# Playing With numbers

**Factors and multiples:**

- Factor of a number is an exact divisor of that number
  6 = 1 x 6 = 2 x 3 : factors of 6 are 1, 2, 3 and 6
- Every number is a factor of itself and 1 is a factor of every number
- Every factor of a number is less than or equal to the given number
- Every number is a multiple of each of its factors
  Factors of 12 are 1, 2, 3, 4, 6 and 12 : 3 x 4 =12, 2 x 6 =12, 12 x 1 = 12
- Every multiple of a given number is greater than or equal to that number.
- Every number is a multiple of itself
- Factors of a given number are finite while its multiples are infinite
- If two given numbers are divisible by a number, then their sum (or difference) is also divisible by that number : 6 is divisible by 3, 18 is divisible by 3. Hence 24 (= 18+ 6) and 12 (= 18-6) are also divisible by 3
- A number for which sum of all its factors is equal to twice the number is called a perfect number :Ex. 6 and 28

**Prime and composite numbers:**

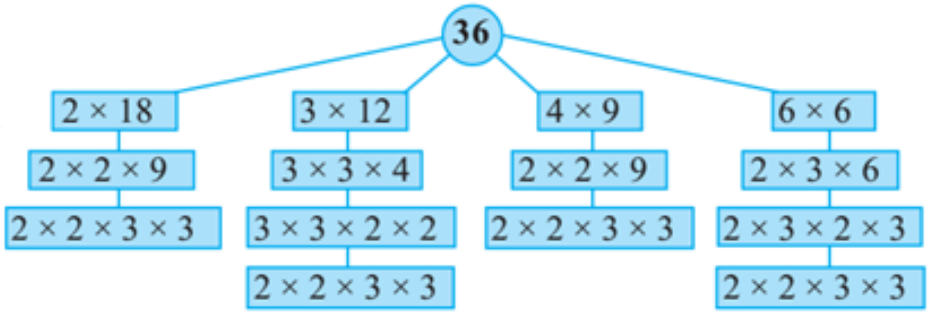
- A number having only 2 factors, namely 1 and the number itself, is a prime number : 2, 3, 5, 7, 11 etc
- Numbers that have more than two factors are called composite numbers : 4, 6, 8, 9, 10, 12 etc
- Number 1 is neither prime nor composite.
- The number 2 is the smallest prime number and is even. Every prime number other than 2 is odd.
- Two numbers with only 1 as a common factor are called co-prime numbers : 15 and 16 , 8 and 9 , 25 and 33 etc
- If a number is divisible by another number then it is divisible by each of the factors of that number
  24 is divisible by 8: it is also divisible by 2 and 4 which are factors of 8
- A number divisible by two co-prime numbers is divisible by their product also 8 and 9 are co-primes and 8 x 9 =72
  144 is divisible 9 : 9 x 16 =144 , 144 is divisible by 8 : 8 x 18 =144
  144 is also divisible by 72 (product of 8 and 9) : 72 x 2 =144

**Test of divisibility:**

| By : | Criteria/ nature of the given number | examples |
|---|---|---|
| 2 | Even numbers (0, 2, 4, 6 or 8 in its ones place) | 12, 148, 36, 100, 14 |
| 3 | sum of the digits is a multiple of 3 | 24: 2 + 4 = 6<br>1566: 1 + 5 + 6 + 6 =18 |
| 4 | Last two digits should be both zeroes or divisible by 4 | 100, 2400, 1936, 7848 |
| 5 | Number should have 0 or 5 in the ones place | 15, 1125, 4560 |
| 6 | Number should be divisible by 2 and 3 | 48 : even and<br>48 : 4 + 8 =12 |
| 8 | Last three digits should be divisible by 8 or should be all zeroes | 1000, 5000, 7024, 3144 |
| 9 | sum of the digits is a multiple of 9 | 144 : 1 + 4 + 4 = 9<br>8631: 8 + 6 + 3 + 1=18 |
| 10 | Number should have 0 in the ones place | 10 20, 30, 56000, 900 |
| 11 | difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) of the number is either 0 or divisible by 11 | 61809 :<br>9 + 8 + 6 = 23 and<br>0 + 1 = 1<br>23 – 1 = 22<br>22 is divisible by 11<br>Other egs : 275, 1331, 6006 |

**Prime factorization:**

Expressing a number as a product of its prime factors




The prime factorisation of 36 is = 2 × 2 × 3 × 3. i.e. the only prime factorisation of 36.

**The Highest Common Factor(HCF)**

- The HCF of two or more given numbers is the highest (or greatest) of their common factors.
- It is also known as Greatest Common Divisor (GCD)

| | | | | |
|---|---|---|---|---|
| 2 | 20 | 2 | 28 | 20 = 2 x 2 x 5<br>28 = 2 x 2 x 7<br>The common factors of 20 and 28 are 2, 2<br>Hence HCF (20, 28) = 2 x 2 = 4 |
| 2 | 10 | 2 | 14 | |
| 5 | 5 | 7 | 7 | |
| | 1 | | 1 | |

**The Lowest Common Multiple (LCM)**

The LCM of two or more given numbers is the lowest (or smallest or least) of their common multiples

Method 1

| | | | | |
|---|---|---|---|---|
| 2 | 12 | 16 | 24 | 36 |
| 2 | 6 | 8 | 12 | 18 |
| 2 | 3 | 4 | 6 | 9 |
| 2 | 3 | 2 | 3 | 9 |
| 3 | 3 | 1 | 3 | 9 |
| 3 | 1 | 1 | 1 | 3 |
| | 1 | 1 | 1 | 1 |

LCM = 2 × 2 × 2 × 2 × 3 × 3 = 144

| | | | |
|---|---|---|---|
| 2 | 20 | 25 | 30 |
| 2 | 10 | 25 | 15 |
| 3 | 5 | 25 | 15 |
| 5 | 5 | 25 | 5 |
| 5 | 1 | 5 | 1 |
| | 1 | 1 | 1 |

LCM = 2 × 2 × 3 × 5 × 5

Method 2 : Finding the LCM of 24 and 90

| | |
|---|---|
| The prime factorisations of 24 is given by :<br><br>24 = 2 × 2 × 2 × 3 | The prime factorisations of 90 is given by :<br><br>90 = 2 × 3 × 3 × 5 |

- In these prime factorisations, the maximum number of times the prime factor 2 occurs is three; this happens for 24.
- Similarly, the maximum number of times the prime factor 3 occurs is two; this happens for 90
- The prime factor 5 occurs only once in 90
- Thus, LCM = (2 × 2 × 2) × (3 × 3) × 5 = 360