark all the correct options.

(*a*) If the probability of Ajay first getting a ticket for the knife throwing show is four times that for Mohan, the ratio between x and y is 3:1

(*b*) If the probability of Ajay first getting a ticket for the knife throwing show is five times that for Mohan, the ratio between y and x is 1:4

(*c*) If the probability of Ajay first getting a ticket for the knife throwing show is two times that for Mohan, the ratio between x and y is 1:1

(*d*) If the probability of Mohan first getting a ticket for the knife throwing show is six times that for Ajay, the ratio between x and y is 5:1

[Based on ITFT, 2006]

27. A medical clinic tests blood for certain disease from which approximately one person in a hundred suffers. People come to the clinic in group of 50. The operator of the clinic wonders whether he can increase the efficiency of the testing procedure by conducting pooled tests. In the pooled test, the operator would pool the 50 blood samples and test them altogether. If the polled test was negative, he could pronounce the whole group healthy. If not, he could then test each person's blood individually. The expected number of tests the operator will have to perform if he pools the blood samples are:

(*a*) 47 (*b*) 25

(*c*) 21 (*d*) None of these

[Based on ITFT, 2008]

28. The game of 'chuck-a-luck' is played at carnivals in some parts of Europe. Its rules are as follows:if you pick a number from 1 to 6 and the operator rolls three dice. If the number you picked comes up on all three dice, the operator pays you € 3; if it comes up on two dice, you are paid € 2; and if it comes up on just one die, you are paid € 1. Only if the number you picked does not comes up at all, you pay the operator € 1. The probability that you will win money playing in this game is:

(*a*) 0.52 (*b*) 0.753

(*c*) 0.42 (*d*) None of these

[Based on ITFT, 2008]

29. Events *X*, *Y* and *Z* are mutually exclusive events such that $P(X) = \frac{3a+1}{3}$, $P(Y) = \frac{1-a}{4}$ and $P(Z) = \frac{1-2a}{2}$. The set of possible values of *a* are in the interval:

(*a*) $\left[\frac{1}{3}, \frac{2}{3}\right]$

(*b*) $\left[\frac{1}{2}, \frac{2}{3}\right]$

(*c*) $\left[\frac{1}{3}, \frac{1}{2}\right]$

(*d*) [0, 1]

30. Sun Life Insurance Company issues standard, preferred and ultra-preferred policies. Among the company's policy holders of a certain age, 50% are standard with a probability of 0.01 of dying in the next year, 30% are preferred with a probability of 0.008 of dying in the next year and 20% are ultra-preferred with a probability of 0.007 of dying in the next year. If a policy holder of that age dies in the next year, what is the probability of the deceased being a preferred policy holder?

(*a*) 0.1591 (*b*) 0.2727

(*c*) 0.375 (*d*) None of these

[Based on IIFT, 2010]

31. A management institute has six senior professors and four junior professors. Three professors are selected at random for a Government project. The probability that at least one of the junior professors would get selected is:

(*a*) 5/6 (*b*) 2/3

(*c*) 1/5 (*d*) 1/6

[Based on XAT, 2007]

32. The supervisor of a packaging unit of a milk plant is being pressurised to finish the job closer to the distribution time, thus giving the production staff more leeway to cater to last minute demand. He has the option of running the unit at normal speed or at 110% of normal 'fast speed'. He estimates that he will be able to run at the higher speed 60% of time. The packet is twice as likely to be damaged at the higher speed which would mean temporarily stopping the process. If a packet on a randomly selected packaging runs has probability of 0.112 of damage, what is the probability that the packet will not be damaged at normal speed?

(*a*) 0.81 (*b*) 0.93

(*c*) 0.75 (*d*) 0.60

[Based on XAT, 2010]

33. The chance of India winning a cricket match against Australia is one-sixth. What is the minimum number of matches India should play against Australia so that there is a fair chance of winning at least one match?

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

[Based on XAT, 2010]

34. There are four machines in a factory. At exactly 8 p.m. when the mechanic is about to leave the factory, he is informed that two of the four machines are not working properly. The mechanic is in a hurry and decides that he will identify the two faulty machines before going home and repair them next morning. It takes him 20 minutes to walk to the bus stop. The last bus leaves at 8:32 p.m. If it takes 6 minutes to identify whether a machine is defective or not and if he decides to check the machines at random, what is the probability that the mechanic will be able to catch the last bus?

(*a*) 0 (*b*) 1/6

(*c*) 1/4 (*d*) 1/3

[Based on XAT, 2011]

35. The scheduling officer for a local police department is trying to schedule additional patrol units in each of two neighbourhoods—Southern and Northern. She knows that on any given day, the probabilities of major crimes and minor crimes being committed in the Northern neighbourhood were 0.418 and 0.612, respectively, and that the corresponding probabilities in the Southern neighbourhood were 0.355 and 0.520. Assuming that all crime occur independent of each other and likewise that crime in the two neighbourhoods are independent of each other, what is the probability that no crime of either type is committed in either neighbourhood on any given day?

(*a*) 0.069 (*b*) 0.225
(*c*) 0.690 (*d*) 0.775

[Based on XAT, 2011]

36. In a game of cards assume King to be greatest card and Ace to be the least value card and the other cards' value increases pinearly from Ace to King. Ram and Shyam are playing a game in which Ram picks a card first and then Shyam picks a card. If the value of Shyam's card is greater than the value of Ram's card then Shyam wins else he loses. What is the probability of Shyam winning the bet. Does he have greater probability if he plays first instead of Ram?

(*a*) 0.47, No (*b*) 0.53, Yes
(*c*) 0.53, No (*d*) 0.47, Yes

37. A blind man lives in an apartment containing 2 rooms. Each day before going to work he enters any one room randomly, picks up a bag and leaves home. One of the rooms contains 3 blue, 4 green and 5 red bags and the other contains 2 blue, 1 green and 3 red bags. What is the probability that he takes a green bag to work?

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$
(*c*) $\frac{1}{3}$ (*d*) $\frac{1}{6}$

38. 7 Indians, 4 Americans and 2 Germans are to be seated on 13 chairs for a photograph. If a photograph is clicked, what is the probability that in the photo no two Indians are together?

(*a*) $\frac{7!4!2!}{13!}$ (*b*) $\frac{7!6!}{13!}$
(*c*) $\frac{7!}{13!}$ (*d*) $\frac{7}{13}$

39. A number is selected at random from all the four digit natural numbers. What is the probability that it is a perfect square?

(*a*) $\frac{16}{2250}$ (*b*) $\frac{17}{2250}$
(*c*) $\frac{9}{9000}$ (*d*) $\frac{17}{2500}$

40. Two people agree to meet on January 9, 2005 between 6.00 p.m. and 7.00 p.m., with the understanding that each will wait no longer than 20 minutes for the other. What is the probability that they will meet?

(*a*) $\frac{5}{9}$ (*b*) $\frac{7}{9}$
(*c*) $\frac{2}{9}$ (*d*) $\frac{4}{9}$

41. Badri has 9 pairs of dark blue socks and 9 pairs of black socks. He keeps them all in the same bag. If he picks out three socks at random, then what is the probability that he will get a matching pair?

(*a*) $(2\,{}^9C_2\,{}^9C_1)/{}^{18}C_3$
(*b*) $({}^9C_3\,{}^9C_1)/{}^{18}C_3$
(*c*) 1
(*d*) None of these

42. In his wardrobe, Timothy has 3 trousers. One of them is black the second blue, and the third brown. In his wardrobe, he also has 4 shirts. One of them is black and the other 3 are white. He opens his wardrobe in the dark and picks out one shirt trouser pair without examining the colour. What is the likelihood that neither the shirt nor the trouser is black?

(*a*) 1/12 (*b*) 1/6
(*c*) 1/4 (*d*) 1/2

43. I forgot the last digit of a 7-digit telephone number. If I randomly dial the final 3 digits after correctly dialling the first four, then what is the chance of dialling the correct number?

(*a*) $\frac{1}{1001}$
(*b*) $\frac{1}{990}$
(*d*) $\frac{1}{999}$
(*d*) $\frac{1}{1000}$

44. There are 6 positive and 8 negative numbers. Four numbers are chosen at random and multiplied. The probability that the product is a positive number is:

(*a*) $\frac{500}{1001}$
(*b*) $\frac{503}{1001}$
(*c*) $\frac{505}{1001}$
(*d*) $\frac{101}{1001}$

45. A programmer noted the results of attempting to run 20 programs. The results showed that 2 programs ran correctly in the first attempt, 7 ran correctly in the second attempt, 5 ran correctly in the third attempt, 4 ran correctly in the fourth attempt, 2 ran correctly in the fifth attempt.

What is the probability that his next program will run correctly on the third run?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{1}{6}$ (d) $\frac{1}{5}$

46. Course materials are sent to students by a distance teaching institution. The probability that they will send a wrong programme's study material is one-fifth.

There is a probability of three-fourths that the package is damaged in transit, and there is a probability of one-third that there is a short shipment. What is the probability that the complete material for the course arrives without any damage in transit?

(a) $\frac{4}{5}$ (b) $\frac{8}{60}$

(c) $\frac{8}{15}$ (d) $\frac{4}{20}$

47. A set A is containing n elements. A subset P of A is chosen at random. The set is reconstructed by replacing the elements of P. A subset of A is again chosen at random. The probability that P and Q have no common elements is:

(a) 5^n (b) $\left(\frac{3}{4}\right)^n$

(c) $\left(\frac{3}{5}\right)^n$ (d) 2^n

48. There are three similar boxes, containing (i) 6 black and 4 white balls; (ii) 3 black and 7 white balls and (iii) 5 black and 5 white balls, respectively. If you choose one of the three boxes at random and from that particular box pick up a ball at random and find that to be black, what is the probability that the ball was picked up from the second box?

(a) $\left(\frac{3}{14}\right)$ (b) $\left(\frac{14}{30}\right)$

(c) $\left(\frac{7}{30}\right)$ (d) $\left(\frac{7}{14}\right)$

[Based on JMET, 2009]

49. Set $A = \{2, 3, 4, 5\}$

Set $B = \{4, 5, 6, 7, 8\}$

Two integers will be randomly selected from the sets above, one integer from set A and one integer from set B. What is the probability that the sum of the two integers will equal 9?

(a) 0.20 (b) 0.25

(c) 0.30 (d) 0.33

[Based on ATMA, 2006]

50. In a charity show tickets numbered consecutively from 101 through 350 are placed in a box. What is the probability that a ticket selected at random (blindly) will have a number with a hundreds digit of 2?

(a) 0.285 (b) 0.40

(c) $\frac{100}{249}$ (d) $\frac{99}{250}$

[Based on ATMA, 2006]

51. A group of $2n$ boys and $2n$ girls is divided at random into two equal batches. The probability that each batch will have equal number of boys and girls is:

(a) $\frac{1}{2}n$ (b) $\frac{1}{4}n$

(c) $\frac{({}^{2n}C_n)^2}{{}^{4n}C_{2n}}$ (d) ${}^{2n}C_n$

[Based on ATMA, 2008]

52. In ΔLMN, LO is the median and the bisector of $\angle MLN$. If $LO = 3$ cm and $LM = 5$ cm, calculate the area of ΔLMN.

(a) 12 cm^2 (b) 10 cm^2

(c) 4 cm^2 (d) 6 cm^2

[Based on CAT, 2009]

53. A garland is to be made from six different flowers and a large pendant that has two different faces. In how many ways, can the garland be made?

(a) 240 (b) 600

(c) 720 (d) None of these

[Based on CAT, 2009]

54. There are four boxes. Each box contains two balls: one red and one blue. You draw one ball from each of the four boxes. What is the probability of drawing at least one red ball?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{16}$ (d) $\frac{15}{16}$

[Based on CAT, 2010]

55. A shopkeeper received a pack of 15 pens, out of which 4 were defective. The shopkeeper decided to examine every pen one by one selecting a pen at random. The pen examined are not put back. What is the probability that the 9th one that had been examined is the last defective pen?

(a) $\frac{11}{195}$ (b) $\frac{16}{195}$

(c) $\frac{8}{195}$ (c) $\frac{17}{195}$

[Based on CAT, 2013]

56. Varun throws two unbiased dice together and gets a sum of 7. If his friend Tarun, then throws the same two dice, what is the probability that the sum is less than 7?

(a) $\frac{1}{6}$ (b) $\frac{7}{12}$

(c) $\frac{1}{2}$ (d) $\frac{5}{12}$

[Based on CAT, 2012]

57. In a factory where toys are manufactured, machines A, B and C produce 25%, 35% and 40% of the total toys, respectively. Of their output, 5%, 4% and 2% respectively, are defective toys. If a toy drawn at random is found to be defective, what is the probability that it is manufactured on machine B?

(a) $\frac{17}{69}$ (b) $\frac{28}{69}$

(c) $\frac{35}{69}$ (d) None of these

[Based on MAT, 2012]

58. A and B alternately throw a pair of dice. A wins if he throws 6 before B throws 7; and B wins if he throws 7 before A throws 6. What are their respective chances of winning, if A throws the dice first?

(a) $\frac{13}{16}, \frac{31}{16}$ (b) $\frac{30}{61}, \frac{31}{61}$

(c) $\frac{31}{61}, \frac{41}{61}$ (d) $\frac{38}{61}, \frac{23}{61}$

[Based on MAT, 2012]

59. A chartered Accountant applies for a job in two firms X and Y. The probability of his being selected in firm X is 0.7 and being rejected at Y is 0.5 and the probability of atleast one of his applications being rejected is 0.6. What is the probability that he will be selected in one of the firns?

(a) 0.2 (b) 0.8

(c) 0.4 (d) 0.7

[Based on MAT, 2012]

60. There are two bags, one of which contains 3 black and 4 white balls, while the other contains 4 black and 3 white balls. A dice is cast, if the face 1 or 3 turns up, a ball is taken from the first bag and if any other face turns up, a ball is chosen from the second bag. The probability of choosing a black ball is:

(a) 10/21 (b) 11/21

(c) 12/21 (d) 9/21

[Based on MAT, 2012]

61. A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots and C can hit twice in 3 shots. When they fire together, what is the probability that atleast two shots hit the target?

(a) 13/30 (b) 1/3

(c) 5/6 (d) None of these

[Based on MAT, 2012]

62. In a defective 6 faced die with number 1 to 6 inscribed, the probability of getting an odd umber is twice the probability of getting an even number. Find the probability of getting a two-digit prime number on adding 2 successive throws of the die:

(a) $\frac{2}{81}$ (b) $\frac{4}{81}$

(c) $\frac{1}{3}$ (d) $\frac{1}{9}$

[Based on MAT, 2013]

63. The probability that a randomly chosen positive divisor of 10^{29} is an integer multiple of 10^{23} is: a^2/b^2, then 'b – a' would be:

(a) 8 (b) 15

(c) 21 (d) 23

[Based on XAT, 2014]

Answer Keys

Difficulty Level-1

1. (*c*)	**2.** (*b*)	**3.** (*d*)	**4.** (*b*)	**5.** (*b*)	**6.** (*c*)	**7.** (*c*)	**8.** (*b*)	**9.** (*b*)	**10.** (*a*)	**11.** (*c*)	**12.** (*d*)	**13.** (*b*)
14. (*d*)	**15.** (*c*)	**16.** (*a*)	**17.** (*a*)	**18.** (*a*)	**19.** (*c*)	**20.** (*b*)	**21.** (*a*)	**22.** (*d*)	**23.** (*a*)	**24.** (*b*)	**25.** (*b*)	**26.** (*c*)
27. (*a*)	**28.** (*d*)	**29.** (*d*)	**30.** (*c*)	**31.** (*a*)	**32.** (*d*)	**33.** (*b*)	**34.** (*d*)	**35.** (*a*)	**36.** (*c*)	**37.** (*d*)	**38.** (*a*)	**39.** (*c*)
40. (*b*)	**41.** (*c*)	**42.** (*a*)	**43.** (*b*)	**44.** (*b*)	**45.** (*c*)	**46.** (*c*)	**47.** (*c*)	**48.** (*b*)	**49.** (*b*)	**50.** (*c*)	**51.** (*a*)	**52.** (*a*)
53. (*b*)	**54.** (*b*)	**55.** (*a*)	**56.** (*a*)	**57.** (*d*)	**58.** (*d*)	**59.** (*d*)	**60.** (*a*)	**61.** (*c*)	**62.** (*d*)	**63.** (*b*)	**64.** (*c*)	**65.** (*c*)
66. (*a*)	**67.** (*b*)	**68.** (*d*)	**69.** (*c*)	**70.** (*d*)	**71.** (*c*)	**72.** (*d*)	**73.** (*a*)	**74.** (*c*)	**75.** (*c*)	**76.** (*a*)	**77.** (*c*)	**78.** (*b*)
79. (*a*)	**80.** (*c*)	**81.** (*c*)	**82.** (*b*)	**83.** (*d*)	**84.** (*b*)	**85.** (*c*)	**86.** (*a*)	**87.** (*a*)	**88.** (*a*)	**89.** (*d*)	**90.** (*a*)	**91.** (*c*)
92. (*c*)												

Difficulty Level-2

1. (*c*)	**2.** (*c*)	**3.** (*b*)	**4.** (*c*)	**5.** (*b*)	**6.** (*a*)	**7.** (*b*)	**8.** (*b*)	**9.** (*d*)	**10.** (*b*)	**11.** (*d*)	**12.** (*d*)	**13.** (*c*)
14. (*d*)	**15.** (*c*)	**16.** (*a*)	**17.** (*b*)	**18.** (*d*)	**19.** (*c*)	**20.** (*b*)	**21.** (*a*)	**22.** (*b*)	**23.** (*a*)	**24.** (*a*)	**25.** (*b*)	**26.** (*a,b,c*)
27. (*c*)	**28.** (*c*)	**29.** (*c*)	**30.** (*b*)	**31.** (*a*)	**32.** (*b*)	**33.** (*b*)	**34.** (*d*)	**35.** (*a*)	**36.** (*d*)	**37.** (*b*)	**38.** (*b*)	**39.** (*b*)
40. (*d*)	**41.** (*c*)	**42.** (*d*)	**43.** (*d*)	**44.** (*c*)	**45.** (*a*)	**46.** (*b*)	**47.** (*a*)	**48.** (*a*)	**49.** (*a*)	**50.** (*b*)	**51.** (*c*)	**52.** (*a*)
53. (*c*)	**54.** (*d*)	**55.** (*c*)	**56.** (*d*)	**57.** (*d*)	**58.** (*b*)	**59.** (*b*)	**60.** (*b*)	**61.** (*c*)	**62.** (*b*)	**63.** (*d*)		

Explanatory Answers

Difficulty Level-1

1. (*c*) Total number of events that would occur by flipping six coins = $2^6 = 64$

Probability that no tail occurs = $\frac{1}{64}$

$\therefore$ Probability of occuring at least one tail

$= 1 - \frac{1}{64} = \frac{63}{64}$.

2. (*b*)

$$P(A \cap B) = P(A) \times P(B) = 0.15 \times P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.45 = 0.15 + P(B) - (0.15) \times P(B)$$

$$= 0.15 + P(B)\,(1 - 0.15)$$

$$= 0.15 + 0.85\,P(B)$$

$$\therefore \quad 0.85\,P(B) = 0.45 - 0.15 = 0.30$$

$$\therefore \quad P(B) = \frac{0.30}{0.85} = \frac{30}{85} = \frac{6}{17}.$$

3. (*d*) Let x be the number of coins showing heads.

$\therefore$ Prob. when $x = 50$ = Prob. when $x = 51$

$$\Rightarrow {}^{100}C_{50} \times p^{50} \times (1-p)^{50} = {}^{100}C_{51} \times p^{51} \times (1-p)^{49}$$

$$\Rightarrow \quad p = \frac{51}{101}.$$

4. (*b*) $P(A) = \frac{1}{5}$, $P(\bar{A}) = 1 - \frac{1}{5} = \frac{4}{5}$

The probability that he will not hit the target in 10 shots is $\left(\frac{4}{5}\right)^{10}$

So, probability that atleast once target will be hit

$$= 1 - \left(\frac{4}{5}\right)^{10}.$$

5. (*b*) The event 'Total score is a prime number when two dice are tossed' occurs in the following 15 ways: (1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5).

$\therefore$ Required probability = $\frac{15}{36} = \frac{5}{12}$.

6. (*c*) Let the four places be 1 2 3 4

Now object i cannot occupy the place i (A)

Suppose Object 2 occupies the place 1. Then other placements can be done in 6 ways as follows:

(1)	2	1	3	4
(2)	2	1	4	3

(3)	2	3	1	4
(4)	2	3	4	1
(5)	2	4	1	3
(6)	2	4	3	1

Here out of six ways, only three are permissible, because (1), (3) and (6) are not permissible because of the non-fulfilment of condition (A). Hence, required probability is $\frac{3}{6} = \frac{1}{2}$. Similarly you can allow objects 3 and 4 to occupy place 1 and in each case you can find that the probability is $\frac{1}{2}$.

7. (*c*) $1-\left(\frac{2}{3}\times\frac{3}{5}\times\frac{7}{12}\right) = \frac{23}{30}$.

8. (*b*) Prob. that it rains on the 1st day $= \frac{1}{2}$

Prob. that it rains on the 2nd day $= \frac{1}{2}$

Prob. that it rains on the 3rd day $= \frac{1}{2}$

Prob. that it rains on the 4th day $= \frac{1}{2}$

Prob. that it rains on the 5th day $= \frac{1}{2}$

Prob. that it rains on any day in a 5-day period

$$= \frac{1}{32}$$

Prob. that it rains on exactly 3 days in a 5-day period

$$= {}^5C_3\times\frac{1}{32} = {}^5C_2\times\frac{1}{32} = \frac{10}{32} = \frac{5}{16}.$$

9. (*b*) Probability of his application being rejected = 0.6

Probability of his application being not rejected

$$= 1 - 0.6$$

∴ Probably of his selection must be less than 0.4. It should be 0.2.

10. (*a*) **Case I:** When one orange fruit is transferred from *A* to *B* and *B* to *A*.

$$\therefore \quad P(E_1) = \frac{{}^5C_1}{{}^{11}C_1}\times\frac{{}^5C_1}{{}^8C_1} = \frac{25}{88}$$

Case II: When one apple is transfered from *A* to *B* and *B* to *A*.

$$\therefore \quad P(E_1) = \frac{{}^6C_1}{{}^{11}C_1}\times\frac{{}^4C_1}{{}^8C_1} = \frac{24}{88}$$

∴ Required probability

$$= P(E_1) + P(E_2)$$

$$= \frac{25}{88} + \frac{24}{88} = \frac{49}{88}.$$

11. (*c*) Let E = Event of getting least six numbers

$$= \{6, 7, 8\}$$

$$\therefore \quad P(E) = \frac{3}{8}$$

∴ Required probability $= \left(\frac{3}{8}\right)^3$.

12. (*d*) Given, $P(A) = 0.25, P(B) = 0.50,$

$$P(A\cap B) = 0.12$$

$$\because \quad P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= 0.25 + 0.50 - 0.12$$

$$= 0.63$$

$$\therefore \quad P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B)$$

$$= 1 - 0.63 = 0.37.$$

13. (*b*) Probability of not successing

$$A = \frac{9}{12}\times\frac{8}{11}\times\frac{7}{10} = \frac{21}{55}$$

Probability of successing $A = 1-\frac{21}{55} = \frac{34}{55}$

Probability of not successing $B = \frac{6}{8}\times\frac{5}{7} = \frac{15}{28}$

∴ Probability of successing $B = 1-\frac{15}{28} = \frac{13}{28}$

∴ Required ratio $= \frac{34}{55}:\frac{13}{28} = 952:715.$

14. (*d*) Here, good bulbs are 6 and defective bulbs are 4.

∴ Required probability $= \frac{{}^6C_3}{{}^{10}C_3}$

$$= \frac{(6\times5\times4)/(3\times2\times1)}{(10\times9\times8)/(3\times2\times1)} = \frac{1}{6}.$$

15. (*c*) Given, $P(A) = \frac{5}{7}$

and, $P(\bar{A}) = 1-\frac{5}{7} = \frac{2}{7}$

Probability of winning atleast one game

$$= 1 - P\text{ (no game winning)}$$

$$= 1-\left(\frac{2}{7}\right)^3$$

$$= 1 - \frac{8}{343}$$

$$= \frac{335}{343}$$

∴ The odds in favour of *A*'s winning atleast one game = 335:8.

16. (*a*) The probability of selecting a same number

$$= \frac{16}{16 \times 16} = \frac{1}{16}$$

$\therefore$ Required probability $= 1 - \frac{1}{16} = \frac{15}{16}$.

17. (*a*) Let, $P(A) = \frac{2}{3}$

and, $P(B) = 1 - \frac{5}{9} = \frac{4}{9}$

Also, $P(A \cup B) = \frac{4}{5}$

$\because$ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$$= \frac{2}{3} + \frac{4}{9} - \frac{4}{5}$$

$$= \frac{30 + 20 - 36}{45} = \frac{14}{45}.$$

18. (*a*) Let, E_1 = Both speak truth

E_2 = Both speak false

and, E = A and B agree in a statement

Given, $P(A) = \frac{75}{100} = \frac{3}{4}, P(B) = \frac{80}{100} = \frac{4}{5}$

$\therefore$ $P(E_1) = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

$P(E_2) = \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

It is clearly,

$$P\left(\frac{E}{E_1}\right) = 1 \text{ and, } P\left(\frac{E}{E_2}\right) = 1$$

$$\therefore \quad P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\,P(E/E_1)}{P(E_1)\,P(E/E_1) + P(E_2)\,P(E/E_2)}$$

$$= \frac{\frac{3}{5} \times 1}{\frac{3}{5} \times 1 + \frac{1}{20} \times 1} = \frac{\frac{3}{5}}{\frac{12+1}{20}}$$

$$= \frac{3}{5} \times \frac{20}{13} = \frac{12}{13}.$$

19. (*c*) Favourable cases $= 2! \times {}^8P_3 \times 5!$

$= 2 \times 8!$

Total number of cases $= 9!$

$\therefore$ Required probability $= \frac{2 \times 8!}{9!} = \frac{2}{9}$.

20. (*b*) Here, $P(A) = \frac{4}{7}$

and, $P(\bar{A}) = 1 - \frac{4}{7} = \frac{3}{7}$

$\therefore$ Probability of winning atleast one game

= 1 – Probability of winning no game

$$= 1 - \left(\frac{3}{7}\right)^3$$

$$= -\frac{27}{343} = \frac{316}{343}$$

$\therefore$ Required odds in favour of A = 316:27.

21. (*a*) Let, E = Event of getting a prime number

= {2, 3, 5, 7}

$\therefore$ Required probability $= \frac{{}^4C_1}{{}^{10}C_1}$

$$= \frac{4}{10} = \frac{2}{5}.$$

22. (*d*) Here we consider AB as a one unit.

$\therefore$ Favourable number of cases = 3!

$\therefore$ Required probability $= \frac{3!}{4!} = \frac{1}{4}$.

23. (*a*) Here non-defective bulbs are 8.

$\therefore$ Required probability $= \frac{{}^8C_3}{{}^{12}C_3}$

$$= \frac{8 \times 7 \times 6}{12 \times 11 \times 10} = \frac{14}{55}.$$

24. (*b*) Let, $P(R_1) = 0.4$, $P(R_2) = 0.5$ and, $P(R_3) = 0.8$

$\Rightarrow$ $P(\bar{R}_1) = 0.6$, $P(\bar{R}_2) = 0.5$ and, $P(\bar{R}_3) = 0.2$

$\therefore$ Required probability

$= P(R_1\, R_2\, \bar{R}_3) + P(R_1\, \bar{R}_2\, R_3) + P(\bar{R}_1\, R_2\, R_3)$

$= P(R_1)\, P(R_2)\, P(\bar{R}_3) + P(R_1)\, (\bar{R}_2)\, P(R_3)$

$+ P(\bar{R}_1)\, P(R_2)\, P(R_3)$

$= 0.4 \times 0.5 \times 0.2 + 0.4 \times 0.5 \times 0.8$

$+ 0.6 \times 0.5 \times 0.8$

$= 0.04 + 0.16 + 0.24 = 0.44.$

25. (*b*) Passed students in Ist examination, $n(E_1) = 60$

and passed students in IInd examination, $n(E_2) = 50$

Also, $n(E_1 \cap E_2) = 30$

$\therefore$ Failed students in 1st examination, $n(\bar{E}_1) = 40$

and failed students in IInd examination, $n(\bar{E}_2) = 50$

Also, $n(\bar{E}_1 \cap \bar{E}_2) = 70$

$\therefore$ Failed students in both subjects,

$$n(\bar{E}_1 \cap \bar{E}_2) = 40 + 50 - 70 = 20$$

$\therefore$ Required probability $= \dfrac{20}{100} = 0.2.$

26. (*c*) Favourable cases $= 2! \times {}^{11}P_3 \times 8!$

$$= 2! \times 11!$$

$\therefore$ Required probability $= \dfrac{2! \times 11!}{12!} = \dfrac{2}{12} = \dfrac{1}{6}.$

27. (*a*) Here, number of boys = 100 – 55 = 45

Boys not studying statistics = 45 – 36 = 9

$\therefore$ Required probability $= \dfrac{9}{45} = \dfrac{1}{5}.$

28. (*d*) Given, $P(A) = \dfrac{4}{5}, P(B) = \dfrac{3}{4}, P(C) = \dfrac{3}{3} = 1$

$\Rightarrow \quad P(\bar{A}) = \dfrac{1}{5}, \quad P(\bar{B}) = \dfrac{1}{4}$ and $P(\bar{C}) = 0$

$\therefore$ Required probability $P(A)\,P(B)\,P(\bar{C}) + P(A)\,P(\bar{B})$ $P(C) + P(\bar{A})\,P(B)\,P(C) + P(A)\,P(B)\,P(C)$

$$= \frac{4}{5} \times \frac{3}{4} \times 0 + \frac{4}{5} \times \frac{1}{4} \times 1 + \frac{1}{5} \times \frac{3}{4} \times 1 + \frac{4}{5} \times \frac{3}{4} \times 1$$

$$= \frac{4}{20} + \frac{3}{20} + \frac{12}{20} = \frac{19}{20}.$$

29. (*d*) Given, $P(A) = \dfrac{3}{5}$ and, $P(B) = \dfrac{5}{8}$

Probability that they are likely to agree each other on an identical point

= *P*(both speak truth) + *P*(both speak false)

$$P(A) + P(B) + P(\bar{A})\,P(\bar{B})$$

$$= \frac{3}{5} \times \frac{5}{8} + \frac{2}{5} \times \frac{3}{8}$$

$$= \frac{15}{40} + \frac{6}{40} = \frac{21}{41} = 0.525$$

$\therefore$ Required percentage = 52.5%

30. (*c*) Let *A* = Event of getting a number multiple of 5

= {5, 10, 15, 20, 25}

and, *B* = Event of getting a number multiple of 7

= {7, 14, 21}

and, $A \cap B = \phi$

$\therefore \quad P(A) = \dfrac{5}{25} = \dfrac{1}{5}, \; P(B) = \dfrac{3}{25}$

and, $P(A \cap B) = 0$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{5} + \frac{3}{25} - 0 = \frac{8}{25}.$$

31. (*a*) Required probability

$$y = 2[P(W)\,P(B)\,P(W)\,P(B)]$$

$$= 2\left[\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5}\right]$$

$$= \frac{2}{14} = \frac{1}{7}$$

[The first ball may be either black or white.]

32. (*d*) Required probability $= \dfrac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{4} + \frac{1}{3}}$

$$= \frac{1}{6} \times \frac{12}{(2+3+4)} = \frac{2}{9}.$$

33. (*b*) If 2 in unit digit, then ten place may be number from 0 to 9 i.e., 10 ways. And if 2 is in ten place, then unit place may be number from 0 to 9. But number 22 is common in both cases.

$\therefore$ Total cases = 20 – 1 = 19

$\therefore$ Required probability $= \dfrac{19}{100}.$

34. (*d*) Required probability $= \dfrac{{}^5C_2 + {}^4C_2}{{}^9C_2}$

$$= \frac{10+6}{36} = \frac{16}{36} = \frac{4}{9}.$$

35. (*a*) Since, all events are independent.

$\therefore$ Required probability $= \dfrac{1}{2}.$

36. (*c*) Since, there are 15 cars in 25 places, total number of selection of places out of (25 – 1) places out of (15 – 1) cars (excepting the owner's car) is

$${}^{25-1}C_{15-1} = {}^{24}C_{14}$$

If neighbouring places are empty, then 14 cars must be parked in (25 – 3 = 22) places. So, the favourable number of cases is

$${}^{25-3}C_{15-1} = {}^{24}C_{14}$$

$\therefore$ Required probability $= \dfrac{{}^{22}C_{14}}{{}^{24}C_{14}}$

$$= \frac{22!}{14! \times 8!} \times \frac{14! \times 10!}{24!}$$

$$= \frac{10 \times 9}{24 \times 23} = \frac{15}{92}.$$

37. (*d*) Required probability

$$= \frac{{}^2C_1 \times {}^3C_1 \times {}^4C_1}{{}^9C_3}$$

$$= \frac{2 \times 3 \times 4}{\frac{9 \times 8 \times 7}{3 \times 2 \times 1}} = \frac{2}{7}.$$

38. (*a*) Given, $P(X) = 0.7, P(Y) = 0.5$

$\Rightarrow \quad P(\overline{X}) = 0.3,\ P(\overline{Y}) = 0.5$

Also, $\quad P(\overline{X} \cup \overline{Y}) = 0.6$

$\Rightarrow \quad 1 - P(X \cap Y) = 0.6$

$\Rightarrow \quad P(X \cap Y) = 0.4$

$\therefore$ Required probability, $P(X \cup Y)$

$= P(X) + P(Y) - P(X \cap Y)$

$= 0.7 + 0.5 - 0.4 = 0.8.$

39. (*c*) Let probability of contract

$$P(A) = \frac{2}{3}$$

and probability of electric contract

$$P(B) = 1 - \frac{5}{9} = \frac{4}{9}$$

Also, $\quad P(A \cup B) = \frac{4}{5}$

$\therefore$ Required probability $= P(A \cap B)$

$= P(A) + P(B) - P(A \cup B)$

$= \frac{2}{3} + \frac{4}{9} - \frac{4}{5}$

$= \frac{30 + 20 - 36}{45}$

$= \frac{14}{45}.$

40. (*b*) Let a coin is tossed n times.

Probability of getting a head in a single coin, $p = \frac{1}{2}$

and probability of getting a tail in a single coin, $q = \frac{1}{2}$

$$\therefore \quad {}^nC_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{n-4} = {}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7}$$

$\Rightarrow \quad {}^nC_4 = {}^nC_7$

$$\Rightarrow \quad \frac{n!}{(n-4)!\,4!} = \frac{n!}{(n-7)!\,7!}$$

$\Rightarrow \quad (n-6)(n-5)(n-4) = 5 \times 6 \times 7$

$\Rightarrow \quad n = 11$

$$\therefore \quad \text{Required probability} = {}^{11}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^9$$

$$= 55 \times \left(\frac{1}{2}\right)^{11} = \frac{55}{2048}.$$

41. (*c*) Total number of ways $= 6^5$

Case I: When numbers are 3, 3, 2, 2, 2.

Number of ways $= \frac{5!}{2! \times 3!} = 10$

Case II: When numbers are 3, 3, 3, 3

[Here '–' denote for blank dice]

Number of ways = 5

$$\therefore \text{ Required probability} = \frac{10 + 5}{6^5} = \frac{15}{6 \times 6^4} = \frac{5}{2592}.$$

42. (*a*) Given, $\quad P(A) = \frac{3}{11}, P(B) = \frac{2}{7}$

Also,

$P(A) + P(B) + P(C) = 1$

$$\Rightarrow \quad P(C) = 1 - \frac{3}{11} - \frac{2}{7}$$

$$= \frac{77 - 21 - 22}{77} = \frac{34}{77}$$

$\therefore$ Odd against C is 43:34.

43. (*b*) $\therefore$ Required probability

$$= \frac{{}^{13}C_2 + {}^{13}C_2}{{}^{52}C_2}$$

$$= \frac{78 + 78}{1326} = \frac{156}{1326}$$

$$= \frac{6}{51} = \frac{2}{17}.$$

44. (*b*) Given, part A is not defective $P(A) = \frac{91}{100}$

and part B is not defective, $P(B) = \frac{95}{100}$

$\therefore$ Required probability $= P(A) \times P(B)$

$= \frac{91}{100} \times \frac{95}{100}$

$= 0.8645$

$= 0.86.$

45. (*c*) Maximum points in four matches be 8 only

$\therefore$ Required probability

$= {}^4C_1\,(0.05)(0.5)^3 + {}^4C_4(0.5)^4$

$= 4 \times 0.000625 + 0.0625$

$= 0.025 + 0.0625 = 0.0875.$

46. (*c*) $\therefore$ Required probability

$$= \left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)$$

$$= \frac{2}{3}\times\frac{3}{4}\times\frac{4}{5} = \frac{2}{5}.$$

47. (c) The required probability $= \frac{1}{4}\times\frac{1}{5}\times\frac{2}{5} = 0.02.$

48. (*b*) Total number of possible arrangement for 4 boys and 3 girls in a queue = 7!

When they occupy alternate position the arrangement would be like

B G B G B G B

Thus, total number of possible arrangements

For boys = $4 \times 3 \times 2$ and for girls = 3×2

$\therefore$ Required probability $= \dfrac{4\times3\times2\times3\times2}{7!}$

$$= \frac{4\times3\times2\times3\times2}{7\times6\times5\times4\times3\times2} = \frac{1}{35}.$$

49. (*b*) Total number of ways in which both of them can select a number each = $5 \times 5 = 25$

Probability that they win the prize

$$= \frac{1\times1}{25} = \frac{1}{25}$$

$\Rightarrow$ Probability that they do not win a prize

$$= 1 - \frac{1}{25} = \frac{24}{25}.$$

50. (*c*) Probability that the new product will be introduced

$= 0.5 \times 0.7 + 0.3 \times 0.6 + 0.2 \times 0.5$

$= 0.35 + 0.18 + 0.10$

$= 0.63.$

51. (*a*) Probability that the article will be defective

$$= \frac{9}{100}\times\frac{95}{100}\times\frac{91}{100}\times\frac{5}{100}\times\frac{9}{100}\times\frac{5}{100}$$

$$= \frac{171}{2000} + \frac{191}{2000} + \frac{9}{2000} = \frac{271}{2000}$$

$\therefore$ Probability that the article will be non-defective

$$= 1 - \frac{271}{2000} = \frac{1729}{2000} = 0.8645$$

Note:

The article will be defective if any one of the part *A* and B is defective or both the parts are defective.

52. (*a*) Total number of events = 20

Probability of running the program correctly in the third run $= \frac{5}{20} = \frac{1}{4}$

Hence the probability of running the next program at the third run is $\frac{1}{4}$.

53. (b) Required Probability $= 1 - \left(\frac{2}{20} + \frac{7}{20} + \frac{5}{20}\right)$

$$= \frac{3}{10}.$$

54. (*b*) Probability of sending the correct material is $\frac{4}{5}$

Probability of the material not being damaged in transit is $\frac{1}{4}$

Probability that there is no short shipment $= \frac{2}{3}$

$\therefore$ Required probability $= \frac{4}{5}\times\frac{1}{4}\times\frac{2}{3} = \frac{2}{15} = \frac{8}{60}.$

55. (*a*) This is the problem of conditional probabilty

Required probability

$$= \frac{\frac{1}{5}\times\frac{1}{5}}{\frac{1}{5}\times\frac{1}{5}+\frac{4}{5}\times\frac{4}{5}+\frac{1}{5}\times\frac{4}{5}+\frac{4}{5}\times\frac{1}{5}} = \frac{\frac{1}{25}}{1} = \frac{1}{25}.$$

56. (*a*) Total number of exhaustive cases when two dice are thrown simultaneously = $6 \times 6 = 36$

Favourable number of cases of getting a sum of

6 = 5 (1, 5; 2, 4; 3, 3; 4, 2; 5, 1)

Hence, required probability $= \frac{5}{36}$.

57. (*d*) When 8 coins are tossed,

Total number of exhaustive cases $= 2^8 = 256$

Since there should be at least 6 heads

i.e., 6, 7 or 8 heads

$\therefore$ Number of favourable cases = 3

Hence, required probability $= \frac{3}{256}$.

58. (*d*) Total number of exhaustive cases of drawing 2 cards from a pack of 52 cards $= {}^{52}C_2$

Also, favourable number of cases of getting both as kings $= {}^4C_2$

Hence, required probability

$$= \frac{^4C_2}{^{52}C_2} = \frac{\frac{4\times 3}{1\times 2}}{\frac{52\times 51}{1\times 2}} = \frac{1}{221}.$$

59. (d) Probability of selecting 3 bulbs so that none is defective

$$= \frac{^{15}C_3}{^{20}C_3} = \frac{\frac{15\times 14\times 13}{1\times 2\times 3}}{\frac{20\times 19\times 18}{1\times 2\times 3}} = \frac{15\times 14\times 13}{20\times 19\times 18} = \frac{91}{228}.$$

60. (*a*) Let P_1 = Probability of drawing one spade in the first chance $= \frac{13}{52} = \frac{1}{4}$

P_2 = Probability of drawing one spade in the second chance $= \frac{13}{51}$

P_3 = Probability of drawing one heart in the first chance $= \frac{1}{4}$ and

P_4 = Probability of drawing one heart in the second chance $= \frac{13}{51}$

$\therefore$ Required Probability = Prob. (first is spade and second is heart) or Prob. (first is heart and second is spade) $= P_1 \times P_4 + P_2 \times P_3$

$$= \frac{1}{4}\times\frac{13}{51} + \frac{13}{51}\times\frac{1}{4} = \frac{13}{102}.$$

61. (*c*) Let P_1 = Probability that the ball drawn from bag A is red $= \frac{4}{9}$

P_2 = Probability that the ball drawn from bag A is black $= \frac{5}{9}$

P_3 = Probability that both the balls drawn from bag B are black $\frac{^7C_2}{^{10}C_2} = \frac{7}{15}$

P_4 = Probability of drawing one red and one black ball from bag $B = \frac{^3C_1 \times {^7C_1}}{^{10}C_2} = \frac{7}{15}$

$\therefore$ Required Probability = Probability (one red ball from bag A and two black balls from bag B) or Probability (one black ball from bag A and one red and one black balls from bag B) $= P_1 \times P_3 + P_2 \times P_4$

$$= \frac{4}{9}\times\frac{7}{15} + \frac{5}{9}\times\frac{7}{15} = \frac{7}{15}.$$

62. (*d*) Let E_1 be the event that the ball is drawn from the first box. E_2 be the event that the ball is drawn from the second box. E_3 be the event that the ball is drawn from the third box. E be the event that the ball is red.

Clearly, we have to find $P\left(\frac{E_1}{E}\right)$

Since all the boxes are equally likely to be selected, therefore

$$P(E_1) - P(E_2) = P(E_3) = 1/3$$

Also, the probability of drawing red ball from the first box $= P(E/E_1) = \frac{6}{10}$

The probability of drawing red ball from the second box $= P(E/E_2) = \frac{4}{10}$

The probability of drawing red ball from the third box $= P(E/E_3) = \frac{5}{10}$

Hence, by Bayes' theorem, we have $P(E_1/E)$

$$= \frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2) + P(E_3)P(E/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{6}{10}}{\frac{1}{3}\times\frac{6}{10} + \frac{1}{3}\times\frac{4}{10} + \frac{1}{3}\times\frac{5}{10}} = \frac{2}{5}.$$

63. (*b*) Let P = Probability of success $= \frac{3}{6} = \frac{1}{2}$ (Since there are 3 odd numbers out of 6 numbers on the dice.)

$\therefore$ Q = Probability of failure $= 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$

Hence the probability of 5 successes

$$= {^6C_5} \times (P)^5 \times (Q)^1$$

$$= 6 \times \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) = \left(\frac{3}{32}\right).$$

64. (*c*) The number of ways in which the series can end for India are (WWW, LWW, WLW, WWL, LLW, LWL, WLL, LLL)

Probability that India wins only 2 consecutive matches

i.e., LWW, WWL $= \frac{2}{8}$ (L-Lose, W-win) (1)

Probability that India wins all the 3 matches i.e., WWW

$$= \frac{1}{8} \quad (2)$$

$\therefore$ Probability that India wins the series by sinning at least 2 consecutive matches = (1) + (2)

$$= \frac{2}{8} + \frac{1}{8} = \frac{3}{8}.$$

65. (*c*) The possible outcomes are as follows: 5H, 5T, (H, 4T), (T, 4H), (2H, 3T), (3H, 2T), i.e., 6 outcomes in all.

Therefore the probability that head appears an odd number of times $= \frac{3}{6} = \frac{1}{2}$ (In only three outcomes out of the six outcomes, head appears an odd number of times).

66. (*a*) Probability that all the balls are white

$$= \frac{5}{5+7+8} = \frac{5}{20} = \frac{1}{4}$$

$\therefore$ Required probability of drawing 4 balls with replacement such that all the balls are white

$$= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{256}.$$

67. (*b*) Chandra hits the target definitely hence required probability that at least 2 shots hit the target is given by as follows. Bhola hits the target and Atal does not hit the target.

or

Bhola does not hit the target and Atal hits the target.

or

Bhola hits the target and Atal hits the target.

$$= \frac{2}{3} \times \frac{3}{6} + \frac{4}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{2}{6} = \frac{24}{36} = \frac{2}{3}$$

68. (*d*) Required probability $= \frac{1}{2}\left[\frac{3}{5} + \frac{2}{6}\right] = \frac{7}{15}$.

69. (*c*) The product of four numbers will be positive only of (1) All the four numbers are positive, (2) All the four numbers are negative, (3) Two numbers are positive and two are negative.

Probability of the above event

$$= \frac{{}^6C_4 + {}^8C_4 + {}^6C_2 \times {}^8C_2}{{}^{14}C_4} = \frac{505}{1001}.$$

70. (*d*) Let x be the number of coins showing heads.

$\therefore$ Prob. when $x = 50$

Prob. when $x = 51$

$$\Rightarrow {}^{100}C_{50} \times P^{50} \times (1-p)^{50}$$

$$= {}^{100}C_{51} \times P^{51} \times (1-p)^{49}$$

$$\Rightarrow \quad p = \frac{51}{101}.$$

71. (*c*) At least one event will happen = total – none of event

$$= 1 - \frac{5}{7} \times \frac{5}{11}$$

$$= 1 - \frac{25}{77} = \frac{52}{77}.$$

72. (*d*) Required probability $= \frac{1}{6}$.

73. (*a*)

$$p(A \cup B) = p(A) + p(B) - p(A) \times p(B)$$

$$\Rightarrow \quad 0.4 = 0.3 + p(B) - 0.3 \times p(B)$$

$$\Rightarrow \quad p(B) = \frac{1}{7}.$$

74. (*c*) In ASSISTANT we have 2*A*'s, 3*S*'s 2*T*'s and one each of 1 and *N*

In STATISTICS, we have 2*I*'s, 3*S*'s, 3*T*'s and one each of *A* and *C*

Here *N* and *C* are not common

Same letters can be *A, I, S, T*

Probability of choosing *A*

$$= \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^1C_1}{{}^{10}C_1} = \frac{2}{9} \times \frac{1}{10} = \frac{1}{45}$$

Probability of choosing *I*

$$= \frac{1}{{}^9C_1} \times \frac{{}^2C_1}{{}^{10}C_1} = \frac{1}{9} \times \frac{2}{10} = \frac{1}{45}$$

Probability of choosing *S*

$$= \frac{{}^3C_1}{{}^9C_1} \times \frac{{}^3C_1}{{}^{10}C_1} = \frac{3}{9} \times \frac{3}{10} = \frac{1}{10}$$

Probability of choosing *T*

$$= \frac{{}^2C_1}{{}^9C_1} \times \frac{{}^3C_1}{{}^{10}C_1} = \frac{2}{9} \times \frac{3}{10} = \frac{1}{15}$$

$\therefore$ Required probability

$$= \frac{1}{45} + \frac{1}{45} + \frac{1}{10} + \frac{1}{15}$$

$$= \frac{2+2+9+6}{90} = \frac{19}{90}.$$

75. (*c*) Let p be the probability that the electricity goes off and q be the probability that it does not go off

$$\Rightarrow \quad p = \frac{1}{2}, q = \frac{1}{2}.$$

Using binomial theorem the required probability = the coefficient of p^3 in the exp. of $(p+q)^7 = {}^7C_3 p^3 q^4$

$$= \frac{35}{128}.$$

76. (*a*) The probability that exactly 4 vessels arrive safely is

$${}^5C_4\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)$$

The probability that all 5 arrive safely is $\left(\frac{9}{10}\right)^5$

$\therefore$ The probability that at least 4 vessels arrive safely

$$= \left(\frac{9}{10}\right)^4\left(\frac{5}{10}+\frac{9}{10}\right) = \frac{(14\times 9^4)}{10^5}.$$

77. (*c*) Total cases of checking in the hotels = 4^3 ways. Cases when 3 men are checking in different hotels

$= (4 \times 3 \times 2)$ ways

Required probability $= \frac{4\times3\times2}{4^3} = \frac{3}{8}$.

78. (*b*) Since, the choices are in terms of *e*, let us apply the Poisson distribution. Since 5% of bulbs are defective, mean, λ = 5% of 100 = 5.

$$P(X=k) = \frac{\lambda^k}{K!\cdot e^{-1}}$$

$$\therefore \quad P(X=0) = \frac{5^0}{0!\cdot e^{-5}} = e^{-5}.$$

79. (*a*) There are total 5 letters. The probability that *B* gets first position is $\frac{1}{5}$. The probability that *G* gets second position is $\frac{1}{4}$. Likewise, probability for 1, *N* and *G* is

$\frac{1}{3}, \frac{1}{2}, \frac{1}{1}$ respectively

Hence, required probability

$$= \frac{1}{5}\times\frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times\frac{1}{1} = \frac{1}{120}.$$

80. (*c*) According to question, $\frac{P(A')}{P(A)} \Rightarrow \frac{8}{3}$,

$$P(A) = \frac{3}{11} \text{ and } P(A') = \frac{8}{11}$$

Also, $\frac{P(B')}{P(B)} = \frac{5}{2} \Rightarrow P(B) = \frac{2}{7}$ and $P(B')\ \frac{5}{7}$

Now, out of *A*, *B* and *C*, one and only one can happen.

$P(A) + P(B) + P(C) = 1$

$$P(C) = 1 - \{P(A) + P(B)\}$$

$$= 1 - \left\{\frac{3}{11}+\frac{2}{7}\right\}$$

$$= 1 - \left(\frac{21+22}{77}\right) = 1 - \frac{43}{77} = \frac{34}{77}$$

So, $\quad P(C') = 1 - \frac{34}{77} = \frac{43}{77}$

So, odd against $\quad C = \frac{P(C')}{P(C)} = \frac{43}{34}$

i.e., 43:34.

81. (*c*) Total number of coins = 4 + 3 + 3 = 10

If 6 coins are drawn from the bag, the probability that the draw yields maximum amount

$$= \frac{{}^4C_4\,{}^3C_2}{{}^{10}C_6} = \frac{1}{70}$$

Then, odds in favour of the draw $= 1 - \frac{1}{70} = \frac{69}{70}$.

82. (*b*) Either the point is on the circumference or inside the circle

Then, probability that point is on the circumference $= \frac{1}{2}$

Also, Probability that point is closer to center than boundary of circle i.e., smaller than radius $= \frac{1}{2}$.

83. (*d*) Let the probability that C wins = x

Then, probability that B wins =$2x$

and probability that A wins = $4x$

Then, sum of all probabilites for an event to happen is equal to 1

$\Rightarrow x + 2x + 4x = 1$

$\Rightarrow 7x = 1 \quad \Rightarrow \quad x = \frac{1}{7}$

$\therefore$ Probability that *C* wins = 1/7

Similarly, probability that *A* wins = 4/7

Therefore, the probability that *A* loses $= 1 - \frac{4}{7} = \frac{3}{7}$.

84. (*b*) Probability that A wins $= \frac{6}{12} = \frac{1}{2}$

Probability that B wins $= \frac{4}{12} = \frac{1}{3}$

Probability that game is drawn $= \frac{1}{6}$

Now, the probability that A wins 1st game in series of 3 games

$$\frac{1}{2}\times\frac{1}{3}\times\frac{1}{2} = \frac{1}{12}$$

Also, the probability that B wins 1st in series of 3 games

$$\frac{1}{3}\times\frac{1}{2}\times\frac{1}{3} = \frac{1}{18}$$

Now, probability that they win alternately

$$= \frac{1}{12}+\frac{1}{18} = \frac{3+2}{36} = \frac{5}{36}.$$

85. (*c*) Let there be 10 positions of the car, where they can be parked i.e., 1, 2 ..., 10

Now, two have exactly 5 cars between the two particular cars their position must be at 1 and 7, 2 and 8, 3 and 9 and 4 and 10.

Now, these two cars can be arranged in $4 \times 2 = 8$ ways as their positions can change.

Remaining 8 cars can be arranged in = 8! ways

Total number of arrangements = 10! ways

$\therefore$ Required probability $= \frac{8 \times 8!}{10!}$.

86. (*a*) Bag I contains 3 black and 4 white balls.

Bag II contains 4 black and 3 white balls.

P(bag I is chosen)

$= \frac{2}{6} = \frac{1}{3}$

P (bag II is selected)

$= \frac{2}{3}$

P (black ball is drawn from bag I) $= \frac{3}{7}$

P (black ball is drawn from bag II) $= \frac{4}{7}$

$\therefore$ *p* (black ball)

$= \frac{1}{3} \times \frac{3}{7} + \frac{2}{3} \times \frac{4}{7}$

$= \frac{3}{21} + \frac{8}{21} = \frac{11}{21}$.

87. (*a*) Total number of integers = 20

Number of ways in which 2 integers can be selected = ${}^{20}C_2 = 190$

For sum to be odd, one integer must be odd and the other even.

$\therefore$ Number of ways in which 1 odd and 1 even integer can be selected = 10 × 10 = 100

$\therefore P(\text{sum is odd}) = \frac{100}{190} = \frac{10}{19}$.

88. (*a*) *P* (getting a tail on the fifth toss) $= \frac{1}{2}$.

89. (*d*) The sum of faces of dice is either 4 or less, so such ways are (1, 1), (2, 1), (1, 2), (2, 2), (3, 1), (1, 3) = 6

Total number of ways in which only one dice shows 2 are (2, 1), (1, 2) = 2

$\therefore$ Required probability that only one dice shows two $= \frac{2}{6} = \frac{1}{3}$.

90. (*a*) There are 13 hearts in the deck of 52 cards.

We are given, a person stops when he draws a heart and for that he make 3 trials.

In the first trial does not get a heart.

So, probability $= \frac{39}{52}$ ($\because$ 39 cards are non-heart cards)

Again in second trial, he does not get a heart, so probability $= \frac{39}{52}$ and in the last trial he picked a heart card so probability $= \frac{13}{52}$

$\therefore$ Required probability, so that he has to make 3 trials

$= \frac{39}{52} \times \frac{39}{52} \times \frac{13}{52} = \frac{9}{64}$.

91. (*c*) Let A be the event that contractor gets the plumbing contract.

Let B be the event that contractor gets the electric contract.

Given, $P(A) = \frac{2}{3}$, $P(B) = \frac{4}{9}$ and, $P(A \cup B) = \frac{4}{5}$

To find: $P(A \cap B) = ?$

We have,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{4}{5} = \frac{2}{3} + \frac{4}{9} - P(A \cap B)$

$\Rightarrow P(A \cap B) = \frac{10}{9} - \frac{4}{5} = \frac{14}{45}$.

92. (*c*) No. of candidates for first post = 5

No. of candidates for 2nd post = 8

No. of candidates for 3rd post = 7

P (getting 1 st post) $= \frac{1}{5}$, P (getting 2nd post) $= \frac{1}{8}$

and P (getting 3 rd post) $= \frac{1}{7}$,

$\therefore$ P (not getting 1 st post) $= \frac{4}{5}$,

P (not getting 2 nd post) $= \frac{7}{8}$ and

P (not getting 3 rd post) $= \frac{6}{7}$

$\therefore$ P (getting at least one pest) $= 1 - \left(\frac{4}{5} \times \frac{7}{8} \times \frac{6}{7}\right) =$

$1 - \frac{3}{5} = \frac{2}{5}$.

Difficulty Level-2

1. (*c*) Total number of ways = $^{10}C_3 = 720$

$$\text{Probability} = \frac{1}{720}.$$

2. (*c*) Probability that the lock is opened in a trial is $1/n$ [Since there is exactly one key, which opens the lock]

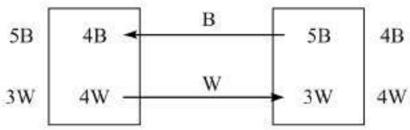
$\therefore$ The chance that the lock is not opened in a particular trial $= 1 - \frac{1}{n}$

P (lock is opened in n trials)

$= 1 - P$ (lock is not opened in n trials)

$$= 1 - \left(1 - \frac{1}{n}\right)^n = 1 - \left(\frac{n-1}{n}\right)^n.$$

3. (*b*) Hence black from bucket 1 and white from bucket 2 will be transferred



$\Rightarrow$ Required probability $= \frac{5}{8} \times \frac{4}{9} = \frac{20}{72} = \frac{5}{18}$.

4. (*c*) The last digit of the product will be 1, 2, 3, 4, 6, 7, 8 or 9 if and only if each of the n positive integers ends in any of these digits. Now the probability of an integer ending 1, 2, 3, 4, 6, 7, 8 or 9 is 8/10. Therefore, the probability that the last digit of the product of n integers is 1, 2, 3, 4, 6, 7, 8, or 9 is $\left(\frac{4}{5}\right)^n$. The probability for an integer to end in 1, 3, 7 or 9 is 4/10 = 2/5. Therefore the probability for the product of n positive integers to end in 1, 3, 7 or 9 is $\left(\frac{12}{5}\right)^n$

Hence, the required probability $= \left(\frac{4}{5}\right)^n - \left(\frac{2}{5}\right)^n$

$$= \frac{4^n - 2^n}{5^n}.$$

5. (*b*) Since there are just four matches to be played. India can get a maximum of 8 points

$\therefore$ P (India gets at least 7 points)

$= P$ (getting exactly 7 points)

$+ P$ (getting exactly 8 points)

$= P$ (getting 2 in each of the 3 matches and 1 in one match) $+ P$ (getting 2 in each of the four matches)

$= {}^4C_3\,(0.5)^3\,(0.05) + {}^4C_4\,(0.5)^4$

$= 0.025 + 0.0625 = 0.0875.$

6. (*a*) The required probability $= P$ (1st is black) $\times P$ (2nd is black)

$$= \frac{^5C_1}{^8C_1} \times \frac{^5C_1}{^8C_1} = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}.$$

7. (*b*) Odds against an event are 5 to 2 means that if total outcomes are (5 + 2), unfavourable ones will be 5

$\Rightarrow$ The chance that the first event fails = 5/7

The chance that the second fails = 5/11

$\Rightarrow$ The chance that both fail $= \frac{5}{7} \times \frac{5}{11} = \frac{25}{77}$ because the two events are independent.

$\Rightarrow$ The probability that both do not fail, i.e., at least one occurs $= 1 - \frac{25}{77} = \frac{52}{77}$.

8. (*b*) Required probability

$= {}^3C_1\,(0.4)\,(0.6)^2 + {}^3C_2\,(0.4)^2\,(0.6) + {}^3C_3\,(0.4)^3$

$= 3\,(0.144) + 3\,(0.096) + 1\,(0.064) = 0.784.$

9. (*d*) February is the only month having 28 or 29 days.

Probability of choosing February $= \frac{1}{12}$.

10. (*b*) Ram may draw 3 prizes in $^3C_3$ ways = 1 way or he may draw 2 prizes and 1 blank in $^3C_2 \times {}^6C_1$ ways

$$= \frac{3 \times 2}{1 \times 2} \times 6 = 18 \text{ ways}$$

or he may draw 1 prize and 2 black in $^3C_1 \times {}^6C_2$

$$= 3 \times \frac{6 \times 5}{1 \times 2} = 45 \text{ ways}$$

Hence, the total number of ways in which Ram can win = 1 + 18 + 45 = 64 ways

The total number of ways when the prices can be won $= {}^9C_3$ ways

$$= \frac{9 \times 8 \times 7}{1 \times 2 \times 3} = 84 \text{ ways}$$

Therefore, Ram's chance of success

$$= \frac{64}{84} = \frac{16}{21}$$

Mohan's chance of success is clearly $\frac{1}{3}$

Therefore Ram's chance of success:Mohan's chance of success $= \frac{16}{21} : \frac{1}{3} = 16{:}7.$

11. (*d*) Probability of forming an incorrect digit = p

$\therefore$ Probability of forming a correct digit = $1 - p$

$\therefore$ Probability of forming 8 correct digits = $(1-p)^8$

Hence, required probability = $1-(1-p)^8$.

12. (*d*) Total names in the lottery

$$= 3 \times 100 + 2 \times 150 + 200 = 800$$

Number of year–II names = $2 \times 15 = 300$

$$\text{Probability} = \frac{300}{800} = \frac{3}{8}.$$

13. (*c*) The probability of hitting in one shot. = $\frac{10}{100} = \frac{1}{10}$

$\therefore$ The probability of not hitting in one shot

$$= \left(1 - \frac{1}{10}\right)$$

If he fires n shots, the probability of hitting at least

once = $1 - \left(1-\frac{1}{10}\right)^n = 1 - \left(\frac{9}{10}\right)^n = \frac{1}{2}$ (as given)

$$\left(\frac{9}{10}\right)^n = \frac{1}{2}$$

$\Rightarrow n(\log 9 - \log 10) = \log 1 - \log 2$

$\Rightarrow \quad n(2\log 3 - 1) = -\log 2$

$$\Rightarrow \quad n = \frac{\log 2}{1 - 2\log 3} = \frac{0.3010}{1 - 2(0.4771)}$$

$$= \frac{0.3010}{1 - 0.9542}$$

$$= \frac{0.3010}{0.0458} = 6.6 \text{ (nearly)}$$

$\therefore$ For 6 shots, the probability is about 53% and for 7 shots it is nearly 48%. Hence $n = 7$.

14. (*d*) The number of ways in which the series can end for India are 12.

D – Draw,	L – Lose,	W – Win
i.e., D L L,	L D L,	L L D,
D L W,	L D W,	L W D,
D W L,	W D L,	W L D,
D W W,	W D W,	W W D,

$\therefore$ Probability that India wins the series by winning 2 consecutive matches

= Probability that India wins 2 consecutive matches

i.e., W W D, D W W = $\frac{2}{12} = \frac{1}{6}$.

15. (*c*) If one angle is 60 degrees then the other will be 30 degrees in the right triangle.

16. (*a*) $\therefore$ Probability that side of the cube is not painted

$$= \frac{8}{64} = \frac{1}{8}.$$

17. (*b*) Number of accidents $x = a + by$

where y is number of overtime hour

$\therefore \quad 8 = a + 1000b$

$5 = a + 400b$

$\Rightarrow \quad b = \frac{1}{200}, a = 3$

For $y = 0, x = a = 3$.

18. (*d*) February is the only month having 3 days less than maximum of 31.

$\therefore$ Probability of choosing February = $\frac{1}{12}$.

19. (*c*) The two equal angles of an isosceles triangle can be 1°, 2°, 3°... ... 89°. Only when the two equal angles are 60°, the triangle will be equilateral.

$\therefore \quad$ Required probability = $\frac{1}{89}$.

20. (*b*) Total no. of cases for which, we get sum as 15 = 10 of these '4' appear in the first roll only in 2 cases

$$\left[\begin{array}{ccc|ccc} 4, & 5, & 6, & 6, & 5, & 4, \\ 4, & 6, & 5, & 3, & 6, & 6, \\ 5, & 4, & 6, & 6, & 3, & 6, \\ 5, & 6, & 4, & 6, & 6, & 3, \\ 6, & 4, & 5, & 5, & 5, & 5, \end{array}\right]$$

$\therefore$ Required probability = $\frac{2}{10} = \frac{1}{5}$.

21. (*a*) Total no. of instances = 120

Numbers divisible by 4 will end with 12, 24, 32 and 52 total 24 such numbers will be there.

22. (*b*) The sum of numbers can be 15 in the following three ways.

Case (1): 15 = 3 + 6 + 6

The first, second and third throws can be (3, 6, 6), (6, 3, 6) and (6, 6, 3) respectively.

$\therefore$ Total number of ways in which 3, 6 and 6 can be obtained is 3.

Case (2): 15 = 4 + 5 + 6

The first, second and third throws can be either of 4, 5 and 6.

$\therefore$ Total number of ways in which 4, 5 and 6 can be obtained is 6.

Case (3): $15 = 5 + 5 + 5$

The first, second and third throws can be 5, 5 and 5.

$\therefore$ Total number of ways in which 5, 5 and 5 can be obtained is 1.

$\therefore$ The total number of ways in which the sum of throws can be 15 is

$$3 + 6 + 1 = 10$$

The total number of ways in which the first roll will be a 4 is 2.

$$\therefore \quad \text{Required chance} = \frac{2}{10} \quad \frac{1}{5}$$

Hence, option (*b*).

23. (*a*) A number divisible by 4 formed using the digits 1, 2, 3, 4 and 5 has to have the last two digits 12 or 24 or 32 or 52.

In each of these cases, the five-digit number can be formed using the remaining 3 digits in

$$3 \times 2 \times 1 = 6 \text{ ways}$$

$\therefore$ A number divisible by 4 can be formed in

$$6 \times 4 = 24 \text{ ways}$$

Total numbers that can be formed using the digits 1, 2, 3, 4 and 5 without repetitions $= 5! = 120$

$$\therefore \quad \text{Required probability} = \frac{24}{120} = \frac{1}{5}$$

Hence, option (*a*).

24. (*a*)
$$P(G \mid E_1, E_2) = P(G \mid E_1) \times P(G \mid E_2)$$
$$= 0.6 \times 0.7 = 0.42.$$

25. (*b*) The probability of any one winning the race given that dead heat is possible is P(A wins, B loses and C loses) + P(B wins, A loses and C loses) + P(C wins, B loses and A loses)

i.e., $\frac{1}{6} \times \frac{3}{5} \times \frac{9}{13} + \frac{5}{6} \times \frac{2}{5} \times \frac{9}{13} + \frac{5}{6} \times \frac{3}{5} \times \frac{4}{13}$

$$= \frac{177}{390} = \frac{59}{130}.$$

26. Options (*a*, *b*, *c*) are correct.

27. (*c*) 1 person in 100 person suffers from disease.

It means 99% persons are healthy.

= Probability that a person is healthy is

$$\left(\frac{99}{100}\right) = (0.99)$$

In a group of 50 people if the test is positive, then we have to perform 50 tests individually on each person and 1 group test

So, it means total 51 tests

So, probability that in a group of 50 people all are healthy $= {}^{50}C_{50} \times (0.99)^{50}$

$$= 1 \times (0.99)^{50} \approx 0.605$$

So, probability that in a group of 50 people at least one people suffers $= 1 - 0.605 = 0.395$

$\therefore$ Expected no. of tests $= 51 \times 0.395 + 0.605 \times 1$

$$= 20.145 + 0.605$$
$$= 20.75$$
$$\approx 21 \text{ tests.}$$

28. (*c*) We know that the probability that a particular number will come on a dice is $\frac{1}{6}$ and the probability that a particular number will not come on a dices is $\frac{5}{6}$

Now in the question there are 3 dices. So, probability that picked number will not come in any of 3 dices is $\left(\frac{5}{6}\right)^3$

And we know that we will lose in only this condition that our picked number will not come on any 3 dices

$$\therefore \quad \text{Probability of losing} = \left(\frac{5}{6}\right)^3 \approx 0.58$$

$$\therefore \quad \text{Probability of winning} = 1 - 0.58$$
$$= 0.42.$$

29. (*c*) For the equation to be satisfied the sum of three probabilities should be less than 1 and each probability should be positive (greater than 0). Putting these equalities we get the set of possible value as $\frac{1}{3}, \frac{1}{2}$.

30. (*b*) The percentage of the three different types of policy holders and the corresponding probability of dying in the next 1 year are as follows:

Type	Standard	Preferred	Ultra - preferred
Precentage	50	30	20
Probability	0.01	0.008	0.007

The expected number of deaths among all the policy holders of the given age (say X) during the next year

$$= T\left[\frac{50}{100}(0.01) + \frac{30}{100}(0.008) + \frac{20}{100}(0.007)\right]$$

T = Total number of policy holders of age X

$$= \frac{T}{100}(0.88)$$

If any of these policy holders (who die during the next year) is picked at random, the probability that he is a preferred policy holder is

$$\frac{30 \times 0.008 \times \frac{T}{100}}{0.88 \times \frac{T}{100}} = \frac{24}{88}$$

$$= \frac{3}{11} = 0.2727.$$

31. (*a*) At least one junior professor is to be selected,

$$n(r) = {}^4C_1 \times {}^6C_2 + {}^4C_2 \times {}^6C_1 + {}^4C_3$$

$$= 60 + 36 + 4 = 100$$

$$n(s) = {}^{10}C_3 = 120$$

$$p(r) = \frac{n(r)}{n(s)} = \frac{100}{120} = \frac{5}{6}.$$

32. (*b*) Let on packets of milk be prepared in unit time at the normal speed

∴ No. of packets of milk produced at fast speed $1.1m$ $(0.6)t = .66mt$

Now, at the normal speed in t time, the number of packets of milk that would be produced = mt

The target for the supervisor is = mt packets

∵ 0.66 mt packets are produced at fast speed the remaining 0.34 mt packets will be produced at show speed (normal speed)

Let probability of a packet being damaged at normal speed be p, ∴ at fast speed be = $2p$

∴ The probability that a pack selected at random will be damaged = $p\frac{(0.34\,mt)}{mt} + \frac{2p(.66\,mt)}{mt}$

$\Rightarrow \quad p\,(0.34) + p\,(1.32) = 0.112$

$\Rightarrow \quad 1.66p = .112$

$\Rightarrow \quad p = 0.0674$

∴ The probability that a packet will not be damaged at normal speed = 1 – 0.0674 = 0.93.

33. (*b*) Let the no. of matches India needs to play is 'n'.

Now, if $1 - (5/6)^1 = 1/2$, we can consider that India has a fair chance of winning a match $(5/6)^n = 1/2$, for $n = 4$, we get $\frac{625}{1296}$ which is less than 2.

34. (*d*) The mechanic can check two machines. The possible outcomes and the corresponding probabilities are tabulated below. A defective machine is denote as d and a non-defective as n.

	Outcome	Probability
1	d d	$\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{6}$
2	d n	$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{1}{3}$
3	n d	$\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{1}{6}$
4	n n	$\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{1}{3}$

In the first and third cases, the mechanic would have identified the defective machines in time to catch the bus. The probability that he is able to catch the last bus

$$= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}.$$

35. (*a*) The probabilities are tabulated below.

Major	Minor	
North	0.418	0.612
South	0.355	0.520

∴ The probability that there is no crime at all is

(1 – 0.418) (1 – 0.612)

(1 – 0.355) (1 – 0520)

= (0582) (0.3880) (0.645) (0.480) ≈ 0.0699.

36. (*d*) Shyam can win only if his card is greater than Ram's card. The required probability is

$$= \left(\frac{48}{51} + \frac{44}{51} + \frac{40}{51} \cdots \frac{4}{51} + \frac{0}{51}\right) \times \frac{1}{13} = 0.47$$

If he plays first, his probability of winning is 0.53 (the difference is due to the fact that in case of cards being equal first player wins).

37. (*b*) The man can enter any room with a probability of $\frac{1}{2}$.

The probability that he picks up a green bag is

(i) $\frac{4}{3+4+5} = \frac{4}{12} = \frac{1}{3}$ in one room,

(ii) $\frac{1}{2+1+3} = \frac{1}{6}$ in the other room

So, the required probability

$$= \frac{1}{2}\left(\frac{1}{3} + \frac{1}{6}\right) = \frac{1}{2}\left(\frac{3}{6}\right) = \frac{1}{4}.$$

38. (*b*) The total number of distinct photographs is the total number of distinct arrangements of these individuals which is 13!

No two Indians will be together, if the arrangement is as below

I × I × I × I × I × I × I

where I stands for an Indian and X for a non-Indian. In such an arrangement, the Indians can be arranged in 7! ways, while the others can be arranged in 6! ways.

So, the required probability = $\frac{7!6!}{13!}$.

39. (*b*) Least 5-digit perfect square = 10000

Least 4-digit perfect square = 322 = 1024

Therefore, 100 – 32 = 68 four-digit integers are perfect squares.

$\Rightarrow \quad \text{Probability} = \frac{68}{9000} = \frac{17}{2250}$.

40. (*d*)

41. (*c*) If he draws any combination of 3 socks he will definitely have the matching pair of either colour.

42. (*d*) Probability that trouser is not black = $\frac{2}{3}$

Probability that shirt is not black = $\frac{3}{4}$

$\therefore$ Required probability = $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$.

43. (*d*) It is given that last 3 digits are randomly dialled. Then, each of the digit can be selected out of 10 digits in 10 ways. Hence, required probability

$$= \left(\frac{1}{10}\right)^3 = \frac{1}{1000}.$$

44. (*c*) The product of four numbers will be positive in the following ways:

1. All the four numbers are positive, hence probability

$$= \frac{{}^6C_4}{{}^{14}C_4}$$

2. All the four numbers are negative hence probability

$$= \frac{{}^8C_4}{{}^{14}C_4}$$

3. Two numbers are positive and two are negative

hence probability = $\frac{{}^6C_2 \times {}^8C_2}{{}^{14}C_4}$

Hence, required probability of the event

$$= \frac{{}^6C_4 + {}^8C_4 + {}^6C_2 \times {}^8C_2}{{}^{14}C_4}$$

$$= \frac{15 + 70 + 15 \times 28}{1001} = \frac{505}{1001}.$$

45. (*a*) Total number of events = 20

Probability of running the program correctly in the third run = $\frac{5}{20} = \frac{1}{4}$.

46. (*b*) Probability of sending a correct programme

$$= 1 - \frac{1}{5} = \frac{4}{5}$$

Probability that package is not damaged

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

Probability that there is not a short shipment

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

$\therefore \quad$ Required probability = $\frac{4}{5} \times \frac{1}{4} \times \frac{2}{3}$

$$= \frac{2}{15} = \frac{8}{60}.$$

47. (*a*)

48. (*a*)

	1	2	3
Black	6	3	5
White	4	7	5

Each box is equally likely to be selected after that each balls is equally likely to be selected

$\therefore$ All the balls have same probability to be selected

$\therefore$ Required probability = $\frac{3}{6+3+5} = \frac{3}{14}$.

49. (*a*) $\quad n(S) = 4 \times 5 = 20$

Here, $\quad n(E) = 4$

$\therefore \quad P(E) = \frac{4}{20} = \frac{1}{5} = 0.20.$

50. (*b*) 250 numbers between 101 and 350 i.e., $n(S) = 250$

$\therefore \quad n(E)$ = 100 digit of 2 = 299 – 199 = 100

$\therefore \quad p(E) = \frac{100}{250} = 0.4.$

51. (*c*) Total number of boys and girls = $4n$

Since, there are two equal batches

So that each batches has $2n$ members

$\therefore$ S = Selection of $2n$ member out of $4n$

$$\therefore \quad n(S) = {}^{4n}C_{2n}$$

Also, in each batch there are 'n' boys and 'n' girls

$\therefore$ E = events of selection of n boys out of $2n$

$$\therefore \quad n(E) = {}^{2n}C_n \times {}^{2n}C_n$$

$$= ({}^{2n}C_n)^2$$

$$\therefore \quad P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{({}^{2n}C_n)^2}{{}^{4n}C_{2n}}.$$

52. (*a*) Since LO is the angle bisector of $\angle MLN$ and the median of MN, by interior angle bisector theorem,

$$\frac{LM}{LN} = \frac{MO}{ON} \text{ and } MO = ON$$

Therefore,

$$\frac{LM}{LN} = 1$$

$\Rightarrow LM = LN$

Therefore, ΔLMN is an isosceles triangle

$\Delta LOM \cong LON$ (s-s-s test)

$\therefore \angle LON = \angle LOM = 90°$

In right-angled ΔLOM with $LM = 5$ and $LO = 3$, $MO = 4$

$$\therefore \text{Area of } \Delta LMN = \frac{1}{2} \times MN \times LO$$

$$= \frac{1}{2} \times 2 \times 4 \times 3 = 12 \text{ cm}^2.$$

53. (*c*) Six different flowers and a large pendant are seven different things that are to be arranged in a circular manner

It can be done in $(7-1)! = 6! = 720$ ways.

54. (*d*) The probability of at least one red ball

= 1 – (probability of all blue balls)

$$= 1 - \frac{1}{16} = \frac{15}{16}.$$

55. (*c*) Let A be the event getting exactly 3 defective pens out of 8 pens examined and B be the event of getting 9th pen defective

Then probability that the 9th pen examined is the last defective pen is

$$P(A \cap B) = P(A)P\left(\frac{B}{A}\right)$$

Now, $P(A) = \dfrac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8}$

and, $P\left(\frac{B}{A}\right)$ = probability that 9th pen examined is defective, given that there were 3 pens defective in first 8 pens examined $= \frac{1}{7}$

Therefore, the required probability

$$= \frac{{}^4C_3 \times {}^{11}C_5}{{}^{15}C_8} \times \frac{1}{7} = \frac{8}{195}.$$

56. (*d*) Probability that Varun gets a sum of 7 = $P(A)$ = $\frac{6}{36} = \frac{1}{6}$

Probability that Tarun gets a sum less than $7 = P(B) = \frac{15}{36} = \frac{5}{12}$

$$\text{Now, } P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)}$$

($\because$ A and B are independents)

$$= P(B) = \frac{5}{12}.$$

57. (*d*) Let A = Event of manufacturing on machine A

B = Event of manufacturing on machine B

C = Event of manufacturing on machine C

D = Event of manufacturing a defective item

$$\therefore \quad P(A) = P(B) = P(C) = \frac{1}{3}$$

$$\text{Now, } P\left(\frac{D}{A}\right) = \frac{5}{25}, P\left(\frac{D}{B}\right) = \frac{4}{35}, P\left(\frac{D}{C}\right) = \frac{2}{40}$$

Now,using-bayes theorem,

$$P\left(\frac{B}{D}\right) = \frac{P\left(\frac{D}{B}\right).P(B)}{P\left(\frac{D}{B}\right).P(A) + P\left(\frac{D}{B}\right).P(B) + P\left(\frac{D}{C}\right).P(C)}$$

$$= \frac{\frac{4}{35} \times \frac{1}{3}}{\frac{5}{25} \times \frac{1}{3} + \frac{4}{35} \times \frac{1}{3} + \frac{2}{40} \times \frac{1}{3}}$$

$$\Rightarrow \quad P\left(\frac{B}{D}\right) = \frac{\frac{4}{35}}{\frac{5}{25} + \frac{4}{35} + \frac{2}{40}} = \frac{\frac{4}{35}}{\frac{280 + 160 + 70}{25 \times 56}}$$

$$= \frac{4 \times 25 \times 56}{35 \times 510} = \frac{16}{51}.$$

58. (*b*) Probability of getting 6 while throwing a pair of dice $= \frac{5}{36}$

Probability of getting 7 while throwing a pair of dice

$= \frac{6}{36}$

Therefore, probability of winning of A

$= \frac{5}{36} + \frac{31}{36} \times \frac{30}{36} \times \frac{5}{36} + \frac{31}{36} \times \frac{30}{36} \times \frac{31}{36} \times \frac{30}{36} \times \frac{5}{36} + \ldots$

$= \frac{5}{36}\left[\frac{1}{1 - \frac{31}{36}\frac{30}{36}}\right] = \frac{5}{36}\left[\frac{1296}{1296 - 930}\right]$

$= \frac{5}{36} \times \frac{1296}{366} = \frac{5 \times 36}{366} = \frac{5 \times 6}{61} = \frac{30}{61}$

And probability of winning of B

$= 1 -$ probability of winning of A

$= 1 - \frac{30}{61} = \frac{31}{61}$.

59. (*b*) $P(X \cup Y) = 0.6$

$\Rightarrow \quad 1 - P(x \cap y) = 0.4$

$\Rightarrow \quad P(x \cap y) = 0.6$

Now, $P(x \cup y) = P(x) + P(x) - P(x \cap y)$

$= 0.7 + 0.5 - 0.4 = 0.8$.

60. (*b*) $P(1 \text{ or } 3) = \frac{1}{6} + \frac{1}{6} + = \frac{2}{6} = \frac{1}{3}$

$P(2 \text{ or } 4 \text{ or } 5 \text{ or } 6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$P(\text{black ball from first bag}) = \frac{3}{7}$

$P(\text{black ball from second bag}) = \frac{4}{7}$

$\therefore$ Required probability $= \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7} = \frac{11}{21}$.

61. (*c*) $P(A)$ = Probability that A hits the target $= \frac{4}{5}$

$P(B)$ = Probability that B hits the target $= \frac{3}{4}$

$P(C)$ = Probability that C hits the target $= \frac{2}{3}$

Required probability

$= P(\bar{A})\,(P(B)\,P(C) + P(A)P(\bar{B})P(C)$
$+ P(A)P(B)P(C) + P(A)P(B)P(C)$

$= \frac{1}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} + \frac{4}{5} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{1}{3} + \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3}$

$= \frac{6 + 8 + 12 + 24}{60}$

$= \frac{50}{60} = \frac{5}{6}$.

62. (*b*) We have, P (getting an odd number) = $2P$ (getting an even number)

$\therefore P(\text{odd number}) = \frac{2}{3}$ and $P(\text{even number}) = \frac{1}{3}$

$\therefore P(\text{getting an odd number}) = \frac{2}{9}$

$P(\text{getting an even number}) = \frac{1}{9}$

$\therefore P(\text{getting a two-digit prime number}) = 2 \times \frac{2}{9} \times \frac{1}{9} = \frac{4}{81}$.

63. (*d*) $10 = 5 \times 2$

$10^{29} = 5^{29} \times 2^{29}$

$\therefore$ Number of divisors of (10)29 = (29 + 1) (29 + 1) = 30 × 30. We need to find all the divisors of m such that $10^{29} = m \times 10^{23}$

$m = 10^6 = 5^6 \times 2^6$

$\therefore$ Number of divisors of m = (6 + 1) (6 + 1) = 7 × 7

The probability that a randomly chosen positive divisor of 10^{29} is an integer multiple of 10^{23}

$\frac{7 \times 7}{30 \times 30} = \frac{a^2}{b^2}$

$\therefore a = 7$ and $b = 30$

$\therefore b - a = 30 - 7 = 23$.