SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA / Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	1(1)	1(2)	1(3)	–	**3(6)**
2.	Inverse Trigonometric Functions	2(2)#	–	–	–	**2(2)**
3.	Matrices	1(1)*	–	–	–	**1(1)**
4.	Determinants	2(2)	1(2)	–	1(5)*	**4(9)**
5.	Continuity and Differentiability	–	1(2)*	2(6)#	–	**3(8)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	2(2)#	1(2)*	1(3)	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)	1(2)	1(3)*	–	**3(6)**
10.	Vector Algebra	3(3)#	1(2)*	–	–	**4(5)**
11.	Three Dimensional Geometry	4(4)#	–	–	1(5)*	**5(9)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	1(4)	2(4)	–	–	**3(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Simplify : $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

OR

If $A^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, then find the value of $A^T - B^T$.

2. Write the projection of the vector $\hat{i}+\hat{j}+\hat{k}$ along the vector $\hat{j}$.

3. Evaluate : $\int[\sin(\log x)+\cos(\log x)]dx$

OR

Evaluate : $\int e^x\left(\tan^{-1}x+\frac{1}{1+x^2}\right)dx$

4. If $A = \{0, 1\}$ and N be the set of all natural numbers. Then, show that the mapping $f:N\rightarrow A$ defined by $f(2n - 1) = 0, f(2n) = 1\ \forall n\in N$, is onto.

5. Write the principal value branch of $\operatorname{cosec}^{-1}x$.

OR

Find the principal values of $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$.

6. The position vectors of points A and B are $\hat{i}+3\hat{j}-7\hat{k}$ and $5\hat{i}-2\hat{j}+4\hat{k}$ respectively, then find the direction cosine of $\overline{AB}$ along Y-axis.

7. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined to x-axis at angles 30° and 120° respectively, then find $|\vec{a}+\vec{b}|$.

OR

Write a unit vector in the direction of the vector $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$.

8. Find the co-factor of each element of the first column of matrix $A=\begin{bmatrix} 2 & 5 & -1 \\ -3 & 0 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

9. A line makes angle $\frac{\pi}{3}$ with X-axis, $\frac{2\pi}{3}$ with Y-axis and $\frac{\pi}{4}$ with Z-axis. Find the direction cosines of the line.

OR

Find the direction cosines of the line joining the points (4, 3, –5) and (–2, 1, –8).

10. Find the equation of the plane passing through (2, 3, –1) and is perpendicular to the vector $3\hat{i}-4\hat{j}+7\hat{k}$.

11. Find the value of $\tan\left(\sin^{-1}\frac{3}{5}\right)$.

12. Evaluate : $\int_1^2 \frac{x^3-1}{x^2}dx$

13. If the points $A(-1, 3, 2)$, $B(-4, 2\ -2)$ and $C(5, 5, \lambda)$ are collinear, then find the value of λ.

14. If $A=\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$, then find adj A.

15. Find the vector equation of the plane whose Cartesian equation is $5x - 7y + 2z = 3$.

16. Find the order and degree of the differential equation $y''' + y^2 + e^{y'} = 0$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. Suppose your friend is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Also, it is given that when it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time.

Based on the above information, answer the following questions :

(i) If an ordinary year is considered, the probability that it rains on wedding day is

(a) $\frac{1}{365}$ (b) $\frac{1}{73}$ (c) $\frac{1}{72}$ (d) $\frac{6}{365}$

(ii) The probability that it does not rain on wedding day is

(a) $\frac{1}{365}$ (b) $\frac{5}{365}$ (c) $\frac{360}{365}$ (d) none of these

(iii) The probability that the weatherman predicts correctly is

(a) $\frac{5}{10}$ (b) $\frac{7}{10}$ (c) $\frac{9}{10}$ (d) $\frac{1}{10}$

(iv) The probability that it will rain on the wedding day, if weatherman predict rain for tomorrow, is

(a) 0.111 (b) 0.222 (c) 0.333 (d) 0.444

(v) The probability that it will not rain on the wedding day, if weatherman predict rain for tomorrow, is

(a) 0.889 (b) 0.778 (c) 0.667 (d) 0.556

18. A real estate company is going to build a new apartment complex. The land they have purchased can hold at most 5000 apartments. Also, if they make x apartments, then the maintenance costs for the building, landscaping etc., would be as follows:

Fixed cost = ₹ 40,00,000

Variable cost = ₹$(140x - 0.04x^2)$

Based on the above information, answer the following questions :

(i) The maintenance cost as a function of x will be

(a) $14x - 0.04x^2$ (b) 4000000

(c) $4000000 + 140x - 0.04x^2$ (d) None of these

(ii) If $C(x)$ denote the maintenance cost function, then maximum value of $C(x)$ occur at $x =$

(a) 0 (b) 1750 (c) 5000 (d) 2000

(iii) The maximum value of $C(x)$ would be

(a) ₹ 5225000 (b) ₹ 4122500 (c) ₹ 5000000 (d) ₹ 4000000

(iv) The number of apartments, that the complex should have in order to minimize the maintenance costs, is

(a) 5000 (b) 4000 (c) 1750 (d) 3500

(v) If the minimum maintenance cost is attain, then the maintenance cost for each apartment would be

(a) ₹ 740 (b) ₹ 540 (c) ₹ 640 (d) ₹ 840

PART - B

Section - III

19. Find the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is increasing.

20. If $|\vec{a}| = 2, |\vec{b}| = 7$ and $(\vec{a} \times \vec{b}) = 3\hat{i} + 2\hat{j} + 6\hat{k}$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find a unit vector in the direction of the resultant of vectors $\hat{i} + 2\hat{j} + 3\hat{k}, -\hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j}$.

21. If $A = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix}$, then find $A + A^{-1}$.

22. Mother, father and son line up at random for a family picture. Find $P(A/B)$, if A and B are defined as follows : A = Son on one end, B = Father in the middle

23. Evaluate : $\int \frac{dx}{3\sin^2 x + 4}$

OR

Evaluate $\int_{\pi/3}^{\pi/4} (\tan x + \cot x)^2 \, dx$ by fundamental theorem of integral calculus.

24. Let $f: R \to R$ be a function defined by $f(x)=\dfrac{e^{|x|}-e^{-x}}{e^{x}+e^{-x}}$, then show that $f(x)$ is many one into function.

25. Solve the differential equation $\dfrac{dy}{dx}=e^{x+y}+e^{y}x^{3}$.

26. Find the area bounded by $y^2 = x$, $y = 0$, $x = 1$ and $x = 3$.

27. In a college, 30% students fail in physics, 25% fail in mathematics and 10% fail in both. One student is chosen at random. Find the probability that she fails in physics if she has failed in mathematics.

28. If $f(x)=\begin{cases}\left(\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}\right), & \text{when } x \neq 0 \\ 0, & \text{when } x = 0\end{cases}$, then show that $f(x)$ is discontinuous at $x = 0$.

OR

If $(\cos x)^y = (\cos y)^x$, then find $\dfrac{dy}{dx}$.

Section-IV

29. Given the sum of the perimeter of a square and a circle. Show that sum of their areas is least when the side of the square is equal to the diameter of the circle.

30. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the X-axis.

31. If $x^p y^q = (x + y)^{p+q}$, then prove that $\dfrac{dy}{dx}=\dfrac{y}{x}$.

OR

If $y=\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$, then show that $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=2$.

32. Evaluate : $\displaystyle\int_0^{\frac{3}{2}} |x\cos \pi x|\,dx$

33. Show that relation 'is congruent to', on the set of all triangles in a plane is an equivalence relation.

34. Solve the differential equation : $(x^2 + 1)\, y' - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 0$.

OR

Solve the differential equation $x(x^2-1)\dfrac{dy}{dx}=1$, given that when $x = 2$, $y = 0$.

35. Find the values of a, b respectively if $f(x)=\begin{cases} x^2+3x+a, & x \le 1 \\ bx+2, & x > 1\end{cases}$ is differentiable at every x.

Section-V

36. Find the matrix P satisfying the matrix equation $\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}P\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

OR

Two factories decided to award their employees for three values of (a) adaptable to new techniques, (b) careful and alert in difficult situations and (c) keeping calm in tense situations, at the rate of ₹ x, ₹ y and ₹ z per person respectively. The first factory decided to honour respectively 2, 4 and 3 employees with a total prize money of ₹ 29,000. The second factory decided to honour respectively 5, 2 and 3 employees with the prize money of ₹ 30,500. If the three prizes per person together cost ₹ 9,500; then

(i) Represent the above situation by a matrix equation and form linear equations using matrix multiplication.

(ii) Solve these equations using matrices.

37. Find the distance of the point (–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

OR

Find the equation of the plane passing through three given points $-2\hat{i}+6\hat{j}-6\hat{k}, -3\hat{i}+10\hat{j}-9\hat{k}, -5\hat{i}-6\hat{k}$.

38. Solve the following linear programming problem (LPP) graphically.

Maximize $Z=\frac{35}{2}x+7y$

Subject to constraints :

$x + 3y \leq 12;$

$3x + y \leq 12;$

$x, y \geq 0$

OR

Solve the following linear programming problem (LPP) graphically.

Maximize $Z = 500\,x + 150\,y$

Subject to constraints :

$2500\,x + 500\,y \leq 50000$

$x + y \leq 60;$

$x, y \geq 0$

SOLUTIONS

1. We have,

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta & \sin\theta\cos\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\cos\theta\sin\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

OR

Given, $A^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$

$$\Rightarrow\ B^T=\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}$$

$$\therefore\ A^T-B^T=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$$

2. The projection of the vector $(\hat{i}+\hat{j}+\hat{k})$ along the vector $\hat{j}$ is $(\hat{i}+\hat{j}+\hat{k})\cdot\left(\dfrac{\hat{j}}{\sqrt{0^2+1^2+0^2}}\right)=1$

3. Let $I=\int[\sin(\log x)+\cos(\log x)]dx$

Put $\log x=t\ \Rightarrow\ x=e^t\Rightarrow dx=e^t dt$

$\therefore\ I=\int(\sin t+\cos t)e^t dt$

$=e^t\sin t+C=x\sin(\log x)+C$

OR

Let $I=\int e^x\left(\tan^{-1}x+\dfrac{1}{1+x^2}\right)dx$

Consider, $f(x)=\tan^{-1}x$ and $f'(x)=\dfrac{1}{1+x^2}$

Integrand is in the form $e^x[f(x)+f'(x)]$

$\therefore\ \int e^x\left(\tan^{-1}x+\dfrac{1}{1+x^2}\right)dx=e^x\tan^{-1}x+C$

4. Given, $A=\{0, 1\}$

Since, $f: N\to A$ such that $f(2n-1)=0, f(2n)=1\ \forall n\in N$

So, A = Range f, which is equal to Co-domain of A

Hence, the mapping $f: N\to A$ is onto.

5. Principal value branch of $\text{cosec}^{-1}x$ is $\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]-\{0\}$.

OR

Let $\cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)=\theta\quad\Rightarrow\cos\theta=\dfrac{-\sqrt{3}}{2}$

$=-\cos\dfrac{\pi}{6}=\cos\left(\pi-\dfrac{\pi}{6}\right)=\cos\dfrac{5\pi}{6}$

$\Rightarrow\ \theta=\dfrac{5\pi}{6}\in[0,\pi]$

$\therefore$ Principal value of $\cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$ is $\dfrac{5\pi}{6}$.

6. Here $\vec{a}=\hat{i}+3\hat{j}-7\hat{k}, \vec{b}=5\hat{i}-2\hat{j}+4\hat{k}$

$\overrightarrow{AB}=\vec{b}-\vec{a}=4\check{i}-5\check{j}+11\check{k}$

Direction cosine along Y-axis $=\dfrac{-5}{\sqrt{16+25+121}}=\dfrac{-5}{\sqrt{162}}$

7. Clearly, angle between $\vec{a}$ and $\vec{b}=\dfrac{\pi}{2}$

$\Rightarrow\ \vec{a}\cdot\vec{b}=0$

$\therefore\ |\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=1+1+0=2$

$\Rightarrow\ |\vec{a}+\vec{b}|=\sqrt{2}$

OR

We have, $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$

$\therefore\ |\vec{a}|=\sqrt{(2)^2+(1)^2+(2)^2}=\sqrt{4+1+4}=\sqrt{9}=3$

Required unit vector is $\hat{a}=\dfrac{\vec{a}}{|\vec{a}|}=\dfrac{2}{3}\hat{i}+\dfrac{1}{3}\hat{j}+\dfrac{2}{3}\hat{k}$

8. $M_{11}=\begin{vmatrix}0 & 1\\ 1 & -1\end{vmatrix}=0-1=-1\Rightarrow C_{11}=M_{11}=-1$

$M_{21}=\begin{vmatrix}5 & -1\\ 1 & -1\end{vmatrix}=-5+1=-4\Rightarrow C_{21}=-M_{21}=4$

$M_{31}=\begin{vmatrix}5 & -1\\ 0 & 1\end{vmatrix}=5-0=5\Rightarrow C_{31}=M_{31}=5$

9. Here, $\alpha=\dfrac{\pi}{3},\quad\beta=\dfrac{2\pi}{3},\quad\gamma=\dfrac{\pi}{4}$

Direction cosines of the line are $\cos\alpha$, $\cos\beta$, $\cos\gamma$

$=\cos\dfrac{\pi}{3},\ \cos\dfrac{2\pi}{3},\ \cos\dfrac{\pi}{4}=\dfrac{1}{2},\ \dfrac{-1}{2},\ \dfrac{1}{\sqrt{2}}$

OR

Direction cosines of the line joining $P(4, 3, -5)$ and $Q(-2, 1, -8)$ is

$l=\dfrac{x_2-x_1}{|PQ|}, m=\dfrac{y_2-y_1}{|PQ|}, n=\dfrac{z_2-z_1}{|PQ|}$

$\because \quad x_2 - x_1 = -6, y_2 - y_1 = -2, z_2 - z_1 = -3$

$|PQ| = \sqrt{36+4+9} = 7$

$\therefore \; l = \frac{-6}{7}, m = \frac{-2}{7}, n = \frac{-3}{7}$

10. The equation of the plane passing through (2, 3, – 1) and perpendicular to the vector $3\hat{i} - 4\hat{j} + 7\hat{k}$ is $3(x - 2) + (-4)(y - 3) + 7(z - (-1)) = 0$

$\Rightarrow \; 3x - 4y + 7z + 13 = 0$

11. Let $\sin^{-1}\frac{3}{5} = \theta \Rightarrow \sin\theta = \frac{3}{5}$ and $\theta \in \left[0, \frac{\pi}{2}\right]$

$\therefore \; \tan\left(\sin^{-1}\frac{3}{5}\right) = \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{3/5}{4/5} = \frac{3}{4}$

12. Here, $\int_1^2 \frac{x^3-1}{x^2}dx = \int_1^2 (x - x^{-2})dx$

$= \left[\frac{x^2}{2} - \frac{x^{-1}}{-1}\right]_1^2 = \left[\frac{x^2}{2} + \frac{1}{x}\right]_1^2 = \left[\frac{4}{2}+\frac{1}{2}\right] - \left[\frac{1}{2}+1\right] = \frac{5}{2} - \frac{3}{2} = 1$

13. Direction ratios of the line $AB = -3, -1, -4$

Direction ratios of the line $BC = 9, 3, \lambda + 2$

Since the points A, B and C are collinear, hence its direction ratios are proportional.

$\therefore \; \frac{-3}{9} = \frac{-1}{3} = \frac{-4}{\lambda+2}$

$\Rightarrow \lambda + 2 = 12 \Rightarrow \lambda = 10$

14. The cofactors of the elements of $|A|$ are given by $A_{11} = 3, A_{12} = -1, A_{21} = -5, A_{22} = 2$

$\therefore \; \text{adj}\,A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

15. The given equation of the plane is $5x - 7y + 2z = 3$

or $(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (5\hat{i} - 7\hat{j} + 2\hat{k}) = 3$

$\Rightarrow \vec{r} \cdot (5\hat{i} - 7\hat{j} + 2\hat{k}) = 3$, which is the required vector equation of plane.

16. The highest order derivative present in the differential equation is y''', so its order is three. The given differential equation is not a polynomial equation in its derivatives and so its degree is not defined.

17. (i) (b) : Since, it rained only 5 days each year, therefore, probability that it rains on wedding day is $\frac{5}{365} = \frac{1}{73}$

(ii) (c) : The probability that it does not rain on wedding day $= 1 - \frac{1}{73} = \frac{72}{73} = \frac{360}{365}$

(iii) (c) : It is given that, when it actually rains, the weatherman correctly forecasts rain 90% of the time.

$\therefore$ Required probability $= \frac{90}{100} = \frac{9}{10}$

(iv) (a) : Let A_1 be the event that it rains on wedding day, A_2 be the event that it does not rain an wedding day and E be the event the weatherman predict rain.

Then we have, $P(A_1) = \frac{5}{365}$, $P(A_2) = \frac{360}{365}$,

$P(E/A_1) = \frac{9}{10}$ and $P(E/A_2) = \frac{1}{10}$

Required probability

$= P(A_1/E) = \frac{P(A_1)\cdot P(E/A_1)}{P(A_1)\cdot P(E/A_1) + P(A_2)\cdot P(E/A_2)}$

$$\frac{\frac{5}{365}\times\frac{9}{10}}{\frac{5}{365}\times\frac{9}{10}+\frac{360}{365}\times\frac{1}{10}} = \frac{45}{405} \approx 0.111$$

(v) (a) : Required probability $= 1 - P(A_1/E) = 1 - 0.111 = 0.889$

18. (i) (b) : Let $C(x)$ be the maintenance cost function, then $C(x) = 4000000 + 140x - 0.04x^2$

We have, $C(x) = 4000000 + 140x - 0.04x^2$

(ii) (b) : Now, $C'(x) = 140 - 0.08\,x$

For maxima/minima, put $C'(x) = 0$

$\Rightarrow \; 140 = 0.08x$

$\Rightarrow \; x = 1750$

(iii) (b) : Clearly, from the problem statement we can see that we only want critical points that are in the interval [0, 5000]

Now, we have $C(0) = 4000000$

$C(1750) = 4122500$

and $C(5000) = 3700000$

$\therefore$ Maximum value of $C(x)$would be ₹ 4122500

(iv) (a) : The complex must have 5000 apartments to minimise the maintenance cost.

(v) (a) : The minimum maintenance cost for each apartment woud be ₹ 740.

19. The given function is

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

$\Rightarrow f'(x) = 4x^3 - 24x^2 + 44x - 24$

$= 4(x^3 - 6x^2 + 11x - 6)$

$= 4(x - 1)(x^2 - 5x + 6)$

$= 4(x - 1)(x - 2)(x - 3)$

Thus, $f'(x) = 0 \Rightarrow x = 1, 2, 3$.

Hence, possible intervals are

$(-\infty, 1), (1, 2), (2, 3)$ and $(3, \infty)$.

In the interval $(-\infty, 1), f'(x) < 0$

In the interval $(1, 2), f'(x) > 0$

In the interval $(2, 3), f'(x) < 0$

In the interval $(3, \infty), f'(x) > 0$

$\therefore \; f$ is increasing in $(1, 2) \cup (3, \infty)$.

20. Given, $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\therefore \; |\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = \sqrt{49} = 7$

Let θ be the angle between $\vec{a}$ and $\vec{b}$.

Now, $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{7}{2\times 7} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

OR

Let $\vec{a}$ be the resultant of given vectors. Then,

$\vec{a} = (\hat{i}+2\hat{j}+3\hat{k}) - \hat{i}+2\hat{j}+\hat{k}+(3\hat{i}+\hat{j}) = 3\hat{i}+5\hat{j}+4\hat{k}$

$\therefore\ |\vec{a}| = \sqrt{3^2+5^2+4^2} = 5\sqrt{2}$

Now, unit vector along $\vec{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{3}{5\sqrt{2}}\hat{i} + \frac{5}{5\sqrt{2}}\hat{j} + \frac{4}{5\sqrt{2}}\hat{k}$.

21. $A = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix}$

$\therefore\ |A| = -14 + 15 = 1 \neq 0$

So, A^{-1} exists.

$\therefore$ adj $A = \begin{bmatrix} -7 & 3 \\ -5 & 2 \end{bmatrix}$

$\Rightarrow\ A^{-1} = \frac{1}{|A|}(\text{adj } A) = \begin{bmatrix} -7 & 3 \\ -5 & 2 \end{bmatrix}$

Thus, $A + A^{-1} = \begin{bmatrix} 2 & -3 \\ 5 & -7 \end{bmatrix} + \begin{bmatrix} -7 & 3 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$

22. Total number of ways in which Mother (M), Father (F) and Son (S) can be lined up at random in one of the following ways:

MFS, MSF, FMS, FSM, SFM, SMF is 6.

We have,

$A = \{MFS, FMS, SMF, SFM\}$ and $B = \{MFS, SFM\}$

$\therefore\ A\cap B = \{MFS, SFM\}$

Clearly, $n(A\cap B) = 2$ and $n(B) = 2$

$\therefore$ Required probability $= P(A/B) = \frac{n(A\cap B)}{n(B)} = \frac{2}{2} = 1$

23. Let $I = \int \frac{dx}{3\sin^2 x + 4} = \int \frac{\sec^2 x}{3\tan^2 x + 4\sec^2 x}dx$

$= \int \frac{\sec^2 x}{4 + 7\tan^2 x}dx$

Put $\tan x = t \ \Rightarrow\ \sec^2 x\, dx = dt$

$\therefore\ I = \int \frac{dt}{4+7t^2} = \frac{1}{2\sqrt{7}}\tan^{-1}\left(\frac{\sqrt{7}\tan x}{2}\right) + c$

OR

Let $I = \int_{\pi/3}^{\pi/4} (\tan x + \cot x)^2 dx$

$= \int_{\pi/3}^{\pi/4} (\tan^2 x + 2 + \cot^2 x)\, dx = \int_{\pi/3}^{\pi/4} (\sec^2 x + \text{cosec}^2\ x)dx$

$= [\tan x - \cot x]_{\pi/3}^{\pi/4} = 1 - 1 - \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{-2}{\sqrt{3}}$

24. $f: R \to R$ such that $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$

$$f(x) = \begin{cases} 0, & x \le 0 \\ \dfrac{e^x - e^{-x}}{e^x + e^{-x}}, & x > 0 \end{cases}$$

$\Rightarrow$ $f(x)$ is many one.

For $x > 0$, $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1} = 1 - \frac{2}{e^{2x}+1}$

Range of $f(x) \in [0, 1) \neq$ codomain of $f(x)$

$\Rightarrow$ $f(x)$ is into. So, $f(x)$ is many one into.

25. We have, $\frac{dy}{dx} = e^{x+y} + e^y x^3 = e^y(e^x + x^3)$

$\frac{dy}{e^y} = (e^x + x^3)dx$

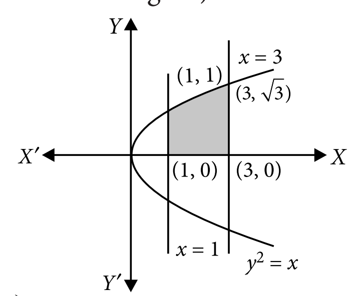
Integrating both sides, we get

$\Rightarrow\ \int \frac{dy}{e^y} = \int (e^x + x^3)dx$

$\Rightarrow\ -e^{-y} = e^x + \frac{x^4}{4} + c_1 \Rightarrow e^x + e^{-y} + \frac{x^4}{4} = c$

26. Given curves are, $y^2 = x$, $y = 0$, $x = 1$ and $x = 3$

Required area = 2 (Area of shaded region)



$= 2\int_1^3 \sqrt{x}\, dx$

$= 2\left[\frac{x^{3/2}}{3/2}\right]_1^3$

$= \frac{4}{3}\left[(3)^{3/2} - (1)^{3/2}\right]$

$= \frac{4}{3}(3\sqrt{3} - 1) = \left(4\sqrt{3} - \frac{4}{3}\right)$ sq. units

27. Let E_1 be the event that a student fails in physics and E_2 be the event that a student fails in mathematics.

Then, $P(E_1) = \frac{30}{100} = \frac{3}{10}$, $P(E_2) = \frac{25}{100} = \frac{1}{4}$

and $P(E_1 \cap E_2) = \frac{10}{100} = \frac{1}{10}$

$\therefore$ Required probability $= P(E_1 | E_2) = \frac{P(E_1\cap E_2)}{P(E_2)}$

$= \frac{1/10}{1/4} = \frac{4}{10} = \frac{2}{5}$

28. Clearly, $f(0) = 0$.

Now, $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} f(h) = \lim_{h\to 0}\left(\frac{e^{1/h} - 1}{e^{1/h} + 1}\right)$

$$= \lim_{h\to 0}\frac{e^{1/h}\left(1-\frac{1}{e^{1/h}}\right)}{e^{1/h}\left(1+\frac{1}{e^{1/h}}\right)} = \lim_{h\to 0}\left(\frac{1-\frac{1}{e^{1/h}}}{1+\frac{1}{e^{1/h}}}\right)=1$$

$$\lim_{x\to 0^-} f(x)=\lim_{h\to 0} f(0-h)=\lim_{h\to 0} f(-h)=\lim_{h\to 0}\left(\frac{e^{\frac{-1}{h}}-1}{e^{\frac{-1}{h}}+1}\right)$$

$$=\lim_{h\to 0}\left(\frac{\frac{1}{e^{1/h}}-1}{\frac{1}{e^{1/h}}+1}\right)=-1.$$

Thus, $\lim\limits_{x\to 0^+} f(x)\neq \lim\limits_{x\to 0^-} f(x)$ and therefore, $\lim\limits_{x\to 0} f(x)$ does not exist. Hence, $f(x)$ is discontinuous at $x = 0$.

OR

We have, $(\cos x)^y = (\cos y)^x$

$\Rightarrow$ $y \log \cos x = x \log \cos y$

$$\Rightarrow\ y\cdot\left(\frac{-\sin x}{\cos x}\right)+(\log\cos x)\frac{dy}{dx}=x\cdot\left(\frac{-\sin y}{\cos y}\right)\cdot\frac{dy}{dx}+(\log\cos y)\cdot 1$$

$$\Rightarrow\ (\log\cos x + x\tan y)\frac{dy}{dx}=(\log\cos y + y\tan x)$$

$$\Rightarrow\ \frac{dy}{dx}=\left(\frac{\log\cos y + y\tan x}{\log\cos x + x\tan y}\right)$$

29. Let side of square = x and radius of circle = r.

Perimeter of square = $4x$ and perimeter of circle = $2\pi r$

Let $4x + 2\pi r = k$

$$\Rightarrow\ 4x = k - 2\pi r \Rightarrow x = \frac{k-2\pi r}{4} \qquad \text{...(i)}$$

Let A = Area of square + Area of circle

$\Rightarrow$ $A = x^2 + \pi r^2$

$$\Rightarrow\ A=\left(\frac{k-2\pi r}{4}\right)^2+\pi r^2 \qquad \text{[From (i)]}$$

Differentiating w.r.t. r, we get

$$\frac{dA}{dr}=2\left(\frac{k-2\pi r}{4}\right)\left(-\frac{2\pi}{4}\right)+2\pi r$$

$$\Rightarrow\ \frac{dA}{dr}=\frac{-k\pi+2\pi^2 r+8\pi r}{4}$$

For finding maxima or minima, $\frac{dA}{dr}=0$

$\Rightarrow$ $-k\pi + 2\pi^2 r + 8\pi r = 0 \Rightarrow -k + 2\pi r + 8r = 0$

$\Rightarrow$ $k = 2\pi r + 8r \Rightarrow k = r(2\pi + 8)$

$$\Rightarrow\ r=\frac{k}{2\pi+8}$$

Now, $\frac{d^2A}{dr^2}=\frac{2\pi^2+8\pi}{4}>0$, so, local minima

From (i), $x=\frac{2\pi r+8r-2\pi r}{4}=2r$

Hence, side of square is equal to diameter of circle when their combined area is least.

30. Given parabola is $y = x^2$ ...(i)

and the given line is $y = x + 2$ ...(ii)

The line and the parabola meet where

$x^2 = x + 2$ (Eliminating y)

$\Rightarrow$ $x^2 - x - 2 = 0$

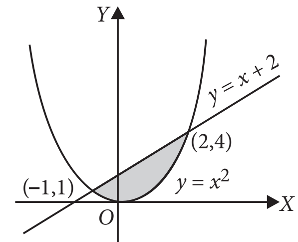
$\Rightarrow$ $(x - 2)(x + 1) = 0$

$\Rightarrow$ $x = 2$ or -1

When $x = 2$, $y = 2 + 2 = 4$

and when $x = -1$,

$y = 2 + (-1) = 1$



$\therefore$ The line and the parabola meet at the points $(-1, 1)$ and $(2, 4)$.

$\therefore$ Required area

$$=\int_{-1}^{2}(x+2)\,dx-\int_{-1}^{2}x^2\,dx=\left[\frac{x^2}{2}+2x\right]_{-1}^{2}-\left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$=\frac{2^2}{2}+2\times 2-\left\{\frac{(-1)^2}{2}+2(-1)\right\}-\frac{1}{3}(2^3-(-1)^3)$$

$$=6-\frac{1}{2}+2-3=\frac{9}{2}\text{ sq. units.}$$

31. Given, $x^p y^q = (x + y)^{p+q}$

Taking log on both sides, we get

$\log (x^p \times y^q) = \log (x + y)^{p+q}$

$\Rightarrow$ $p \log x + q \log y = (p + q) \log (x + y)$

Differentiating w.r.t. x, we get

$$\frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=(p+q)\left(\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)\right)$$

$$\Rightarrow\ \frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}+\frac{p+q}{x+y}\frac{dy}{dx}$$

$$\Rightarrow\ \frac{dy}{dx}\left(\frac{q}{y}-\frac{(p+q)}{x+y}\right)=\left(\frac{p+q}{x+y}-\frac{p}{x}\right)$$

$$\Rightarrow\ \frac{dy}{dx}\left(\frac{qx+qy-py-qy}{y(x+y)}\right)=\left(\frac{px+qx-px-py}{x(x+y)}\right)$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{y}{x}$$

OR

Given, $y=\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=2\log\left(x+\sqrt{x^2+1}\right)\cdot\frac{1}{x+\sqrt{x^2+1}}\cdot\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{2\log\left(x+\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}$$

$$\Rightarrow \sqrt{x^2+1}\,\frac{dy}{dx} = 2\log\left(x+\sqrt{x^2+1}\right)$$

Squaring both sides, we get

$$(x^2+1)\left(\frac{dy}{dx}\right)^2 = 4\left[\log\left(x+\sqrt{x^2+1}\right)\right]^2$$

$$\Rightarrow (x^2+1)\left(\frac{dy}{dx}\right)^2 = 4y$$

Differentiating w.r.t. x, we get

$$(x^2+1)\,2\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + 2x\left(\frac{dy}{dx}\right)^2 = 4\frac{dy}{dx}$$

$$\Rightarrow (x^2+1)\frac{d^2y}{dx^2} + x\cdot\frac{dy}{dx} = 2$$

32. Let $I = \int_0^{\frac{3}{2}} |x\cos\pi x|\,dx$

$$\because\ |x\cos\pi x| = \begin{cases} x\cos\pi x\ ; & 0<x<\frac{1}{2} \\ -x\cos\pi x\ ; & \frac{1}{2}<x<\frac{3}{2} \end{cases}$$

$$\therefore\ I = \int_0^{\frac{1}{2}}(x\cos\pi x)dx - \int_{\frac{1}{2}}^{\frac{3}{2}} x\cos\pi x\,dx$$

$$= \left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_0^{\frac{1}{2}} - \left[\frac{x\sin\pi x}{\pi}+\frac{\cos\pi x}{\pi^2}\right]_{\frac{1}{2}}^{\frac{3}{2}}$$

$$= \left[\left(\frac{1}{2\pi}+0\right)-\left(0+\frac{1}{\pi^2}\right)\right] - \left[\left(\frac{-3}{2\pi}+0\right)-\left(\frac{1}{2\pi}+0\right)\right]$$

$$= \frac{1}{2\pi}-\frac{1}{\pi^2}+\frac{4}{2\pi} = \frac{5}{2\pi}-\frac{1}{\pi^2} = \frac{5\pi-2}{2\pi^2}$$

33. Let S be the set of all triangles in a plane R and $R = \{(\Delta_1, \Delta_2) : \Delta_1$ is congruent to $\Delta_2\}$. Then the congruence relation on S is

(i) Reflexive, since $\Delta \cong \Delta$ for all $\Delta \in S$

(ii) Symmetric, since $(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2 \Rightarrow \Delta_2 \cong \Delta_2$ $\Rightarrow (\Delta_2, \Delta_1) \in R$

(iii) Transitive, since $(\Delta_1, \Delta_2), (\Delta_2, \Delta_3) \in R \Rightarrow \Delta_1 \cong \Delta_2$ and $\Delta_2 \cong \Delta_3 \ \Rightarrow\ \Delta_1 \cong \Delta_3$

$\Rightarrow (\Delta_1, \Delta_3) \in R$

Hence, above relation on S is an equivalence relation.

34. We have, $(x^2+1)y' - 2xy = (x^4+2x^2+1)\cos x$

$$\Rightarrow \frac{dy}{dx} - \frac{2x}{x^2+1}y = (x^2+1)\cos x \qquad ...(i)$$

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$ where $P = \frac{-2x}{x^2+1}$ and $Q = (x^2+1)\cos x$.

$$\therefore\ \text{I.F.} = e^{\int\frac{-2x}{x^2+1}dx} = e^{-\log(x^2+1)} = (x^2+1)^{-1}$$

Thus, solution of given differential equation is

$$y\times\frac{1}{x^2+1} = \int\cos x\,dx + C \Rightarrow \frac{y}{x^2+1} = \sin x + C \qquad ...(ii)$$

It is given that $y(0) = 0$ *i.e.*, $y = 0$ when $x = 0$.

Putting $x = 0$, $y = 0$ in (ii), we get $C = 0$

Putting $C = 0$ in (ii), we get

$\frac{y}{x^2+1} = \sin x \quad \Rightarrow y = (x^2+1)\sin x$, which is the required particular solution.

OR

We have $x(x^2-1)\frac{dy}{dx} = 1$

$$\Rightarrow dy = \frac{1}{x(x^2-1)}dx$$

Integrating both sides, we get

$$\int dy = \int\frac{1}{x(x^2-1)}dx = \int\left(-\frac{1}{x}+\frac{x}{x^2-1}\right)dx$$

$$\Rightarrow y = -\log|x| + \frac{1}{2}\log|x^2-1| + \log c$$

It is given that $y = 0$ when $x = 2$

$$\Rightarrow 0 = -\log|2| + \frac{1}{2}\log|3| + \log c$$

$$\Rightarrow \log c = \log 2 - \frac{1}{2}\log 3$$

$$\therefore\ y = \frac{1}{2}\log|x^2-1| - \log|x| + \log 2 - \frac{1}{2}\log 3$$

35. As f is derivable at $x = 1$, therefore, f is continuous too at $x = 1$.

$$\therefore\ \lim_{x\to1^-} f(x) = f(1) = \lim_{x\to1^+} f(x)$$

$$\Rightarrow \lim_{x\to1^-}(x^2+3x+a) = 1+3+a = \lim_{x\to1^+}(bx+2)$$

$\Rightarrow 4+a = 4+a = b+2$

$\Rightarrow a-b = -2 \qquad ...(i)$

Also, f is derivable at $x = 1$

$\Rightarrow Lf'(1) = Rf'(1)$

$$\Rightarrow \lim_{h\to0}\frac{f(1-h)-f(1)}{-h} = \lim_{h\to0}\frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow \lim_{h\to0}\frac{(1-h)^2+3(1-h)+a-(4+a)}{-h}$$

$$= \lim_{h\to0}\frac{b(1+h)+2-(4+a)}{h}$$

$$\Rightarrow \lim_{h\to0}\frac{h^2-5h}{-h} = \lim_{h\to0}\frac{bh}{h} \qquad (\because\ b = a+2)$$

$\Rightarrow 5 = b$

$\therefore$ From (i), $a = b-2 = 5-2 = 3$

36. Let $A=\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}$, then

$$|A|=\begin{vmatrix}2 & 1\\ 3 & 2\end{vmatrix}=4-3=1\neq 0$$

and $|B|=\begin{vmatrix}-3 & 2\\ 5 & -3\end{vmatrix}=9-10=-1\neq 0$

So, A and B are non-singular and invertible matrices.

$\therefore$ adj $A=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$

and $A^{-1}=\dfrac{1}{|A|}\text{adj } A=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}$ ...(i)

$\therefore$ adj $B=\begin{bmatrix}-3 & -2\\ -5 & -3\end{bmatrix}$

and $B^{-1}=\dfrac{1}{|B|}\text{adj } B=\dfrac{1}{(-1)}\begin{bmatrix}-3 & -2\\ -5 & -3\end{bmatrix}=\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix}$ ...(ii)

The given matrix equation is

$$\begin{bmatrix}2 & 1\\ 3 & 2\end{bmatrix}P\begin{bmatrix}-3 & 2\\ 5 & -3\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix} \Rightarrow APB=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$$

$$\Rightarrow A^{-1}APB=A^{-1}\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$$

$$\Rightarrow (A^{-1}A)(PB)=\begin{bmatrix}2 & -1\\ -3 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix} \quad \text{[Using (i)]}$$

$$\Rightarrow I(PB)=\begin{bmatrix}2-2 & 4+1\\ -3+4 & -6-2\end{bmatrix} \quad [\because A^{-1}A=I=AA^{-1}]$$

$$\Rightarrow PB=\begin{bmatrix}0 & 5\\ 1 & -8\end{bmatrix} \Rightarrow (PB)B^{-1}=\begin{bmatrix}0 & 5\\ 1 & -8\end{bmatrix}B^{-1}$$

$$\Rightarrow P(BB^{-1})=\begin{bmatrix}0 & 5\\ 1 & -8\end{bmatrix}\begin{bmatrix}3 & 2\\ 5 & 3\end{bmatrix} \quad \text{[Using (ii)]}$$

$$\Rightarrow PI=\begin{bmatrix}0+25 & 0+15\\ 3-40 & 2-24\end{bmatrix} \Rightarrow P=\begin{bmatrix}25 & 15\\ -37 & -22\end{bmatrix}$$

OR

(i) According to question, we have

$x+y+z=9500$

$2x+4y+3z=29000$

$5x+2y+3z=30500$

The system of equations can be written as $AX=B$,

where $A=\begin{bmatrix}1 & 1 & 1\\ 2 & 4 & 3\\ 5 & 2 & 3\end{bmatrix}$; $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$; $B=\begin{bmatrix}9500\\ 29000\\ 30500\end{bmatrix}$

(ii) $|A|=\begin{vmatrix}1 & 1 & 1\\ 2 & 4 & 3\\ 5 & 2 & 3\end{vmatrix}$

$=1(12-6)-1(6-15)+1(4-20)$

$=6+9-16=-1\neq 0$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution and it is given by $X=A^{-1}B$.

Now, $A_{11}=6, A_{12}=9, A_{13}=-16,$

$A_{21}=-1, A_{22}=-2, A_{23}=3,$

$A_{31}=-1, A_{32}=-1, A_{33}=2$

$\therefore$ adj $A=\begin{bmatrix}6 & -1 & -1\\ 9 & -2 & -1\\ -16 & 3 & 2\end{bmatrix}$

Now, $A^{-1}=\dfrac{1}{|A|}\cdot\text{adj } A=(-1)\begin{bmatrix}6 & -1 & -1\\ 9 & -2 & -1\\ -16 & 3 & 2\end{bmatrix}$

$$=\begin{bmatrix}-6 & 1 & 1\\ -9 & 2 & 1\\ 16 & -3 & -2\end{bmatrix}$$

Now, $X=A^{-1}B=\begin{bmatrix}-6 & 1 & 1\\ -9 & 2 & 1\\ 16 & -3 & -2\end{bmatrix}\begin{bmatrix}9500\\ 29000\\ 30500\end{bmatrix} \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2500\\ 3000\\ 4000\end{bmatrix}$

$\Rightarrow x=2500, y=3000$ and $z=4000$

37. Let $\dfrac{x+2}{3}=\dfrac{2y+3}{4}=\dfrac{3z+4}{5}=\lambda$

Any point on the line is given by

$\left(3\lambda-2, \dfrac{4\lambda-3}{2}, \dfrac{5\lambda-4}{3}\right)$.

Now, direction ratios of a line joining $(-2, 3, -4)$ and $\left(3\lambda-2, \dfrac{4\lambda-3}{2}, \dfrac{5\lambda-4}{3}\right)$ are $3\lambda, \dfrac{4\lambda-9}{2}, \dfrac{5\lambda+8}{3}$

Now, the distance is measured parallel to the plane

$4x+12y-3z+1=0.$

$\therefore$ $4\times 3\lambda+12\times\left(\dfrac{4\lambda-9}{2}\right)-3\times\left(\dfrac{5\lambda+8}{3}\right)=0$

$\Rightarrow 12\lambda+24\lambda-54-5\lambda-8=0$

$\Rightarrow 31\lambda-62=0 \Rightarrow \lambda=2$

$\therefore$ The required point is $\left(4, \dfrac{5}{2}, 2\right)$.

So, required distance$=\sqrt{(4+2)^2+\left(\dfrac{5}{2}-3\right)^2+(2+4)^2}$

$$=\sqrt{36+36+\frac{1}{4}}=\sqrt{\frac{289}{4}}=\frac{17}{2} \text{ units}$$

OR

Let $\vec{a}=-2\hat{i}+6\hat{j}-6\hat{k}$, $\vec{b}=-3\hat{i}+10\hat{j}-9\hat{k}$ and $\vec{c}=-5\hat{i}-6\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 6 & -6\\ -3 & 10 & -9\end{vmatrix}$

$= \hat{i}\,(-54+60) - \hat{j}\,(18-18) + \hat{k}\,(-20+18) = 6\hat{i}-2\hat{k}$

$\vec{b}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -3 & 10 & -9\\ -5 & 0 & -6\end{vmatrix}$

$= \hat{i}\,(-60+0) - \hat{j}\,(18-45) + \hat{k}\,(0+50)$

$= -60\hat{i} + 27\hat{j} + 50\hat{k}$

$\vec{c}\times\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -5 & 0 & -6\\ -2 & 6 & -6\end{vmatrix}$

$= \hat{i}(0+36)-\hat{j}(30-12)+\hat{k}(-30+0)=36\hat{i}-18\hat{j}-30\hat{k}$

Hence, equation of the plane,

$(\vec{r}-\vec{a})\cdot(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=0$

$\Rightarrow\ \vec{r}-(-2\hat{i}+6\hat{j}-6\hat{k})\cdot\left[(-18\hat{i}+9\hat{j}+18\hat{k})\right]=0$

$\Rightarrow\ \vec{r}\cdot(-18\hat{i}+9\hat{j}+18\hat{k})=36+54-108$

$\Rightarrow\ \vec{r}\cdot(2\hat{i}-\hat{j}-2\hat{k})=2$

38. We have, Maximize $Z=\dfrac{35}{2}x+7y$

subject to constraints

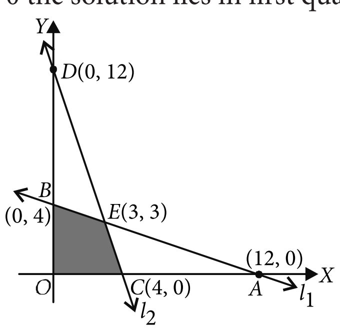
$x + 3y \le 12$

$3x + y \le 12$

$x, y \ge 0$

We draw the graph of lines $L_1 : x + 3y = 12$ and $l_2 : 3x + y = 12$.

As $x \ge 0, y \ge 0$ the solution lies in first quadrant



Vertices of feasible region are $O(0, 0)$, $C(4, 0)$ $E(3, 3)$ and $B(0, 4)$

Corner points	Value of $Z=\dfrac{35}{2}x+7y$
O (0, 0)	0
C(4, 0)	70
E(3, 3)	73.5 (Maximum)
B(0, 4)	28

Hence the maximum value is 73.50 which is attained at $E(3, 3)$.

OR

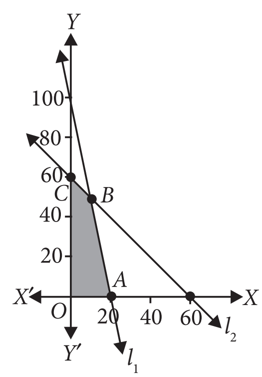
We have, Maximize, $Z = 500x + 150y$

Subject to constraints, $2500x + 500y \le 50000$,

$x + y \le 60$, $x \ge 0$, $y \ge 0$

We draw the graph of lines $l_1 : 2500x + 500y = 50000$, $l_2 : x + y = 60$

As $x \ge 0$, $y \ge 0$ the solution lies in the first quadrant.



The vertices of the feasible region are $O(0, 0)$, $A(20, 0)$, $B(10, 50)$ and $C(0, 60)$.

Corner points	Value of Z = 500 x + 150 y
O(0, 0)	0
A(20, 0)	10000
B(10, 50)	12500 (Maximum)
C(0, 60)	9000

$\therefore$ Maximum value of Z is 12500 which is attained at $B(10, 50)$.

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