CHAPTER 16

Linear Equations in One and Two Variables

Linear Equations in One Variable

Equation An equation is a statement of equality of two algebraic polynomial (monomials) involving one or more variables.

Linear Equations The expression of the form $Ax+B$, where A and B are real numbers and $A \neq 0$, is a linear polynomial and equation involving only linear polynomial are called as linear equations.

- Graph of linear equation is a straight line.
- Linear equations involving only one variable is called a linear equation in that variable.

e.g., (i) $5x+8=9-x$

(ii) $\frac{2}{3}y+7=\frac{y}{2}$

(iii) $t+3t=9-t$

all are linear equations in one variable.

Rules for Solving a Linear Equation

1. If same number is added to both the sides of an equation, the equality remains same.
2. If same number is subtracted to both the sides of an equation, the equality remains the same.
3. If same number is multiplied to both the sides of the equation, the equality remains the same.
4. If both sides are divided by a some non-zero number the equality remains the same.

- Any term can be taken to the other side with its sign changed, without affecting the equality.

Example 1. Solve

$$2(x-3)-(5-3x)=3(x+1)-4(2+x)$$

and the value of x is

(a) 1 (b) −1 (c) 0 (d) 3

Sol. (*a*) $2(x-3)-(5-3x)=3(x+1)-4(2+x)$

$\Rightarrow \quad 2x-6-5+3x=3x+3-8-4x$

$\Rightarrow \quad 5x-11=-x-5 \Rightarrow 6x=6 \Rightarrow x=1$

Example 2. Solve $\frac{2}{x-3}+\frac{3}{x-4}=\frac{5}{x}$, where $x\neq 3, x\neq 4$ and $x\neq 0$ and the value of x is

(a) $2\frac{1}{3}$ (b) $3\frac{1}{3}$ (c) $3\frac{1}{2}$ (d) $3\frac{1}{4}$

Sol. (*b*) $\frac{2}{x-3}+\frac{3}{x-4}=\frac{5}{x}$

$\Rightarrow \quad \frac{2(x-4)+3(x-3)}{(x-3)(x-4)}=\frac{5}{x}$

$\Rightarrow \quad [2(x-4)+3(x-3)]x=5[(x-3)(x-4)]$

$\Rightarrow \quad (2x-8+3x-9)x=5(x^2-4x-3x+12)$

$\Rightarrow \quad (5x-17)x=5(x^2-7x+12)$

$\Rightarrow \quad 5x^2-17x=5x^2-35x+60$

$\Rightarrow \quad -17x+35x=60$

$\Rightarrow \quad 18x=60$

$\Rightarrow \quad x=\frac{60}{18}=\frac{10}{3}=3\frac{1}{3}$

So, $x=3\frac{1}{3}$ is solution of the given equation.

Applications of Linear Equation

1. Problems on Mensuration

Example 3. The length of a rectangle is 8 cm more than its breadth. If the perimeter of the rectangle of 68 cm, then its length and breadth are

(a) 13 cm, 21 cm (b) 14 cm, 23 cm
(c) 19 cm, 20 cm (d) 9 cm, 15 cm

Sol. (*a*) Let the breadth of the rectangle be 'x'.

Then, its length $=(x+8)$ cm

$\therefore$ Perimeter of rectangle $=2[x+(x+8)]=2[2x+8]=4x+16$

$\therefore \quad 4x+16=68$

$4x=68-16=52$

$\Rightarrow \quad x=13$

Breadth of rectangle = 13 cm and length $=13+8=21$ cm

Example 4. The length of a room exceeds its breadth by 5 m. If the length be increased by 5 m and breadth decreased by 3 m, the area remains the same. Then, the length and breadth of the room are

(a) (20, 15) (b) (25, 20) (c) (16, 11) (d) (15, 10)

Sol. (*a*) Let the breadth of the room $= x$ m

$\therefore$ Length of the room $= (x+5)$ m

Length after increase $= [(x+5)+5] = (x+10)$ m

Breadth after decrease $= (x-3)$ m

According to question, $x(x+5) = (x+10)(x-3)$

$$x^2 + 5x = x^2 + 7x - 30$$

or $$x = 15\text{ m}$$

So, breadth $= 15$ m or length $= 15 + 5 = 20$ m

2. Problem on Ages

Example 5. A man is 30 yr older than his son. After 12 yr, the man be twice as old as his son. Then, their present ages.

(a) 18 yr and 48 yr (b) 16 yr and 42 yr
(c) 20 yr and 42 yr (d) None of these

Sol. (*a*) Let age of son $= x$ yr. Then, age of man $= x + 30$ yr

After 12 yr, Age of son $= (x+12)$ yr

Age of man $= (x + 30 + 12)$ yr

By condition, $x + 30 + 12 = 2(x+12)$

$$x + 42 = 2x + 24$$

$$42 - 24 = 2x - x \Rightarrow x = 18\text{ yr}$$

So, age of the son $= 18$ yr and age of the man $= 30 + 18 = 48$ yr

Example 6. A boy is now one-third as old as his father. Twelve years hence he will be half as old as his father. The present age of the boy and of his father is

(a) 6 yr and 22 yr (b) 8 yr and 21 yr
(c) 12 yr and 36 yr (d) None of these

Sol. (c) Let the present age of the father be x yr and that of the son be $\frac{1}{3}x$ yr.

After 12 yr,

Age of father $= (x+12)$ yr

Age of son $= \left(\frac{x}{3} + 12\right)$ yr

According to question,

$$\frac{1}{3}x + 12 = \frac{1}{2}(x+12) \Rightarrow \frac{x+36}{3} = \frac{x+12}{2}$$

$$\Rightarrow \quad 2(x+36) = 3(x+12)$$

$\Rightarrow 2x + 72 = 3x + 36 \Rightarrow x = 36$

$\therefore$ Present age of father $= 36$ yr

$\therefore$ Present age of son $= \frac{1}{3} \times 36 = 12$ yr

3. Problems on Time and Work

Example 7. Ram and Shyam together can do a piece of work in 8 days, which Ram alone can do in 12 days. In how many days can. Shyam alone will do the same work?

(a) 21 (b) 23
(c) 25 (d) None of these

Sol. (*d*) Let Shyam alone can do the work in x days.

Then, Shyam's one day's work $= \frac{1}{x}$

Ram's one day's work $= \frac{1}{12}$

Ram and Shyam's one day's work $= \frac{1}{8}$

Now, Ram's one day work + Shyam's one day work
= (Ram and Shyam)'s one day work

$$\therefore \quad \frac{1}{12} + \frac{1}{x} = \frac{1}{8} \Rightarrow \frac{1}{x} = \frac{1}{8} - \frac{1}{12}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{24} \Rightarrow x = 24$$

So, Shyam can finish the work in 24 days.

Example 8. *A* can do a work in 3 days less time than *B*. *A* works it alone for 4 days and then *B* takes over and completes it. If altogther 14 days were needed to complete the work, how many days does each of them take to do the work alone?

(a) 12 and 15 (b) 18 and 14
(c) 10 and 16 (d) None of these

Sol. (*a*) Suppose, *A* takes x days to complete the work and *B* takes $(x+3)$ days.

$\therefore$ *A*'s one day work $= \frac{1}{x}$

B's one day work $= \frac{1}{x+3}$

$\therefore$ *A*'s four days work $= \frac{4}{x}$

B's 10 days work $= \frac{10}{x+3}$

According to the condition given $\frac{4}{x} + \frac{10}{x+3} = 1$

$$\therefore \quad 4(x+3) + 10x = x^2 + 3x$$

$$\Rightarrow \quad x^2 - 11x - 12 = 0$$

$$\Rightarrow \quad (x-12)(x+1) = 0$$

$$\Rightarrow \quad x = 12 \text{ or } x = -1 \text{ (not possible)}$$

$\therefore$ *A* can do the work in 12 days and *B* can do the work in $12 + 3 = 15$ days.

4. Problems on Time and Distance

Example 9. A cycled from *P* to *Q* at 10 km/h and returned at the rate of 9 km/h *B* cycled both ways at 12 km/h. It was discovered that for the total journey, *B* took 10 km/h minutes less than *A*. The distance between *P* and *Q* is

(a) 3.0 km (b) 3.75 km
(c) 4.0 km (d) 4.75 km

Sol. (*b*) Let the distance between P and Q be x km.

We know that, Time $= \dfrac{\text{Distance}}{\text{Speed}}$

According to question,

Total time taken by $A = \dfrac{x}{10} + \dfrac{x}{9} = \dfrac{9x + 10x}{90} = \dfrac{19x}{90}$ h

Total time taken by $B = \dfrac{x}{12} + \dfrac{x}{12} = \dfrac{2x}{12} = \dfrac{x}{6}$ h

$\therefore \quad \dfrac{19x}{90} - \dfrac{x}{6} = \dfrac{10}{60}$ (by condition)

$\Rightarrow \quad \dfrac{19x}{90} - \dfrac{x}{6} = \dfrac{1}{6}$

$\Rightarrow \quad \dfrac{8x}{180} = \dfrac{1}{6} \Rightarrow x = \dfrac{1}{6} \times \dfrac{180}{8}$

$\Rightarrow \quad x = \dfrac{30}{8} = \dfrac{15}{4} = 3.75$ km

$\therefore$ Distance between P and Q is 3.75 km.

Example 10. A streamer goes downstream from one port to another in 4 h. It covers the same distance upstream in 5 h. If the speed of the steam be 2 km/h, the distance between the two ports is

(a) 10 km (b) 40 km
(c) 75 km (d) 80 km

Sol. (*d*) Let the speed of the streamer be x km/h

Speed of the stream = 2 km/h

$\therefore$ Speed of the streamer downstream = $(x + 2)$ km/h

Speed of the stream upstream = $(x - 2)$ km/h

Time $= \dfrac{\text{Distance}}{\text{Speed}}$ or Distance = Speed × Time

According to question,

During downstream the distance covered

$= (x + 2) \times 4 = (4x + 8)$ km

Also, during upstream the distance covered

$= (x - 2) \times 5 = (5x - 10)$ km

But distance coverd is the same

$4x + 8 = 5x - 10$

$4x - 5x = -18 \Rightarrow x = 18$ km/h

$\therefore$ Distance between the two ports $= (18 + 2) \times 4 = 80$ km

5. Problems on Numbers

Example 11. A number when added to its two-thirds is equal to 35. Then, the number is

(a) 16 (b) 21 (c) 12 (d) 18

Sol. (*b*) Let the number be x.

Then, according to question,

$$x + \frac{2}{3}x = 35 \Rightarrow \frac{3x + 2x}{3} = 35 \Rightarrow x = \frac{35 \times 3}{5} = 21$$

$\therefore$ The required number is 21.

Example 12. Divide 300 into two parts so that half of one part may be less than the other part by 48. The parts are

(a) 122 and 178 (b) 108 and 192
(c) 155 and 145 (d) 132 and 168

Sol. (*d*) Let the first part be x, then the second part is $(300 - x)$

According to question,

$\dfrac{1}{2}x = (300 - x) - 48 \Rightarrow \dfrac{1}{2}x = 300 - x - 48 \Rightarrow \dfrac{1}{2}x = 252 - x$

$\Rightarrow \quad 3x = 504 \Rightarrow x = \dfrac{504}{3} = 168$

$\therefore$ First part = 168 and,
Second part = 300 − 168 = 132

6. Problems on Interest, Profit and Loss

Example 13. By selling a car for ₹ 72000, a person made a profit of 20%. The cost price of the car was

(a) ₹ 40000 (b) ₹ 47000
(c) ₹ 55000 (d) ₹ 60000

Sol. (*d*) Let the cost price of the car be ₹ x and profit per cent = 20%

Selling price $= x \times \dfrac{(100 + 20)}{100} = \dfrac{120x}{100} = ₹\ \dfrac{6x}{5}$

But selling price = ₹ 72000

$\therefore \quad \dfrac{6}{5}x = 72000 \Rightarrow x = 60000$

Hence, the cost price of the car is ₹ 60000.

Example 14. On the occasion of Diwali, khadi bhandar allows a discount of 20% on all textiles and 25% on readymade garments. Hari paid ₹ 180 for a gown. The marked price of the gown was

(a) ₹ 220 (b) ₹ 270 (c) ₹ 320 (d) ₹ 240

Sol. (d) Let the marked price of the gown be ₹ x.

Discount allowed = 25%

(because gown is a readymade garments)

Selling price $= \dfrac{(100 - 25)}{100} \times x = ₹\ \dfrac{3}{4}x$

But selling price of the gown = ₹ 180

$\dfrac{3}{4}x = 180$

$x = \dfrac{180 \times 4}{3} \Rightarrow x = 240$

Hence, the marked price of the gown is ₹ 240.

Example 15. Aman invested ₹ 35000, a part of it at an annual interest rate of 12% and the rest at 14%. If he receives a total annual interest of ₹ 4460, how much did he invested at each rate?

(a) at 12% ₹ 20000 and at 14% ₹ 14000
(b) at 12% ₹ 22000 and at 14% ₹ 13000
(c) at 12% ₹ 25000 and at 14% ₹ 27000
(d) None of the above

Sol. (*b*) Let the amount invested at 12% be ₹ x, then the amount invested at 14% be ₹ $(35000 - x)$.

Interest $= \dfrac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$

Interest on ₹ x at 12% for 1 yr $= \dfrac{x \times 12 \times 1}{100} = ₹\ \dfrac{3x}{25}$

Interest on ₹ $(35000-x)$ at 14% for 1 yr

$$= \frac{(35000-x)\times 14\times 1}{100} = ₹\ \frac{(35000-x)7}{50}$$

$\therefore$ Total interest $= \frac{3x}{25} + \frac{7(35000-x)}{50}$

But total interest = ₹ 4460

$$\therefore \quad \frac{3x}{25} + \frac{7(35000-x)}{50} = 4460$$

$$\therefore \quad \frac{6x+245000-7x}{50} = 4460$$

$$245000 - x = 4460\times 50$$

$$-x = 223000 - 245000$$

$$-x = -22000 \Rightarrow x = 22000$$

Here, the amount invested at 12% = ₹ 22000

and the amount invested at 14% = ₹ $(35000-22000)$ = ₹ 13000

7. Miscellaneous Problems

Example 16. How much pure alcohol should be added to 600 mL of a 15% solution to make its strength 32%?

(a) 120 mL (b) 133 mL (c) 150 mL (d) 163 mL

Sol. (c) Quality of alcohol in 600 mL of 15% solution

$$= \left(\frac{15}{100}\times 600\right)\text{mL} = 90\,\text{mL}$$

Total quality of 32% alcohol solution $= (600+x)$ mL

Quality of alcohol in $(600+x)$ solution of 32%

$$= (600+x)\times\frac{32}{100}\text{mL} = \frac{(600+x)}{25}\times 8$$

$$= \left(\frac{4800+8x}{25}\right)\text{mL} = \left(192+\frac{8x}{25}\right)\text{mL}$$

Quality of alcohol needed $= 128 + \frac{8x}{25} - 90$

But $\quad 192 + \frac{8x}{25} - 90 = x \quad$ or $\quad 102 = x - \frac{8x}{25} = \frac{17x}{25}$

$$x = \frac{102\times 25}{75} = 150\,\text{mL}$$

Here, the quality of alcohol to be added = 150 mL

Example 17. A man leaves half of his property to his wife, one-third to his son and the remaining to his daughter. If the daughter's share is ₹ 15000, how much money did the man leave? How much money did his wife get? What is his son's share?

(a) ₹ 60000, ₹ 45000 and ₹ 30000
(b) ₹ 80000, ₹ 40000 and ₹ 20000
(c) ₹ 90000, ₹ 45000 and ₹ 30000
(d) None of the above

Sol. (c) Let the total property of a man be ₹ x.

Wife's share = ₹ $\frac{x}{2}$, son's share = ₹ $\frac{x}{3}$

And $\quad$ daughter's share $= x - \left(\frac{x}{2}+\frac{x}{3}\right) = x - \frac{5x}{6} = ₹\ \frac{x}{6}$

But $\quad$ daugther's share = ₹ 15000

$$\frac{x}{6} = 15000 \quad \text{(given)}$$

$$x = ₹\ 15000\times 6 = ₹\ 90000$$

Here, total property of the man = ₹ 90000

Wife's share = ₹ $\frac{90000}{2}$ = ₹ 45000

Son's share = ₹ $\frac{90000}{3}$ = ₹ 30000

Linear Equations in Two Variables

An equation of the form $ax+by+c=0$, where $a, b, c \in R$, $a \neq 0$, $b \neq 0$ and here x and y are variables. It is called as linear equation in two variables.

e.g., $2x+3y=5$, $\sqrt{2}x+\sqrt{3}y=0$ and $2a+3b=0$ are linear equations in two variables.

- The linear equation $ax+by+c=0$ has an infinite number of solutions.
- The graph of equation $ax+by+c=0$ is a straight line so it is called as linear equation.
- Every point on graph of $ax+by+c=0$ gives its solution.

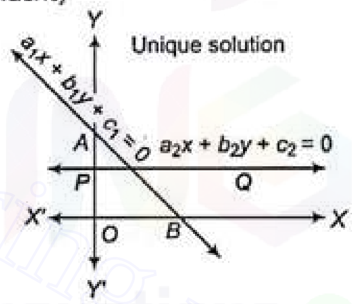
System of linear equations in two variables

When we draw the graph of each of the two equations, we have the following three possibilities

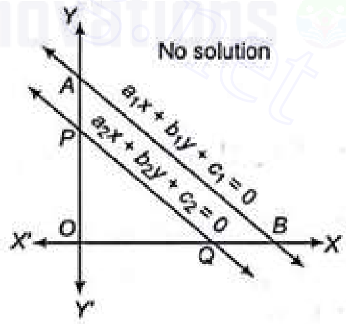
(i) Either the lines intersect (Consistent)
(ii) The lines are parallel (Inconsistent)
(iii) The lines coincide (Dependent)

***Case* I. When two lines intersect** As two intersecting straight lines always intersect in a point. So, there is only one point which lies on both the lines.

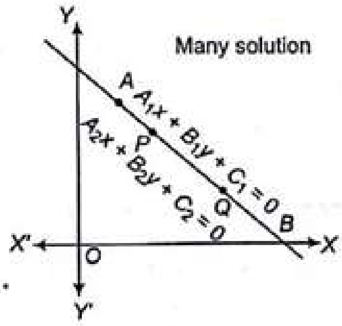
- If the straight lines in the graph intersect each other at a single point, then system has unique solution.



***Case* II. When two lines are parallel** When two lines are parallel they do not meet at any point. So, there is no common point on both of the straight lines. So, thus system has no solution.



***Case* III. When the lines conicide** When two lines coincide, the points lying on one also belong to the other. Thus, all points are common so the system has infinite number of solution.



Algebraic Methods of Solutions

1. Substitution Method

- From either equation find the value of one of the unknown in terms of the other.
- Substitute the value thus found in the other equation.
- Solve the resulting equation involving only one unknown.
- Substitute the value of this unknown in the equation obtained in step first to find the other unknown.

Example 18. The following system of equations are a/an

$$x+3y=4 \quad \text{...(i)}$$
$$2x+6y=8 \quad \text{...(ii)}$$

(a) Unique solution (b) Infinite solution
(c) No solution (d) None of these

Sol. (*b*) From Eq. (i) $x=4-3y$

Put the value of x in Eq. (ii), $2(4-3y)+6y=8$

$\Rightarrow \quad 8-6y+6y=8$

$\Rightarrow \quad 8=8$

Which is always true. So, the given system of equations is dependent and thus has an infinite number of solution.

Example 19. The following system of equations have a/an

$$3x-2y=5 \quad \text{...(i)}$$
$$6x-4y=12 \quad \text{...(ii)}$$

(a) Unique solution (b) Infinite solution
(c) No solution (d) None of these

Sol. (*c*) Here, from Eq. (i),

$$3x=5+2y$$
$$x=\frac{5+2y}{3}$$

Put the value of x in Eq. (ii),

$$\frac{6(5+2y)}{3}-4y=12$$
$$10+4y-4y=12$$

$10=12$, which is not true.

So, the given system of equations is inconsistent.

So, there is no solution.

2. Elimination Method

- Multiply the coefficients of the equation with some constant so as to make the coefficients of one of the variables to be equal to the coefficients of same variable in other equation. (If coefficients are equal move to next step).
- If the coefficients of x (or y) have the same sign, then subtract, if opposite signs, then add the resulting equations.
- The resulting equation will have only one unknown y (or x) and can be solved easily.
- Substitute value of unknown found in above step in either of the equation and find the other unknown.

Example 20. The solutions of the following system of equations

$$2x+5y=11 \quad \text{...(i)}$$
$$3x+4y=13 \quad \text{...(ii)}$$

(a) {4, 2} (b) {3, 1} (c) {5, 2} (d) {1, 1}

Sol. (*b*) Multiplying Eq. (i) by 3 and Eq. (ii) by 2, we get

$$6x+15y=33 \quad \text{...(iii)}$$
$$6x+8y=26 \quad \text{...(iv)}$$

Subtracting Eq. (iv) from Eq. (iii), we get

$$(6x+15y)-(6x+8y)=33-26$$
$$7y=7$$

$\Rightarrow y=1$ put in Eq. (i), $2x+5\times1=11$

$\Rightarrow \quad 2x=11-5$

$\Rightarrow 2x=6 \Rightarrow x=3$ and $y=1$

Example 21. The following system of equations have the solution

$$4x+3y=18xy \quad \text{...(i)}$$
$$2x-5y=-4xy \quad \text{...(ii)}$$

(a) $x=2, y=3$ (b) $x=3, y=2$
(c) $x=\frac{1}{3}, y=\frac{1}{2}$ (d) $x=\frac{1}{2}, y=\frac{1}{3}$

Sol. (*d*) Divide both the equations by xy

$$\frac{4x}{xy}+\frac{3y}{xy}=\frac{18xy}{xy}$$
$$\frac{2x}{xy}-\frac{5y}{xy}=\frac{-4xy}{xy}$$

or
$$\frac{4}{y}+\frac{3}{x}=18$$
$$\frac{2}{y}-\frac{5}{x}=-4$$

Step Put $\frac{1}{y}=A$ and $\frac{1}{x}=B$

$$4A+3B=18 \quad \text{...(iii)}$$
$$2A-5B=-4 \quad \text{...(iv)}$$

Multiplying Eq. (iv) by 2 and subtracting

$$\begin{aligned} 4A+3B&=18\\ 4A-10B&=-8\\ -\quad +\quad &\quad + \\ \hline 13B&=26 \Rightarrow B=2 \end{aligned}$$

Put the value of B in Eq. (iii)

$\Rightarrow \quad 4A+6=18$

$\Rightarrow \quad 4A=12 \Rightarrow A=13$

But $\quad \frac{1}{x}=B=2$

$\Rightarrow \quad x=\frac{1}{2}$

$\quad \frac{1}{y}=A=3$

$\Rightarrow \quad y=\frac{1}{3}$

$\therefore \quad x=\frac{1}{2}$ and $y=\frac{1}{3}$ is solution.

Example 22. The system of equations have the solution

$$\sqrt{2}x - \sqrt{3}y = 0 \quad \text{...(i)}$$
$$\sqrt{5}x + \sqrt{2}y = 0 \quad \text{...(ii)}$$

(a) $x = y = 1$ (b) $x = y = -1$
(c) $x = y = 0$ (d) $x = 1, y = 2$

Sol. (c) Multiplying Eq. (i) by $\sqrt{2}$ and Eq. (ii) $\sqrt{3}$, we get

$$\sqrt{2}(\sqrt{2}x - \sqrt{3}y) = 0 \Rightarrow 2x - \sqrt{6}y = 0 \quad \text{...(iii)}$$
$$(\because \sqrt{a}\sqrt{b} = \sqrt{ab})$$
$$\sqrt{3}(\sqrt{5}x + \sqrt{2}y) = 0 \Rightarrow \sqrt{15}x + \sqrt{6}y = 0 \quad \text{...(iv)}$$

On adding Eqs. (iii) and (iv), we get

$$(2 + \sqrt{15})x = 0$$
$$(2 + \sqrt{15}) \neq 0 \Rightarrow x = 0$$

Put in Eq. (ii),

$$\sqrt{5}(0) + \sqrt{2}y = 0 \Rightarrow \sqrt{2}y = 0 \Rightarrow y = 0$$

$\therefore$ (0, 0) is the required solution.

3. Method of Comparison

- From each of the equations, find the value of one of the variables (the same in both case) in terms of the other.
- Equate the results; solve the resulting equation.
- Substitute the values in either of the results obtained is first step and find the value of other variable.

Example 23. The system of equations have the solution

$$3x - 5y = 1 \quad \text{...(i)}$$
$$4x + 3y = 11 \quad \text{...(ii)}$$

(a) (1, 2) (b) (2, 1) (c) (1, 3) (d) (4, 1)

Sol. (b) As $3x - 5y = 1 \Rightarrow x = \frac{1+5y}{3}$

Also, $4x + 3y = 11 \Rightarrow 4x = 11 - 3y \Rightarrow x = \frac{11-3y}{4}$

Equating both values of x, we get

$$\frac{1+5y}{3} = \frac{11-3y}{4} \Rightarrow 4(1+5y) = 3(11-3y)$$
$$\Rightarrow \quad 4 + 20y = 33 - 9y \Rightarrow 29y = 29 \Rightarrow y = 1$$
$$\therefore \quad x = \frac{1+5y}{3} = \frac{1+5}{3} \Rightarrow x = 2$$

$\therefore$ (2, 1) is the required solution.

4. Cross-Multiplication Method

Let a General System of two simultaneous linear equations be

$$a_1x + b_1y + c_1 = 0 \quad \text{...(i)}$$
$$a_2x + b_2y + c_2 = 0 \quad \text{...(ii)}$$

Then, the solution is

$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} \text{ and}$$
$$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$

To memorise the above solution keep in mind the following diagram.

The arrows between two numbers indicates that the numbers are to be multiplied. The downward arrow are to be multiplied first and from their product, the product of numbers with upward arrows is to be subtracted.

Then, equate values of x and + 1 to get value of x.

And values of y and +1 to get value of y.

Example 24. The following system of equations have the solution

$$ax - by = a^2 - b^2 \quad \text{...(i)}$$
$$x + y = a + b \quad \text{...(ii)}$$

(a) $x = a$ and $y = b$ (b) $x = -a$ and $y = -b$
(c) $x = b$ and $y = a$ (d) $x = -b$ and $y = -a$

Sol. (a) Here, in Eq. (i),

$$a_1 = a, b_1 = -b, c_1 = -(a^2 - b^2) \text{ and}$$

In Eq. (ii), $a_2 = 1, b_2 = 1, c_2 = -(a+b)$

So, $$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1} = \frac{(b)(a+b) + (1)(a^2 - b^2)}{a(1) - (1)(-b)}$$
$$x = \frac{ab + b^2 + a^2 - b^2}{a+b} = \frac{a(a+b)}{(a+b)} = +a$$

and $$y = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$$
$$y = \frac{-(a^2 - b^2)(1) + (a+b)(a)}{a+b} = \frac{b(a+b)}{a+b} = b$$

So, $x = a$ and $y = b$ is solution.

Conditions of Solvability

The system of equations

$$a_1x + b_1y + c_1 = 0$$
and $$a_2x + b_2y + c_2 = 0$$
has

Case I. no solution, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Case II. an infinite number of solutions, if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Case III. a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

A solution can be obtained by any of the above four stated methods.

Homogeneous System of Equations

The system of equations

$$a_1x + b_1y = 0 \text{ and } a_2x + b_2y = 0$$

has only one solution $x = 0, y = 0$ when

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

and an infinite number of solutions when $\frac{a_1}{a_2} = \frac{b_1}{b_2}$

- In both the cases equations are consistent.

Example 25. The value of k for which the system

$$kx + 2y = 5$$
$$3x + y = 1$$

has (a) unique solution. (b) no solution

(a) $k \neq 6$ and $k = 6$ (b) $k \neq 3$ and $k = 3$
(c) $k \neq 2$ and $k = 2$ (d) None of these

Sol. (a) (a) Here, the equation have a unique solution, if

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

i.e., if $\frac{k}{3} \neq \frac{2}{1}$

$\Rightarrow$ If $k \neq 6$

(b) The equations has no solution, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore \frac{k}{3} = \frac{2}{1} \Rightarrow k = 6$

Example 26. The value or values of k for which the system of equations $kx - y = 2$, $6x - 2y = 3$ has infinitely many solutions is

(a) 3 (b) 4
(c) 12 (d) No such value of k

Sol. (d) Here, for the system of equations to have an infinite number of solutions.

We must have, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

But here $\frac{a_1}{a_2} = \frac{k}{6}, \frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-2}{-3} = \frac{2}{3}$

So, here $\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ in any case.

Thus, the system has no such value of k, for which the given system has infinitely many solutions.

Example 27. A man when asked how many hens and buffaloes he has told that his animals have 120 eyes and 180 legs. How many hens has he?

(a) 30 (b) 40 (c) 45 (d) 60

Sol. (a) Let number of buffaloes $= x$ The number of hens $= y$

$\therefore$ Total eyes $= 2x + 2y = 120$ ...(i)

$\therefore$ Total legs $= 4x + 2y = 180$ ...(ii)

Subtracting Eq. (ii) from Eq. (i),

$$\begin{aligned} 2x + 2y &= 120 \\ 4x + 2y &= 180 \\ - \quad - \quad & \quad - \\ \hline -2x &= -60 \Rightarrow x = 30 \end{aligned}$$

Put the value of x in Eq. (i), $60 + 2y = 120$

$$2y = 60 \Rightarrow y = 30$$

Hence, number of hens $= 30$

Example 28. Ratio between the girls and boys in a class of 40 students is 2 : 3. Five new students joined the class. How many of them be boys so that the ratio between girls and boys becomes 4 : 5?

(a) 5 (b) 4 (c) 3 (d) 1

Sol. (d) The total number of students $= 40$

Since, girls : boys $= 2 : 3$

$$\text{Number of girls} = \frac{40}{5} \times 2 = 16$$

$$\text{Number of boys} = \frac{40}{5} \times 3 = 24$$

Let the number of boys among five new-comers $= x$

Number of girls among the 5 new-comers $= y$

$\therefore x + y = 5$ ...(i)

also $\frac{16 + y}{24 + x} = \frac{4}{5}$

$\Rightarrow 96 + 4x = 80 + 5y \Rightarrow 4x - 5y = -16$ ...(ii)

On multiplying Eq. (i) by 5 and adding Eq. (ii), we get

$$\begin{aligned} 4x - 5y &= -16 \\ 5x + 5y &= 25 \\ \hline 9x &= 9 \Rightarrow x = 1 \end{aligned}$$

$\therefore$ Number of boys among the 5 new-comers is 1.

Example 29. If we add 1 to the numerator and subtract 1 from the denominator a fraction becomes 1. It also becomes $\frac{1}{2}$, if we only add 1 to the denominator. The fraction is

(a) 4/5 (b) 3/5 (c) 2/5 (d) 1/5

Sol. (b) Let the numerator of the fraction $= x$

And the denominator of the fraction $= y$

$\therefore$ The fraction $= \frac{x}{y}$

Case I. $\frac{x+1}{y-1} = 1 \Rightarrow x + 1 = y - 1 \Rightarrow x - y = -2$ ...(i)

Case II. $\frac{x}{y+1} = \frac{1}{2} \Rightarrow 2x = y + 1 \Rightarrow 2x - y = 1$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$\begin{aligned} x - y &= -2 \\ 2x - y &= 1 \\ - \quad + \quad & \quad - \\ \hline -x &= -3 \Rightarrow x = 3 \end{aligned}$$

Put the value of x in Eq. (i),

$$3 - y = -2 \Rightarrow y = 5$$

Hence, the fractions is

$$\frac{x}{y} = \frac{3}{5}$$

Exercise

1. $\sqrt{3}x - 2 = 2\sqrt{3} + 4$, then value of x is
   (a) $2(1-\sqrt{3})$ (b) $2(1+\sqrt{3})$ (c) $1+\sqrt{3}$ (d) $1-\sqrt{3}$

2. Find x, if $25x - 19 - [3 - \{4x - 5\}] = 3x - (6x - 5)$
   (a) $x = 1$ (b) $x = -1$ (c) $x = \frac{1}{2}$ (d) $x = 2$

3. If $(x, y) = (4, 1)$ is the solution of the pair of linear equations $mx + y = 2x + ny = 5$, then what is value of $m + n$? **(CDS 2011 II)**
   (a) -2 (b) -1 (c) 2 (d) 1

4. If $\frac{x^2 - 3x + 2}{x^2 - 5x + 4} = \frac{x^2 - 6x + 8}{x^2 - 9x + 14}$, then the value of x is
   (a) $2\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) -2

5. If $a(x - a^2) - b(x - b^2) = 0$, then x is equal to
   (a) $\frac{(-a+b)(a^2+ab+b^2)}{(a+b)}$ (b) $\frac{a^3+b^3}{(a-b)}$
   (c) $\frac{a^3-b^3}{a+b}$ (d) a^2+ab+b^2

6. Under what condition do the equations $kx - y = 2$ and $6x - 2y = 3$ have a unique solution? **(CDS 2010 II)**
   (a) $k = 3$ (b) $k \neq 3$ (c) $k = 0$ (d) $k \neq 0$

7. Sum of two numbers is 21 and their difference is 11, then the greatest number is
   (a) 5 (b) 16 (c) 9 (d) 10

8. Which of the following equations have $x = 2$ and $y = 1$ as a solution
   I. $2x + 5y = 9$ II. $5x + 3y = 14$
   III. $2x + 3y = 7$ IV. $2x - 3y = 1$
   (a) I and IV only (b) II and III only
   (c) I only (d) I, III and IV

9. The line $3x - 5y = -10$ cuts y-axis at
   (a) (0, 2) (b) (0, 1) (c) (0, 3) (d) (0, 4)

10. **Assertion (A)** The equations $2x - 3y = 5$ and $6y - 4x = 11$ cannot be solved graphically.
    Reason (R) The equations given above represent parallel lines. **(CDS 2007 I)**
    (a) A and R are correct and R is correct explanation of A.
    (b) A and R are correct but R is not correct explanation of A.
    (c) A is correct but R is wrong.
    (d) A is wrong but R is correct.

11. If $x + y = 7$ and $3x - 2y = 11$, then
    (a) $x = 2, y = 5$ (b) $x = 5, y = 5$
    (c) $x = 5, y = 2$ (d) $x = 0, y = 3$

12. If $2x + 3y = \frac{11}{3}$ and $5x - 7y = \frac{31}{3}$, then x and y are respectively.
    (a) $\frac{107}{87}, \frac{7}{87}$ (b) $\frac{107}{87}, \frac{-7}{87}$ (c) $\frac{160}{78}, \frac{-7}{87}$ (d) $\frac{170}{87}, \frac{-7}{87}$

13. If $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$ and $\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$, where $x \neq 0$ and $y \neq 0$, then what is the value of $x + y$? **(CDS 2010 I)**
    (a) 2 (b) 3 (c) 4 (d) 8

14. If $7x : 63 = 1 : 9$, then x is equal to
    (a) 1 (b) 2 (c) 3 (d) -1

15. If 5 is added to twice of a number it becomes 6, then the number is
    (a) 0.5 (b) 5
    (c) 0.25 (d) None of these

16. The sum of the two number is 11 and their product is 30, then the numbers are
    (a) 8, 3 (b) 9, 2 (c) 7, 4 (d) 6, 5

17. If one number is thrice the other and their sum is 20, then the numbers are
    (a) 5, 15 (b) 4, 12
    (c) 3, 9 (d) None of these

18. What is the solution of the equation $x - y = 0.9$ and $11(x + y)^{-1} = 2$? **(CDS 2009 I)**
    (a) $x = 3.2$ and $y = 2.3$ (b) $x = 1$ and $y = 0.1$
    (c) $x = 2$ and $y = 1.1$ (d) $x = 1.2$ and $y = 0.3$

19. The solution of the system of linear equations $0.4x + 0.3y = 1.7$ and $0.7x - 0.2y = 0.8$ is
    (a) $x = 3, y = 2$ (b) $x = 2, y = -3$
    (c) $x = 2, y = 3$ (d) None of these

20. The solution of the pair of equation $\frac{x}{2} + y = 0.8$ and $\frac{7}{x + \frac{y}{2}} = 10$ is
    (a) $x = \frac{2}{5}, y = \frac{3}{5}$ (b) $x = \frac{2}{3}, y = 5$
    (c) $x = \frac{2}{5}, y = \frac{5}{3}$ (d) $x = \frac{3}{5}, y = \frac{2}{5}$

21. A system of two simultaneous linear equations in two variables has a unique solution if their graphs
    (a) are coincident (b) are parallel
    (c) intersect in one point (d) None of these

22. Consider the linear equation
    $$p_1x + q_1y = r_1$$
    $$p_2x + q_2y = r_2$$
    where $\frac{p_1}{p_2} = \frac{q_1}{q_2} = \frac{r_1}{r_2}$,
    then gives system of equation is known as
    (a) consistent (b) inconsistent
    (c) dependent (d) None of these

23. The system of equations $6x + 5y = 11$ and $9x + \frac{15}{2}y = 21$, then there is
    (a) a unique solution (b) many solution
    (c) no solution (d) None of these

24. The sum of two numbers is 10 and their product is 20. What is the sum of their reciprocals? **(CDS 2011 II)**
(a) 1/10 (b) 1/2 (c) 1 (d) 2

25. The distance between two stations is 340 km. Two trains start simultaneously from these stations on parallel tracks to cross each other. The speed of one of them is greater than that of other by 5 km/h. If the distance between the two tains after 2 h of their start is 30 km, the speed of each train is
(a) 75 km/h, 80 km/h (b) 60 km/h, 65 km/h
(c) 80 km/h, 85 km/h (d) None of these

26. A streamer goes downstream and covers the distance between two ports in 4 h while it covers the same distance upstream in 5 h. If the speed of the stream is 2 km/h, then the speed of the streamer in still water is
(a) 20 km/h (b) 19 km/h (c) 18 km/h (d) 19.5 km/h

27. The sum of two numbers is 2490 and if 6.5% of one number is equal to 8.5% of the other, then numbers are
(a) 1414, 1076 (b) 1411, 1079
(c) 1412, 1078 (d) None of these

28. If $2x + y = 35$ and $3x + 4y = 65$, then the value of $\frac{x}{y}$ is
(a) $\frac{12}{5}$ (b) $\frac{15}{4}$ (c) 3 (d) 4

29. If $x + y = a + b$ and $ax - by = a^2 - b^2$, then x and y are respectively.
(a) $-a, b$ (b) a, b (c) $\frac{1}{a}, b$ (d) $-a, -b$

30. When in two linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the graph is
(a) parallel (b) intersection at one point
(c) coincident (d) None of these

31. The equation $px + q = 0$ and $rx + s = 0$ are consistent, if
(a) $ps = qr$ (b) $ps + qr = 0$
(c) $pq - rs = 0$ (d) $pq + rs = 0$

32. What is the value of k for which the system of equations $x + 2Y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution? **(CDS 2009 II)**
(a) $-3/14$ (b) $-14/3$ (c) 1/10 (d) 10

33. For what value of p, the following equations will be inconsistent?

$$4x + 6y = 11 \text{ and } 2x + ky = 7$$

(a) $k = -3$ (b) $k = \frac{12}{5}$ (c) $k = 12$ (d) $k = 3$

34. For what value(s) of k, the system of equations has infinitely many solutions $2x - ky = 4$ and $3x + 2y = 6$.
(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

35. For what value of α, the system of equation $\alpha x + 3y = \alpha - 3$, $12x + \alpha y = \alpha$ will have a unique solution?
(a) $\alpha = \pm 6$ (b) $\alpha = 6$ (c) $\alpha \neq \pm 6$ (d) $\alpha = -6$

36. A horse and two cows together cost ₹ 680. If a horse costs ₹ 80 more than a cow, then the cost of horse is
(a) ₹ 170 (b) ₹ 280 (c) ₹ 200 (d) ₹ 220

37. Sunita has 10 paise and 50 paise coins in her purse. If the total number of coins is 17 and their total values is ₹ 4.50, then number of 10 paise coins is
(a) 9 (b) 7 (c) 10 (d) 5

38. Korna has pens and pencils which together are 40 in number. If she had 5 more pencils and 5 less pens, the number of pencils would have become 4 times the number of pens. Then, the original number of pencils Korna had
(a) 19 (b) 27 (c) 13 (d) 17

39. If a scooterist drives at the rate of 24 km/h, he reaches his destination 5 min too late; if he drives at the rate of 30 km/h, he reaches his destination 4 min too soon. Then, his destination is at a distance of
(a) 20 km (b) 30 km (c) 25 km (d) 18 km

40. A pharmacist needs to strengthen a 15% alcohol solution to one of 32% alcohol. How much pure alcohol should be added to 400 mL of the 15% solution?
(a) 90 mL (b) 100 mL (c) 30 mL (d) 110 mL

41. The solution of the equation

$$\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5} = \frac{3x + 2y + 1}{6}$$

is given by which one of the following? **(CDS 2008 II)**
(a) $x = 2, y = 1$ (b) $x = 1, y = 1$
(c) $x = -1, y = -1$ (d) $x = 1, y = 2$

42. In a ΔABC, $\angle C = 3$, $\angle B = 2(\angle A + \angle B)$, then $\angle C$ is
(a) 120° (b) 40° (c) 20° (d) 90°

43. In ΔABC, $\angle A = x°$, $\angle B = (3x - 2)°$, $\angle C = y°$, also $\angle C - \angle B = 9°$, then $\angle B$ is
(a) 32° (b) 73° (c) 25° (d) 78°

44. The value of $x + y$ in the solution of equations $3x + 2y = 11$ and $x - y = 2$ is
(a) 2 (b) 7 (c) 4 (d) 6

45. If $\sqrt{2}x - \sqrt{3}y = 0$ and $\sqrt{7}x + \sqrt{2}y = 0$, then the value of $x + y$.
(a) 1 (b) 2 (c) 3 (d) 0

46. If $ax + by + p = 0$ and $ax - cy - q = 0$, then the value of x is
(a) $-\frac{p+q}{b+c}$ (b) $\frac{bq - pc}{a(b+c)}$ (c) $\frac{bq + pc}{a(b+c)}$ (d) $\frac{a(b+c)}{bq + pc}$

47. The sum of digits of a two-digit number is 8 and the difference between the number and that formed by reversing the digits is 18. What is the difference between the digits of the number? **(CDS 2011 II)**
(a) 1 (b) 2 (c) 3 (d) 4

48. One of the angle of a triangle is equal to the sum of the other two angles. If the ratio of the other two angles is 4 : 5, then the angles of triangles are
(a) 90°, 40°, 50° (b) 15°, 60°, 105°
(c) 30°, 60°, 90° (d) 30°, 40°, 110°

49. Two planes start from a city and fly in opposite directions, one averaging a speed of 40 km/h greater than the others. If they are 3400 km apart after 5 h, the average speeds, respectively are
(a) 330, 370 km/h (b) 320, 360 km/h
(c) 250, 290 km/h (d) 300, 340 km/h

50. If 1 is added to the denominator of a fraction, it becomes $\frac{1}{2}$ and if 1 is added to the numerator, the fraction becomes 1. What is the fraction? **(CDS 2010 II)**
(a) $\frac{5}{9}$ (b) $\frac{2}{3}$ (c) $\frac{4}{7}$ (d) $\frac{10}{11}$

51. A railway ticket for a child costs half the full fare but the reservation charge is the same on half tickets as much as on full ticket. One reserved first class ticket for a journey between two stations is ₹ 362; one full and one half reserved first class tickets costs ₹ 554. What is the reservation charge? **(CDS 2009 II)**
(a) ₹ 18 (b) ₹ 22 (c) ₹ 38 (d) ₹ 46

52. Consider the following sets of equations
I. $2x - y = 0$ and $6x - 3y = 0$
II. $3x - 4y = 0$ and $12x - 20y = 0$
Then,
(a) Both sets I and II possess unique solution.
(b) Set I possesses unique solution and set II has infinitely many solutions.
(c) Set I possesses infinitely many solutions and set II possess unique solution.
(d) None of the sets I and II possesses a unique solution.

53. A fraction becomes $\frac{4}{5}$, if 1 is added to both numerator and denominator. If however 5 is subtracted from both numerator and denominator the fraction becomes $\frac{1}{2}$. Then, the fractions is
(a) $\frac{7}{9}$ (b) $\frac{9}{7}$ (c) $\frac{3}{5}$ (d) $\frac{4}{3}$

54. The value of y in the solution of $2^{x+y} = 2^{x-y} = 16$ is
(a) 4 (b) 2 (c) 1 (d) 0

55. A number consists of two digits, whose sum is 10. If 18 is subtracted from the number, digits of the number are reserved. What is the product? **(CDS 2009 II)**
(a) 15 (b) 18 (c) 24 (d) 32

56. For what value of k, the following system of equations $3x + 4y = 6$, $6x + 8y = k$ represents coincident lines?
(a) 12 (b) 11 (c) 13 (d) 10

57. If $\frac{44}{x+y} + \frac{30}{x-y} = 10$, $\frac{55}{x+y} + \frac{40}{x-y} = 13$, then value of x and y are respectively.
(a) $x = 2, y = 8$ (b) $x = 8, y = 3$
(c) $x = 8, y = 2$ (d) $x = 3, y = 8$

58. If $\frac{a}{x} - \frac{b}{y} = 0$, $\frac{ab^2}{x} + \frac{a^2b}{y} = a^2 + b^2$; $(x, y \neq 0)$ the value of $x + y$ equal to
(a) a (b) $2a$
(c) $a + b$ (d) $a - b$

59. A number consists of two digits whose sum is 9. If 27 is added to the number, digits change their place, then the number is
(a) 36 (b) 27 (c) 72 (d) 63

60. In a triangle the sum of two angles equal to the third angle. If the difference between the two angles is 30°, then that greatest angle is
(a) 88° (b) 90° (c) 80° (d) 120°

61. A person bought a certain number of books for ₹ 80. If he had bought 4 more books for the same sum, each book would have cost ₹ 1 less. What is the price of each book? **(CDS 2008 I)**
(a) ₹ 10 (b) ₹ 8 (c) ₹ 5 (d) ₹ 4

62. A and B are friends and their ages differ by 2 yr. A's father D is twice as old as A and B is twice as old as his sister C. The ages of D and C differ by 40 yr. Then, the age of A is
(a) 25 yr (b) 24 yr (c) 26 yr (d) 28 yr

63. After covering a distance of 30 km with a uniform speed there is some defect in a rail engine and therefore, its speed is reduced to $\frac{4}{5}$ of its original speed. Consequently, the train reaches its destination late by 45 min. Had it happened after covering 18 km more, the train would have reached 9 min ealier. Then, the speed of the train (in km/h) is
(a) 25 (b) 30 (c) 45 (d) 40

64. The course of an enemy submarine as plotted on a set of axes given the equation $2x + 3y = 5$. On the same axes a distroyer's course is indicated by the graph of $x - y = 10$. At what point do the paths of the destroyer and submarine intersect?
(a) (3, 7) (b) (7, 3) (c) (7, −3) (d) (−7, −3)

65. A man invested ₹ 3500, part of it at a yearly interest rate of 40% and the rest at 5%. He received a total annual interest of ₹ 153. How much did he invest at rate 5%?
(a) ₹ 2200 (b) ₹ 1300 (c) ₹ 2000 (d) ₹ 1800

66. The system of equations $x + 2y = 3$ and $3x + 6y = 9$ has **(CDS 2011 II)**
(a) unique solution
(b) no solution
(c) infinitely many solutions
(d) finite number of solutions

67. The area of a rectangle remains the same if the length is increased by 7 m and the breadth is decreased by 3 m. The area remains unaffected in the length is decreased by 7 m and the breadth is increased by 5 m, then area of rectangle is
(a) 280 m^2 (b) 320 m^2 (c) 420 m^2 (d) 400 m^2

68. The graph of the equation $x = 9$ is a straight line
(a) parallel to x-axis (b) parallel to y-axis
(c) not perpendicular to x-axis
(d) perpendicular to y-axis

69. When the graph of a linear equation plotted, it always comes to be a
(a) parabola (b) ellipse
(c) straight line (d) circle

70. The value of x in the solution of the equation $2^{x+y} = 2^{x-y} = \sqrt{8}$ is

(a) 0 (b) $\frac{3}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

71. A person bought 5 tickets from a station P to a station Q and 10 tickets from the station P to a station R. He paid ₹ 350. If the sum of a ticket from P to Q and a ticket from P to R is ₹ 42, then what is the fare from P to Q? **(CDS 2009 I)**

(a) ₹ 12 (b) ₹ 14 (c) ₹ 16 (d) ₹ 18

72. The set of homogeneous simultaneous equations $4x + 2y = 0$ and $6x + 3y = 0$ has

(a) $x = 0$, $y = 0$ as solution

(b) $x = 0$, $y = 0$ and $x = 1$, $y = -2$ as solutions

(c) $x = 0$, $y = 0$; $x = -1$, $y = 2$ and $x = 1$, $y = -2$ as solutions

(d) as infinite number of solutions

73. Soldiers of a company are made to stand in rows. If one soldier is extra in a row, there would be 2 rows less. If one soldier is less in a row there would be 3 rows more. Then, the number of soldiers in company is

(a) 60 (b) 50 (c) 30 (d) 100

74. The average score of boys in an examination of a school is 71 and that of the girls is 73. The average score of the school in the examination is 71.8. Then, the ratio of the number of boys to the number of girls appeared in the examination is

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{2}{3}$

75. Pooja started her job with certain monthly salary and gets a fixed increment every year. If her salary was ₹ 4200 after 3 yr and ₹ 6800 after 8 yr of service, then what are her initial salary and the annual increment, respectively?

(a) ₹ 2640, ₹ 320 (b) ₹ 2460, ₹ 320
(c) ₹ 2460, ₹ 520 (d) ₹ 2640, ₹ 520

76. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, then the numbers are

(a) 6, 9 (b) 10, 5 (c) 8, 7 (d) 7, 8

77. A man starts his job with a certain monthly salary and earns a fixed increment every year. If his salary was ₹ 1500 after 4 yr of services and ₹ 1800 after 10 yr of service. What was his starting salary?

(a) ₹ 1300 (b) ₹ 1200 (c) ₹ 50 (d) ₹ 1100

78. A and B each have a certain number of mangoes. A says to B : "If you give 30 of your mangoes, I will have twice as many as left with you" B replies "If you give me 10, I will have thrice as many as left with you". How many mangoes did A has?

(a) 41 (b) 62 (c) 34 (d) 32

79. There are two examination rooms A and B. If 10 candidates are sent from A to room B, the number of students in each room is the same. If 20 candidates are sent from B to A, the number of students in A is double the number of students in B. Then, number of students in room B is

(a) 40 (b) 100 (c) 80 (d) 60

80. A train started from a station with a certain number of passengers. At the first halt, 1/3rd of its passengers got down and 120 passengers got in. At the second halt, half of the passengers got down and 100 persons got in. Then, the train left for its destination with 240 passengers. How many passengers were there in the train when it started? **(CDS 2008 I)**

(a) 540 (b) 480 (c) 360 (d) 240

81. Seven times a two-digit number is equal to four times the number obtained by reversing the order of digits and the difference of the digits is 3, then the number is

(a) 51 (b) 36 (c) 15 (d) 24

82. Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction they meet in 9 h and if they go in opposite direction they meet in $\frac{9}{7}$ h. Then, their speeds are respectively

(a) 40 km/h, 30 km/h (b) 50 km/h, 60 km/h
(c) 30 km/h, 40 km/h (d) None of these

83. If $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}, \frac{3}{x} + \frac{2}{y} = 0$, then find the value of 'a' for which $y = ax + 4$

(a) $-\frac{4}{3}$ (b) $\frac{4}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

84. A man went to the Resere Bank of India with ₹ 1000. He asked the cashier to give him ₹ 5 and ₹ 10 notes only in return. The man got 175 notes in all. How many notes of ₹ 10 did he receive?

(a) 150 (b) 25
(c) 35 (d) 70

85. The ratio of income of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them saves ₹ 200 per month. Their monthly income are respectively.

(a) ₹ 1800, ₹ 1400 (b) ₹ 1600, ₹ 12000
(c) ₹ 700, ₹ 2100 (d) None of these

86. The cost of 4 books and 3 pencils is same as that of 8 books and 1 pencil. This cost will be same as that of which one of the following?

(a) 2 books and 6 pencils (b) 5 books and 5 pencils
(c) 6 books and 2 pencils (d) 12 books and 4 pencils

Answers

1. (b)	2. (a)	3. (a)	4. (d)	5. (d)	6. (b)	7. (b)	8. (d)	9. (a)	10. (a)
11. (c)	12. (d)	13. (c)	14. (a)	15. (a)	16. (d)	17. (a)	18. (a)	19. (c)	20. (a)
21. (c)	22. (c)	23. (c)	24. (b)	25. (a)	26. (c)	27. (b)	28. (c)	29. (b)	30. (b)
31. (a)	32. (d)	33. (d)	34. (b)	35. (c)	36. (b)	37. (c)	38. (b)	39. (d)	40. (b)
41. (b)	42. (a)	43. (b)	44. (c)	45. (d)	46. (b)	47. (b)	48. (a)	49. (b)	50. (b)
51. (b)	52. (c)	53. (a)	54. (d)	55. (d)	56. (a)	57. (b)	58. (c)	59. (a)	60. (b)
61. (c)	62. (c)	63. (b)	64. (c)	65. (b)	66. (c)	67. (c)	68. (b)	69. (c)	70. (b)
71. (b)	72. (d)	73. (a)	74. (c)	75. (d)	76. (b)	77. (a)	78. (c)	79. (c)	80. (d)
81. (b)	82. (a)	83. (a)	84. (b)	85. (a)	86. (c)				

Hints and Solutions

1. $\sqrt{3}x - 2 = 2\sqrt{3} + 4$

$$\sqrt{3}x = 2\sqrt{3} + 6$$

$$x = \frac{2\sqrt{3}+6}{\sqrt{3}} \Rightarrow x = \frac{2\sqrt{3}+6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{6(1+\sqrt{3})}{3} \Rightarrow x = 2(1+\sqrt{3})$$

2. $25x - 19 - [3 - \{4x - 5\}] = 3x - (6x - 5)$

$\Rightarrow$ $25x - 19 - [3 - 4x + 5] = 3x - 6x + 5$

$\Rightarrow$ $25x - 19 + 4x - 8 = -3x + 5$

$\Rightarrow$ $29x + 3x = 5 + 27$

$\Rightarrow$ $32x = 32$

$\Rightarrow$ $x = \frac{32}{32} = 1 \Rightarrow x = 1$

3. Given, $(x, y) = (4, 1)$

and $mx + y = 2x + ny = 5$

$\therefore$ $m(4) + 1 = 2 \times 4 + n = 5$

$\therefore$ $4m + 1 = 5$ and $8 + n = 5$

$\Rightarrow$ $m = 1$ and $n = -3$

$\therefore$ $m + n = 1 - 3 = -2$

4. $\frac{(x-2)(x-1)}{(x-4)(x-1)} = \frac{(x-2)(x-4)}{(x-2)(x-7)} \Rightarrow \frac{x-2}{x-4} = \frac{x-4}{x-7}$

$\Rightarrow$ $x^2 - 9x + 14 = x^2 - 8x + 16 \Rightarrow x = -2$

5. $ax - a^3 - bx + b^3 = 0$

$\Rightarrow$ $x = \frac{a^3 - b^3}{a - b} = a^2 + ab + b^2$

6. Since, the equations $kx - y = 2$ and $6x - 2y = 3$ have a unique solution.

$\therefore$ $\frac{k}{6} \neq \frac{1}{2} \Rightarrow k \neq 3$

7. Let numbers be x and y, then by given condition is

$$x + y = 21 \quad \text{...(i)}$$
$$x - y = 11 \quad \text{...(ii)}$$

On adding Eqs. (i) and (ii), we get $2x = 32$

$$x = \frac{32}{2} = 16 \text{ and } y = 5$$

8. Put $x = 2$ and $y = 1$ in each equation

$2x + 5y = 9 \Rightarrow 2(2) + 5(1) = 9$

$9 = 9$, it is true.

$2x + 3y = 7 \Rightarrow 2(2) + 3(1) = 7$

$7 = 7$ it is true.

$2x - 3y = 1 \Rightarrow 2(2) - 3(1) = 1$

$1 = 1$, it is true.

So, $x = 2$ and $y = 1$ is solution of above equations but for

$5x + 3y = 14, 5(2) + 3(1) \neq 14$

$13 \neq 14$

$x = 2$ and $y = 1$ is not a solution.

9. Here, $3x - 5y = 10$

At y-axis $x = 0$, put in equation $0 - 5y = -10 \Rightarrow y = \frac{10}{5} = 2$

$\therefore$ Point on y-axis is $(0, 2)$.

10. The given equations are $2x - 3y = 5$ ...(i)

and $-4x + 6y = 11$ ...(ii)

Here, $a_1 = 2, b_1 = -3, c_1 = 5, a_2 = -4, b_2 = 6, c_2 = 11$

Also, $\frac{a_1}{a_2} = \frac{2}{-4} = \frac{-1}{2}$

$\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

The given equations represent parallel lines, so it is not solved graphically.

Hence, (A) and (R) are individually true and (R) is correct explanation of (A).

11. Multiply Eq. (i) by 2 and adding, we get

$$2x + 2y = 14 \quad \text{...(i)}$$
$$3x - 2y = 11 \quad \text{...(ii)}$$
$$5x = 25 \quad \Rightarrow x = 5$$

Put the value of x in Eq. (i), $5 + y = 7$

$\Rightarrow$ $y = 2$

So, $x = 5$ and $y = 2$

12. $2x + 3y = \frac{11}{3}$ and $5x - 7y = \frac{31}{3}$

$\Rightarrow$ $6x + 9y = 11$ ...(i)

and $15x - 21y = 31$ ...(ii)

On multiplying Eq. (i) by 7 and Eq. (ii) by 3 and adding, we get

$$\begin{array}{r} 42x + 63y = 77 \\ 45x - 63y = 93 \\ \hline 87x \quad = 170 \Rightarrow x = \frac{170}{87} \end{array}$$

Put the value of x in Eq. (i),

$$6\left(\frac{170}{87}\right) + 9y = 11$$

$$9y = 11 - \frac{1020}{87} \Rightarrow y = \frac{-7}{87}$$

$\therefore \quad x = \frac{170}{87}$ and $y = \frac{-7}{87}$

13. Given, $\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$

$\Rightarrow \quad 2y + 3x = 9 \quad$ ...(i)

and $\quad \frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$

$\Rightarrow \quad 4y + 9x = 21 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get $x = 1$ and $y = 3$

$\therefore \quad x + y = 1 + 3 = 4$

14. $\frac{7x}{63} = \frac{1}{9} \Rightarrow x = 1$

15. $2x + 5 = 6 \quad$ (by condition)

$\Rightarrow \quad 2x = 1 \Rightarrow x = 0.5$

16. Let the numbers are x and $11 - x$.

$\Rightarrow \quad x(11 - x) = 30 \quad$ (by condition)

$\Rightarrow \quad (x-5)(x-6) = 0 \Rightarrow x = 5, 6$

17. $x + 3x = 20 \Rightarrow x = 5 \therefore 3x = 3 \times 5 = 15$

18. Given, $\quad x - y = 0.9 \quad$ ...(i)

and $11(x+y)^{-1} = 2 \Rightarrow 2x + 2y = 11 \quad$ ...(ii)

On multiplying Eq. (i) by 2 and adding Eqs. (i) and (ii), we get

$$4x = 12.8 \Rightarrow x = 3.2$$

From Eq. (i), $\quad y = 3.2 - 0.9 = 2.3$

19. Here, $\frac{4x}{10} + \frac{3y}{10} = \frac{17}{10}$ and $\frac{7x}{10} - \frac{2y}{10} = \frac{8}{10}$

or $\quad 4x + 3y = 17 \quad$ ...(i)

and $\quad 7x - 2y = 8 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get $x = 2$ and $y = 3$

20. Here, $x + 2y = 1.6$ and $x + \frac{y}{2} = \frac{7}{10}$

or $\quad 10x + 20y = 16 \quad$ ...(i)

and $\quad 10x + 5y = 7 \quad$ ...(ii)

On subtracting, we get

$$\begin{array}{r} 10x + 20y = 16 \\ 10x + 5y = 7 \\ - \quad - \quad - \\ \hline 5y = 9 \end{array} \Rightarrow y = \frac{3}{5}$$

From Eq. (i), $\quad x + 2y = \frac{8}{5}$

Put $y = \frac{3}{5}$, $\quad x + \frac{6}{5} = \frac{8}{5} \Rightarrow x = \frac{2}{5}$

21. As for unique solution the lines have one common point.

22. Dependent system (see theory).

23. Here, $\frac{a_1}{a_2} = \frac{6}{9} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{5}{15/2} = \frac{2}{3}$ and $\frac{c_1}{c_2} = \frac{11}{21}$

$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ So, the system has no solution.

24. Let the two numbers are x and y, then by condition

$\therefore \quad x + y = 10$ and $xy = 20$

$\therefore \quad \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{10}{20} = \frac{1}{2}$

25. Let speed of first train $= x$ km/h and

Speed of second train $= x + 5$ km/h

Distance travelled in 2 h by first train $= 2x$ km

Distance travelled in 2 h by second train

$$= (x+5) \times 2 \text{ km}$$

$\because 2x + (2x + 10) + 30 = 340$

$$4x = 300 \Rightarrow x = 75$$

Speed of first train = 75 km/h and

Speed of second train = 80 km/h

26. Let the speed of the streamer in still water $= x$ km/h

Speed of streamer downstream $= (x+2)$ km/h

Speed of streamer upstream $= (x-2)$ km/h

Distance travelled by streamer in downstream in

$$4\text{h} = 4(x+2) \text{ km}$$

So, $\quad 4(x+2) = 5(x-2)$

$\Rightarrow \quad x = 18$ km/h is the speed in still water.

27. Let the numbers be x and $2490 - x$.

$$6.5\% \text{ of one} = \frac{6.5}{100} \times x = \frac{13x}{200}$$

$$8.5\% \text{ of other number} = \frac{8.5}{100}(2490 - x) = \frac{17}{200}(2490 - x)$$

By condition, $\quad \frac{13x}{200} = \frac{17(2490 - x)}{200}$

$\Rightarrow \quad 13x = 17(2490 - x)$

$\Rightarrow \quad 13x + 17x = 42330$

$\Rightarrow \quad x = \frac{42330}{30} = 1411$

Second number $= 2490 - 1411 = 1079$

28. Here, $\quad 2x + y = 35 \quad$ ...(i)

And $\quad 3x + 4y = 65 \quad$ ...(ii)

On multiply Eq.(i) by 4 and Eq. (ii) subtracting from in Eq (i) we get

$$\begin{array}{r} 8x + 4y = 140 \\ 3x + 4y = 65 \\ - \quad - \quad - \\ \hline 5x \quad = 75 \end{array} \Rightarrow x = 15$$

Put the value of x in Eq. (i), $2\times 15+y=35$

$\Rightarrow \quad y=35-30=5$

Then, $\quad \frac{x}{y}=\frac{15}{5}=3$

29. $x+y=a+b$ ...(i)

and $\quad ax-by=a^2-b^2$ ...(ii)

On multiplying Eq. (i) by b and adding, we get

$$bx+by=ab+b^2$$
$$ax-by=a^2-b^2$$
$$(a+b)x=a(a+b) \Rightarrow x=a$$

Put the value of x in Eq. (i), $a+y=a+b$

$\Rightarrow \quad y=b$

So, $x=a$ and $y=b$ is solution.

30. As this is case of unique solution $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$, so the graph of equations will intersect in a point.

31. $px+q=0$ and $rx+s=0$

$\Rightarrow \quad x=\frac{-q}{p}$ and $x=\frac{-s}{r}$

So, $\quad \frac{-q}{p}=\frac{-s}{r} \Rightarrow ps=qr$

32. Since, given system of equations

$$x+2y-3=0 \text{ and } 5x+ky+7=0$$

has no solution,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

$\therefore \quad \frac{1}{5}=\frac{2}{k}\neq\frac{-3}{7} \Rightarrow k-10=0 \Rightarrow k=10$

33. Here, $\frac{a_1}{a_2}=\frac{4}{2}$ and $\frac{b_1}{b_2}=\frac{6}{k}$

As system will be inconsistent, so $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$

$$\frac{4}{2}=\frac{6}{k} \Rightarrow 4k=12 \Rightarrow k=3$$

34. Here, $\frac{a_1}{a_2}=\frac{2}{3}, \frac{b_1}{b_2}=\frac{-k}{2}$ and $\frac{c_1}{c_2}=\frac{4}{6}$

The system will have infinitely may solutions

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$
$$\frac{2}{3}=\frac{-k}{2}=\frac{2}{3} \Rightarrow -3k=4 \Rightarrow k=\frac{-4}{3}$$

35. System will have unique solution, if

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

if $\quad \frac{\alpha}{12}\neq\frac{3}{\alpha}$

$\Rightarrow \quad \alpha^2\neq 36 \Rightarrow \alpha\neq\pm 6$

36. Let cost of one horse be ₹ x.

Cost of one cow be ₹ y.

So, $\quad x+2y=680$ ...(i)

$\quad x-y=80$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$3y=600 \Rightarrow y=200$$

$\therefore$ Cost of one horse $=200+80=$ ₹ 280

37. Let number of 10 paise coins be x and number of 50 paise coins be y.

Then, $\quad x+y=17$ ...(i)

and $\quad 10x+50y=450$ ...(ii)

From Eq. (ii), $\quad x+5y=45$ ...(iii)

On subtracting Eq. (i) from Eq. (iii), we get

$$4y=28 \Rightarrow y=\frac{28}{4}=7$$

$\therefore$ Number of 10 paise coins $=x=17-y=17-7=10$

38. Let the original number of pens be x and original number of pencils be y.

$\because \quad x+y=40$ ...(i)

and $\quad (y+5)=4(x-5)$ ...(ii)

From Eq. (i), $y=40-x$

Put the value of of y in Eq. (ii), $(40-x)+5=4(x-5)$

$$45-x=4x-20$$
$$-5x=-65 \Rightarrow x=13$$

$\therefore$ Original number of pencils $=40-13=27$

39. Let the required distance be x km.

With the two speeds, the difference of time taken = 9 min

$\therefore \quad \frac{x}{24}-\frac{x}{30}=\frac{9}{60} \Rightarrow \frac{x}{120}=\frac{9}{60}$

$$\frac{x}{2}=9 \Rightarrow x=18 \text{ km}$$

40. Let the pure alcohol to be added be x mL.

Quantity of alcohol in 15% of 400 mL solution

$$=\frac{15\times 400}{100}=60 \text{ mL}$$

Total quality of 32% alcohol solution $=(400+x)$ mL

$\therefore$ Quality of alcohol in $(400+x)$ mL solution of 32%

$$=(400+x)\times\frac{32}{100} \text{ mL}=\frac{(400+x)\times 8}{25}=\left(128+\frac{8x}{25}\right) \text{mL}$$

Quantity of alcohol needed $=128+\frac{8x}{25}-60$

But $\quad 68+\frac{8x}{25}=x$

$$68=\frac{17x}{25} \Rightarrow \frac{68\times 25}{17}=x$$

$\Rightarrow \quad x=100$ mL

41. Given, equations are

$$\frac{3x-y+1}{3}=\frac{2x+y+2}{5}=\frac{3x+2y+1}{6}$$

Taking Ist and IInd terms,

$$5(3x-y+1)=3(2x+y+2)$$

$\Rightarrow \quad 9x-8y=1$ ...(i)

Taking IInd and IIIrd terms

$$6(2x+y+2)=5(3x+2y+1)$$

$\Rightarrow \quad 3x+4y=7$ ...(ii)

On solving Eqs. (i) and (ii), we get $y=1$ and $x=1$

42. Let $\angle A = x$ and $\angle B = y$

$\because \quad \angle C = 3\angle B \Rightarrow \angle C = 3y$

But $\quad \angle A + \angle B + \angle C = 180°$

$x + y + 3y = 180°$

$x + 4y = 180° \quad \text{...(i)}$

Also, $\quad \angle C = 2(\angle A + \angle B)$

$3y = 2(x + y)$

$3y = 2x + 2y \Rightarrow y = 2x \quad \text{...(ii)}$

Put the value of y in Eq. (i),

$x + 4(2x) = 180°$

$9x = 180° \Rightarrow x = 20° \Rightarrow y = 40°$

$\therefore \quad \angle C = 3y = 3 \times 40° = 120°$

43. $\because \angle A + \angle B + \angle C = 180°$

$x + (3x - 2) + y = 180°$

$4x + y = 182 \quad \text{...(i)}$

Also, $\quad \angle C - \angle B = 9°$

$y - (3x - 2) = 9°$

$y - 3x = 7° \quad \text{...(ii)}$

On subtracting Eq. (ii) from Eq. (i), we get

$4x + y = 182°$

$-3x + y = 7°$

$+ \quad - \quad -$

$7x = 175° \Rightarrow x = 25°$

$\angle B = (3x - 2) = 3 \times 25° - 2 = 73°$

44. $3x + 2y = 11 \quad \text{...(i)}$

and $\quad x - y = 2 \quad \text{...(ii)}$

Multiply Eq. (ii) by 2 and adding, we get

$5x = 15 \Rightarrow x = 3$

Put the value of x in Eq. (ii), $x - y = 2$

$\Rightarrow \quad -y = -1$

$\Rightarrow \quad y = 1$

So, $\quad x + y = 3 + 1 = 4$

45. As the equations are homogeneous equations and also $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. So, equation has one solution $x = y = 0$

$\Rightarrow \quad x + y = 0$

46. By cross-multiplication method,

$$\frac{x}{-bq + pc} = \frac{y}{ap + aq} = \frac{1}{-ac - ab}$$

$$\Rightarrow \quad x = \frac{-bq + pc}{-ac - ab}$$

$$x = \frac{bq - pc}{a(b + c)}$$

47. Let x be the first digit and y be the second digfit of two-digit number.

By given codition,

$x + y = 8 \quad \text{...(i)}$

and $\quad (10x + y) - (10y + x) = 18 \Rightarrow 9x - 9y = 18$

$\Rightarrow \quad x - y = 2 \quad \text{...(ii)}$

On solving Eqs. (i) and (ii), we get $x = 5$ and $y = 3$

$\therefore \quad$ Required digit $= 10x + y = 50 + 3 = 53$

$\therefore$ Required difference of digits: $x - y = 5 - 3 = 2$

48. Let the two angles be $4x$ and $5x$, then

third angle $= 4x + 5x = 9x$

So, $\quad 4x + 5x + 9x = 180°$

$18x = 180° \Rightarrow x = 10°$

So, angles are $4x = 4 \times 10 = 40°$

$5x = 5 \times 10 = 50°$ and $9x = 9 \times 10 = 90°$

49. Let average speed of one plane be x km/h.

Then, average speed of other plane be $(x + 40)$ km/h.

Distance travelled by first plane in 5 h $= 5x$ km

Distance travelled by second plane in 5 h $= 5(x + 40)$ km

So, $\quad 5x + 5(x + 40) = 3400 \Rightarrow 10x + 200 = 3400$

$\Rightarrow 10x = 3200 \Rightarrow x = \frac{3200}{10} = 320$ km/h

So, average speed of second plane $= 320 + 40 = 360$ km/h

50. Let the numerator $= x$ and denominator $= y$

By given condition, $\frac{x}{y + 1} = \frac{1}{2}$

$\Rightarrow \quad 2x - y = 1 \quad \text{...(i)}$

and $\quad \frac{x + 1}{y} = 1$

$\Rightarrow \quad x - y = -1 \quad \text{...(ii)}$

On solving Eqs. (i) and (ii), we get $x = 2$ and $y = 3$

$\therefore \quad$ Fraction $= \frac{x}{y} = \frac{2}{3}$

51. Let full fare $=$ ₹ x and reservation charges $=$ ₹ y

$\therefore \quad x + y = 362 \quad \text{...(i)}$

and $\quad 1\frac{1}{2}x + 2y = 554$

$\Rightarrow \quad 3x + 4y = 1108 \quad \text{...(ii)}$

On solving Eqs. (i) and (ii), we get $x = 340$ and $y = 22$

Hence, reservation charge is ₹ 22.

52. **Set I.** $2x - y = 0$ and $6x - 3y = 0$

as $\quad \frac{2}{6} = \frac{1}{3}$

So, system of equations have infinitely many solutions.

Set II. $3x - 4y = 0$ and $12x - 20y = 0$, here $\frac{3}{12} \neq \frac{4}{20}$

So, system has unique solution.

53. Let the fraction be $\frac{x}{y}$.

Case I. $\frac{x + 1}{y + 1} = \frac{4}{5}$ (by condition)

$\Rightarrow \quad 5(x + 1) = 4(y + 1)$

$\Rightarrow \quad 5x - 4y = -1 \quad \text{...(i)}$

Case II. $\frac{x - 5}{y - 50} = \frac{1}{2}$ (by condition)

$\Rightarrow \quad 2(x - 5) = (y - 5)$

$\Rightarrow \quad 2x - y = 5 \quad \text{...(ii)}$

Multiplying Eq. (ii) by 4 and subtracting, we get

$5x - 4y = -1$

$8x - 4y = 20$

$- \quad + \quad -$

$-3x = -21 \Rightarrow x = 7$

Put the value of x in Eq. (i),

$$35-4y=-1 \Rightarrow y=9$$

$\therefore$ The fraction is $\frac{7}{9}$.

54. $\because \quad 2^{x+y}=2^{x-y}=16=2^4$

On comparing, $x+y=4$ and $x-y=4$

Adding the equation

$$2x=8 \Rightarrow x=4, \text{ so } y=0$$

55. Let the two-digits number be $10y+x$.

By given condition,

$$x+y=10 \quad \text{...(i)}$$

and $\quad 10y+x-18=10x+y$

$\Rightarrow \quad 9x-9y=-18$

$\Rightarrow \quad x-y=-2 \quad \text{...(ii)}$

On solving Eqs. (i) and (ii), we get $x=4$ and $y=6$

$\therefore \quad$ Product $=xy=4\times 6=24$

56. The system of equations represents concident line, if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

i.e., $\quad \frac{3}{6}=\frac{4}{8}=\frac{6}{k}$

$\Rightarrow \quad 4k=6\times 8 \Rightarrow k=\frac{48}{4}=12$

57. Put $\frac{1}{x+y}=A$ and $\frac{1}{x-y}=B$

$\Rightarrow \quad 44A+30B=10 \quad \text{...(i)}$

and $\quad 55A+40B=13 \quad \text{...(ii)}$

Multiplying Eq. (i) by 4 and Eq. (ii) by 3 and subtracting

$$176A+120B=40$$
$$165A+120B=39$$
$$- \quad - \quad -$$
$$11A \quad =1 \Rightarrow A=\frac{1}{11}$$

Put the value of A in Eq. (i)

$$4+30B=10$$

$$30B=6 \Rightarrow B=\frac{1}{5}$$

$\therefore \quad x+y=11$ and $x-y=5$

On adding, we get $\quad 2x=16$

$$x=8 \text{ and } y=3$$

58. Put $\frac{1}{x}=A$ and $\frac{1}{y}=B$

$\Rightarrow \quad aA-bB=0 \quad \text{...(i)}$

and $\quad ab^2A+a^2bB=a^2+b^2 \quad \text{...(ii)}$

Multiplying Eq. (i) by a^2 and adding in Eq. (ii), we get

$$a^3A+ab^2A=a^2+b^2$$

$$aA(a^2+b^2)=(a^2+b^2) \Rightarrow A=\frac{1}{a}$$

Put the value of A in Eq. (i)

$$1-bB=0 \Rightarrow B=\frac{1}{b}$$

$\Rightarrow \quad x=a$ and $y=b$

$\Rightarrow \quad x+y=a+b$

59. Let units digit be x and ten's digit be y.

$\Rightarrow \quad$ Sum of the digits $=x+y=9 \quad \text{...(i)}$

and number is $=10y+x$

Here, $\quad 10y+x+27=10x+y \quad$ (by condition)

$\Rightarrow \quad 9y-9y=-27 \Rightarrow x-y=3 \quad \text{...(ii)}$

Adding Eqs. (i) and (ii), we get $2x=12 \Rightarrow x=6$

Put the value of x in Eq. (i) $6+y=9 \Rightarrow y=3$

$\therefore$ The two-digit number is

$$=10y+x=10\times 3+6=30+6=36$$

60. Let the two angles be x and y, also $x>y$,

then third angle $=(x+y)$

$\Rightarrow \quad x+y+(x+y)=180°$

$$2x+2y=180° \Rightarrow x+y=90° \quad \text{...(i)}$$

$$x-y=30° \quad \text{(given) ...(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2x=120 \Rightarrow x=60°$$

Put the value of x in Eq. (i) $y=90°-x=30°$

$\therefore$ Greatest angle $=90°$

61. Let the price of each book is ₹ x and the number of books is y.

$\therefore \quad xy=80 \quad \text{...(i)}$

and $\quad (y+4)(x-1)=80 \quad$ (by condition)

$\Rightarrow \quad xy-y+4x-4=80$

$\Rightarrow \quad 80-y+4x=84 \quad$ [using Eq. (i)]

$\Rightarrow \quad 4x-y=4 \Rightarrow y=4(x-1)$

On putting the value of y in Eq. (i), we get

$$4(x-1)x=80$$

$\Rightarrow \quad x^2-x-20=0$

$\Rightarrow \quad x^2-5x+4x-20=0$

$\Rightarrow \quad (x-5)(x+4)=0 \Rightarrow x=5 \quad (\because x\neq -4)$

Hence, price of each is ₹ 5.

62. Let age of $A=x$ yr and age of $B=y$ yr

So, $\quad x-y=2 \quad \text{...(i)}$

Age of $D=2x$ and Age of $C=\frac{y}{2}$

Also, $\quad 2x-\frac{y}{2}=40 \Rightarrow 4x-y=80 \quad \text{...(ii)}$

On subtracting Eq. (i) from Eq. (ii), we get

$$4x-y=80$$
$$x-y=2$$
$$- \quad + \quad -$$
$$3x \quad =78y \quad x=26$$

$\therefore$ Age of $A=26$ yr

63. Let original speed of train be x km/h and length of journey be y km. $\therefore \quad$ Time taken $=\frac{y}{x}$ h

Case I When defect in engine occurs after covering 30 km.

$\therefore \quad$ Speed for 30 km $=x$

$$\text{Speed from } (y-30) \text{ km}=\frac{4x}{5}$$

$\therefore \quad$ Time taken to cover 30 km $=\frac{30}{x}$ h

$$\text{Time taken to cover } (y-30) \text{ km}=\left[\frac{y-30}{4x/5}\right]=\frac{5y-150}{4x}$$

$\therefore \quad \frac{30}{x}+\frac{5y-150}{4x}=\frac{y}{x}+\frac{45}{60}$ or $y-3x=30$ ...(i)

Case II When defect occurs after 48 km.
Speed from 48 km = x km/h

Speed from $(y-48)$ km $=\frac{4x}{5}$

Time taken to cover 48 km $=\left[\frac{48}{x}\right]$ h

Time taken to cover $(y-48)$ km $=\left[\frac{y-48}{4x/5}\right]=\frac{5y-240}{4x}$

$\therefore \quad \frac{48}{x}+\frac{5y-240}{4x}=\frac{y}{x}+\frac{36}{60} \Rightarrow 5y-12x=240$ ...(ii)

On solving Eqs. (i) and (ii), we get $x=30$ km/h and $y=120$ km

64. The required point where the paths of the destroyer and submarine intersect is given by the solution of the system of equation.

$$2x+3y=5 \quad \text{...(i)}$$
$$x-y=10 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get $x=7$ and $y=-3$
$\therefore$ Two path intersects at $(7,-3)$.

65. Let amount invested at the rate of 4% be ₹ x and the amount at the rate of 5% be ₹ y.

Annual interest at 4% $=\frac{x\times4\times1}{100}=\left[\frac{4x}{100}\right]$

Amount interest at 5% $=\frac{y\times5\times1}{100}=\frac{5y}{100}$

So, $\frac{4x}{100}+\frac{5y}{100}=153 \Rightarrow 4x+5y=15300$

and $x+y=3500$ (by condition)
On solving, we get $x=2200$ and $y=1300$
Hence, amount invested at 5% = ₹ 1300

66. Given, system of equations are

$$x+2y=3 \text{ and } 3x+6y=9 \Rightarrow x+2y=3$$

Here, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Hence, given system of equation has infinitely many solutions.

67. Let the length of rectangle be x m and breadth be y m.

$\therefore$ Area of rectangle $=xy$

Case I Length $=(x+7)$ m and Breadth $=(y-3)$ m

$\therefore$ Area $=(x+7)(y-3)$

$xy=xy-3x+7y-21$ (by condition)

$3x-7y+21=0$ ...(i)

Case II Length $=(x-7)$ and Breadth $=(y+5)$

$\therefore$ Area of rectangle $=(x-7)(y+5)$

$xy=xy+5x-7y-35$ (by condition)

$5x-7y-35=0$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$\begin{array}{r} 3x-7y+21=0 \\ 5x-7y-35=0 \\ -\quad+\quad+ \\ \hline -2x\quad+56=0 \end{array}$$

$x=28$ m and $y=15$ m

$\therefore$ Area of rectangle $=15\times28=420\text{m}^2$

68. Equation of a straight line parallel to y-axis at a distance 9 is given by $x=9$

69. Linear equation always represents a straight line.

70. $2^{x+y}=\sqrt{8}, 2^{x-y}=\sqrt{8}$ (given)

$$2^{x+y}=2\sqrt{2}, 2^{x-y}=2\sqrt{2}$$
$$2^{x+y}=2^{3/2}, 2^{x-y}=2^{3/2}$$

On comparing

So, $x+y=\frac{3}{2}, x-y=\frac{3}{2}$

On adding, we get $2x=3$

$\Rightarrow \quad x=\frac{3}{2}$ and $y=0$

71. Let the fare from station P to station Q is ₹ x and the fore from station P to station R is ₹ y.
By given condition, $x+y=42$ ...(i)
and $5x+10y=350$ ...(ii)
On solving Eqs. (i) and (ii), we get $x=14$ and $y=28$
Hence, fare from station P to station Q is ₹ 14.

72. The set of homogeneous equation has an infinite number of solutions when

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

73. Let the number of soldiers be x and number of rows be y.

$\therefore$ Number of soldiers in each row $=\frac{x}{y}$

Case I When number of soldiers each row $=\frac{x}{y}+1$, then number of rows $=y-2$

$\therefore \quad \left(\frac{x}{y}+1\right)(y-2)=x$ ...(i)

Case II When number of soldiers in each row $=\frac{x}{y}-1$

Number of rows $=y+3$

$\therefore \quad \left(\frac{x}{y}-1\right)(y+3)=x$ ...(ii)

So, from Eqs. (i) and (iii) $\left(\frac{x}{y}+1\right)(y-2)=\left(\frac{x}{y}-1\right)(y+3)$

$\Rightarrow \quad 2y+1=\frac{5x}{y} \Rightarrow \frac{x}{y}=\frac{2y+1}{5}$ ...(iii)

Also from Eq. (i),

$$\left(\frac{x}{y}+1\right)(y-2)=x \Rightarrow 2\frac{x}{y}=y-2$$

From Eq. (iii), $2\left(\frac{2y+1}{5}\right)=y-2 \Rightarrow 4y+2=5y-10$

$\Rightarrow \quad y=12$
Put $y=12$ in Eq. (iii), we get $x=5\times12$
$\therefore$ Number of soldiers $=5\times12=60$

74. Let the numbers of boys and girls be x and y respectively, then

$$71x+73y=71.8(x+y) \Rightarrow 0.8x=1.2y$$

$\therefore \quad \frac{x}{y}=\frac{1.2}{0.8}=\frac{3}{2}$

75. Let pooja initial salary is ₹ x and fixed increment every year is ₹ y.
By given condition, $x+3y=4200$ ...(i)
and $x+8y=6800$ ...(ii)
On solving Eqs. (i) and (ii), we get $x=$ ₹ 2640 and $y=$ ₹ 520

76. Let the number be x and y.
$\therefore \quad x+y=15$ and $\frac{1}{x}+\frac{1}{y}=\frac{3}{10}$ (by condition)
$$\therefore \quad \frac{x+y}{xy}=\frac{3}{10}$$
$$\frac{15}{xy}=\frac{3}{10} \Rightarrow xy=50$$
$$x-y=\sqrt{(x+y)^2-4xy}=\sqrt{(15)^2-4\times 20}=\sqrt{25}=5$$
Sloving $x-y=5$ and $x+y=15$,
we get $x=10$ and $y=5$

77. Let the starting salary be ₹ x and fixed annual increment be ₹ y.
By condition, $x+4y=1500$ ...(i)
$x+10y=1800$ ...(ii)
On solving Eqs. (i) and (ii), we get $x=$ ₹ 1300

78. Let A has x mangoes and B has y mangoes.
Case I $x+30=2(y-30) \Rightarrow x+30=2y-60$
$$x-2y=-90 \quad ...(i)$$
Case II $(y+10)=3(x-10)$
$$y=3x-30-10$$
$$3x-y=40 \quad ...(ii)$$
On solving Eqs. (i) and (ii), we get $x=34, y=62$
$\therefore$ A has 34 mangoes.

79. Let number of students in room $A=x$ and in room $B=y$
So, by condition $x-10=y+10 \Rightarrow x-y=20$ ...(i)
But $x+20=2(y-20)$
$x-2y=-60$ ...(ii)
On solving Eqs. (i) and (ii), we get $x=100, y=80$
$\therefore$ Student in room $B=80$

80. Let the number of passengers in the starting $=x$
Number of passengers after first halt
$$=\left[x-\frac{x}{3}\right]+120=\frac{2}{3}x+120$$
and number of passengers after second halt
$$=\frac{1}{2}\left[\frac{2}{3}x+120\right]+100$$
But number of passengers after second halt $=240$
$$\therefore \quad \frac{1}{2}\left[\frac{2}{3}x+120\right]+100=240$$
$$\Rightarrow \quad \frac{2}{3}x+120=280 \Rightarrow \frac{2}{3}x=160 \Rightarrow x=240$$

81. Let the number be $10y+x$ and the new number obtained by reversing the order is $10x+y$.
Given that, $x-y=3$ ...(i)
According to question,
$$7(10y+x)=4(10x+y)$$
$$\Rightarrow \quad 70y+7x=40x+4y$$
$$\Rightarrow \quad 33x-66y=0 \Rightarrow x-2y=0 \quad ...(ii)$$
On solving Eqs. (i) and (ii), we get $x=6, y=3$
$\therefore$ Number $=10\times 3+6=36$

82. Let speed of car at A be x km/h and speed of car at B be y km/h.
$$\therefore \quad 9x-9y=90$$
$$x-y=10 \quad ...(i)$$
and $\frac{9}{7}x+\frac{9}{7}y=90 \Rightarrow x+y=70$ ...(ii)
On solving Eqs. (i) and (ii), we get $x=40$ km/h and $y=30$ km/h

83. Given, $\frac{2}{x}+\frac{2}{3y}=\frac{1}{6}$ ...(i)
$\frac{3}{x}+\frac{2}{y}=0$ ...(ii)
Let $A=\frac{1}{x}$ and $B=\frac{1}{y}$
$2A+\frac{2}{3}B=\frac{1}{6}$ ...(iii)
$3A+2B=0$ ...(iv)
Multiply Eq. (iii) by 6 and Eq. (iv) by 2 and subtracting Eq. (iv) from Eq. (iii)
$$12A+4B=1$$
$$6A+4B=0$$
$$- \quad - \quad -$$
$$6A=1 \Rightarrow A=\frac{1}{6} \Rightarrow x=6$$
and $2B=-\frac{1}{2} \Rightarrow B=-\frac{1}{4} \Rightarrow y=-4$
Put in relation
$$y=ax+4$$
$$-4=6a+4 \Rightarrow a=\frac{-8}{6}=\frac{-4}{3}$$

84. Let number of ₹ 5 and ₹ 10 notes be x and y.
Then, $x+y=175$, also $5x+10y=1000$
On solving these, we get $x=150$ and $y=25$
$\therefore$ Number of ₹ 10 notes is 25.

85. Let income be $9x$ and $7x$ and their expenditure is $4y$ and $3y$, respectively.
$\Rightarrow \quad 9x-4y=200$ ...(i)
$7x-3y=200$ ...(ii)
On multiply Eq. (i) by 3 and Eq. (ii) by 4 and subtracting
$$\Rightarrow \quad 27x-12y=600$$
$$28x-12y=800$$
$$- \quad + \quad -$$
$$-x=-200 \Rightarrow x=200$$
$\therefore$ Income of first person $=9\times 200=1800$
Income of second person $=7\times 200=1400$

86. Let cost of one book $=$ ₹ x
and cost of one pencil $=$ ₹ y
By given condition,
$$4x+3y=8x+y \Rightarrow 2y=4x \Rightarrow y=2x$$
$\therefore$ Cost of 4 books and 3 pencils $=4x+3y$
$=4x+6x=10x$
Also, cost of 6 books and 2 pencils $=6x+2y$
$=6x+4x=10x$