Real Numbers

Start From Here

Irrational Numbers

$\pi, \sqrt{2}$

Can't be represented as $x = p/q$, Where p & q are integers and $q \neq 0$. Decimal representation is non terminating & non-recurring.

Rational Numbers

1, 1/2, –7/5...

Can be represented as $x = p/q$, Where p & q are integers and $q \neq 0$. Decimal representation is either terminating or non-terminating & recurring

Terminating

$x = p/q$ and $q \neq 0$ (q is in the form of $2^m 5^n$ where m, n are non-negative integers, then it is terminating decimal)

e.g. $\frac{3}{2^2 \times 5} = 0.15$

Non-Terminating

$x = p/q$ and $q \neq 0$ (q is not in the form of $2^m 5^n$ where m, n are non-negative integers, then it is non- terminating decimal)

e.g. $\frac{10}{3} = 3.\overline{3}$

Euclid division Lemma

Given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$, $0 \leq r < b$.

Euclid division Algorithm

If 'a' and 'b' are positive integers such that $a = bq + r$, then every common divisor of 'a' and 'b' is a common divisor of 'b' and 'r', and vice-versa.

e.g. HCF of 420 & 48

$420 = 48 \times 8 + 36$

$48 = 36 \times 1 + 12$

$36 = 12 \times 3 + 0$

$\Rightarrow$ H.C.F. of 420 & 48 is 12

Fundamental theorem of Arithmetic

Every composite number can be expressed or factorized as a product of primes.

e.g. $48 = 2 \times 2 \times 2 \times 2 \times 3$

$420 = 2 \times 2 \times 3 \times 5 \times 7$

Application

H.C.F

e.g. $48 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{3}$

$420 = \underline{2} \times \underline{2} \times \underline{3} \times 5 \times 7$

H.C.F $= 2 \times 2 \times 3$

L.C.M

e.g. $48 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{3}$

$420 = 2 \times 2 \times 3 \times \underline{5} \times \underline{7}$

L.C.M $= 2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 1680$

H.C.F. (48, 420) × L.C.M. (48,420) = 48 × 420

(This is true for 2 numbers only)

Some Important Results :

1. Let 'p' be a prime number and 'a' be a positive integer. If 'p' divides a^2, then 'p' divides 'a'.
2. HCF × LCM = Product of two numbers.
3. LCM is always divisible by HCF.

e.g. Check whether 6^n can end with the digit 0 for any natural number 'n'.

Sol. Any positive integer ending with the digit zero is divisible by 5 and so its prime factorization must contain the prime 5.

$6^n = (2 \times 3)^n = 2^n \times 3^n$

$\Rightarrow$ The prime in the factorization of 6^n is 2 and 3.

$\Rightarrow$ 5 does not occur in the prime factorization of 6^n for any n.

$\Rightarrow$ 6^n does not end with the digit zero for any natural number n.