Sample Question Paper - 21
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Write the modal class for the following frequency distribution.

Class-interval	10-15	15-20	20-25	25-30	30-35	35-40
Frequency	30	35	75	40	30	15

2. If 18^{th} and 11^{th} term of an A.P. are in the ratio 3 : 2, then find the ratio of its 21^{st} and 5^{th} terms.

OR

Find the sum of all 2-digit numbers.

3. AP and AQ are tangents drawn from a point A to a circle with centre O and radius 9 cm. If $OA = 15$ cm, then find AP.

4. Solve the given quadratic equation
$12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

5. The rainwater from a roof 44 m × 10 m drain into a conical vessel having diameter of base as 1 m and height 7 m. If the vessel is just full, find the rainfall (in cm).

OR

A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal, then find the ratio of its radius and the slant height of the conical part.

6. Find the value of mode, using an empirical relation, when it is given that mean and median are 10.5 and 9.6 respectively.

SECTION - B

7. Find two consecutive positive integers, the sum of whose squares is 61.

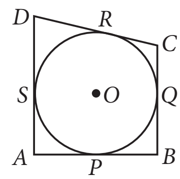
8. If p, q, r are in A.P., then find the value of $p^3 + r^3 - 8q^3$ in terms of pqr.

OR

Which term of the A.P. 4, 7, 10, 13, ..., is 49?

9. Two men on either side of a 75 m high building and in line with base of building observe the angles of elevation of the top of the building as 30° and 60°. Find the distance between the two men.

10. In the given figure, $\angle DAB = 90^\circ$, $AD = 40$ cm, $CD = 35$ cm and $CQ = 18$ cm. Find the radius of the circle.



SECTION - C

11. Draw a circle of radius 7 cm and then draw a tangent to this circle making angle of 45° with a line passing through the centre.

OR

Draw a pair of tangents to a circle of radius 3 cm, which are inclined to each other at an angle of 90°.

12. Find the mean marks of students from the following cumulative frequency distribution:

Marks	**Number of students**
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

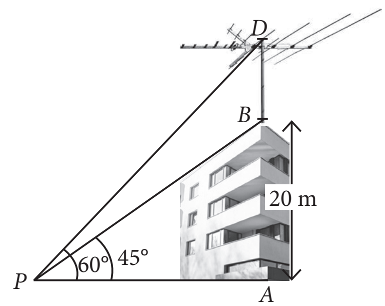
Case Study - 1

13. Soumya made some orange juice in a cylindrical jug of radius 14 cm to a height of 25 cm. Then she added 11 ice cubes, some slices of orange into jug.

(i) Find the volume of juice in the jar.
(ii) If each ice cube is of side 5.6 cm, then what is the volume of each ice cube?

Case Study - 2

14. A building stands on a horizontal plane and is surmounted by a vertical antenna. At a point on a plane an observer notices that the angles of elevation of the top and the bottom of the antenna are 60° and 45° respectively. The height of the building is 20 m. (Take $\sqrt{3}$ = 1.732)



(i) Find the distance of foot of building from P.
(ii) Find the height from the top of antenna to ground level.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. The highest frequency is 75 corresponds to class 20-25. So, the modal class is 20-25.

2. Given, $\frac{a_{18}}{a_{11}} = \frac{3}{2} \Rightarrow \frac{a+17d}{a+10d} = \frac{3}{2}$

$\Rightarrow$ $2a + 34d = 3a + 30d \Rightarrow a = 4d$ ... (i)

Now, $\frac{a_{21}}{a_5} = \frac{a+20d}{a+4d} = \frac{4d+20d}{4d+4d}$ [Using (i)]

$= \frac{24d}{8d} = \frac{3}{1}$

$\therefore$ Required ratio = 3 : 1

OR

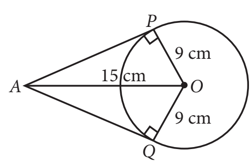
All two digit numbers are 10, 11,....., 99.

Here, $a = 10$, $d = 1$, $n = 90$

$\therefore$ Required sum, $S_n = \frac{n}{2}(10+99) = \frac{90}{2}(109)$

$= 45 \times 109 = 4905$

3. Since, tangents drawn from an external point of a circle are equal.



$\therefore$ $AP = AQ$

Also, $OP \perp AP$ and $OQ \perp AQ$

[$\because$ Tangent at any point of a circle is perpendicular to the radius through the point of contact.]

$\therefore$ In ΔAOP,

$AP^2 = AO^2 - OP^2$ [By Pythagoras theorem]

$= 15^2 - 9^2 = 225 - 81 = 144$

$\Rightarrow$ $AP = 12$ cm

4. We have, $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

$\Rightarrow$ $12abx^2 - 9a^2x + 8b^2x - 6ab = 0$

$\Rightarrow$ $3ax(4bx - 3a) + 2b(4bx - 3a) = 0$

$\Rightarrow$ $(4bx - 3a)(3ax + 2b) = 0$

$\Rightarrow$ $4bx - 3a = 0$ or $3ax + 2b = 0$

$\Rightarrow$ $x = \frac{3a}{4b}$ or $x = \frac{-2b}{3a}$

5. Let the rainfall be x.

Now, volume of water on roof = volume of cone

$\Rightarrow$ $44 \times 10 \times x = \frac{1}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 7$

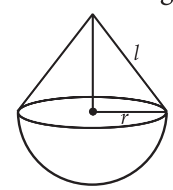
$\Rightarrow$ $x = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{1}{44} \times \frac{1}{10}$

$\Rightarrow$ $x = \frac{1}{240}$ m $= \frac{1}{240} \times 100$ cm $= \frac{5}{12}$ cm

Hence, required rainfall is 5/12 cm.

OR

Let r be the radius of hemisphere and conical part. Also, let l be the slant height of conical part.



Given, Surface area of hemisphere

= Surface area of conical part

$\Rightarrow$ $2\pi r^2 = \pi rl \Rightarrow 2r = l$

$\Rightarrow$ $\frac{r}{l} = \frac{1}{2}$

$\therefore$ Required ratio = 1 : 2

6. We know, the empirical relationship is

Mode = 3 Median – 2 Mean

= 3(9.6) – 2(10.5) [$\because$ Median = 9.6 and Mean = 10.5]

= 28.8 – 21.0 = 7.8

7. Let the two consecutive positive integers be x and $x + 1$.

According to question, $x^2 + (x + 1)^2 = 61$

$\Rightarrow$ $x^2 + x^2 + 2x + 1 = 61$

$\Rightarrow$ $2x^2 + 2x = 60 \Rightarrow x^2 + x = 30$

Adding $\left(\frac{1}{2}\right)^2$ on both sides, we get

$$x^2 + x + \frac{1}{4} = 30 + \frac{1}{4}$$

$\Rightarrow$ $\left(x+\frac{1}{2}\right)^2 = \frac{121}{4} \Rightarrow x + \frac{1}{2} = \pm\frac{11}{2} \Rightarrow x = \pm\frac{11}{2} - \frac{1}{2}$

$\Rightarrow$ $x = \frac{11}{2} - \frac{1}{2}$ or $x = -\frac{11}{2} - \frac{1}{2}$

$\Rightarrow$ $x = 5$ or $x = -6$

$\Rightarrow$ $x = 5$ [Since x is a positive integer]

And $x + 1 = 6$

$\therefore$ The two consecutive positive integers are 5 and 6.

8. Since p, q, r are in A.P.

$\therefore$ $q - p = r - q \Rightarrow 2q = p + r \Rightarrow p + r - 2q = 0$

$\Rightarrow$ $p^3 + r^3 + (-2q)^3 = 3 \times p \times r \times (-2q)$

[$\because$ If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$]

$\Rightarrow$ $p^3 + r^3 - 8q^3 = -6pqr$

OR

The given A.P. is 4, 7, 10, 13, …

Here, $a = 4$, $d = 7 - 4 = 3$

Let the n^{th} term of the A.P. be 49.

Then, $a_n = a + (n - 1)d \Rightarrow 49 = 4 + (n - 1)(3)$

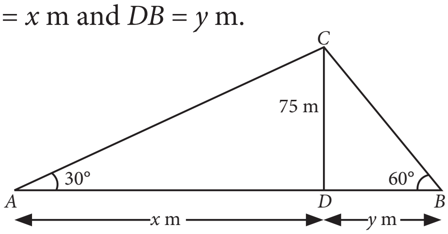
$\Rightarrow\ 45 = 3(n - 1) \Rightarrow n - 1 = 15\ \Rightarrow\ n = 16$

Hence, 16^{th} term of the A.P. is 49.

9. Let $CD = 75$ m be the height of the building. Let A and B be the points of observations such that the angle of elevation at A is 30° and the angle of elevation at B is 60°.

$\therefore\ \angle CAD = 30°$ and $\angle CBD = 60°$

Let $AD = x$ m and $DB = y$ m.



In right angled ΔADC,

$$\tan 30° = \frac{CD}{AD}\ \Rightarrow\ \frac{1}{\sqrt{3}} = \frac{75}{x}\ \Rightarrow\ x = 75\sqrt{3}\text{ m} \quad \ldots\text{(i)}$$

In right angled ΔBDC,

$$\tan 60° = \frac{CD}{DB}\ \Rightarrow\ \sqrt{3} = \frac{75}{y} \Rightarrow y = \frac{75}{\sqrt{3}}\text{ m} \quad \ldots\text{(ii)}$$

The distance between two men is AB,

i.e., $AB = AD + DB = x + y$

$$\Rightarrow\ AB = \left(75\sqrt{3} + \frac{75}{\sqrt{3}}\right) \quad \text{[Using (i) and (ii)]}$$

$$\Rightarrow\ AB = \left(\frac{225 + 75}{\sqrt{3}}\right) = \frac{300}{\sqrt{3}} = \frac{300\sqrt{3}}{3}$$

$= 100\sqrt{3} = 100 \times 1.73 \Rightarrow AB = 173$ m

10. Join OP and OS.

Since, length of tangents drawn from an external point to a circle are equal.

$\therefore\ AP = AS$ [Tangents from A] ...(i)

$CQ = CR$ [Tangents from C] ...(ii)

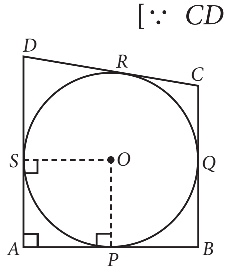
$DR = DS$ [Tangents from D] ...(iii)

Now, $CQ = CR\ \Rightarrow\ CR = 18$ cm

[$\because\ CQ = 18$ cm (given)]

$DR = DC - CR = 35 - 18 = 17$ cm

[$\because\ CD = 35$ cm (given)]



$\therefore\ DS = 17$ cm [Using (iii)]

$AS = AD - DS = 40 - 17 = 23$ cm

[$\because\ AD = 40$ cm (given)]

$\therefore\ AP = 23$ cm [Using (i)]

Now, $OP \perp AP$ and $OS \perp AS$

[$\because$ Tangent at any point of circle is perpendicular to the radius through the point of contact]

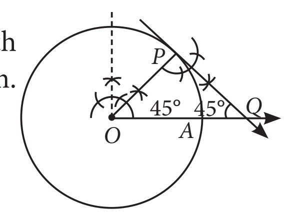
Also, $\angle DAB = 90°$ [Given]

Since, all angles are of 90° and adjacent sides are equal in $APOS$, so $APOS$ is a square.

$\therefore\ OP = OS = AS = AP = 23$ cm

Thus, radius of the circle is 23 cm.

11. Steps of construction :



Step-I : Draw a circle with centre O and radius, $OP = 7$ cm.

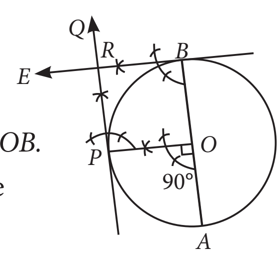
Step-II : Construct an angle AOP equal to complement of 45° *i.e.*, $\angle AOP = 45°$.

Step-III : Draw perpendicular to OP at P which meets OA produced at Q.

$\therefore\ PQ$ is the required tangent such that $\angle OQP = 45°$.

OR

Steps of construction :



Step-I : Draw a circle with centre O and radius 3 cm.

Step-II : Draw any diameter AOB.

Step-III : Take a point P on the circle such that $\angle AOP = 90°$.

Step-IV : Draw $PQ \perp OP$ and $BE \perp OB$. Let PQ and BE intersect at R.

Hence, RB and RP are the required tangents inclined at an angle of 90°.

12. Here we have, the cumulative frequency distribution more than type. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured greater than or equal to 10. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3.

Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on.

Thus, we obtain the following frequency distribution.

Marks	Frequency (f_i)	Class Mark (x_i)	f_ix_i
0-10	80 – 77 = 3	5	15
10-20	77 – 72 = 5	15	75

20-30	72 – 65 = 7	25	175
30-40	65 – 55 =10	35	350
40-50	55 – 43 = 12	45	540
50-60	43 – 28 = 15	55	825
60-70	28 – 16 = 12	65	780
70-80	16 – 10 = 6	75	450
80-90	10 – 8 = 2	85	170
90-100	8 – 0 = 8	95	760
Total	$\Sigma f_i = 80$		$\Sigma f_i x_i = 4140$

$\therefore$ Mean, $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{4140}{80} = 51.75$

Hence, mean marks scored by the students is 51.75.

13. (i) We have, $r = 14$ cm, $h = 25$ cm

Volume of juice in the jar $= \pi r^2 h$

$$= \frac{22}{7} \times (14)^2 \times 25 = 15400 \text{ cu. cm}$$

(ii) Side of ice cube = 5.6 cm

$\therefore$ Volume of each ice cube $= (5.6)^3 = 175.616$ cu. cm

14. (i) In ΔPAB, $\tan 45° = \frac{AB}{PA} \Rightarrow 1 = \frac{20}{PA}$

$\Rightarrow PA = 20$ m

So, required distance between foot of building and P is 20 m.

(ii) Let h be the height of antenna from the top of the building.

Then, in ΔPAD, $\tan 60° = \frac{AD}{PA}$

$\Rightarrow \sqrt{3} = \frac{AB + BD}{PA}$

$\Rightarrow \sqrt{3} \times 20 = 20 + h$

$\Rightarrow h = 20(\sqrt{3} - 1) = 20\,(1.732 - 1)$

$= 20 \times 0.732 = 14.64$ m

$\therefore$ Required height $= AD = AB + BD$

$= 20 + 14.64 = 34.64$ m