CBSE Board

Class X Mathematics

Sample Paper 1 (Standard)

Time: 3 hrs **Total Marks: 80**

General Instructions:

1. This question paper contains **two parts** A and B.
2. Both **Part A** and **Part B** have internal choices.

Part – A:

1. It consists **two sections** - I and II.
2. **Section I** has **16 questions** of **1 mark** each. Internal choice is provided in **5 questions**.
3. **Section II** has **4 questions** on **case study**. Each case study has **5 case-based sub-parts**. An examinee is to attempt any **4 out of 5 sub-parts**. Each subpart carries **1 mark.**

Part – B:

1. It consists **three sections** – III, IV and V
2. **Section III: Question No 21 to 26** are **Very short answer** Type questions of **2 marks** each.
3. **Section IV: Question No 27 to 33** are **Short Answer Type** questions of **3 marks** each.
4. **Section V: Question No 34 to 36** are **Long Answer Type** questions of **5 marks** each.
5. Internal choice is provided in **2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks**

Part A

Section I

Section I has 16 questions of 1 mark each.

(Internal choice is provided in 5 questions)

1. What is the LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$?

OR

What is the largest number that divides each one of 1152 and 1664 exactly?

2. If $\frac{2x}{3} - \frac{y}{2} + \frac{1}{6} = 0$ and $\frac{x}{2} + \frac{2y}{3} = 3$ then

3. What is the value of tan 5° tan 25° tan 30° tan 65° tan 85° = ?

4. Find the value of cos 1° cos 2° cos 3° …… cos 180°.

5. Find the distance of the point (–3, 4) from x-axis.

6. The area of a square field is 6050 m^2, then what will be the length of its diagonal?

7. If one zero of $3x^2 + 8x + k$ be the reciprocal of the other then find the value of k?

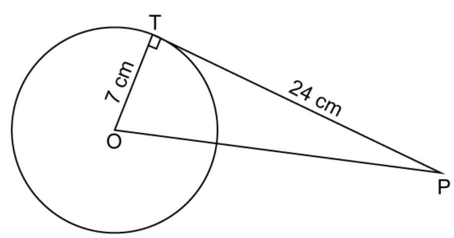
8. The shadow of a 5 m long stick is 2 m long. At the same time what will be the length of the shadow of a 12.5 m high tree (in m)?

9. Find the common difference of the AP, if the sum of first n terms of an AP is $(3n^2 + 6n)$.

OR

Which term of the AP 21, 18, 15, … is –81?

10. In a circle of radius 7 cm, tangent PT is drawn from a point P such that PT = 24 cm. If O is the center of the circle, then what is the length of OP?



OR

The chord of a circle of radius 10 cm subtends a right angle at its center. Find the length of the chord (in cm)?

11. If the probability of occurrence of an event is p then find the probability of non-happening of this event.

OR

A digit is chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 then find the probability that it is odd.

12. What are the Zeroes of $p(x) = x^2 – 2x – 3$?

13. Is the pair of equations y = 0 and y = –5 has solutions?

14. Determine the values of p for which the quadratic equation $2x^2 + px + 8 = 0$ has real and equal roots.

OR

If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 + 2x + 1$, find the value value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

15. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 81, find the other.

16. If $\Delta ABC \sim \Delta DEF$ such that $2AB = DE$ and $BC = 6$ cm, find EF.

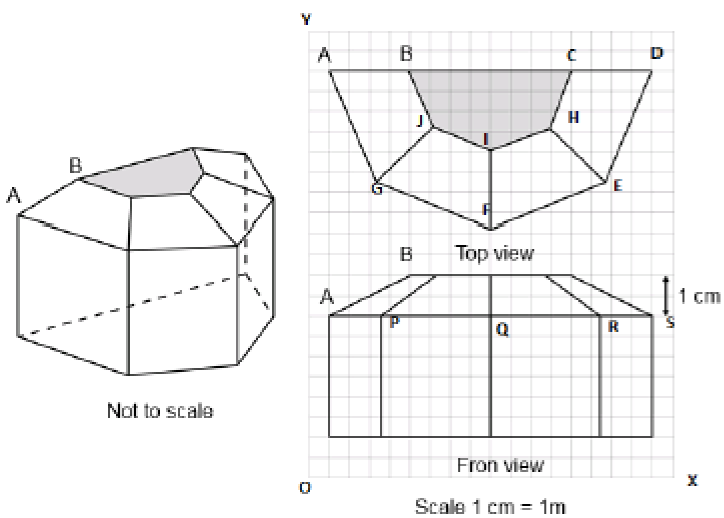
Section II

(Q 17 to Q 20 carry 4 marks each)

Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-1
SUN ROOM

The diagrams show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sunroom are square clear glass panels. The roof is made using
• Four clear glass panels, trapezium in shape, all the same size
• One tinted glass panel, half a regular octagon in shape



(a) **Refer to Front View**
Find the mid-point of the segment joining the points J (6, 17) and I (9, 16).
(i) (33/2, 15/2)
(ii) (3/2, 1/2)
(iii) (15/2, 33/2)
(iv) (1/2, 3/2)

(b) **Refer to Front View**
The distance of the point P from the y-axis is
(i) 4 (ii) 15 (iii) 19 (iv) 25

(c) **Refer to Front View**
The distance between the points A and S is
(i) 4 (ii) 8 (iii)16 (iv)20

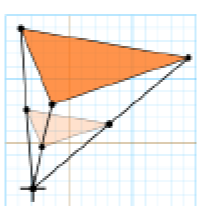
(d) **Refer to Front View**
Find the co-ordinates of the point which divides the line segment joining the points A and B in the ratio 1:3 internally.
(i) (8.5, 2.0) (ii) (2.0, 9.5) (iii) (3.0, 7.5) (iv) (2.0, 8.5)

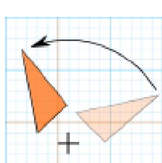
(e) **Refer to Front View**
If a point (x, y) is equidistant from the Q(9, 8) and S(17, 8), then
(i) $x + y = 13$ (ii) $x - 13 = 0$ (iii) $y - 13 = 0$ (iv) $x - y = 13$

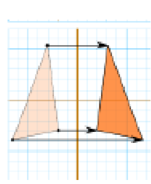
18. Case Study Based- 2
SCALE FACTOR AND SIMILARITY
SCALE FACTOR
A scale drawing of an object is the same shape as the object but a different size.
The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio.
SIMILAR FIGURES
The ratio of two corresponding sides in similar figures is called the scale factor.

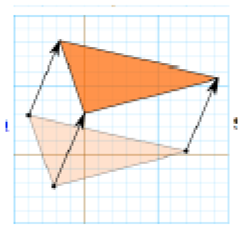
$$\text{Scale factor} = \frac{\text{length in image}}{\text{corresponding length in object}}$$

If one shape can become another using Resizing then the shapes are Similar




Rotation or Turn

Reflection or Flip




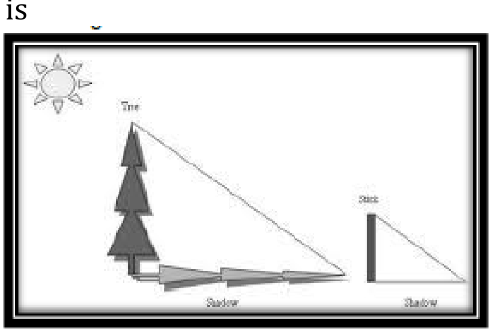
Hence, two shapes are Similar when one can become the other after a resize, flip, slide or turn.

(a) A model of a boat is made on the scale of 1:4. The model is 120cm long. The full size of the boat has a width of 60cm. What is the width of the scale model?
(i) 20cm
(ii) 25cm
(iii) 15cm
(iv) 240cm

(b) What will affect the similarity of any two polygons?
(i) They are flipped horizontally
(ii) They are dilated by a scale factor
(iii) They are translated down
(iv) They are not the mirror image of one another

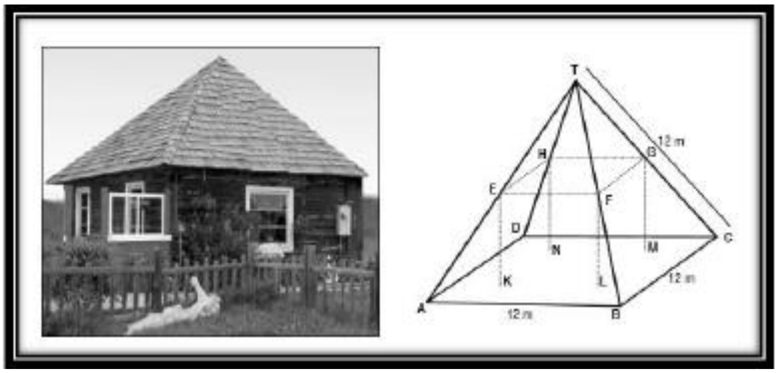
(c) If two similar triangles have a scale factor of a: b. Which statement regarding the two triangles is true?
(i) The ratio of their perimeters is 3a: b
(ii) Their altitudes have a ratio a: b
(iii) Their medians have a ratio $a/2$: b
(iv) Their angle bisectors have a ratio a^2: b^2

(d) The shadow of a stick 5m long is 2m. At the same time the shadow of a tree 12.5m high is



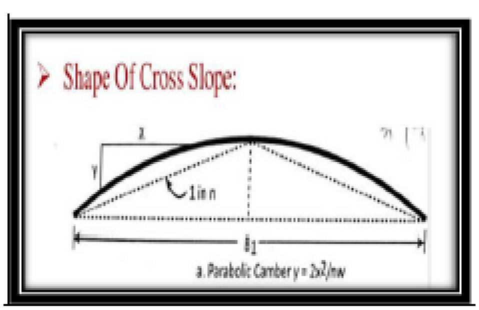
(i) 3m
(ii) 3.5m
(iii) 4.5m
(iv) 5m

(e) Below you see a student's mathematical model of a farmhouse roof with measurements.The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have length of 12 m.



What is the length of EF, where EF is one of the horizontal edges of the block?
(i) 24m
(ii) 3m
(iii) 6m
(iv) 10m

19. Case Study Based- 3
Applications of Parabolas-Highway Overpasses/Underpasses
A highway underpass is parabolic in shape.



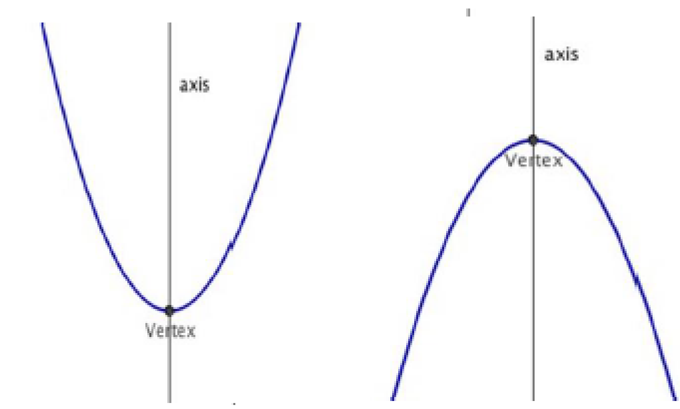
Parabola

A parabola is the graph that results from $p(x)=ax^2+bx+c$

Parabolas are symmetric about a vertical line known as the Axis of Symmetry.

The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the

Vertex



(a) If the highway overpass is represented by x^2-2x-8. Then its zeroes are
(i) (2, –4) (ii) (4, –2) (iii) (–2, –2) (iv) (–4, –4)

(b) The highway overpass is represented graphically.
Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial
(i) Intersects x-axis
(ii) Intersects y-axis
(iii) Intersects y-axis or x-axis
(iv) None of the above

(c) Graph of a quadratic polynomial is a
(i) straight line
(ii) circle
(iii) parabola
(iv) ellipse

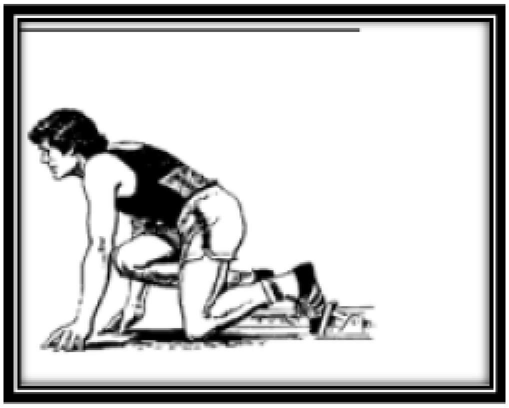
(d) The representation of Highway Underpass whose one zero is 6 and sum of the zeroes is 0, is
(i) $x^2 - 6x + 2$
(ii) $x^2 - 36$
(iii) $x^2 - 6$
(iv) $x^2 - 3$

(e) The number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is:
(i)1
(ii) 2
(iii) 0
(iv) 3

20. Case Study Based- 4

100m RACE

A stopwatch was used to find the time that it took a group of students to run 100m.



Time(in sec)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
No. of students	8	10	13	6	3

(a) Estimate the mean time taken by a student to finish the race.
(i) 54
(ii) 63
(iii) 43
(iv) 50

(b) What will be the upper limit of the modal class?
(i) 20
(ii) 40
(iii) 60
(iv) 80

(c) The construction of cumulative frequency table is useful in determining the
(i) Mean
(ii) Median
(iii) Mode
(iv) All of the above

(d) The sum of lower limits of median class and modal class is
(i) 60
(ii) 100
(iii) 80
(iv) 140

(e) How many students finished the race within 1 minute?
(i)18
(ii)37
(iii)31
(iv)8

Part B

All questions are compulsory. In case of internal choices, attempt any one.

Section III

(Q 21 to Q 26 carry 2 marks each)

21. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segments joining the points of contact at the centre.

22. A ladder 15 m long just reaches the top of a vertical wall. If the ladder makes an angle of 60° with the wall then find the height of the wall.

OR

Find the value of $\dfrac{2\sin^2 63^\circ + 1 + 2\sin^2 27^\circ}{3\cos^2 17^\circ - 2 + 3\cos^2 73^\circ}$.

23. Find the coordinates of the point on x-axis which is equidistant from points A(−1, 0) and B(5, 0).

OR

If R(5, 6) is the midpoint of the line segment AB joining the points A(x, 5) and B(4, y) then find the values of x and y.

24. If one zero of the quadratic polynomial $(k - 1)x^2 + kx + 1$ is −4, then find the value of k

25. Find:

i. If a, a − 2, 3a are in A.P. then a = ____

ii. If a = 8, $T_n = 62$ and $S_n = 210$ then n = ____

26. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

Section IV

(Q 27 to Q 33 carry 3 marks each)

27. In a trapezium ABCD, it is given that AB ∥ CD and AB = 2CD. Its diagonals AC and BD intersect at the point O such that ar(ΔAOB) = 84 cm^2. Find ar(ΔCOD).

OR

Two triangle ABC and PQR are such that AB = 3 cm, AC = 6 cm, ∠A = 70°, PR = 9 cm, ∠P = 70° and PQ = 4.5 cm. Show that ΔABC ∼ ΔPQR and state the similarity criterion.

28. Find the number of solid spheres, each of diameter 6 cm that could be molded to form a solid metallic cylinder of height 45 cm and diameter 4 cm.

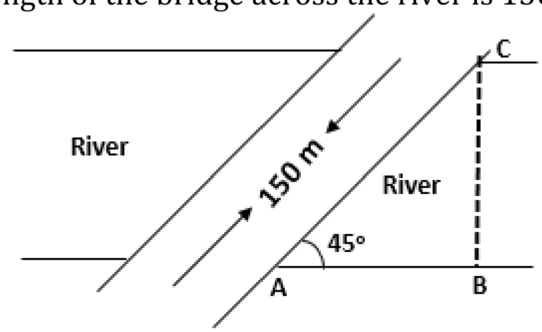
29. Prove that $\frac{2}{\sqrt{7}}$ is irrational.

30. The arithmetic mean of the following frequency distribution is 25.

Class	0-10	10-20	20-30	30-40	40-50
Frequency	16	p	30	32	14

Find the value of p.

31. Bridge across a river makes an angle of 45° with the river bank as shown in the figure. If the length of the bridge across the river is 150 m, what is the width of the river?



32. Find the mean of following distribution by the step deviation method.

Daily Expenditure:	100-150	150-200	200-250	250-300	300-350
No. of householders:	4	5	12	2	2

33. Solve: $23x + 29y = 98$, $29x + 23y = 110$

OR

Solve: $6x + 3y = 7xy$ and $3x + 9y = 11xy$

Section V
(Q 34 to Q 36 carry 5 marks each)

34. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km/hr, in how much time will the tank be filled completely?

35. An electrician has to repair an electric fault on a pole of height 4 metres. He needs to reach a point 1 metre below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use, which when inclined at an angle of 60° to the horizontal would enable him to reach the required position?
$\left[\text{Take } \sqrt{3} = 1.732\right]$

OR

A straight highway leads to foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30^o, which is approaching the foot of the tower with a uniform speed. Six seconds later the angle of depression of the car is found to be 60^o. Find the time taken by the car to reach the foot of the tower from this point.

36. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

CBSE Board

Class X Mathematics

Sample Paper 1 (Standard) – Solution

Part A
Section I

1.

$(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$

$\text{LCM} = 2^4 \times 3 \times 5 \times 7 = 1680$

OR

Prime factorization of 1152 = $2^7 \times 3^2$

Prime factorization of 1664 = $2^7 \times 13$

HCF(1152, 1664) = 2^7 = 128

Hence, the largest number is 128, which divides 1152 and 1664 exactly.

2.

$\frac{2x}{3} - \frac{y}{2} + \frac{1}{6} = 0$

Multiply by the LCM, 6.

$\Rightarrow 4x - 3y + 1 = 0$

$\Rightarrow 4x - 3y = -1 \quad ....(i)$

$\frac{x}{2} + \frac{2y}{3} = 3$

Multiply by the LCM, 6.

$\Rightarrow 3x + 4y = 18 \quad ....(ii)$

Multiply equation (i) and (ii) by 4 and 3 respectively.

$16x - 12y = -4 \quad ....(iii)$

$9x + 12y = 54 \quad ....(iv)$

Adding equations (iii) and (iv), we get

$25x = 50$

$\Rightarrow x = 2$

Substituting $x = 2$ in (ii), we get $y = 3$.

3.

$\tan 5° \tan 25° \tan 30° \tan 65° \tan 85°$

$= \tan 5° \times \tan 25° \times \frac{1}{\sqrt{3}} \times \tan(90° - 25°) \times \tan(90° - 5°)$

$= \tan 5° \times \tan 25° \times \frac{1}{\sqrt{3}} \times \cot 25° \times \cot 5°$

$= \tan 5° \times \cot 5° \times \tan 25° \times \cot 25° \times \frac{1}{\sqrt{3}}$

$= 1 \times 1 \times \frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

4.

Since $\cos 90° = 0$

$\cos 1° \cos 2° \cos 3° ... \cos 90° ... \cos 180° = 0$

5.

The distance of the point $P(-3, 4)$ from the x-axis

= y-coordinate of the point

= 4 units

6.

We know that all the sides of a square are equal.

Let each side of the square = x m

Area of the square $= (\text{side})^2$

$\Rightarrow 6050 = x^2$

$\Rightarrow x = 77.78$

$\Rightarrow$ Each side of the square = 77.8 m

We know that,

Length of the diagonal $= \sqrt{2}x$

$= 1.414 \times 77.8$

$= 110$ m

7.

Let α and $\frac{1}{\alpha}$ be the zeros of $3x^2 + 8x + k$.

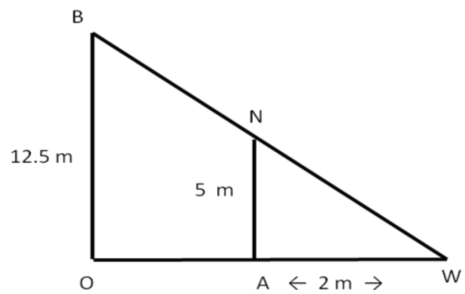
Then, we have

$\alpha \times \frac{1}{\alpha} = \frac{k}{3}$

$\Rightarrow 1 = \frac{k}{3}$

$\Rightarrow k = 3$

8.



Let AN be the long stick and AW be its shadow.

Let OB be the tree and OW be its shadow.

$AW = 2\ m$

$AN = 5\ m$

$OB = 12.5\ m$

Ratio of actual lengths = Ratio of their shadows

$\Rightarrow \frac{OB}{AN} = \frac{OW}{AW}$

$\Rightarrow \frac{12.5}{5} = \frac{OW}{2}$

$\Rightarrow OW = \frac{12.5 \times 2}{5}$

$\Rightarrow OW = 5.0\ m$

So, the length of the shadow is 5.0 m

9.

The sum of first n terms of an AP is $\left(3n^2 + 6n\right)$.

$S_n = 3n^2 + 6n$

$\Rightarrow S_{n-1} = 3\left(n-1\right)^2 + 6\left(n-1\right)$

$= 3\left(n^2 - 2n + 1\right) + 6\left(n-1\right)$

$= 3n^2 - 6n + 3 + 6n - 6$

$= 3n^2 - 3$

$a_n = S_n - S_{n-1}$

$= 3n^2 + 6n - 3n^2 + 3$

$= 6n + 3$

Let d be the common difference of the AP.

$d = a_n - a_{n-1}$

$= (6n+3) - [6(n-1)+3]$

$= (6n+3) - 6(n-1) - 3$

$= 6$

OR

The given AP is 21, 18, 15,...

$a = 21$ and $d = 18 - 21 = -3$

$a_n = a + (n-1)d$

$\Rightarrow -81 = 21 + (n-1)(-3)$

$\Rightarrow -81 = 21 + (n-1)(-3)$

$\Rightarrow -81 = 21 - 3n + 3$

$\Rightarrow 3n = 105$

$\Rightarrow n = 35$

So, -81 is the 35th term.

10.

$PT = 24$ cm

$OT = 7$ cm

Since PT is a tangent to the circle at T.

$\angle PTO = 90^\circ$ ....(tangent is perpendicular to the radius of a circle)

In ΔPTO,

By Pythagoras theorem,

$OP^2 = PT^2 + OT^2$

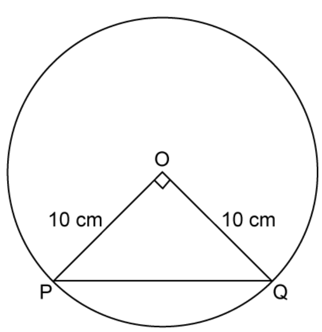
$\Rightarrow OP^2 = 24^2 + 7^2$

$\Rightarrow OP^2 = 576 + 49$

$\Rightarrow OP^2 = 625$

$\Rightarrow OP = 25$ cm

OR



In ΔPOQ,

By Pythagoras theorem,

$PQ^2 = PO^2 + OQ^2$

$\Rightarrow PQ^2 = 10^2 + 10^2$

$\Rightarrow PQ^2 = 100 + 100$

$\Rightarrow PQ^2 = 200$

$\Rightarrow PQ = 10\sqrt{2}$ cm

So, the length of the chord is $10\sqrt{2}$ cm.

11.

Let E be the event.

So, the probability of the event happening will be P(E).

Thus, the probability of the event not happening will be P(E').

Given that, $P(E) = p$

We know that, $P(E) + P(E') = 1$

$\Rightarrow p + P(E') = 1$

$\Rightarrow P(E') = 1 - p$

OR

Let A be the event of getting a number which is odd.
S = {1, 2, 3, 4, 5, 6, 7, 8, 9} and A = {1, 3, 5, 7, 9}
n(S) = 9 and n(A) = 5
P(A) = 5/9

12.

$x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow x(x - 3) + (x - 3) = 0$

$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3$ or $x = -1$

13.

y = 0 is the x-axis.
y = –5 is the line parallel to x-axis at a distance of 5 units.
Both the lines are parallel to each other. So, they don't meet anywhere.
Hence, no solution exists.

14.

$2x^2 + px + 8 = 0$

$\Rightarrow a = 2, b = p$ and $c = 8$

The given quadratic equation has real and equal roots.

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow p^2 - 4 \times 2 \times 8 = 0$

$\Rightarrow p^2 = 64$

$\Rightarrow p = \pm 8$

OR

It is given that α and β are the zeros of the quadratic polynomial $f(x) = x^2 + 2x + 1$

$\therefore \alpha + \beta = -\frac{2}{1} = -2$ and $\alpha\beta = \frac{1}{1} = 1$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-2}{1} = -2$

15.

Let the numbers be a and 81.

$\text{HCF} \times \text{LCM} =$ product of the two numbers

$\Rightarrow 27 \times 162 = 81a$

$\Rightarrow a = 54$

So, the other number is 54.

16.

$\Delta ABC \sim \Delta DEF$

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{1}{2} = \frac{6}{EF}$

$\Rightarrow EF = 12$ cm

Section II

17.

(a) Mid-point of J and I = $\left(\frac{6+9}{2}, \frac{17+16}{2}\right) = \left(\frac{15}{2}, \frac{33}{2}\right)$

(b) The distance of the point P from the y – axis is 4m.

(c) The distance between A and S is 16m.

(d) Coordinates of A are (1, 8) and that of B are (5, 10)
Coordinates of a point dividing AB in the ratio 1:3 is

$\left(\frac{1\times5+3\times1}{1+3}, \frac{1\times10+3\times8}{1+3}\right) = \left(2, \frac{17}{2}\right) = (2.0, 8.5)$

(e) (x, y) is equidistant from Q(9, 8) and S(17, 8).

$\Rightarrow (9 - x)^2 + (8 - y)^2 = (17 - x)^2 + (8 - y)^2$

$\Rightarrow 81 - 18x = 289 - 34x$

$\Rightarrow 16x = 208$
$\Rightarrow x = 13$
$\Rightarrow x - 13 = 0$

18.

(a) Width of the scale model = $\frac{1}{4} \times \text{width of the boat} = \frac{1}{4} \times 60 = 15\text{cm}$

(b) If any two polygons are not the mirror image of one another then there similarity will effect.

(c) We know that,

Scale factor = $\frac{\text{length in image}}{\text{corresponding length in object}}$

If two similar triangles have a scale factor of a: b then their altitudes have a ratio a: b

(d) This is an example of similarity.

$\Rightarrow \frac{5}{2} = \frac{12.5}{\text{Shadow of a tree}}$

$\Rightarrow \text{Shadow of a tree} = \frac{25}{5} = 5\text{m}$

(e) Here, ΔTEF and ΔTAB are similar triangles as they form the equal angles
Therefore, the ratio of their corresponding sides is same.
As E and F are the midpoints TA and TB, so TE = 6m and TF = 6m

$\frac{EF}{AB} = \frac{TE}{TA} = \frac{1}{2} \Rightarrow EF = 6\text{m}$

19.

(a) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = 0$
$\Rightarrow (x - 4)(x + 2) = 0$
$\Rightarrow x = 4$ or $x = -2$

(b) Zeroes of a polynomial can be expressed graphically. Number of zeroes of polynomial is equal to number of points where the graph of polynomial **Intersects x – axis.**

(c) Graph of a quadratic polynomial is a Parabola.

(d) A highway underpass is parabolic in shape and a parabola is the graph that results from $p(x)=ax^2+bx+c$ which has two zeroes. (As it is a quadratic polynomial)
Sum of zeroes = 0 and one of the zero = 6 $\Rightarrow$ other zero = –6
x^2 – (sum of zeroes)x + product of zeroes
$= x^2 - 36$

(e) $f(x) = (x - 2)^2 + 4 = x^2 - 4x + 8$ is a Quadratic Polynomial.
The number of zeroes that f(x) can have is 2

20.

(a)

Time (in sec)	No. od students(f)	x	fx
0 – 20	8	10	80
20 – 40	10	30	300
40 – 60	13	50	650
60 – 80	6	70	420
80 – 100	3	90	270
	Σf = 40		**Σ**fx = 1720

Mean time taken by a student to finish the race = 1720/40 = 43 seconds

(b) The modal class is 40 – 60 as it has the highest frequency i.e 13.
Upper limit of the modal class = 60

(c) The construction of cumulative frequency table is useful in determining the Median.

(d)

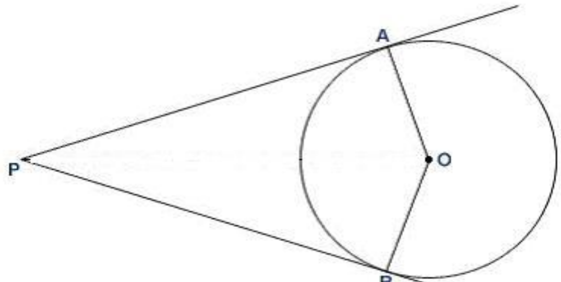
Time (in sec)	**No. od students(f)**	**cf**
0 – 20	8	8
20 – 40	10	18
40 – 60	13	31
60 – 80	6	37
80 – 100	3	40
	N= **Σ**f = 40	

Here N/2 = 40/2 = 20, Median Class = 40 – 60, Modal Class = 40 – 60
Sum of lower limits of median class and modal class = 40 + 40 = 80

(e) Number of students who finished the race within 1 minute
= 8 + 10 + 13 = 31

Part B
Section III

21.



Given: PA and PB are the tangents drawn from a point P to a circle with center O.

Also, the line segments OA and OB are drawn.

To prove: $\angle APB + \angle AOB = 180°$

Proof:

We know that the tangent is perpendicular to the radius through the point of contact.

$\therefore PA \perp OA \Rightarrow \angle OAP = 90°$

$\therefore PB \perp OB \Rightarrow \angle OBP = 90°$

$\therefore \angle OAP + \angle OBP = 90° + 90° = 180°$ ....(i)

But, we know that the sum of all the angles of a quadrilateral is 360°.

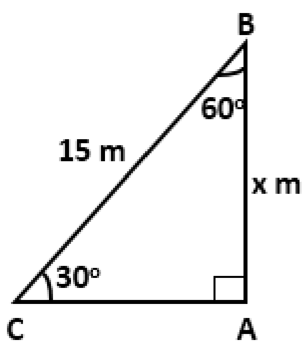
$\therefore \angle OAP + \angle OBP + \angle APB + \angle AOB = 360°$ ....(ii)

From (i) and (ii), we get

$\angle APB + \angle AOB = 180°$

Hence proved.

22.



Let BC be the ladder and AB be the wall.

Then, $BC = 15$ m

$\angle ABC = 60°$

$\Rightarrow \angle ACB = 90° - 60° = 30°$

Let the height of the wall $AB = x$ m

Now, $\sin 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{2} = \frac{x}{15}$

$\Rightarrow x = \frac{15}{2}$ m

OR

$$\frac{2\sin^2 63° + 1 + 2\sin^2 27°}{3\cos^2 17° - 2 + 3\cos^2 73°}$$

$$= \frac{2\sin^2 63° + 2\sin^2 27° + 1}{3\cos^2 17° + 3\cos^2 73° - 2}$$

$$= \frac{2\sin^2 63° + 2\cos^2 63° + 1}{3\cos^2 17° + 3\sin^2 17° - 2}$$

$$=\frac{2(\sin^2 63^\circ+\cos^2 63^\circ)+1}{3(\cos^2 17^\circ+\sin^2 17^\circ)-2}$$
$$=\frac{2\times1+1}{3\times1-2}$$
$$=\frac{2+1}{3-2}$$
$$=3$$

23.

Since the point lies on the x-axis, let the point be P

and its coordinates be $(x,0)$.

Given that the point is equidistant from the points A and B.

$\Rightarrow PA=PB$

$\Rightarrow \sqrt{(x+1)^2}=\sqrt{(x-5)^2}$

$\Rightarrow (x+1)^2=(x-5)^2$

$\Rightarrow x^2+2x+1=x^2-10x+25$

$\Rightarrow 2x+1=-10x+25$

$\Rightarrow 12x=24$

$\Rightarrow x=2$

Hence, the point is $(2,0)$.

OR

Given that R is the mid-point of the line segment AB.

The $x-$coordinate of $R=\frac{x+4}{2}$ and the y-coordinate of $R=\frac{5+y}{2}$

$\Rightarrow 5=\frac{x+4}{2}$ and $6=\frac{5+y}{2}$

$\Rightarrow 10=x+4$ and $12=5+y$

$\Rightarrow x=6$ and $y=7$

24.

Since -4 is a zero of $f(x)=(k-1)x^2+kx+1$, we have

$f(-4)=0$

$\Rightarrow (k-1)(-4)^2+k(-4)+1=0$

$\Rightarrow (k-1)16-4k+1=0$

$\Rightarrow 16k-16-4k+1=0$

$\Rightarrow 12k-15=0$

$\Rightarrow 12k=15$

$\Rightarrow k=\frac{15}{12}=\frac{5}{4}$

25.

i. a, a – 2, 3a are in A.P.

$\Rightarrow 2(a - 2) = a + 3a$

$\Rightarrow 2a - 4 = 4a$

$\Rightarrow 2a = -4$

$\Rightarrow a = -2$

ii. a = 8, T_n = 62 and S_n = 210

S_n = 210

$$\Rightarrow \frac{n}{2}(a + T_n) = 210$$

$$\Rightarrow \frac{n}{2}(8 + 62) = 210$$

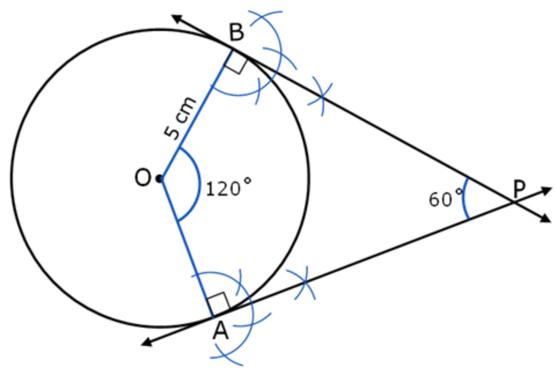
$$\Rightarrow 35n = 210 \Rightarrow n = 6$$

26.

i. Draw circle with centre O and radius OA = 5 cm. Mark B on the circle such that ∠AOB = 120°.

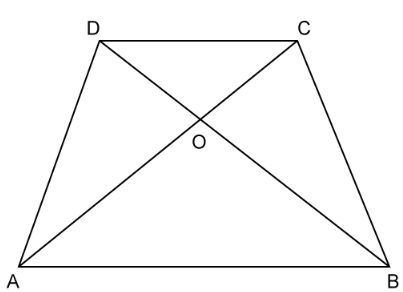
ii. Construct angles of 90° at A and B and extend the lines so as to intersect at point P.

iii. Thus, AP and BP are the required tangents to the circle.



Section IV

27.



The diagonals of a trapezium divide each other proportionally.

$\angle CDO = \angle OBA$ ....(alternate angles)

$\angle COD = \angle AOB$ ....(vertically opposite angles)

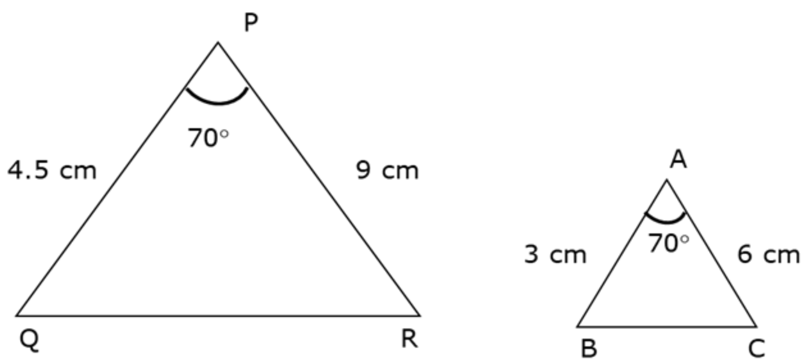
$\Rightarrow \Delta COD \sim \Delta AOB$ ...(AA criterion for similarity)

$$\Rightarrow \frac{ar(\Delta COD)}{ar(\Delta AOB)} = \frac{CD^2}{AB^2}$$

$$\Rightarrow \frac{ar(\Delta COD)}{84} = \frac{1^2}{2^2}$$

$$\Rightarrow ar(\Delta COD) = 21 \text{ cm}^2$$

OR



In ΔABC and ΔPQR,

$\angle A = \angle P = 70°$ ....(Given)

$$\frac{AB}{PQ} = \frac{3}{4.5} = \frac{2}{3}$$

$$\frac{AC}{PR} = \frac{6}{9} = \frac{2}{3}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AC}{PR}$$

So, $\Delta ABC \sim \Delta PQR$ ....(SAS criterion for similarity)

28.

Let the number of solid spheres be n.

Given Diameter of sphere $= 6$ cm $\Rightarrow$ radius $= 3$ cm

Diameter of cylinder $= 4$ cm

$\Rightarrow$ radius $= 2$ cm and height of the cylinder $= 45$ cm

Now,

Volume of the cylinder $=$ Volume of the sphere $\times$ n

$\Rightarrow \pi r^2 h = \frac{4}{3}\pi r^3 \times n$

$\Rightarrow \pi \times 2 \times 2 \times 45 = \frac{4}{3} \times \pi \times 3 \times 3 \times 3 \times n$

$\Rightarrow 45 = 9 \times n$

$\Rightarrow n = \frac{45}{9}$

$\Rightarrow n = 5$

Hence, number of solid spheres is 5.

29.

We have $\frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$ ...(i)

Let $\frac{2}{\sqrt{7}}$ be rational.

Then, from (i), $\frac{2}{7}\sqrt{7}$ is rational.

Now, $\frac{7}{2}$ is rational, $\frac{2}{7}\sqrt{7}$ is rational.

$\Rightarrow \left(\frac{7}{2} \times \frac{2}{7}\sqrt{7}\right)$ is rational.

$\Rightarrow \sqrt{7}$ is rational.

Thus, from (i), it follows that $\sqrt{7}$ is rational.

This contradicts the fact that $\sqrt{7}$ is irrational.

The contradiction arises by assuming that $\frac{2\sqrt{7}}{7}$ is rational.

Hence, $\frac{2\sqrt{7}}{7}$ is irrational.

30.

We have,

Class interval	Frequency f_i	Mid-value x_i	$f_i \times x_i$
$0-10$	16	5	80
$10-20$	p	15	15p
$20-30$	30	25	750
$30-40$	32	35	1120
$40-50$	14	45	630
	$\sum f_i = 92 + p$		$\sum f_i x_i = 2580 + 15p$

$$\text{Now, Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\Rightarrow 25 = \frac{2580 + 15p}{92 + p}$$

$$\Rightarrow 25(92 + p) = 2580 + 15p$$

$$\Rightarrow 2300 + 25p = 2580 + 15p$$

$$\Rightarrow 10p = 280$$

$$\Rightarrow p = 28$$

31.

In right triangle ABC,

$$\sin 45^\circ = \frac{BC}{AC}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{150}$$

$$\Rightarrow BC = \frac{150}{\sqrt{2}}$$

$$\Rightarrow BC = 75\sqrt{2} \text{ m}$$

Thus, the width of the river is $75\sqrt{2}$ metres.

32.

Consider the following table:

Let A = 225

$d_i = \frac{x_i - 225}{50}$

C.I.	fi	xi	di	fidi
100 - 150	4	125	−2	−8
150 - 200	5	175	−1	−5
200 - 250	12	225	0	0
250 - 300	2	275	1	2
300 – 350	2	325	2	4
Total	25			−7

$\text{Mean} = \bar{x} = A + \frac{\sum f_i d_i}{\sum f_i} \times h = 225 - \frac{7}{\cancel{25}} \times \cancel{50}^{2} = 225 - 14 = 211$

33.

$23x + 29y = 98$ .....(i) and

$29x + 23y = 110$ .....(ii)

Adding (i) and (ii), we get

$52x + 52y = 208$

$\Rightarrow x + y = 4$ .....(iii)

Subtract (i) from (ii), we get

$6x - 6y = 12$

$\Rightarrow x - y = 2$ .....(iv)

Adding (iii) and (iv), we get

$2x = 6$

$\Rightarrow x = 3$

Substituting $x = 3$ in (iii), we get $y = 1$.

Hence, $x = 3$ and $y = 1$.

OR

$6x + 3y = 7xy$ and $3x + 9y = 11xy$

Dividing throughout by xy, we get

$\frac{6}{y} + \frac{3}{x} = 7$ and $\frac{3}{y} + \frac{9}{x} = 11$

$\frac{3}{x} + \frac{6}{y} = 7$ and $\frac{9}{x} + \frac{3}{y} = 11$

Put $\frac{1}{x} = u$ and $\frac{1}{y} = v$

So, we get

$3u + 6v = 7 \ldots (i)$ and $9u + 3v = 11 \ldots (ii)$

Multiply (i) by 3 and subtract (ii) from the resultant.

$\Rightarrow 9u + 18v = 21 \quad \text{and} \quad 9u + 3v = 11$

$\Rightarrow 15v = 10$

$\Rightarrow v = \dfrac{2}{3}$

Substituting $v = \dfrac{2}{3}$ in (i), we get $u = 1$.

$\Rightarrow \dfrac{1}{x} = 1 \ \text{and} \ \dfrac{1}{y} = \dfrac{2}{3}$

$\Rightarrow x = 1 \ \text{and} \ y = \dfrac{3}{2}$

Section V

34.

Let x hours be the time taken by the pipe to fill the tank.

$\because$ The water is flowing at the rate of 4 km/hr,

$\therefore$ Length of the water column in x hours is $4x\ \text{km} = 4000x\ \text{m}$.

$\therefore$ The length of the pipe is 4000x m

The diameter of the pipe $= 20$ cm

$\Rightarrow$ radius $= 10$ cm

$= \dfrac{10}{100}$ m

$= 0.1$ m

$\therefore$ Volume of the water flowing through the pipe in x hours $= V_1$

$= \pi r^2 h$

$= \pi \times (0.1)^2 \times 4000x \quad ...(i)$

Given Diameter of the cylindrical tank $= 10$ m

$\Rightarrow$ radius $= 5$ m and

Volume of the water that falls into the tank in x hours $= V_1$

$= \pi r^2 h$

$= \pi \times (5)^2 \times 2 \quad ...(ii)$

$\therefore$ Volume of the water flowing through the pipe in x hours

$=$ Volume of the water that falls into the tank in x hours

$\Rightarrow \pi \times (0.1)^2 \times 4000x = \pi \times (5)^2 \times 2$

$\Rightarrow 40x = 50$

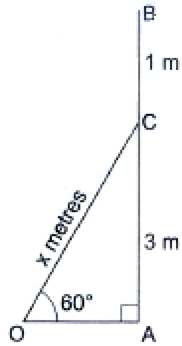
$\Rightarrow x = \dfrac{50}{40}$ hour

$\Rightarrow x = \dfrac{50}{40} \times 60$ minutes

$\Rightarrow x = 75$ minutes $= 1$ hour 15 mins

Thus, the water in the tank will be filled in 1 hour 15 minutes.

35.



Let AB be the electric pole such that $AB = 4$ m.

Let C be a point 1 m below B.

$\Rightarrow AC = 4\text{ m} - 1\text{ m} = 3\text{ m}$

Let OC be the ladder $= x$ metres.

Then, $\angle AOC = 60°$.

In right ΔOAC,

$\operatorname{cosec} 60° = \dfrac{OC}{AC}$

$\Rightarrow \dfrac{2}{\sqrt{3}} = \dfrac{x}{3}$

$\Rightarrow x = \dfrac{6}{\sqrt{3}}$

On rationalising we get,

$x = \dfrac{6}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow x = \dfrac{6\sqrt{3}}{3}$

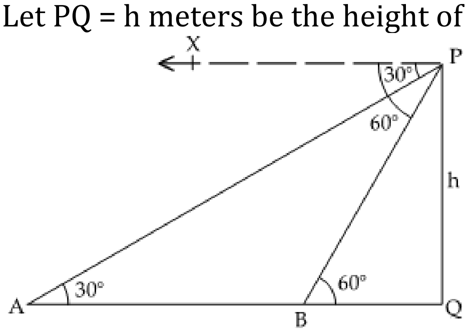
$\Rightarrow x = 2\sqrt{3}$

$\Rightarrow x = 2 \times 1.73 = 3.46$ m

Hence, the length of the ladder should be 3.46 m.

OR

Let PQ = h meters be the height of the tower. P is the top of the tower.



The first and second positions of the car are at A and B respectively.

$\angle APX = 30° \Rightarrow \angle PAQ = 30°$

$\angle BPX = 60° \Rightarrow \angle PBQ = 60°$

Let the speed of the car be x m/second

Then, distance AB = 6x meters

Let the time taken from B to Q be 'n' seconds

$\therefore$ BQ = nx metres

In $\triangle$ PAQ,

$$\frac{h}{6x+nx} = \tan 30^o = \frac{1}{\sqrt{3}}$$

$$\therefore h = \frac{(n+6)x}{\sqrt{3}} \quad ...(1)$$

In $\triangle$ PBQ,

$$\frac{h}{nx} = \tan 60^o = \sqrt{3}$$

$$\therefore h = nx\left(\sqrt{3}\right) \quad ...(2)$$

From (1) and (2),

$$\frac{(n+6)x}{\sqrt{3}} = nx\left(\sqrt{3}\right)$$

$$nx + 6x = 3nx \Rightarrow n = 3$$

Hence, the time taken by the car to reach the foot of the tower from B is 3 seconds.

36.

Suppose B alone takes x days to finish the work.

Then, A alone can finish it in $(x-10)$ days.

Now, $(\text{A's one day's work}) + (\text{B's one day work}) = \frac{1}{x-10} + \frac{1}{x}$

And, $(A+B)\text{'s one day's work} = \frac{1}{12}$

$$\therefore \frac{1}{x-10} + \frac{1}{x} = \frac{1}{12}$$

$$\Rightarrow \frac{x+x-10}{x(x-10)} = \frac{1}{12}$$

$$\Rightarrow 12(2x-10) = x(x-10)$$

$$\Rightarrow 24x - 120 = x^2 - 10x$$

$$\Rightarrow x^2 - 34x + 120 = 0$$

$$\Rightarrow x^2 - 30x - 4x + 120 = 0$$

$\Rightarrow x(x-30)-4(x-30)=0$

$\Rightarrow (x-30)(x-4)=0$

$\Rightarrow x-30=0 \text{ or } x-4=0$

$\Rightarrow x=30 \text{ or } x=4$

Since x cannot be less than 10, $x \neq 4$.

$\Rightarrow x=30$

Hence, B alone can finish the work in 30 days.