CHAPTER 17

Working Stress Method.

CONTENTS.

17. Working Stress Method

17.1 Introduction:

WSM design of RCC member is the oldest method of design. Main advantages of WSM are following

i) Less deflection due to larger section size

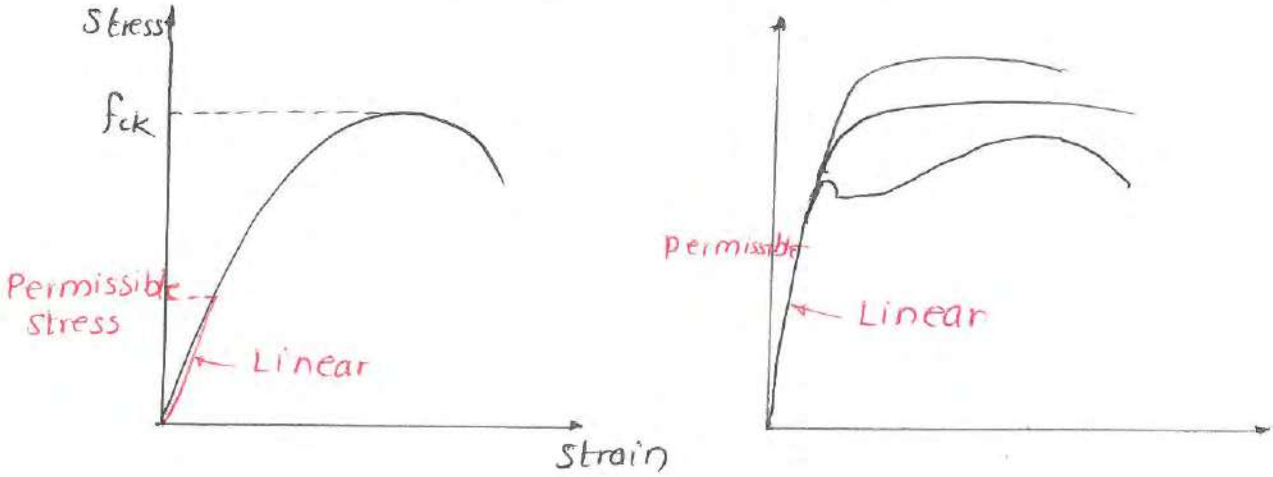
ii) Less crack width due to lower stress level of steel.

iii) Low % of steel because of larger section size.

*Note:-
LSM is economical than WSM.

17.2 Assumptions:

- Plane section remains plane after bending.
- Tensile strength of concrete is ignored.
- Modular ratio is $\frac{280}{3\sigma_{cbc}}$.
- Both materials are assumed to be linearly elastic.



17.3 Permissible Stress of Material:

A> Concrete:

Grade	σ_{cc}	σ_{cbc}	σ_t
M10	2.5	3.0	1.2
M15	4.0	5.0	2.0
M20	5.0	7.0	2.8
M25	6.0	8.5	3.2
M 30	8.0	10.0	3.6
M 35	9.0	11.5	4.0
M 40	10.0	13.0	4.4

σ_{cc} = Permissible stress under direct compression

σ_{cbc} = Permissible stress under bending compression.

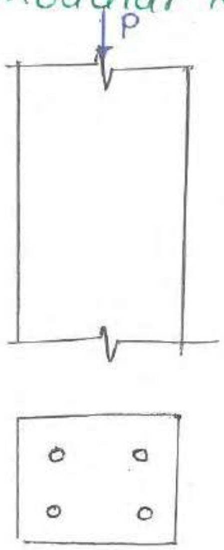
σ_t = Under tension.

B> Steel (σ_{st})

Types of Stress	Fe 250	Fe 415	Fe 500
• Tension:			
$\phi \leq 20mm$	140	230	275
$\phi > 20mm$	130	230	275
• Compression	130	190	190

* Note:
  - σ_{cc} and σ_{cbc} are approximately $\frac{f_{ck}}{4}$ and $\frac{f_{ck}}{3}$ respe.
  - Above values are increased by 33.33 % for structure subjected to wind load or earthquake load.

17.4 Use of Modular Ratio:



$$P = P_c + P_s$$

$$\Rightarrow P = f_c \cdot A_c + f_s \cdot A_s \quad \cdots\cdots (i)$$

From strain compatibility.

$$\epsilon_s = \epsilon_c$$

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = \left(\frac{E_s}{E_c}\right) f_c$$

$$f_s = m f_c \quad \cdots\cdots (ii)$$

from (i) and (ii)

$$P = f_c A_c + (m f_c) A_s$$

$$\Rightarrow P = f_c A_c + f_c (m A_s)$$

- From above expression, it is clear that area of steel can be converted into equivalent area of concrete by multiplying modular ratio to area of steel.

Ex. Compare modular ratio of WSM with short term and long term modular ratio. M30 concrete and creep coefficient 1.6.

$\Rightarrow$

- WSM

$$m = \frac{280}{3\sigma_{cbc}} = \frac{280}{3 \times 10} = 9.33$$

- Short Term:

$$m = \frac{E_s}{E_c} = \frac{2 \times 10^5}{5000\sqrt{30}} = 7.3$$

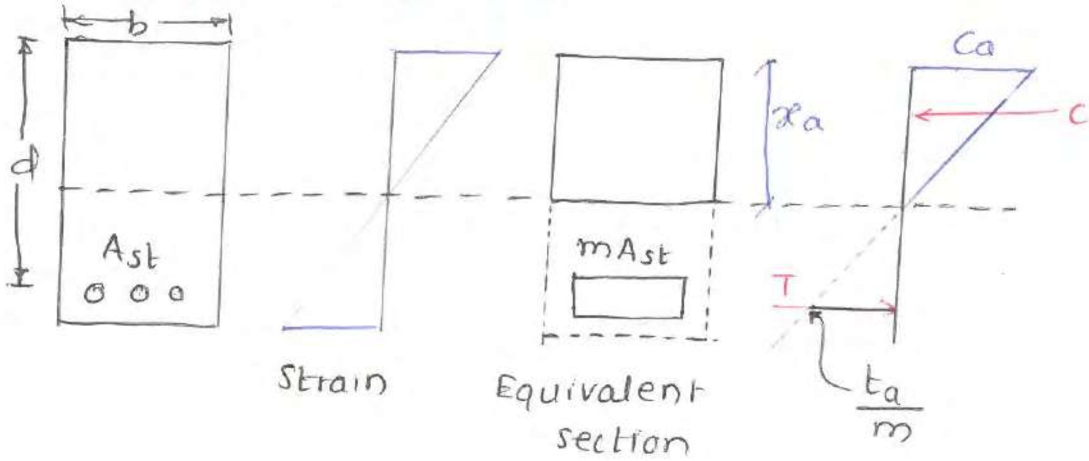
- Long Term

$$m = \frac{E_s}{E_{ce}} = \frac{2 \times 10^5}{\frac{5000\sqrt{30}}{1+1.6}} = 18.98$$

Since, m of WSM lies between short term & long term modular ratio so it can be concluded that m of WSM is partially incorporating the effect of creep.

17.5 Analysis of Singly Reinforced Section:

17.5.1 Position of Neutral Axis:



For position of NA

$$C = T$$

$$\Rightarrow \quad \frac{1}{2} \times x_a \times C_a \times b = \frac{t_a}{m} \times m A_{st} \quad \text{---- (i)}$$

from stress diagram:

$$\frac{C_a}{x_a} = \frac{t_a/m}{d - x_a}$$

$$\Rightarrow \quad C_a = \left(\frac{x_a}{d - x_a}\right) \cdot \frac{t_a}{m} \quad \text{---- (ii)}$$

from (i) and (ii)

$$\Rightarrow \frac{1}{2} \times x_a \times \left(\frac{x_a}{d - x_a}\right) \frac{t_a}{m} \times b = t_a \cdot A_{st}$$

$$\Rightarrow \quad b \, x_a \cdot \frac{x_a}{2} = m A_{st} (d - x_a)$$

$\Rightarrow$ Moment of area of comp. zone about NA = Moment of area of tension zone about NA.

Above expression shows that position of NA can be directly calculated by equating moment of area of comp. and tension zone about NA. This cannot be applied in LSM because section is of two diff. materials.

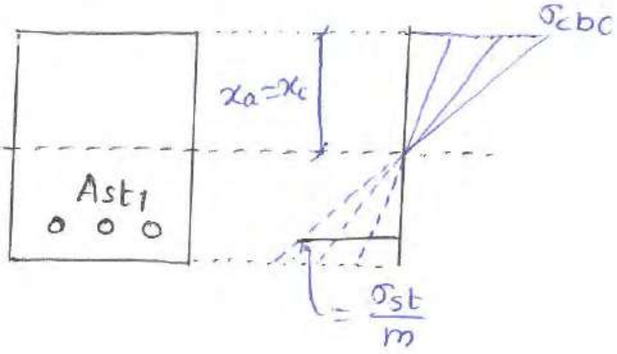
In WSM steel is converted into concrete so above expression is valid.

17.5.2 Types of Section:

Based on quantity of steel present in section, three types of sections are defined.

1) Balanced Section:

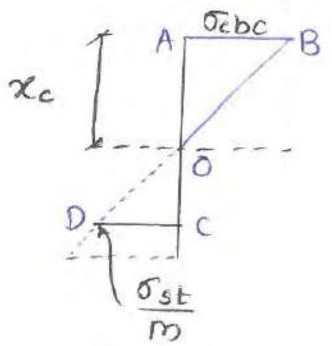
Amount of steel in section is such that concrete and steel both attain their permissible stress simultaneously.



$c_a = \sigma_{cbc}$ $\quad t_a = \sigma_{st}$

$x_a = x_c$

For $x_c \Rightarrow$



$$\frac{OA}{AB} = \frac{OC}{CD}$$

$$\frac{x_c}{\sigma_{cbc}} = \frac{d - x_c}{\sigma_{st}/m}$$

$$\Rightarrow x_c = \left(\frac{m\,\sigma_{cbc}}{m\,\sigma_{cbc} + \sigma_s}\right) d$$

$$\Rightarrow x_c = \left(\frac{mc}{mc + t}\right) \cdot d$$

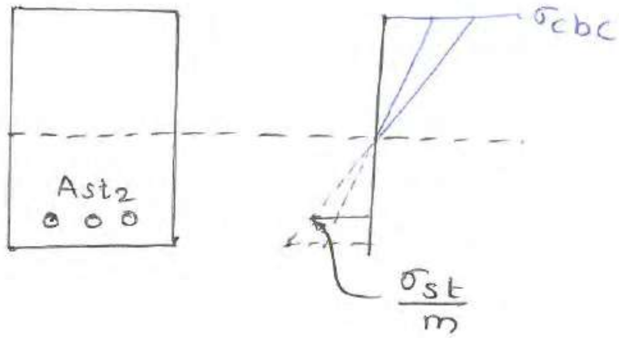
$$\Rightarrow x_c = kd$$

*** Note:**

Position of NA for balanced section (x_c) depends on grade of steel only.

2) Under Reinforced Section:

Amount of steel in section is such that steel attains its permissible stress before concrete.



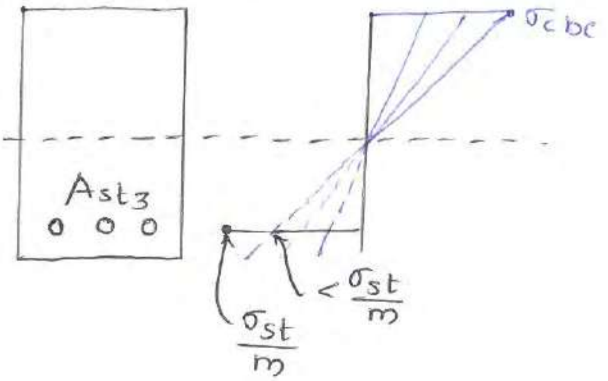
$C_a < \sigma_{cbc}$, $t_a = \sigma_{st}$

$x_a < x_c$

- Failure of under reinforced section is tension failure
- ~~Fail~~ Under reinforced section gives sufficient warining before failure so it is preferable.

3) Over Reinforced Section:

Amount of steel in section is such that concrete attains its permissible stress before steel

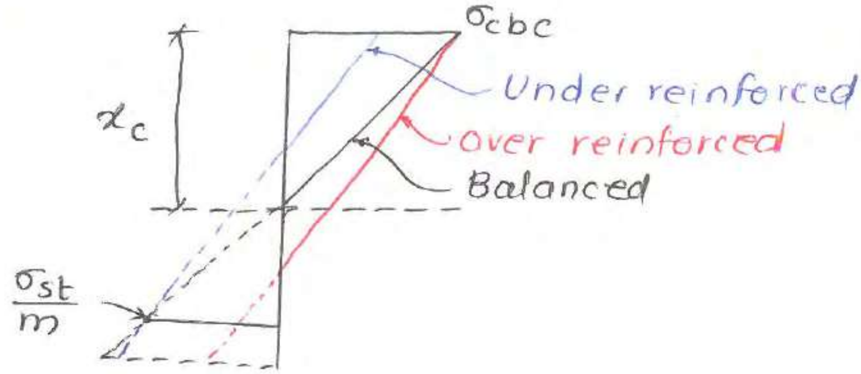


$Ca = \sigma_{cbc}$ $\quad t_a < \sigma_{st}$

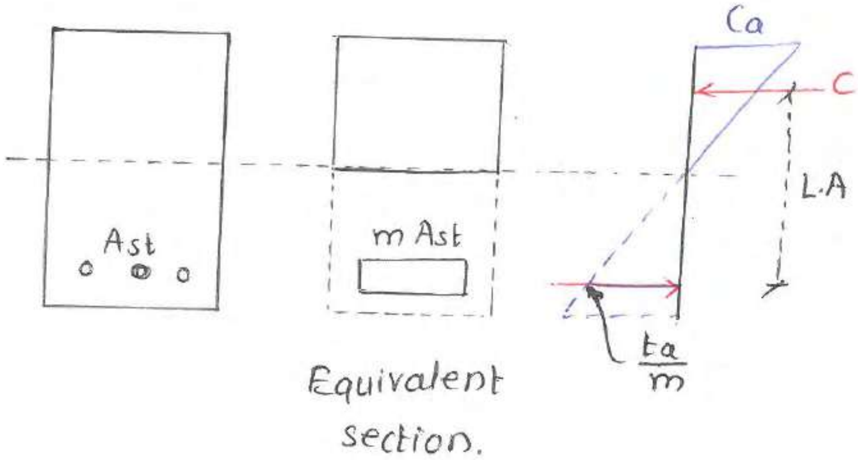
$x_a > x_c$

- Failure of over reinforced section is compression failure
- Failure of over reinforced section is sudden (without warning) so it is undesirable.

* Comparing failure stress diagram of all three types of section.



17.5.3 MR of Section:



$$MR = C \times LA$$

$$= \frac{1}{2} \times x_a \times C_a \times b \left(d - \frac{x_a}{3}\right)$$

$$MR = T \times LA$$

$$= t_a \times A_{st} \times \left(d - \frac{x_a}{3}\right)$$

1) Balanced Section:

$$MR_{bal} = C \times LA$$

$$= \frac{1}{2} \times x_c \times \sigma_{cbc} \times b \left(d - \frac{x_c}{3}\right)$$

$$= \frac{1}{2} \times (kd) \times \sigma_{cbc} \times b \left(d - \frac{kd}{3}\right)$$

$$= \frac{1}{2} \times k \left(1 - \frac{k}{3}\right) \sigma_{cbc}\ bd^2$$

$$= \underbrace{\frac{1}{2} \cdot k \cdot j \cdot \sigma_{cbc}}_{Q} \cdot bd^2$$

$$MR_{bal} = Q\,bd^2$$

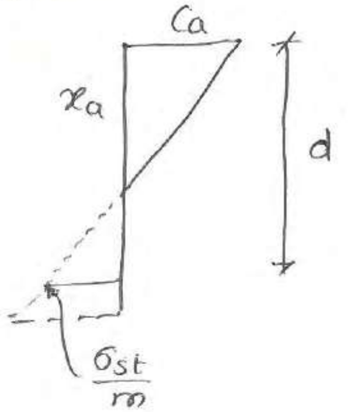
$$MR_{bal} = T \times LA$$
$$= \sigma_{st} A_{st} \left(d - \frac{kd}{3}\right)$$
$$= \sigma_{st} \cdot A_{st} \left(1 - \frac{k}{3}\right) d$$
$$MR_{bal} = \sigma_{st} \cdot A_{st} \cdot (jd)$$

~~jb~~ jd = LA of balanced section.

2> Under Reinforced Section:

$$MR = C \times LA$$
$$= \frac{1}{2} \times x_a \times C_a \times b \left(d - \frac{x_a}{3}\right)$$

C_a can be calculated from stress diagram as follows—



$$C_a = \left(\frac{x_a}{d - x_a}\right) \frac{\sigma_{st}}{m}$$

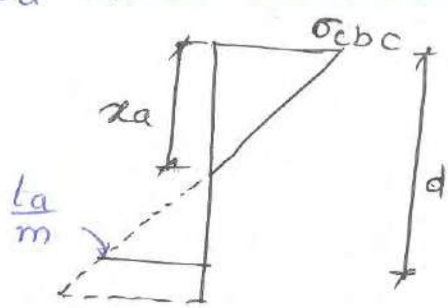
$$MR = T \times LA$$
$$= \sigma_{st} A_{st} \left(d - \frac{x_a}{3}\right) \quad \text{(Preferable)}$$

3> Over Reinforced Section.

$$MR = C \times LA$$
$$= \frac{1}{2} \times x_a \times \sigma_{cbc} \times b \left(d - \frac{x_a}{3}\right) \quad \text{(Preferable)}$$

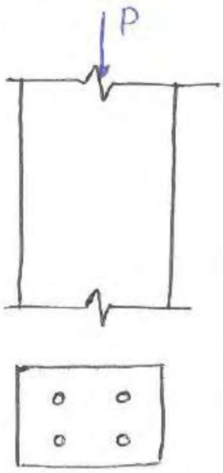
$$MR = T \times LA$$
$$= t_a \times A_{st} \cdot \left(d - \frac{x_a}{3}\right)$$

t_a can be calculated from stress diagram as follows:-



$$t_a = \left(\frac{d - x_a}{x_a}\right) m \cdot \sigma_{cbc}$$

17.6 Axial Load Carrying Capacity of Column:



$$P_{short} = P_c + P_s$$

$$P_{short} = \sigma_{cc} \cdot A_c + \sigma_{sc} \cdot A_{sc}$$

$$= \sigma_{cc}(A_g - A_{sc}) + \sigma_{sc} \cdot A_{sc}$$

$$P_{short} = \sigma_{cc} \cdot A_g + (\sigma_{sc} - \sigma_{cc}) \cdot A_{sc}$$

Now,

$$P_{Long} = C_r \cdot P_{short}$$

where, C_r = Reduction coefficient

$$C_r = 1.25 - \frac{L_{eff}}{48 b}$$

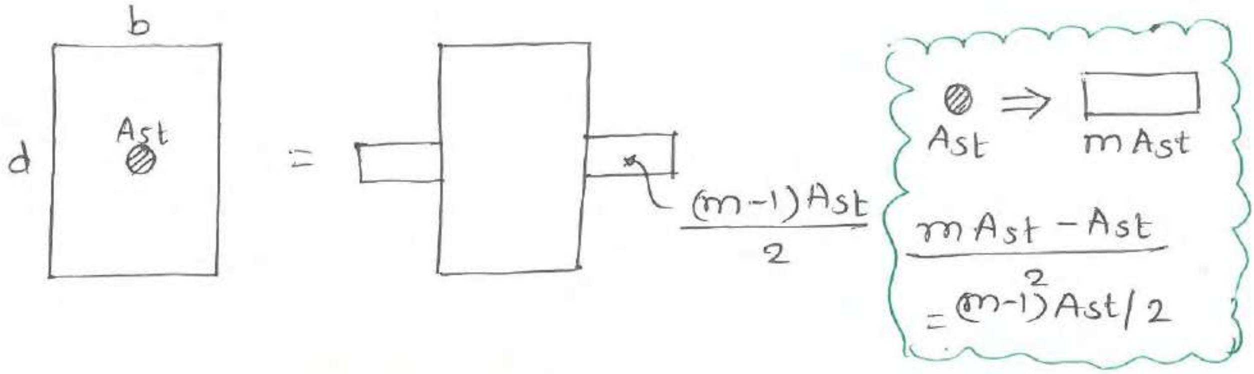
* Note:-

- Modular ratio of compression stee is 1.5 times modular ratio of tension steel This enhancement is done to take care of creep effect of concrete

$$m' = 1.5 m$$

$$\text{where, } m = \frac{280}{3\sigma_{cbc}}$$

Ex. Calculate equivalent area of section given below.



$A_{eq} = bd + (m-1) \cdot A_{st}$

Ex. Calculate

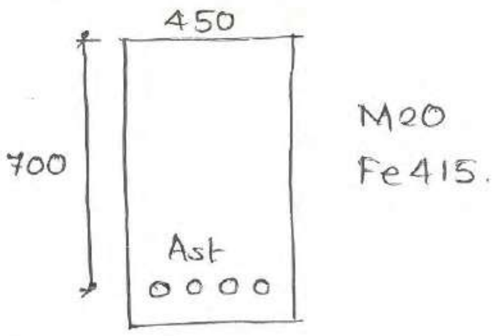
i) Position of critical N.A.

ii) Ast required for balanced section.

iii) MR of balanced section.

iv) If Ast is 3-20ϕ then calculate MR, also calculate stress of concrete and steel for BM 100 kNm. (WSM special)

v) If Ast is 5-20ϕ the calculate MR



M20

Fe415.

⇒ (i) $m = \dfrac{280}{3\sigma_{cbc}} = \dfrac{280}{3 \times 7} = 13.33$

$k = \dfrac{mc}{mc + t} = \dfrac{13.33 \times 7}{13.33 \times 7 + 230}$

$k = 0.288$

$x_c = kd$

$= 0.288 \times 700$

$x_c = 201.6$ mm

(ii) $C = T$

$\frac{1}{2} \times x_c \times \sigma_{cbc} \times b = \sigma_{st} \cdot A_{st}$

$\frac{1}{2} \times 201.6 \times 7 \times 450 = 230 \times A_{st}$

$A_{st} = 1380.52 mm$

(iii) $MR_{bal} = \frac{1}{2} k \cdot j \cdot \sigma_{cbc} \cdot bd^2$

$= \frac{1}{2} \times 0.288 \times \left(1 - \frac{0.288}{3}\right) \times 7 \times 450 \times 700^2$

$MR_{bal} = 200.92$ kN·m

(iv) For position of N.A.

$b \cdot x_a \cdot \frac{x_a}{2} = m A_{st} (d - x_a)$

$450 \times \frac{x_a^2}{2} = 13.33 \times 3 \times \frac{\pi}{4} \times 20^2 (700 - x_a)$

$x_a = 171.74 mm$

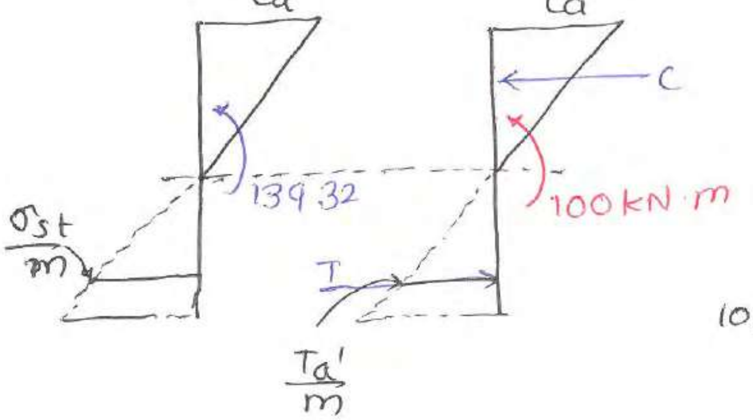
Since, $x_a < x_c$ so section is under reinforced.

$MR = T \times LA$

$= \sigma_{st} \cdot A_{st} \cdot \left(d - \frac{x_a}{3}\right)$

$= 230 \times 3 \times \frac{\pi}{4} \times 20^2 \times \left(700 - \frac{171.74}{3}\right)$

$MR = 139.32$ kN·m



BM = Resistance of section

$BM = C \times LA$

$BM = \frac{1}{2} \cdot x_a C_a' \cdot b\left(d - \frac{x_a}{3}\right)$

$100 \times 10^6 = \frac{1}{2} \times 171.74 \times C_a' \times 450 \times \left(700 - \frac{171.74}{3}\right)$

$C_a' = 4.03 N/mm^2$

From stress diagram:

$$t_a' = \left(\frac{d - x_a}{x_a}\right) m \cdot c_a' = \left(\frac{700 - 171.74}{171.74}\right) \times 13.33 \times 4.03$$

$$t_a' = 165.23 \ N/mm^2$$

v) For position of N.A.

$$b \cdot x_a \cdot \frac{x_a}{2} = m \ A_{st} \cdot (d - x_a)$$

$$450 \times \frac{x_a^2}{2} = 13.33 \times 5 \times \frac{\pi}{4} \times 20^2 \times (700 - x_a)$$

$$x_a = 212.906 mm$$

Since, $x_a > x_c$ so section is over reinforced.

$$MR = C \times LA = \frac{1}{2} x_a \cdot \sigma_{cbc} \times b \left(d - \frac{x_a}{3}\right)$$

$$=$$

$$MR = 210.93 \ kN \cdot m$$

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