- System of linear homogeneous equation —

Vectors:

$v \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$

$v \rightarrow \begin{bmatrix} 1 \\ 10 \\ 100 \\ 11 \end{bmatrix}$ $\quad v \rightarrow \begin{array}{c} x_1 = \\ x_2 = \\ x_3 = \end{array} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 10 & 12 & 13 & 14 \\ 22 & 20 & 17 & 19 \end{bmatrix}$ 4×3

properties of ~~vectors~~ addition and scalar multiplication of vectors —

(i) $(x+y) = (y+x)$

(ii) $x+(y+z) = (x+y)+z$

(iii) $p(x+y) = px+py$

(iv) $(p+q)x = px+qx$

(v) $p(qx) = (pq)x$.

- linearly dependent of vector & linearly independent of vectors —

Ex. show that vectors $x_1 = (1,2,4)$
$x_2 = (3,6,12)$ are linearly dependent.

$\Rightarrow \quad 3x_1 = x_2$

$3x_1 + (-x_2) = 0$

here,

$k_1 x_1 + k_2 x_2 = 0$, k_1 and k_2 are not '0' (ex) ~~$x_1 = (1,2,3), x_2 = (4,-2,7)$~~

$x_1 = (0,0,0,0)$

$(1)x_1 = 0$ $\longrightarrow$ linearly ~~Bo~~ dependent.

$\boxed{k_1 x_1 = 0}$

(ex) $x_1 = (1,2,3)$, $x_2 = (4,-2,7)$

$k_1 x_1 + k_2 x_2 = 0$

$k_1(1,2,3) + k_2(4,-2,7) = (0,0,0)$

$(k_1 + 4k_2), (2k_1 - 2k_2), (3k_1 + 7k_2) = (0,0,0)$

$\boxed{k_1 = 0 \text{ and } k_2 = 0}$

$\hookrightarrow$ Linearly Independent

(ex) $x_1 = (1,0,0)$
$x_2 = (0,1,0)$
$x_3 = (0,0,1)$

$k_1 x_1 + k_2 x_2 + k_3 x_3 = 0$

$\Rightarrow k_1(1,0,0) + k_2(0,1,0) + k_3(0,0,1) = (0,0,0)$

$\Rightarrow (k_1 + 0 + 0), (0 + k_2 + 0), (0 + 0 + k_3) = (0,0,0)$

$k_1 = k_2 = k_3 = 0$

$\hookrightarrow$ ~~not~~ Linear Indep.

~~Column rank and row rank~~ =

- 1A Homogeneous equation -

ex-1
$$\left.\begin{aligned} x + 2y + 3z &= 0 \\ 3x + 4y + 4z &= 0 \\ 7x + 10y + 12y &= 0 \end{aligned}\right\} \Rightarrow$$

$$\underset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{O}{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & -4 & -9 \end{bmatrix}_{3\times3} \qquad \begin{aligned} R_2 &\to R_2 - 3R_1 \\ R_3 &\to R_3 + R_2 \end{aligned}$$

$$|A| = -2 \neq 0, \quad \text{rank}(A) = 3$$

** $n > r$

order $n = 3$ $\qquad n - r = 3 - 3 = 0$

rank $r = 3$ $\qquad n = r \to$ no. of linearly dependent solu 'o'

$$\boxed{x = 0, \ y = 0, \ z = 0}$$

Ex-2

$x + 3y - 2z = 0$
$2x - y + 4z = 0$
$x - 11y + 14z = 0$

$$\Rightarrow \underbrace{\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

→ clm transformation not possi.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix}_{3\times3} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$16 \times 7 = 112$ $14 \times 8 = 112$

$|A| = -112 + 112 = 0$

$$\begin{vmatrix} 1 & 3 \\ 0 & -7 \end{vmatrix} = -7 + 0 = -7 \neq 0$$

rank $= 2$
$n = 3$

** Linearly Independent

$r < n \Rightarrow n - r = 3 - 2 = 1 \rightarrow$ no. of solution '1'

$x + 3y - 2z = 0$
$-7y + 8z = 0$

$$\boxed{\begin{aligned} y &= \frac{8}{7}z, \\ x &= \frac{-10}{7}z \end{aligned}} \quad \begin{aligned} &= \frac{8}{7}c \\ &= -\frac{10}{7}c \end{aligned}$$

$z = c \rightarrow$ random value.