9. Line

- **Equation of a line through a given point and parallel to a given vector:**
  - **Vector form:** Equation of a line that passes through the given point whose position vector is $\vec{a}$ and which is parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda \vec{b}$, where λ is a constant.
  - **Cartesian form:**
    - Equation of a line that passes through a point (x_1, y_1, z_1) having d.r.'s as a, b, c is given by $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
    - Equation of a line that passes through a point (x_1, y_1, z_1) having d.c.'s as l, m, n is given by $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

- **Equation of a line passing through two given points:**
  - **Vector form:** Equation of a line passing through two points whose position vectors are $\vec{a}$ and $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$, where $\lambda \in \mathbf{R}$
  - **Cartesian form:** Equation of a line that passes through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by, $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

- **Angle between two Non-skew lines:**
  - **Cartesian form:**
    - If l_1, m_1, n_1, and l_2, m_2, n_2 are the d.c.'s of two lines and θ is the acute angle between them, then $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$
    - If a_1, b_1, c_1 and a_2, b_2, c_2 are the d.r.'s of two lines and θ is the acute angle between them, then $\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}\right|$
  - **Vector form:** If θ is the acute angle between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_1}$, then $\cos\theta = \left|\frac{b_1 . b_2}{|b_1||b_2|}\right|$

- Two lines with d.r.'s a_1, b_1, c_1 and a_2, b_2, c_2 are
  - perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$
  - parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- Two lines in space are said to be skew lines, if they are neither parallel nor intersecting. They lie in different planes.
- Angle between two skew lines is the angle between two intersecting lines drawn from any point (preferably from the origin) parallel to each of the skew lines.

- **Shortest Distance between two skew lines:** The shortest distance is the line segment perpendicular to both the lines.
  - **Vector form:** Distance between two skew lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is given by,

    $$d = \left| \frac{\left(\vec{b_1} \times \vec{b_2}\right) \cdot \left(\vec{a_2} - \vec{a_1}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|} \right|$$

  - **Cartesian form:** The shortest distance between two lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is given by,

    $$d = \left| \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\left(b_1c_2 - b_2c_1\right)^2 + \left(c_1a_2 - c_2a_1\right)^2 + \left(a_1b_2 - a_2b_1\right)^2}} \right|$$

- The shortest distance between two parallel lines $\vec{r} = \vec{a_1} + \lambda \vec{b}$ and $\vec{r} = \vec{a_2} + \mu \vec{b}$ is given by,

  $$d = \left| \frac{\vec{b} \times \left(\vec{a_2} - \vec{a_1}\right)}{\left|\vec{b}\right|} \right|$$