

# Previous Years Paper

23<sup>rd</sup> May 2023 (Shift 1)

- Q1.** Let  $A$  be a square matrix of order 3 then  $|3A|$  is equal to  
 (a)  $3|A|$   
 (b)  $3^2|A|$   
 (c)  $|A|^3$   
 (d)  $3^3|A|$

- Q2.** In a LLP, let  $R$  be the feasible region.  
 A. If  $R$  is unbounded then a max./min. value of objective function may not exist.  
 B. If  $R$  is bounded then a max. and min. value of objective function will always exist.  
 C. If a solution exists, it must occur at a corner point.  
 D. If  $R$  is bounded then max. will exist but min. may or may not exist for an objective function.

Choose the correct answer from the options given below:

- (a) A, B, C only  
 (b) B only  
 (c) A, C only  
 (d) D, C only

- Q3.** The volume of a cube is increasing at the rate of  $27 \text{ cm}^3/\text{s}$ . How fast is the surface area increasing when the length of the cube is 12 cm?  
 (a)  $9 \text{ cm}^2/\text{s}$   
 (b)  $\frac{9}{4} \text{ cm}^2/\text{s}$   
 (c)  $\frac{4}{9} \text{ cm}^2/\text{s}$   
 (d)  $\frac{9}{2} \text{ cm}^2/\text{s}$

- Q4.** If a fair coin is tossed 10 times, then the probability of obtaining at least one head is:  
 (a)  $\frac{1}{1024}$   
 (b)  $\frac{17}{1024}$   
 (c)  $\frac{1023}{1024}$   
 (d)  $\frac{23}{1024}$

- Q5.** In the context of differential equation Match List I with List II

| List I                                |      | List II                     |
|---------------------------------------|------|-----------------------------|
| $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ | I.   | Not a differential equation |
| $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ | II.  | Linear first order          |
| $\sin x + y = \cos(x + y)$            | III. | Variable separable          |
| $(x + y) \frac{dy}{dx} = 1$           | IV.  | Homogenous                  |

Choose the correct answer from the options given below:

- (a) A-I, B-II, C-III, D-IV  
 (b) A-II, B-IV, C-III, D-I  
 (c) A-III, B-IV, C-I, D-II  
 (d) A-IV, B-I, C-III, D-II

- Q6.** Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Then variance of the number of kings is

- (a)  $\frac{680}{(221)^3}$   
 (b)  $\frac{6080}{(221)^2}$   
 (c)  $\frac{680}{221}$   
 (d)  $\frac{6800}{(221)^2}$

- Q7.** If  $y = x^x$ ,  $\frac{dy}{dx}$  will be  
 (a)  $x^x$   
 (b)  $x^x(1 + \log x)$   
 (c)  $x^{x-1}$   
 (d)  $x^{x+1}$

- Q8.** The inverse of the matrix  $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$  is

- (a)  $\begin{bmatrix} -2 & 3 \\ 1 & -2 \end{bmatrix}$   
 (b)  $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$   
 (c)  $\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$   
 (d)  $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

- Q9.** The value of  $\int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$  is

- (a)  $\frac{x^2}{2} + \log|x| + C$ , (where  $C$  is constant of integration)  
 (b)  $\frac{x^2}{2} + \log|x| + 2x + C$ , (where  $C$  is constant of integration)  
 (c)  $\frac{3}{2} + \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right) + C$ , (where  $C$  is constant of integration)  
 (d)  $\frac{2}{3}x^{\frac{3}{2}} + 2\sqrt{x} + C$ , (where  $C$  is constant of integration)

- Q10.** The integral  $\int e^x \left( \frac{x-1}{2x^2} \right) dx$  is equal to

- (a)  $\frac{e^x}{x} + C$ , where  $C$  is constant of integration  
 (b)  $\frac{e^x}{2x} + C$ , where  $C$  is constant of integration  
 (c)  $e^x x + C$ , where  $C$  is constant of integration  
 (d)  $x^2 e^x + C$ , where  $C$  is constant of integration

- Q11.** The order of a null matrix is

- (a) 0  
 (b) 1  
 (c) 2  
 (d) any order

- Q12.** Which of the following differential equation represents the family of circles touching the  $x$ -axis at the origin?

- (a)  $(x^2 - y^2)dy - 2xydx = 0$   
 (b)  $(x^2 + y^2)dy + 2xydx = 0$

- (c)  $(x^2 - y^2)dx + 2xydy = 0$   
 (d)  $(x^2 + y^2)dy - 2xydx = 0$

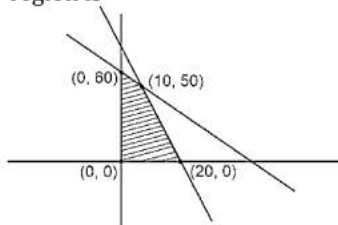
**Q13.** Let  $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$ , then adjoint (A) is

- (a)  $\begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$   
 (b)  $\begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$   
 (c)  $\begin{bmatrix} -4 & 1 \\ -2 & -3 \end{bmatrix}$   
 (d)  $\begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$

**Q14.** The critical points of  $f(x) = x^3 + x^2 + x + 1$  are

- (a) 2, 1  
 (b) -2, -1  
 (c) 2, -1  
 (d) do not exist

**Q15.** If  $5x + y \leq 100, x + y \leq 60, x \geq 0, y \geq 0$ . Then one of the corner points of the feasible region is



Corner point :  $\{(0, 60), (20, 0), (10, 50), (0, 0)\}$

- (a) (60, 0)  
 (b) (0, 100)  
 (c) (10, 50)  
 (d) (0, 20)

**Q16.**  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \sqrt{\tan x}}$  is equal to:

- (a)  $\frac{\pi}{6}$   
 (b)  $\frac{\pi}{12}$   
 (c)  $\frac{\pi}{4}$   
 (d)  $\frac{\pi}{3}$

**Q17.** The value of  $\lambda$ , so that lines  $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2}$  and  $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$  are perpendicular is:

- (a)  $-\frac{70}{11}$   
 (b)  $\frac{70}{11}$   
 (c)  $\frac{11}{70}$   
 (d)  $-\frac{11}{70}$

**Q18.** The value of  $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$  is

- (a) 0  
 (b)  $(a+b+c)$   
 (c)  $(a-b)(b-c)(c-a)$   
 (d)  $a^2 + b^2 + c^2$

**Q19.** If  $y = \sin^{-1} \left( \frac{1-x^2}{1+x^2} \right)$  then  $\frac{dy}{dx} =$

- (a)  $-\frac{2}{1+x^2}$   
 (b)  $\frac{2}{1+x^2}$

- (c)  $-\frac{1}{1+x^2}$   
 (d)  $\frac{1}{1+x^2}$

**Q20.** The intervals for which  $f(x) = x^4 - 2x^2$  is increasing are

- (a)  $(-\infty, 1)$   
 (b)  $(-1, \infty)$   
 (c)  $(-\infty, -1) \cup (0, 1)$   
 (d)  $(-1, 0) \cup (1, \infty)$

**Q21.** The simplest form of  $\tan^{-1} \left\{ \frac{x}{\sqrt{a^2 - x^2}} \right\}$  is, where  $-a < x < a$ .

- (a)  $\tan^{-1} \frac{x}{a}$   
 (b)  $\tan^{-1} ax$   
 (c)  $a \tan^{-1} \frac{x}{a}$   
 (d)  $\sin^{-1} \frac{x}{a}$

**Q22.** The maximum value of  $\sin x + \cos x, x \in R$  is:

- (a) 2  
 (b)  $\sqrt{2}$   
 (c)  $\frac{1}{\sqrt{2}}$   
 (d) Not known

**Q23.** Which one of the following options is incorrect? For a square matrix A in the matrix equation  $AX = B$ .

- (a) If  $|A| \neq 0$ , then there exists a unique solution  
 (b) If  $|A| = 0$  and  $(\text{adj } A)B \neq 0$  then there is no solution  
 (c) If  $|A| \neq 0$  and  $(\text{adj } A)B \neq 0$  then there is no solution  
 (d) If  $|A| = 0$  and  $(\text{adj } A)B = 0$  then system has infinitely many solutions

**Q24.** The simplest form of  $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}, x \neq 0$  is:

- (a)  $\tan^{-1} x$   
 (b)  $x$   
 (c)  $\frac{1}{2} \tan^{-1} x$   
 (d)  $\sqrt{1+x^2}$

**Q25.** If  $x, y$  and  $z$  are non-zero real numbers, the inverse

of matrix  $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$  is

- (a)  $\begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$   
 (b)  $xyz \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$   
 (c)  $\frac{1}{xyz} \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$   
 (d)  $\frac{1}{xyz} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**Q26.** Match List-I with List-II

| List-I |                       | List-II |                                     |
|--------|-----------------------|---------|-------------------------------------|
| A.     | $lx + my + nz = d$ is | I.      | Equation of plane passing through a |

|    |                                                                      |      |                                                      |
|----|----------------------------------------------------------------------|------|------------------------------------------------------|
|    |                                                                      |      | given point and normal to given vector               |
| B  | $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$                        | II.  | Equation of plane in normal form                     |
| C. | $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$                              | III. | Plane passing through the intersection of two planes |
| D. | $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$ | IV.  | Intercept from of plane                              |

Choose the correct answer from the options given below:

- (a) A-I, B-III, C-IV, D-II  
 (b) A-IV, B-III, C-I, D-II  
 (c) A-II, B-IV, C-I, D-III  
 (d) A-I, B-II, C-III, D-IV

Q27. If  $\begin{vmatrix} 2x & 2 \\ 4 & x \end{vmatrix} = 10$ , then x is

- (a)  $\pm 2$   
 (b)  $\pm 3$   
 (c)  $\pm 4$   
 (d) 0

Q28. The minimum value of  $f(x) = |2x - 1|$  is

- (a)  $-\infty$   
 (b) 0  
 (c)  $\frac{1}{2}$   
 (d) 1

Q29. If  $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$  and  $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$  the angle Between  $\vec{a} + \vec{b}$  and  $\vec{a} - \vec{b}$  is:

- (a)  $60^\circ$   
 (b)  $90^\circ$   
 (c)  $120^\circ$   
 (d)  $30^\circ$

Q30. The relation  $R = \{(a, b) : a \leq b^2\}$  on the set of real numbers is:

- (a) Reflexive and symmetric  
 (b) Neither reflexive nor symmetric  
 (c) Transitive  
 (d) Reflexive but not symmetric

Q31. The solution of  $y' - y'' = 2x$  is:

- (a)  $y = x^2 + 2x + 2$   
 (b)  $y = x^2 + 2x + 1$   
 (c)  $y = x + 2$   
 (d)  $y = x^2 - 2x + 1$

Choose the correct answer from the options given below:

- (a) A and B only  
 (b) B only  
 (c) C only  
 (d) A and D only

Q32. The slope of the normal to the curve  $y = 2x^2 - 4$  at  $P(1, -2)$  is

- (a) 4  
 (b) -4  
 (c)  $-\frac{1}{4}$

(d) 0

Q33. The area of the region  $\{(x, y) : y \geq x^2 \text{ and } y \leq |x|\}$  is

- (a) 2  
 (b) 1  
 (c)  $\frac{1}{2}$   
 (d)  $\frac{1}{3}$

Q34. The area enclosed between the curves  $y = x^2$  and  $x = y^2$  is

- (a) 1  
 (b)  $\frac{1}{2}$   
 (c)  $\frac{1}{3}$   
 (d)  $\frac{1}{4}$

Q35. The matrix  $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  is a

- (a) Zero matrix  
 (b) Identity matrix  
 (c) Scalar matrix  
 (d) Diagonal matrix

Q36. The principal value of  $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

- (a)  $\frac{\pi}{3}$   
 (b)  $-\frac{\pi}{6}$   
 (c)  $\frac{1}{\sqrt{3}}$   
 (d)  $\frac{2\pi}{3}$

Q37. If  $y = x^{(x \sin x)}$  then  $\frac{dy}{dx} = ?$

- (a)  $x^{x \cos x}$   
 (b)  $x^{(x \sin x)} [\sin x + \sin \log x]$   
 (c)  $x^{(x \sin x)} [\sin(1 + \log x) + x \log x \cos x]$   
 (d)  $x^{(x \cos x)} [\sin \log x + x \log x \cos x]$

Q38. The angle between the lines

$\vec{r} = 3\hat{i} + 2\hat{j} + 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$  and  $\vec{r} = 5\hat{j} - 2\hat{k} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$  is

- (a)  $\sin^{-1}\left(\frac{19}{21}\right)$   
 (b)  $\cos^{-1}\left(\frac{19}{23}\right)$   
 (c)  $\cos^{-1}\left(\frac{19}{21}\right)$   
 (d)  $\sin^{-1}\left(\frac{19}{23}\right)$

Q39. A doctor is to visit a patient. It is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively  $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$  and  $\frac{2}{5}$ . The probabilities that he will be late are  $\frac{1}{4}, \frac{1}{3}$  and  $\frac{1}{12}$ , if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he arrives late. The probability that he comes by bus is:

- (a)  $\frac{4}{9}$   
 (b)  $\frac{1}{18}$   
 (c)  $\frac{1}{3}$   
 (d)  $\frac{1}{2}$

- Q40.** The number of all onto functions from the set  $\{1, 2, \dots, n\}$  to itself is  
 (a)  $2^n$   
 (b)  $n^2$   
 (c)  $n!$   
 (d)  $(2n)!$
- Q41.** The unit vector in the direction of  $\vec{a} + \vec{b}$  if  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$  &  $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$  is:  
 (a)  $\hat{i} + 0\hat{j} + \hat{k}$   
 (b)  $\hat{i} - \hat{j} + \hat{k}$   
 (c)  $\hat{i} + \hat{j} + \hat{k}$   
 (d)  $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$
- Q42.** Points of discontinuity of the greatest integer function  $f(x) = [x]$ , where  $[x]$  denotes integer less than or equal to  $x$ , are  
 (a) all natural numbers  
 (b) all rational numbers  
 (c) all integers  
 (d) all real numbers
- Q43.** Let  $A$  be the square matrix of order 3, then  $|kA|$ , where  $k$  is a scalar, is equal to:  
 (a)  $3k|A|$   
 (b)  $k^3|A|$   
 (c)  $k^2|A|$   
 (d)  $k|A|$
- Q44.** If a fair coin is tossed 10 times the probability of at least 6 heads is:  
 (a)  $\frac{105}{512}$   
 (b)  $\frac{53}{128}$   
 (c)  $\frac{53}{64}$   
 (d)  $\frac{193}{512}$
- Q45.** The general solution of  $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$  is :  
 (given that  $C$  is the constant of integration)  
 (a)  $\tan^{-1} x = y + \frac{y^3}{3} + C$   
 (b)  $\tan^{-1} y = x + \frac{x^3}{3} + C$   
 (c)  $\tan^{-1} x = \tan^{-1} y + C$   
 (d)  $\tan^{-1} x + \tan^{-1} y = C$
- Q46.** The value of integral  $\int \sqrt{4x^2 + 9} dx$  is  
 (a)  $\frac{x}{2}\sqrt{4x^2 + 9} + \frac{9}{2}\log|2x + \sqrt{4x^2 + 9}| + C$   
 (b)  $\frac{x}{2}\sqrt{4x^2 + 9} + \frac{3}{2}\log|2x + \sqrt{4x^2 + 9}| + C$   
 (c)  $2x\sqrt{4x^2 + 9} + \frac{9}{2}\log|2x + \sqrt{4x^2 + 9}| + C$   
 (d)  $x\sqrt{4x^2 + 9} + \frac{9}{4}\log|2x + \sqrt{4x^2 + 9}| + C$
- Q47.** Let  $f(x) = x^3$  be a function with domain  $\{0, 1, 2, 3\}$  then domain of  $f^{-1}$  is:  
 (a)  $\{3, 2, 1, 0\}$   
 (b)  $\{0, -1, -2, -3\}$   
 (c)  $\{0, 1, 8, 27\}$   
 (d)  $\{0, -1, -8, -27\}$
- Q48.** The area of the parallelogram determined by the vectors  $\hat{i} + 2\hat{j} + 3\hat{k}$  and  $3\hat{i} - 2\hat{j} + \hat{k}$  is  
 (a)  $8\sqrt{3}$   
 (b)  $4\sqrt{3}$   
 (c)  $16\sqrt{3}$   
 (d)  $2\sqrt{3}$
- Q49.** A manufacturing company makes two models  $M_1$  and  $M_2$  of a product. Each piece of  $M_1$  requires 9 labour hours for fabricating and one labour hour for finishing. Each piece of  $M_2$  require 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 800 on each piece of  $M_1$  and Rs. 1200 on each piece of  $M_2$ .  
 The above Linear Programming Problem [LPP] is given by  
 (a) Maximize  $Z = 800x + 1200y$   
 Subject to constraints,  
 $3x + 4y \leq 60$   
 $x + 3y \leq 30$   
 $x, y \geq 0$   
 (b) Maximize  $Z = 800x + 1200y$   
 Subject to constraints,  
 $3x + 4y \geq 60$   
 $x + 3y \geq 30$   
 $x, y \geq 0$   
 (c) Minimize  $Z = 800x + 1200y$   
 Subject to constraints,  
 $3x + 4y \leq 60$   
 $x + 3y \geq 30$   
 $x, y \geq 0$   
 (d) Minimize  $Z = 800x + 1200y$   
 Subject to constraints,  
 $3x + 4y \geq 60$   
 $x + 3y \leq 30$   
 $x, y \geq 0$
- Q50.** A manufacturing company makes two models  $M_1$  and  $M_2$  of a product. Each piece of  $M_1$  requires 9 labour hours for fabricating and one labour hour for finishing. Each piece of  $M_2$  require 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs. 800 on each piece of  $M_1$  and Rs. 1200 on each piece of  $M_2$ .  
 The maximum profit will be at the point  
 (a)  $(0, 10)$   
 (b)  $(20, 0)$   
 (c)  $(12, 6)$   
 (d)  $(0, 0)$

## SOLUTIONS

**S1. Ans. (d)**

**Sol.**  $|3A| = 3^3 \times |A|$   
Option (d) is correct.

**S2. Ans. (a)**

**Sol.** If R is bounded, then the objective function Z has both a max. and a min. value on R and each of these occurs at a corner point of R.  
If the feasible region R is unbounded, then the max. and min. value of the objective function may or may not exist.  
Option (a) is correct.

**S3. Ans. (a)**

**Sol.**  $V = a^3$   
 $\frac{dv}{dt} = 3a^2 \frac{da}{dt}$   
 $\Rightarrow \frac{da}{dt} = \frac{27}{3a^2} = \frac{27}{3 \times (12)^2} = \frac{1}{16}$   
 $S = 6a^2$   
 $\frac{ds}{dt} = 6 \times 2a \frac{da}{dt}$   
 $= 12 \times 12 \times \frac{1}{16}$   
 $= 9 \text{ cm}^2/\text{s}$   
Option (a) is correct.

**S4. Ans. (c)**

**Sol.**  $P(\text{obtaining at least one head})$   
 $= 1 - P(\text{Obtaining no head})$   
 $= 1 - \frac{1}{2^{10}}$   
 $= \frac{1023}{1024}$   
Option (c) is correct.

**S5. Ans. (c)**

**Sol.** A:  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2} \rightarrow$  Variable separable type  
B:  $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$   
 $\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2} \rightarrow$  Homogeneous  
C:  $\sin x + y = \cos(x + y)$   
 $\rightarrow$  Not a differential equation  
D:  $(x + y) \frac{dy}{dx} = 1$   
 $\Rightarrow \frac{dx}{dy} = x + y \rightarrow$  Linear differential equation  
Option (c) is correct.

**S6. Ans. (d)**

**Sol.** Let X denote the number of kings in a draw of two cards  
 $\Rightarrow P(X = 0) = \frac{48C_2}{52C_2} = \frac{188}{221}$   
 $\Rightarrow P(X = 1) = \frac{4C_1 \times 48C_1}{52C_2} = \frac{32}{221}$   
 $\Rightarrow P(X = 2) = \frac{4C_2}{52C_2} = \frac{1}{221}$   
 $\Rightarrow \text{Mean of } X = \sum_{i=1}^2 x_i P(x_i)$   
 $= 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2}{221} = \frac{34}{221}$   
&  $\Rightarrow E(x^2) = \sum_{i=1}^2 x_i^2 P(x_i)$   
 $= 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221}$   
 $= \frac{36}{221}$

$$\Rightarrow \text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{36}{221} - \left(\frac{34}{221}\right)^2$$

$$= \frac{6800}{(221)^2}$$

Option (d) is correct

**S7. Ans. (b)**

**Sol.**  $y = x^x$   
 $\frac{dy}{dx} = x^x(1 + \log x)$

**S8. Ans. (b)**

**Sol.**  $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$   
 $\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$   
 $|A| = 4 - 3 = 1$   
 $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

**S9. Ans. (b)**

**Sol.**  $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 dx$   
 $= \int \left(x + \frac{1}{x} + 2\right) dx$   
 $= \frac{x^2}{2} + \log|x| + 2x + C$

**S10. Ans. (b)**

**Sol.**  $\int e^x \left(\frac{x-1}{2x^2}\right) dx$   
 $\int e^x \left(\frac{1}{2x} - \frac{1}{2x^2}\right) dx$   
 $= \frac{e^x}{2x} + C$

**S11. Ans. (d)**

**Sol.** Order of null matrix is any order

**S12. Ans. (a)**

**Sol.**  $x^2 + (y - a)^2 = a^2$   
 $x^2 + y^2 - 2ay = 0$   
 $\frac{x^2 + y^2}{y} = 2a$   
 $\frac{y(2x + 2yy') - (x^2 + y^2)y'}{y^2} = 0$   
 $2xy + 2y^2 \frac{dy}{dx} - (x^2 + y^2) \frac{dy}{dx} = 0$   
 $(x^2 - y^2) \frac{dy}{dx} - 2xy = 0$   
 $(x^2 - y^2)dy - 2xydx = 0$

**S13. Ans. (d)**

**Sol.**  $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$   
 $\text{adj } A = \begin{bmatrix} 4 & -2 \\ 1 & 3 \end{bmatrix}'$   
 $= \begin{bmatrix} 4 & 1 \\ -2 & 3 \end{bmatrix}$

**S14. Ans. (d)**

**Sol.**  $f(x) = x^3 + x^2 + x + 1$   
 $f'(x) = 3x^2 + 2x + 1 = 0$   
 $\therefore D < 0$   
 $\therefore$  No solution

**S15. Ans. (c)**

**Sol.** Corner points:  $\{(0, 60), (20, 0), (10, 50), (0, 0)\}$

**S16. Ans. (b)**

**Sol.**  $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$

$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$

$I = \frac{\pi}{12}$

Option (b) is correct.

**S17. Ans. (b)**

**Sol.**  $\vec{v}_1: -3\hat{i} + \frac{2\lambda}{7}\hat{j} + 2\hat{k}$

$\vec{v}_2: \frac{-3\lambda}{7}\hat{i} + \hat{j} - 5\hat{k}$

$\vec{v}_1 \cdot \vec{v}_2 = 0$

$\Rightarrow \frac{9\lambda}{7} + \frac{2\lambda}{7} - 10 = 0$

$\Rightarrow \lambda = \frac{+70}{11}$

Option (b) is correct.

**S18. Ans. (a)**

**Sol.**  $\begin{bmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{bmatrix}$

$C_2 \rightarrow C_2 + C_3$

$\begin{bmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{bmatrix}$

$= (a+b+c) \begin{bmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{bmatrix}$

$= 0$

**S19. Ans. (a)**

**Sol.**  $y = \sin^{-1} \left( \frac{1-x^2}{1+x^2} \right)$

$y = \frac{\pi}{2} - 2 \tan^{-1} x$

$\frac{dy}{dx} = \frac{-2}{1+x^2}$

Option (a) is correct.

**S20. Ans. (d)**

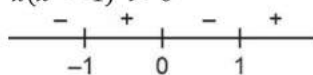
**Sol.**  $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x$

$= 4(x^3 - x)$

$x^3 - x > 0$

$x(x^2 - 1) > 0$



$x \in (-1, 0) \cup (1, \infty)$

Option (d) is correct.

**S21. Ans. (d)**

**Sol.**  $x = a \sin \theta$

$\tan^{-1} \left( \frac{a \sin \theta}{\sqrt{a^2 - a^2 \sin^2 \theta}} \right)$

$= \tan^{-1}(\tan \theta)$

$= \theta = \sin^{-1} \left( \frac{x}{a} \right)$

**S22. Ans. (b)**

**Sol.**  $E = \sin x + \cos x$

$E \in [-\sqrt{2}, \sqrt{2}]$

$E_{\max} = \sqrt{2}$

**S23. Ans. (c)**

**Sol.**  $AX = B$

$X = A^{-1}B$

$X = \frac{(\text{adj } A)}{|A|} \cdot B$

If  $|A| \neq 0 \Rightarrow$  unique solution

If  $|A| = 0$

and  $(\text{adj } A)B \neq 0 \Rightarrow$  No solution

If  $|A| \neq 0$

and  $(\text{adj } A)B \neq 0 \Rightarrow$  unique solution

If  $|A| = 0$

and  $(\text{adj } A)B = 0 \Rightarrow$  Infinite solution.

**S24. Ans. (c)**

**Sol.**  $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$

Let  $x = \tan \theta$

$\tan \left( \frac{\sec \theta - 1}{\tan \theta} \right)$

$= \tan \left( \frac{1 - \cos \theta}{\sin \theta} \right)$

$= \tan \left( \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} \right)$

$= \tan \left( \tan \frac{\theta}{2} \right) = \frac{\theta}{2}$

$= \frac{1}{2} \tan^{-1} x$

**S25. Ans. (a)**

**Sol.**  $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$

$A^{-1} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$

**S26. Ans. (c)**

**Sol.**

(A)  $lx + my + nz = d$  Equation of plane in normal (II)

(B)  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$  Intercept form of plane (IV)

(C)  $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$  Equation of plane passing through a given point and normal to given vector (I)

(D)  $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$  Plane passing through the intersection of two planes (III)

**S27. Ans. (b)**

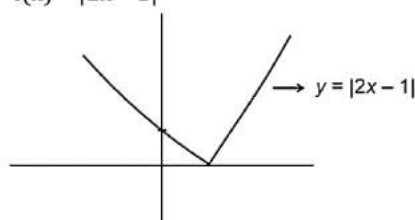
**Sol.**  $\begin{vmatrix} 2x & 2 \\ 4 & x \end{vmatrix} = 10$

$2x^2 - 8 = 10$

$x = \pm 3$

**S28. Ans. (b)**

**Sol.**  $f(x) = |2x - 1|$



$\therefore \text{Range} = [0, \infty]$

Min value = 0

**S29. Ans. (b)**

**Sol.**  $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$$\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$$

$$\vec{a} + \vec{b} = 6\hat{i} - 4\hat{j} + 2\hat{k} = \vec{c}$$

$$\vec{a} - \vec{b} = 4\hat{i} + 2\hat{j} - 8\hat{k} = \vec{d}$$

Angle between  $\vec{c}$  and  $\vec{d}$

$$\Rightarrow \cos \theta = \frac{\vec{c} \cdot \vec{d}}{|\vec{c}| |\vec{d}|}$$

$$= \frac{24 - 8 - 16}{\sqrt{36 + 16 + 4} \sqrt{16 + 4 + 64}}$$

$$= 0$$

$$\theta = 90^\circ$$

**S30. Ans. (b)**

**Sol.**  $R = \{(a, b) : a \leq b^2\}$

For reflexive  $(a, a) \in R$

If  $a \in (0, 1)$  then  $a > a^2$

$\Rightarrow R$  is not reflexive

For symmetric

If  $(a, b) \in R \Rightarrow a \leq b^2$

$\nRightarrow (b, a) \in R \nRightarrow b \leq a^2$

$\therefore R$  is not symmetric

$\therefore R$  is neither Reflexive nor symmetric

**S31. Ans. (a)**

**Sol.**

$$(a) \quad y = x^2 + 2x + 2$$

$$y' = 2x + 2$$

$$\therefore y' - y'' = 2x$$

$$y'' = 2$$

$$(b) \quad y = x^2 + 2x + 1$$

$$y' = 2x + 2$$

$$y' - y'' = 2x$$

$$y'' = 2$$

$$(c) \quad y = x + 2$$

$$y' = 1$$

$$y' - y'' = 1$$

$$y'' = 0$$

$$(d) \quad y = x^2 + 2x + 1$$

$$y' = 2x + 2$$

$$y'' = 2 \Rightarrow y' - y'' = 2x - 4$$

$\therefore A$  is  $B$  only are solution

**S32. Ans. (c)**

**Sol.**  $y = 2x^2 - 4$

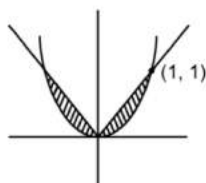
$$y' = 4x$$

$$y'_T = 4$$

$$y'_N = -\frac{1}{4}$$

**S33. Ans. (d)**

**Sol.**  $\{(x, y) : y \geq x^2 \text{ and } y \leq |x|\}$



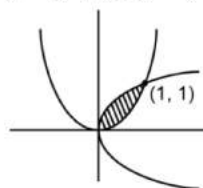
$$A = 2 \int_0^1 (x - x^2) dx$$

$$= 2 \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1$$

$$= 2 \left[ \frac{1}{2} - \frac{1}{3} \right] = \frac{2}{6} = \frac{1}{3} \text{ sq unit.}$$

**S34. Ans. (c)**

**Sol.**  $y = x^2$  and  $x = y^2$



$$\text{Area} = \int_0^1 (x^2 - \sqrt{x}) dx$$

$$\left[ \frac{x^3}{3} - \frac{2x^{3/2}}{3} \right]_0^1$$

$$\left[ \frac{1}{3} - \frac{2}{3} \right] = \frac{1}{3}$$

**S35. Ans. (c)**

$$\text{Sol.} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Is scalar matrix

**S36. Ans. (d)**

$$\text{Sol.} \cot^{-1} \left( \frac{-1}{\sqrt{3}} \right)$$

$$= \frac{2\pi}{3}$$

**S37. Ans. (\*)**

**Sol.**  $y = x^{x \sin x}$

$$\log y = x \sin x \log x$$

$$\frac{1}{y} y' = \sin x + (\log x)(\sin x + x \cos x)$$

$$y' = x^{x \sin x} (\sin x + (\sin x + x \cos x)(\log x))$$

$$y' = x^{x \sin x} ((1 + \log x) \sin x + x \cos x \log x)$$

**S38. Ans. (c)**

$$\text{Sol.} \frac{(i+2j+2k) \cdot (3i+2j+6k)}{|i+2j+2k| |3i+2j+6k|}$$

$$= \frac{19}{3.7}$$

$$\theta = \cos^{-1} \left( \frac{19}{21} \right)$$

**S39. Ans. (a)**

**Sol.**

$$P(\text{Comes by train}) = \frac{3}{10} P(\text{late by train}) = \frac{1}{4}$$

$$P(\text{Comes by bus}) = \frac{1}{5} P(\text{late by bus}) = \frac{1}{3}$$

$$P(\text{Comes by scooter}) = \frac{1}{10} P(\text{late by scooter}) = \frac{1}{12}$$

$$P(\text{Comes by other means}) = \frac{2}{5} P(\text{late by other}) = 0$$

$$P(\text{by bus / late}) = \frac{\frac{1}{5} \times \frac{1}{3}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0}$$

$$= \frac{\frac{1}{15}}{\frac{3}{40} + \frac{1}{15} + \frac{1}{120}}$$

$$= \frac{\frac{1}{15}}{\frac{9+8+1}{120}} = \frac{120}{15 \times 18} = \frac{8}{18} = \frac{4}{9}$$

**S40. Ans. (c)**

**Sol.**  $S: \{1, 2, \dots, n\}$

Number of onto functions =  $n!$

**S41. Ans. (d)**

$$\text{Sol.} \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{i + \hat{k}}{\sqrt{2}} = \frac{i}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$$

**S42. Ans. (c)**

**Sol.**  $[x]$  is discontinuous at all integers.

**S43. Ans. (b)**

**Sol.**  $|kA| = k^3 |A|$

By property of determinants

**S44. Ans. (d)**

**Sol.**  $P(\text{at least 6 heads}) = p(6H) + p(7H) + p(8H) + p(9H) + p(10H)$

$$= {}^{10}C_6 \left(\frac{1}{2}\right)^{10} + {}^{10}C_7 \left(\frac{1}{2}\right)^{10} + {}^{10}C_8 \left(\frac{1}{2}\right)^{10} + {}^{10}C_9 \left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10}$$

$$= \left(\frac{1}{2}\right)^{10} ({}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10})$$

$$= \left(\frac{1}{2}\right)^{10} \left(1 + 10 + 45 + \frac{10 \times 9 \times 8}{6} + \frac{10 \times 9 \times 8 \times 7}{24}\right)$$

$$= \left(\frac{1}{2}\right)^{10} (1 + 10 + 45 + 120 + 210)$$

$$= \left(\frac{1}{2}\right)^{10} (386)$$

$$= \frac{193}{512}$$

**S45. Ans. (b)**

**Sol.**  $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

$$\Rightarrow \frac{dy}{dx} = (1 + x^2)(1 + y^2)$$

$$\Rightarrow \frac{dy}{1+y^2} = (1+x^2)dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + C$$

**S46. Ans. (\*)**

**Sol.** Using  $\int \sqrt{(ax)^2 + b^2} dx$

$$= \frac{ax}{2} \sqrt{(ax)^2 + b^2} + \frac{b^2}{2} \log |ax + \sqrt{(ax)^2 + b^2}| + C$$

$$\therefore \int \sqrt{(2x)^2 + (3)^2} dx$$

$$= \frac{2x}{2} \sqrt{4x^2 + 9} + \frac{9}{2} \log |2x + \sqrt{4x^2 + 9}| + C$$

$$= \frac{x}{2} \sqrt{4x^2 + 9} + \frac{9}{4} \log |2x + \sqrt{4x^2 + 9}| + C$$

**S47. Ans. (c)**

**Sol.**  $f(x) = x^3$

$$f(1) = 1$$

$$f(0) = 0$$

$$f(2) = 8$$

$$f(3) = 27$$

$$\therefore f^{-1}(x) \text{ domain} = f(x) \text{ range} = \{0, 1, 8, 27\}$$

$$\therefore \text{Domain of } f^{-1}(x) = \{0, 1, 8, 27\}$$

**S48. Ans. (a)**

**Sol.** Area of a parallelogram whose adjacent edges are  $\vec{a}$  and  $\vec{b}$  is  $|\vec{a} \times \vec{b}|$ .

Given:  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k} \Rightarrow |\vec{a}| = \sqrt{14}$

$$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k} \Rightarrow |\vec{b}| = \sqrt{14}$$

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 \cdot |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$$

$$= 14 \cdot 14 - (3 - 4 + 3)^2$$

$$= 196 - 4$$

$$= 192$$

$$|\vec{a} \times \vec{b}| = \sqrt{192}$$

$$8\sqrt{3}$$

**S49. Ans. (a)**

**Sol.**

|             | $M_1$ (x units) | $M_2$ (y units) | Available |
|-------------|-----------------|-----------------|-----------|
| Fabricating | 9 labour hours  | 12 labour hours | 180       |
| Finishing   | 1 labour hours  | 3 labour hours  | 30        |
| Unit profit | Rs. 800         | Rs. 1200        |           |

Maximize :  $Z = 800x + 1200y$

Constraints :  $9x + 12y \leq 180$

$$\Rightarrow 3x + 4y \leq 60 \dots (i)$$

$$x + 3y \leq 30 \dots (ii)$$

$$x, y \geq 0 \dots (iii)$$

**S50. Ans. (c)**

**Sol.**

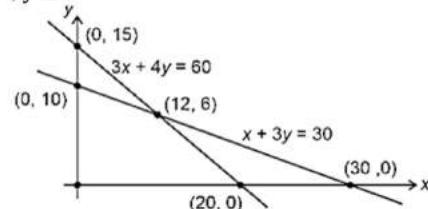
|             | $M_1$ (x units) | $M_2$ (y units) | Available |
|-------------|-----------------|-----------------|-----------|
| Fabricating | 9 labour hours  | 12 labour hours | 180       |
| Finishing   | 1 labour hours  | 3 labour hours  | 30        |
| Unit profit | Rs. 800         | Rs. 1200        |           |

Maximize:  $Z = 800x + 1200y$

Constraints:  $3x + 4y \leq 60$

$$x + 3y \leq 30$$

$$x, y \geq 0$$



Profit at (12, 6) is 16800

Profit at (20, 0) is 16000

$\therefore$  Maximum profit will be at (12, 6).