Exercise 4.5

Answer 1E.

There are various techniques to evaluate the integral.

One of the techniques is the method of substitution.

In the method of substitution, an expression is substituted for one variable in order to simplify the integrand so that it is brought into a form that is easy to evaluate.

Consider the integral:

$$\int \sin(\pi x)dx$$

Make the substitution as shown below:

$$\begin{aligned} \pi x &= u \\ \pi dx &= du \\ dx &= \frac{1}{\pi}du \end{aligned}$$

So, the integral becomes:

$$\begin{aligned} \int \sin(\pi x)dx &= \int(\sin u)\left(\frac{1}{\pi}du\right) \\ &= \frac{1}{\pi}\int \sin(u)du \\ &= \frac{1}{\pi}(-\cos u)+C \\ &= -\frac{1}{\pi}\cos \pi x + C \end{aligned}$$

Hence, the final expression is $\boxed{-\frac{1}{\pi}\cos \pi x + C}$.

Answer 2E.

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Consider the integral,

$\int x^3(2+x^4)^5\,dx, u=2+x^4$.

Use substitution rule: Let $u=2+x^4$

Differentiate to each side, then the derivative of u is,

$$du = 4x^3dx$$
$$\frac{1}{4}du = x^3dx$$

So, the integral can be evaluated as,

$$\int x^3(2+x^4)^5\,dx = \int u^5\frac{1}{4}du \quad \text{Use } u=2+x^4 \text{ and } \frac{1}{4}du = x^3dx$$
$$=\frac{1}{4}\int u^5du$$
$$=\frac{1}{4}\left(\frac{u^6}{6}\right)+C$$
$$=\frac{u^6}{24}+C$$
$$=\frac{1}{24}(2+x^4)^6+C$$

Therefore, $\int x^3(2+x^4)^5\,dx = \boxed{\frac{1}{24}(2+x^4)^6+C}$.

Answer 3E.

We have to evaluate $\int x^2\sqrt{x^3+1}dx$

Put $u=x^3+1$ then $du=3x^2dx$ or $\frac{du}{3}=x^2dx$

Thus, $\int x^2\sqrt{x^3+1}dx = \frac{1}{3}\int u^{1/2}du$

$$=\frac{1}{3}\left(\frac{u^{1/2+1}}{1/2+1}\right)+C$$
$$=\frac{1}{3}\times\frac{2}{3}u^{3/2}+C$$
$$=\boxed{\frac{2}{9}(x^3+1)^{3/2}+C}$$

Answer 4E.

Given $\int \frac{dt}{(1-6t)^4}$

Let $1-6t=u \Rightarrow -6dt=du$

$$\Rightarrow dt = \frac{-1}{6}du$$

Substituting in above integral

$$\Rightarrow \int \frac{dt}{(1-6t)^4} = \frac{-1}{6}\int \frac{1}{u^4}du$$

$$= \frac{-1}{6}\int u^{-4}du$$

$$= \frac{-1}{6}\frac{u^{-4+1}}{-4+1}+C \qquad \left(\text{Since } \int x^n dx = \frac{x^{n+1}}{n+1}+C, n \neq -1\right)$$

$$= \frac{-1}{6}\frac{u^{-3}}{-3}+C$$

$$= \frac{1}{18}\frac{1}{u^3}+C$$

$$= \frac{1}{18}\frac{1}{(1-6t)^3}+C \quad (\because u = 1-6t)$$

Therefore $\boxed{\int \frac{dt}{(1-6t)^4} = \frac{1}{18}\frac{1}{(1-6t)^3}+C}$

Answer 5E.

We have to evaluate $\int \cos^3\theta \sin\theta d\theta$

Let $\cos\theta = u$ then $-\sin\theta d\theta = du$ or $\sin\theta d\theta = -du$

Thus $\int \cos^3\theta\sin\theta d\theta = -\int (u)^3 du$

$$= -\left(\frac{u^4}{4}\right)+C$$

$$= \boxed{-\frac{1}{4}\cos^4\theta + C}$$

Answer 6E.

Given $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,\mathrm{d}x$

x^2 can be brought up to the numerator as x^{-2} giving you

$\int \sec^2\left(\frac{1}{x}\right)x^{-2}\,\mathrm{d}x$ -------(1)

let u = 1/x or x^{-1} , therefore $\mathrm{d}u = -x^{-2}\,\mathrm{d}x$ or $-\mathrm{d}u = x^{-2}\,\mathrm{d}x$

substituting u and du in (1)

$$= -\int \sec^2(u)\,\mathrm{d}u$$

$$= -\tan(u) + C$$

$$= -\tan\left(\frac{1}{x}\right) + C$$ answer

Answer 7E.

We have to evaluate $\int x\sin(x^2)dx$

Let $x^2 = u$ then $2xdx = du$

Thus $\int x\sin(x^2)dx = \frac{1}{2}\int \sin u du$

$$= -\frac{1}{2}\cos u + C$$

$$= \boxed{-\frac{1}{2}\cos(x^2)+C}$$

Answer 8E.

Given $\int x^2 \cos(x^3)dx$

Let $x^3 = u \Rightarrow 3x^2dx = du$

$$\Rightarrow x^2dx = \frac{1}{3}du$$

$$\Rightarrow \int x^2 \cos(x^3)dx = \frac{1}{3}\int \cos u du$$

$$= \frac{1}{3}.\sin u + C \quad \left(\because \int \cos x dx = \sin x + C\right)$$

$$\boxed{\int x^2 \cos(x^3)dx = \frac{\sin(x^3)}{3} + C} \quad \left(\because u = x^3\right)$$

Answer 9E.

Given $\int (1-2x)^9 dx$

Let $1-2x = u \Rightarrow 0 - 2dx = du$

$$\Rightarrow dx = \frac{-1}{2}du$$

$$\Rightarrow \int (1-2x)^9 dx = \frac{-1}{2}\int u^9 du$$

$$= \frac{-1}{2}.\frac{u^{9+1}}{9+1} + C \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$$

$$= \frac{-1}{2}.\frac{u^{10}}{10} + C$$

$$\boxed{\int (1-2x)^9 dx = \frac{-(1-2x)^{10}}{20} + C} \quad (\because u = 1-2x)$$

Answer 10E.

Given $\int (3t+2)^{2.4} dx$

Let $3t+2 = u \Rightarrow 3dt = du$

$$\Rightarrow dt = \frac{1}{3}du$$

$$\Rightarrow \int (3t+2)^{2.4} dx = \frac{1}{3}\int (u)^{2.4} du$$

$$= \frac{1}{3}\frac{u^{2.4+1}}{2.4+1} + C \quad \left(\text{Since } \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$$

$$= \frac{1}{3}\frac{u^{3.4}}{3.4} + C$$

$$= \frac{(3t+2)^{3.4}}{10.2} + C \quad (\text{Since } u = 3t+2)$$

Therefore $\boxed{\int (3t+2)^{2.4} dx = \frac{(3t+2)^{3.4}}{10.2} + C}$

Answer 11E.

Consider the following integral:

$$\int (x+1)\sqrt{2x+x^2}\,dx$$

Let, $u = 2x + x^2$.

Then, $du = (2+2x)\,dx$, so $(x+1)\,dx = \frac{1}{2}du$ and $\int (x+1)\sqrt{2x+x^2}\,dx = \frac{1}{2}\int \sqrt{u}\,du.$

$$= \frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right] + C \quad \text{Since } \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$

$$= \frac{1}{2}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$$

$$= \frac{1}{3}u^{\frac{3}{2}} + C$$

$$= \frac{1}{3}\left(2x+x^2\right)^{\frac{3}{2}} + C \quad \text{Since } u = 2x + x^2$$

Therefore, the result is $\boxed{\int (x+1)\sqrt{2x+x^2}\,dx = \frac{1}{3}\left(2x+x^2\right)^{\frac{3}{2}} + C}$.

Answer 12E.

Consider the following integral:

$$\int \sec^2 2\theta\,d\theta$$

Evaluate the indefinite integral by using the substitution method.

Substitution rule: If $u = g(x)$ is a differentiable function whose range is an interval *I* and *f* is continuous on *I*, then

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

State the given integral.

$$\int \sec^2 2\theta\,d\theta$$

Apply the substitution rule.

Let $u = 2\theta \Rightarrow du = 2d\theta$

$$\Rightarrow d\theta = \frac{1}{2}du$$

Substitute the values and solve the integral.

$$\int \sec^2 2\theta\,d\theta = \frac{1}{2}\int \sec^2 u\,du \quad \text{Since } u = 2\theta \text{ and } d\theta = \frac{1}{2}du$$

$$= \frac{1}{2}.\tan u + C \quad \text{Use } \int \sec^2 x\,dx = \tan x + C$$

$$= \frac{1}{2}.\tan 2\theta + C \quad \text{Re-substitute } u = 2\theta$$

$$\boxed{\int \sec^2 2\theta\,d\theta = \frac{\tan 2\theta}{2} + C}$$

Answer 13E.

Given $\int \sec 3t \tan 3t dt$

Let $3t = u \Rightarrow 3dt = du$

$$\Rightarrow dt = \frac{1}{3} du$$

$$\Rightarrow \int \sec 3t \tan 3t dt = \frac{1}{3} \int \sec u \tan u du$$

$$= \frac{1}{3} . \sec u + C \quad \left(\because \int \sec x \tan x dx = \sec x + C \right)$$

$$\boxed{\int \sec 3t \tan 3t dt = \frac{\sec 3t}{3} + C} \quad (\because u = 3t)$$

Answer 14E.

Given $\int u\sqrt{1-u^2} du$

Let $1-u^2 = t \Rightarrow 0 - 2udu = dt$

$$\Rightarrow udu = \frac{-1}{2} dt$$

$$\Rightarrow \int u\sqrt{1-u^2} du = \frac{-1}{2} \int \sqrt{t} dt$$

$$= \frac{-1}{2} . \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1 \right)$$

$$= \frac{-1}{2} . \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{-1}{2} . \frac{2}{3} t^{\frac{3}{2}} + C$$

$$= \frac{-1}{3} \left(1-u^2\right)^{\frac{3}{2}} + C \quad \left(\because t = 1-u^2 \right)$$

$$\boxed{\int u\sqrt{1-u^2} du = \frac{-1}{3} \left(1-u^2\right)^{\frac{3}{2}} + C}$$

Answer 15E.

Consider the indefinite integral $\int \frac{a+bx^2}{\sqrt{3ax+bx^3}} dx$.

Evaluate the given indefinite integral.

The substitution rule for integrals states that if $u = g(x)$ is a differentiable function on interval I and f is continuous on I then $\int f(g(x)) g'(x) dx = \int f(u) du$.

Let $u = 3ax + bx^3$ then

$$du = d\left(3ax + bx^3\right)$$

$$= 3a + 3bx^2 \, dx$$

$$= \left(3a + 3bx^2\right) dx$$

$$\frac{du}{3} = \left(a + bx^2\right) dx$$

The indefinite integral becomes after substituting above values,

$$\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\,dx = \int \frac{1}{\sqrt{u}}\left(\frac{du}{3}\right)$$

$$= \frac{1}{3}\int u^{-1/2}\,du$$

$$= \frac{1}{3}\left[\frac{u^{-1/2+1}}{\frac{-1}{2}+1}\right]+C \qquad \left(\text{Since} \int x^n\,dx = \frac{x^{n+1}}{n+1}+C\right)$$

$$= \frac{1}{3}\left[\frac{u^{1/2}}{\frac{1}{2}}\right]+C$$

$$= \frac{2}{3}u^{1/2}+C$$

$$= \frac{2}{3}\sqrt{u}+C$$

$$= \frac{2}{3}\sqrt{3ax+bx^3}+C \quad \left(\text{Since}\, u = 3ax+bx^3\right)$$

Hence, the value of indefinite integral is $\boxed{\frac{2}{3}\sqrt{3ax+bx^3}+C}$.

Answer 16E.

Given $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

Let $\sqrt{x} = u \Rightarrow \frac{1}{2\sqrt{x}}dx = du$

$$\Rightarrow \frac{1}{\sqrt{x}}dx = 2du$$

$$\Rightarrow \int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \int \sin u.2du$$

$$= 2\int \sin u du$$

$$= 2.-\cos u + C \qquad \left(\because \int \sin x dx = -\cos x + C\right)$$

$$= -2\cos u + C$$

$$= -2\cos\sqrt{x} + C \qquad \left(\because u = \sqrt{x}\right)$$

$$\boxed{\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = -2\cos\sqrt{x}+C}$$

Answer 17E.

Given $\int \sec^2\theta \tan^3\theta d\theta$

Let $\tan\theta = u \Rightarrow \sec^2 d\theta = du$

$$\Rightarrow \int \sec^2\theta\tan^3\theta d\theta = \int u^3 du$$

$$= \frac{u^{3+1}}{3+1}+C \qquad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}+C, n \neq -1\right)$$

$$= \frac{u^4}{4}+C$$

$$= \frac{\tan^4\theta}{4}+C \qquad (\because u = \tan\theta)$$

$$\boxed{\int \sec^2\theta\tan^3\theta d\theta = \frac{\tan^4\theta}{4}+C}$$

Answer 18E.

Given $\int \cos^4\theta \sin\theta d\theta$

Let $\cos\theta = u \Rightarrow -\sin\theta d\theta = du$

$\sin\theta d\theta = -du$

$\Rightarrow \int \cos^4\theta \sin\theta d\theta = -\int u^4 du$

$= -\frac{u^{4+1}}{4+1} + C \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$

$= -\frac{u^5}{5} + C$

$= -\frac{\cos^5\theta}{5} + C \quad (\because u = \cos\theta)$

$$\boxed{\therefore \int \cos^4\theta \sin\theta d\theta = -\frac{\cos^5\theta}{5} + C}$$

Answer 19E.

Given $\int (x^2+1)(x^3+3x)^4 dx$

Let $x^3 + 3x = u \Rightarrow (3x^2+3)dx = du$

$3(x^2+1)dx = du$

$(x^2+1)dx = \frac{du}{3}$

$\Rightarrow \int (x^2+1)(x^3+3x)^4 dx = \frac{1}{3}\int u^4 du$

$= \frac{1}{3} \cdot \frac{u^{4+1}}{4+1} + C \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$

$= \frac{1}{3} \cdot \frac{u^5}{5} + C$

$= \frac{u^5}{15} + C$

$= \frac{(x^3+3x)^5}{15} + C \quad (\because u = x^3 + 3x)$

$$\boxed{\therefore \int (x^2+1)(x^3+3x)^4 dx = \frac{(x^3+3x)^5}{15} + C}$$

Answer 20E.

We have to evaluate $\int \sqrt{x}\sin(1+x^{3/2})dx$.

Let $1+x^{3/2} = u$ then $\frac{3}{2}x^{1/2}dx = du$ or $\sqrt{x}dx = \frac{2}{3}du$

Then $\int \sqrt{x}\sin(1+x^{3/2})dx = \frac{2}{3}\int \sin u\, du$

Using the formula $\int \sin u\, dx = -\cos u + C$, where C is any constant.

Then $\int \sqrt{x}\sin(1+x^{3/2})dx = -\frac{2}{3}\cos u + C$

Putting $u = 1 + x^{3/2}$ we have

$\boxed{\int \sqrt{x}\sin(1+x^{3/2})dx = -\frac{2}{3}\cos(1+x^{3/2}) + C}$, where C is any constant.

Answer 21E.

Consider the following integral:

$$\int \frac{\cos x}{\sin^2 x} dx$$

The objective is to evaluate the integral using substitution rule.

Let $u = \sin x$

Differentiate u with respect to x.

$$du = \cos x dx$$

Use the above details to evaluate the integral.

$$\int \frac{\cos x}{\sin^2 x} dx = \int \frac{du}{u^2}$$

$$= \int u^{-2} du$$

$$= \frac{u^{-2+1}}{-2+1} + C \qquad \left(\text{Use } \int x^n dx = \frac{x^{n+1}}{n+1} + C \right)$$

$$= \frac{u^{-1}}{-1} + C$$

$$= \frac{-1}{u} + C$$

$$= \frac{-1}{\sin x} + C \qquad (\text{Resubstitute } u = sinx)$$

Hence, the integral is $\boxed{\frac{-1}{\sin x} + C}$.

Answer 22E.

We have to evaluate $\int \frac{\cos(\pi / x)}{x^2} dx$.

Let $\frac{\pi}{x} = t$ then $-\frac{\pi}{x^2} dx = dt$ or $\frac{1}{x^2} dx = -\frac{1}{\pi} dt$

So we have $\int \frac{\cos(\pi / x)}{x^2} dx = -\frac{1}{\pi} \int \cos t \, dt$

$$= -\frac{1}{\pi} \sin t + C$$

Putting back the value of t, we have

$$\boxed{\int \frac{\cos(\pi / x)}{x^2} dx = -\frac{1}{\pi} \sin(\pi / x) + C}$$

Answer 23E.

Given

$$\int \frac{z^2}{\sqrt[3]{1+z^3}} dz = \int \frac{z^2}{(1+z^3)^{1/3}} dz$$

Let $u = 1+z^3$

Then $du = 3z^2 dz$

$$\frac{du}{3} = z^2 dz$$

Then the integral becomes

$$\int \frac{z^2}{\sqrt[3]{1+z^3}}dz = \int \frac{z^2}{(1+z^3)^{1/3}}dz$$
$$= \int \frac{du}{3u^{1/3}}$$
$$= \frac{1}{3}\int u^{-1/3}du$$
$$= \frac{1}{3}\cdot\left(\frac{u^{-1/3+1}}{-1/3+1}\right)+C$$
$$= \frac{1}{3}\cdot\left(\frac{u^{2/3}}{2/3}\right)+C$$
$$= \left(\frac{u^{2/3}}{2}\right)+C$$

Now substituting the value of u in the above equation, we get

$$\int \frac{z^2}{\sqrt[3]{1+z^3}}dz = \frac{1}{2}(1+z^3)^{2/3}+C$$

Answer 24E.

Consider the following integral:

$$\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}}$$

Rewrite the integral as follows:

$$\int \frac{\sec^2 t dt}{\sqrt{1+\tan t}} \qquad \text{Since } \frac{1}{\cos^2 t} = \sec^2 t$$

To evaluate the integral value, use Substitution rule.

Let $1+\tan t = x$

$\sec^2 t dt = dx$

Compute the integral using substitution rule as follows:

$$\int \frac{dt}{\cos^2 t\sqrt{1+\tan t}} = \int \frac{\sec^2 t dt}{\sqrt{1+\tan t}}$$
$$= \int \frac{dx}{\sqrt{x}}$$
$$= \int x^{-1/2}dx$$
$$= \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+c \qquad \text{since } \int x^n dx = \frac{x^{n+1}}{n+1}+c$$

$$= \frac{x^{1/2}}{1/2}+c$$
$$= 2x^{1/2}+c$$
$$= 2\sqrt{1+\tan t}+c \qquad \text{since } x = 1+\tan t$$

Therefore, the value of the integral is $\boxed{2\sqrt{1+\tan t}+c}$.

Answer 25E.

We have to evaluate $\int \sqrt{\cot x}\csc^2 x\, dx$.

Let $\cot x = u$ then $-\csc^2 x\, dx = du$ or $\csc^2 x dx = -du$

Then we have

$$\int \sqrt{\cot x}\csc^2 x\, dx = -\int \sqrt{u}\, du$$
$$= -\int u^{1/2}\, du$$

Using the formula $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, where C is any constant and $n \neq -$

So $\int \sqrt{\cot x} \csc^2 x dx = -\frac{u^{1/2+1}}{\frac{1}{2}+1} + C$

$= -\frac{2}{3} u^{3/2} + C$

Putting $u = \cot x$, we have

$\boxed{\int \sqrt{\cot x} \csc^2 x\, dx = -\frac{2}{3}(\cot x)^{3/2} + C}$, where C is any constant.

Answer 26.

We have to evaluate $\int \sin t \sec^2(\cos t)\, dt$.

Let $\cos t = x$ then $-\sin t\, dt = dx$ or $\sin t\; dt = -dx$

Then $\int \sin t \sec^2(\cos t)\, dt = -\int \sec^2 x\, dx$

Using the formula $\int \sec^2 x dx = \tan x + C$, where C is any constant.

We have

$\int \sin t \sec^2(\cos t) dt = -\tan x + C$

Putting $x = \cos t$, we have

$\boxed{\int \sin t \sec^2(\cos t) dt = -\tan(\cos t) + C}$, where C is any constant.

Answer 27E.

Consider the integral $\int \sec^3 x \tan x\; dx$.

Evaluate the indefinite integral as shown below:

Rewrite the integral as follows:

$\int \sec^3 x \tan x\; dx = \int \sec^2 x . \sec x \tan x\; dx$

Let $\sec x = u$ then $\sec x \tan x dx = du$

Substitute these values in the above integral.

$\int \sec^3 x \tan x\; dx = \int u^2 du$ Substitute $u = \sec x, du = \sec x \tan x$

$= \frac{u^3}{3} + C$ Since $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

$= \frac{1}{3}\sec^3 x + C$ Resubstitute $u = \sec x$

Therefore, $\int \sec^3 x \tan x dx = \boxed{\frac{1}{3}\sec^3 x + C}$

Answer 28E.

Given $\int x^2 \sqrt{2+x} dx$

Let $2 + x = u \Rightarrow x = u - 2$

$dx = du$

$\Rightarrow \int x^2 \sqrt{2+x} dx = \int (u-2)^2 \sqrt{u} du$

$= \int (u^2 - 4u + 4) u^{\frac{1}{2}} du$

$= \int \left(u^{\frac{5}{2}} - 4u^{\frac{3}{2}} + 4u^{\frac{1}{2}} \right) du$

$$= \frac{u^{\frac{5}{2}+1}}{\frac{5}{2}+1} - 4.\frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 4.\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$$

$$= \frac{2}{7}u^{\frac{7}{2}} - 4.\frac{2}{5}u^{\frac{5}{2}} + 4.\frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{2}{7}u^{\frac{7}{2}} - \frac{8}{5}u^{\frac{5}{2}} + \frac{8}{3}u^{\frac{3}{2}} + C$$

$$= \frac{2}{7}(2+x)^{\frac{7}{2}} - \frac{8}{5}(2+x)^{\frac{5}{2}} + \frac{8}{3}(2+x)^{\frac{3}{2}} + C \quad (\because u = 2+x)$$

$$\boxed{\therefore \int x^2\sqrt{2+x}\,dx = \frac{2}{7}(2+x)^{\frac{7}{2}} - \frac{8}{5}(2+x)^{\frac{5}{2}} + \frac{8}{3}(2+x)^{\frac{3}{2}} + C}$$

Answer 29E.

Consider the following integration

$$\int x(2x+5)^8\,dx$$

Need to find the value of the given integration.

Use an appropriate substitution

Let $2x+5=u$

$2x = u-5$ Subtract on both sides with 5

$x = \frac{u-5}{2}$ Divide on both sides by 2

Differentiating with respect to x on both sides, get

$dx = \frac{du}{2}$ Since $\frac{d}{dx}(x)=1, \frac{d}{dx}(k)=0$

Substitute $x = \frac{u-5}{2}, dx = \frac{du}{2}$ and $2x+5=u$ in $\int x(2x+5)^8\,dx$

$$\int x(2x+5)^8\,dx = \int\left(\frac{u-5}{2}\right)u^8\frac{du}{2}$$

$= \frac{1}{4}\int\left(u^9 - 5u^8\right)du$ Multiply

Continuation to the above steps,

$= \frac{1}{4}\left(\frac{u^{9+1}}{9+1} - 5\frac{u^{8+1}}{8+1}\right) + C$ Since $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$

$$= \frac{1}{4}\left(\frac{u^{10}}{10} - 5\frac{u^9}{9}\right) + C$$

$= \frac{u^{10}}{40} - \frac{5u^9}{36} + C$ Multiply

$= \frac{(2x+5)^{10}}{40} - \frac{5(2x+5)^9}{36} + C$ Since $2x+5=u$

Therefore,

$$\int x(2x+5)^8\,dx = \boxed{\frac{(2x+5)^{10}}{40} - \frac{5(2x+5)^9}{36} + C}.$$

Answer 30E.

Given $\int x^3\sqrt{x^2+1}dx$

Let $x^2+1=u \Rightarrow x^2=u-1$

$$\Rightarrow 2xdx = du$$

$$\Rightarrow xdx=\frac{1}{2}du$$

$$\text{Now } \int x^3\sqrt{x^2+1}dx=\int x^2\sqrt{x^2+1}xdx$$

$$=\int (u-1)\sqrt{u}\frac{du}{2}$$

$$=\frac{1}{2}\int\left(u^{1+\frac{1}{2}}-u^{\frac{1}{2}}\right)du$$

$$=\frac{1}{2}\int\left(u^{\frac{3}{2}}-u^{\frac{1}{2}}\right)du$$

$$=\frac{1}{2}\left(\frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1}-\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)+C$$

$$\left(\text{Since } \int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq -1\right)$$

$$=\frac{1}{2}\left(\frac{u^{\frac{5}{2}}}{\frac{5}{2}}-\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\left(\frac{u^{\frac{5}{2}}}{5}-\frac{u^{\frac{3}{2}}}{3}\right)+C$$

$$=\frac{\sqrt{(x^2+1)^5}}{5}-\frac{\sqrt{(x^2+1)^3}}{3}+C \qquad \left(\text{Since } u=x^2+1\right)$$

Therefore $\boxed{\int x^3\sqrt{x^2+1}dx=\frac{\sqrt{(x^2+1)^5}}{5}-\frac{\sqrt{(x^2+1)^3}}{3}+C}$

Answer 31E.

Consider the integral $\int x\left(x^2-1\right)^3dx$

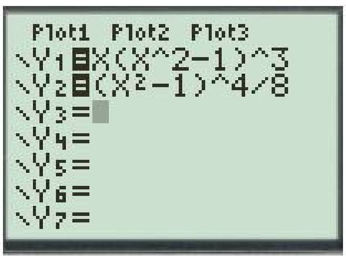
Evaluate the indefinite integral:

Let $u=x^2-1$. Then the differential is $du=2xdx$, so

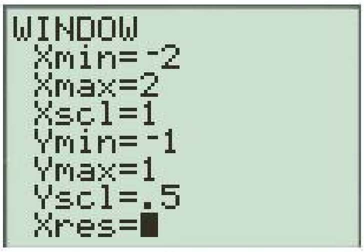
Thus, the substitution rule gives

Sketch the graph of $x\left(x^2-1\right)^3$ and $\frac{\left(x^2-1\right)^4}{8}$ is as follows:

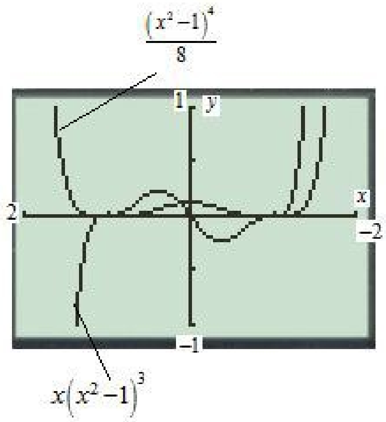
Now enter the two equations in the Y=window.



Here, we're using the window settings,



Press GRAPH to graph the equations



Answer 32E.

We have to evaluate $\int \tan^2\theta \sec^2\theta \, d\theta$

Let $\tan\theta = t$ then $\sec^2\theta \, d\theta = dt$

Then we have $\int \tan^2\theta \sec^2\theta \, d\theta = \int t^2 \, dt$

Using the formula $\int t^n dt = \frac{t^{n+1}}{n+1} + C$ where C = constant and $n \neq -1$

We have $\int \tan^2\theta \sec^2\theta \, d\theta = \frac{t^3}{3} + C$

Putting $t = \tan\theta$, we have

$$\boxed{\int \tan^2\theta \sec^2\theta \, d\theta = \frac{\tan^3 x}{3} + C}$$

Putting $C = 0$, we have $\int \tan^2\theta \sec^2\theta \, d\theta = \frac{\tan^3 x}{3}$

Let $F(\theta) = \frac{\tan^3\theta}{3}$ *and* $f(\theta) = \tan^2\theta \sec^2\theta$

So $F(\theta)$ is an anti derivative of the function $f(\theta)$ or in other words we can say that $f(\theta)$ is the derivative of the function $F(\theta)$

Now we sketch the curves of $F(\theta)$ and $f(\theta)$ on the same axis we see that in figure 1, the function $F(\theta)$ is an increasing function in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so the value of $f(\theta)$ is positive in this interval and at $\theta = \pm\frac{\pi}{2}$, $F(\theta)$ and $f(\theta)$ both area not defined. Where $F(\theta)$ has a horizontal tangent, $f(\theta) = 0$. This shows that $F(\theta)$ is an anti derivative of $f(\theta)$ and our answer is reasonable

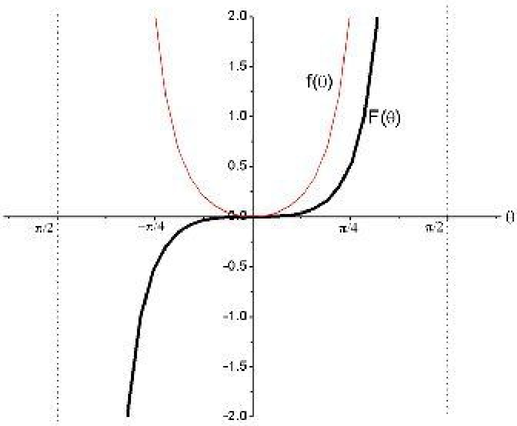


Fig.1

Answer 33E.

We have to evaluate $\int \sin^3 x \cos x \, dx$

Let $\sin x = t$ then $\cos x \, dx = dt$

So we have

$$\int \sin^3 x \cos x \, dx = \int t^3 dt$$

We use the formula $\int t^n dt = \frac{t^{n+1}}{n+1} + C$ where C = constant and $n \neq -1$

Then $\int \sin^3 x \cos x \, dx = \frac{t^4}{4} + C$

Or $\boxed{\int \sin^3 x \cos x \, dx = \frac{\sin^4 x}{4} + C}$

Putting C = 0, we have $\int \sin^3 x \cos x \, dx = \frac{\sin^4 x}{4}$

Let $F(x) = \frac{\sin^4 x}{4}$ *and* $f(x) = \sin^3 x \cos x$

So $F(x)$ is an anti derivative of the function $f(x)$ or in other words we can say that $f(x)$ is the derivative of $F(x)$.

Now we sketch the curves of $F(x)$ and $f(x)$ on the same axis in figure 1, we see that where $F(x)$ is increasing, $f(x)$ is positive and where $F(x)$ is decreasing, $f(x)$ has negative value. Where $F(x)$ has horizontal tangent, $f(x)$=0. This shows that $F(x)$ is an anti derivative of $f(x)$ and our answer is reasonable.

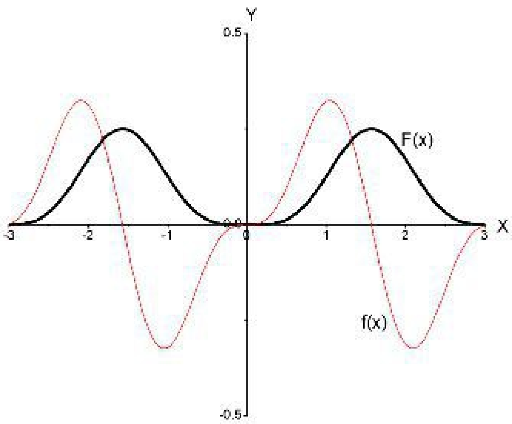


Fig.1

Answer 34E.

Given $\int \sin x \cos^4 x dx$

Let $\cos x = u \Rightarrow -\sin x dx = du$

$\sin x dx = -du$

$\Rightarrow \int \sin x \cos^4 x dx = -\int u^4 du$

$$= -\frac{u^{4+1}}{4+1} + C \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1 \right)$$

$$= -\frac{u^5}{5} + C$$

$$= -\frac{\cos^5 x}{5} + C \quad (\because u = \cos x)$$

$$\boxed{\therefore \int \sin x \cos^4 x dx = -\frac{\cos^5 x}{5} + C}$$

Answer 35E.

Consider the integral $\int_0^1 \cos\left(\frac{\pi t}{2}\right) dt$

Let $\frac{\pi t}{2} = u$ then $\frac{\pi}{2} dt = du$

$\Rightarrow dt = \frac{2}{\pi} du$

Now the lower limit $t = 0$ then $u = \frac{\pi t}{2} = \frac{\pi \cdot 0}{2} = 0$

And the upper limit $t = 1$ then $u = \frac{\pi t}{2} = \frac{\pi \cdot 1}{2} = \frac{\pi}{2}$

Now the given definite integral is

$$\int_0^1 \cos\left(\frac{\pi t}{2}\right) dt = \int_0^{\frac{\pi}{2}} \cos(u) \cdot \frac{2}{\pi} du$$

$$= \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos u \, du$$

$$= \frac{2}{\pi} (\sin u)_0^{\frac{\pi}{2}} \quad \left(\because \int \cos x dx = \sin x + C \right)$$

$$= \frac{2}{\pi} \left(\sin \frac{\pi}{2} - \sin 0 \right)$$

$$= \frac{2}{\pi} (1 - 0)$$

$$= \frac{2}{\pi}$$

Thus, $\int_0^1 \cos\left(\frac{\pi t}{2}\right) dt = \boxed{\frac{2}{\pi}}$.

Answer 36E.

Given $\int_0^1 (3t-1)^{50} dt$

Let $3t - 1 = u \Rightarrow 3dt = du$

$dt = \frac{1}{3} du$

Now the lower limit $t = 0$ then $u = 3t - 1 = 3.0 - 1 = -1$

The upper limit $t = 1$ then $u = 3t - 1 = 3.1 - 1 = 2$

Now the given definite integral is

$$\Rightarrow \int_0^1 (3t-1)^{50}\,dt = \frac{1}{3}\int_{-1}^{2} (u)^{50}\,du$$

$$= \frac{1}{3}\left(\frac{u^{50+1}}{50+1}\right)_{-1}^{2} \qquad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$$

$$= \frac{1}{3}\left(\frac{u^{51}}{51}\right)_{-1}^{2}$$

$$= \frac{1}{3}\left(\frac{2^{51}}{51} - \frac{(-1)^{51}}{51}\right)$$

$$= \frac{1}{3}\left(\frac{2^{51}}{51} - \frac{-1}{51}\right)$$

$$= \frac{1}{3}\left(\frac{2^{51}+1}{51}\right)$$

$$= \frac{1}{153}\left(2^{51}+1\right)$$

$$\boxed{\therefore \int_0^1 (3t-1)^{50}\,dt = \frac{1}{153}\left(2^{51}+1\right)}$$

Answer 37E.

Given $\int_0^1 \sqrt[3]{1+7x}\,dx$

Let $1+7x = u \Rightarrow 7dx = du$

$$dx = \frac{1}{7}du$$

Now the lower limit $t = 0$ then $u = 1+7x = 1+7.0 = 1$

The upper limit $t = 1$ then $u = 1+7x = 1+7.1 = 8$

Now the given definite integral is

$$\Rightarrow \int_0^1 \sqrt[3]{1+7x}\,dx = \int_1^8 \sqrt[3]{u}\,\frac{1}{7}du$$

$$= \frac{1}{7}\int_1^8 u^{\frac{1}{3}}du$$

$$= \frac{1}{7}\left(\frac{u^{\frac{1}{3}+1}}{\frac{1}{3}+1}\right)_1^8 \qquad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$$

$$= \frac{1}{7}\left(\frac{u^{\frac{4}{3}}}{\frac{4}{3}}\right)_1^8$$

$$= \frac{1}{7}\cdot\frac{3}{4}\left(8^{\frac{4}{3}} - 1^{\frac{4}{3}}\right)$$

$$= \frac{3}{28}(16-1)$$

$$= \frac{3}{28}(15)$$

$$= \frac{45}{28}$$

$$\boxed{\therefore \int_0^1 \sqrt[3]{1+7x}\,dx = \frac{45}{28}}$$

Answer 38E.

We have to evaluate $\int_0^{\sqrt{\pi}} x\cos\left(x^2\right)dx$

Since $\cos\left(x^2\right)$ is defined for all values of x, so the function $f(x)=x\cos\left(x^2\right)$ is continuous on $\left[0,\sqrt{\pi}\right]$, and then $\int_0^{\sqrt{\pi}} x\cos\left(x^2\right)dx$ exists and we can apply fundamental theorem

Let $x^2=t$ then $2xdx=dt$ or $xdx=\frac{1}{2}dt$

Now when x = 0, t = 0

And when $x=\sqrt{\pi},\ t=\left(\sqrt{\pi}\right)^2=\pi$

Then we have $\int_0^{\sqrt{\pi}} x\cos\left(x^2\right)dx=\frac{1}{2}\int_0^{\pi}\cos t\,dt$

Using fundamental theorem as $\int_a^b f(x)dx=F(x)\Big]_a^b$ where $F'=f$

Since an anti derivative of $\cos t$ is $F=\sin t$ then we have

$$\int_0^{\sqrt{\pi}} x.\cos\left(x^2\right)dx=\frac{1}{2}.\sin t\Big]_0^{\pi}$$
$$=\frac{1}{2}[\sin\pi-\sin 0]$$
$$=\frac{1}{2}[0-0]$$
$$=0$$

Then $\boxed{\int_0^{\sqrt{\pi}} x.\cos\left(x^2\right)dx=0}$

Answer 39E.

We have to evaluate $\int_0^{\pi}\sec^2(t/4)dt$

Let $\frac{t}{4}=u$ then $\frac{dt}{4}=du\Rightarrow dt=4du$

When $t=0,\ u=0$, and when $t=\pi,\ u=\frac{\pi}{4}$

Thus $\int_0^{\pi}\sec^2(t/4)dt=4\int_0^{\pi/4}\sec^2 u\,du$
$$=4[\tan u]_0^{\pi/4}$$
$$=4\left[\tan\frac{\pi}{4}-\tan 0\right]$$
$$=4[1-0]$$

Then, $\boxed{\int_0^{\pi}\sec^2(t/4)dt=4}$

Answer 40E.

Consider the definite integral,

$$\int_{\frac{1}{6}}^{\frac{1}{2}}\csc\pi t\cot\pi t\ dt \quad ......\ (1)$$

Use substitution to solve the integral (1).

Let the substitution be $\pi t=\theta$

Then differentiate this on each side.

$$\pi dt=d\theta \quad \text{or} \quad dt=\frac{d\theta}{\pi}$$

Change the limits of integration.

When $t=\frac{1}{6}$ then $\theta=\frac{\pi}{6}$

And when $t=\frac{1}{2}$ then $\theta=\frac{\pi}{2}$

Apply this substitution and limits to integral (1). Then the integral (1) becomes,

$$\int_{\frac{1}{6}}^{\frac{1}{2}} \csc \pi t \cot \pi t \; dt = \frac{1}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc\theta.\cot\theta \; d\theta$$

Since $\csc\theta$ and $\cot\theta$ are continuous on the interval $\left[\frac{\pi}{6},\frac{\pi}{2}\right]$

So, use fundamental theorem of calculus to evaluate the limit.

From fundamental theorem of Calculus, Suppose *f* is continuous on $[a,b]$, then $\int_a^b f(x)dx = F(b)-F(a)$ where F is any anti derivative of *f*, that is, $F' = f$

The anti-derivative of $(\csc\theta.\cot\theta)$ is $F(\theta) = -\csc\theta$. Then

$$\frac{1}{\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc\theta.\cot\theta \; d\theta = -\frac{1}{\pi}\left[-\csc\left(\frac{\pi}{2}\right)-\left(-\csc\left(\frac{\pi}{6}\right)\right)\right]$$ Use fundamental theorem

$$= \frac{1}{\pi}\left[\csc\frac{\pi}{6} - \csc\frac{\pi}{2}\right]$$

$$= \frac{1}{\pi}[2-1]$$

$$= \frac{1}{\pi}$$

Therefore, the value of the integral (1) is $\boxed{\frac{1}{\pi}}$.

Answer 41E.

Given $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \left(x^3 + x^4 \tan x\right)dx$

Let $f(x) = x^3 + x^4 \tan x$

$$f(-x) = (-x)^3 + (-x)^4 \tan(-x)$$

$$f(-x) = -(x)^3 + -(x)^4 \tan(x)$$

$$f(-x) = -f(x)$$

Therefore the given function is odd function. And we have the theorem

If f is continuous $[-a,a]$

(a) if f is even $[f(-x) = f(x)]$ then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

(b) if f is odd $[f(-x) = -f(x)]$ then $\int_{-a}^{a} f(x)dx = 0$

So the given definite integral value is 0.

$$\boxed{\therefore \int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} \left(x^3 + x^4 \tan x\right)dx = 0}$$

Answer 42E.

We have to evaluate $\int_0^{\pi/2} \cos x \sin(\sin x)\, dx$.

Since $\cos x$ and $\sin x$ functions are defined for all real values of x.

The function $f(x) = \cos x \sin x(\sin x)$ is also defined for all x, so $f(x)$ is continuous on $\left[0,\frac{\pi}{2}\right]$, hence we can use fundamental theorem of calculus.

Let $\sin x = t$ then $\cos x\, dx = dt$
When $x = 0$, $t = 0$
And when $x = \frac{\pi}{2}$, $t = 1$
Then we have

$$\int_0^{\pi/2} \cos x \sin(\sin x) dx = \int_0^1 \sin t\, dt$$

Using fundamental theorem of calculus as $\int_a^b f(x)dx = F(x)]_a^b$ when $F' = f$
Since an anti-derivative of $\sin t$ is $F(t) = -\cos t$.
So we have

$$\int_0^{\pi/2} \cos x \sin(\sin x) dx = -\cos t]_0^1$$
$$= -[\cos 1 - \cos 0]$$
$$= -[\cos 1 - 1]$$

So $\boxed{\int_0^{\pi/2} \cos x \sin(\sin x) dx = 1 - \cos 1}$

Answer 43E.

Consider the following definite integral:

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}.$$

The objective is to find the integral value by using substitution rule and fundamental theorem of calculus.

Let $1 + 2x = t^3$. Then,

$$2dx = 3t^2 dt$$
$$dx = \frac{3}{2} t^2 dt$$

The change in limits of integration is as follows:

If $x = 0$, then $t = \sqrt[3]{1+0} = 1$

If $x = 13$, then $t = \sqrt[3]{(1+2(13))} = 3$.

Compute the integral value as follows:

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^3 \frac{1}{t^2}\left(\frac{3}{2} t^2 dt\right)$$
$$= \frac{3}{2}\int_1^3 (1) dt$$
$$= \frac{3}{2}[t]_1^3$$
$$= \frac{3}{2}(3-1)$$
$$= 3$$

Therefore, the value of the definite integral is $\boxed{3}$.

Answer 44E.

We have to evaluate $\int_0^a x\sqrt{a^2 - x^2}\, dx$

Let $f(x) = x\sqrt{a^2 - x^2}$, since $f(x)$ is defined when $x \le a$
So $f(x)$ is continuous on [0, a] and we can use fundamental theorem of calculus
Let $a^2 - x^2 = t^2$ then $-2x dx = 2t\, dt$ or $x\, dx = -t\, dt$
When x = 0, t = a
And when x = a, t = 0

So we have $\int_0^a x\sqrt{a^2-x^2}\,dx = -\int_a^0 t.t\,dt$

$$= -\int_a^0 t^2\,dt$$

Using the property of definite integral $\int_a^b f(t)\,dt = -\int_b^a f(t)\,dt$

We have $\int_0^a x\sqrt{a^2-x^2}\,dx = \int_0^a t^2\,dt$

Using fundamental theorem of calculus as $\int_a^b f(x)dx = F(x)\Big]_a^b$ where $F' = f$

Since an anti derivative of $f(t) = t^2$ *is* $F(t) = \frac{t^3}{3}$

So $\int_0^a x\sqrt{a^2-x^2}\,dx = \frac{t^3}{3}\Big]_0^a$

Or $\boxed{\int_0^a x\sqrt{a^2-x^2}\,dx = \frac{a^3}{3}}$

Answer 45E.

Consider the following integral:

$$\int_0^a x\sqrt{a^2-x^2}\,dx$$

The objective is to find the definite integral by substitution method.

Let $u = \sqrt{a^2-x^2}$

Squaring on both sides,

$$u^2 = a^2 - x^2$$

Differentiate u with respect to x.

$$2udu = -2xdx$$
$$udu = -xdx$$
$$xdx = -udu$$

When $x = 0$, then $u = \sqrt{a^2-0^2}, u = a$

When $x = a$, then $u = \sqrt{a^2-a^2}, u = 0$

Using the above details find the definite integral.

$$\int_0^a x\sqrt{a^2-x^2}\,dx = \int_0^a \left(\sqrt{a^2-x^2}\right)xdx$$
$$= \int_a^0 u\cdot(-u)\,du$$
$$= -\int_a^0 u^2 du$$

$$= -\left[\frac{u^3}{3}\right]_a^0$$
$$= -\left[0 - \frac{a^3}{3}\right]$$
$$= \frac{a^3}{3}$$

So, $\int_0^a x\sqrt{a^2-x^2}\,dx = \frac{a^3}{3}$

Therefore, the definite integral is $\boxed{\int_0^a x\sqrt{a^2-x^2}\,dx = \frac{a^3}{3}}$.