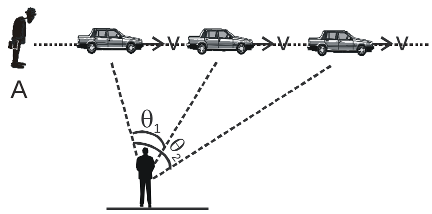
CIRCULAR MOTION

1. CIRCULAR MOTION

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as the circular motion with respect to that fixed (or moving) point. That fixed point is called centre and the distance between fixed point and particle is called radius.



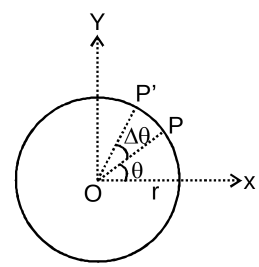
The car is moving in a straight line with respect to the man A. But the man B continuously rotate his face to see the car.

2. KINEMATICS OF CIRCULAR MOTION :

2.1 Variables of Motion :

(a) Angular Position :

The angle made by the position vector with given line (reference line) is called angular position Circular motion is a two dimensional motion or motion in a plane. Suppose a particle P is moving in a circle of radius r and centre O. The position of the particle P at a given instant may be described by the angle θ between OP and OX. This angle θ is called the angular position of the particle. As the particle moves on the circle its angular position θ change. Suppose the point rotates an angle $\Delta\theta$ in time Δt.



(b) Angular Displacement :

Definition :

Angle rotated by a position vector of the moving particle in a given time interval with some reference line is called its angular displacement.

(c) Angular Velocity μ

Average Angular Velocity

$$\omega_{av} = \frac{\text{Total Angle of Rotation}}{\text{Total time taken}} ;$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2 respectively.

Important points :

- It is an axial vector with dimensions $[T^{-1}]$ and SI unit rad/s.
- For a rigid body as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about its own axis is $(2\pi/24)$ rad/hr.
- If a body makes 'n' rotations in 't' seconds then angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{t}$$

If T is the period and 'f' the frequency of uniform circular motion

$$\omega_{av} = \frac{2\pi \times 1}{T} = 2\pi f$$

Ex.1 If ¯ depends on time t in following way

¯ = 2t² + 3 then

(a) Find out µ average upto 3 sec.

Sol. $\omega_{avg} = \dfrac{\text{Total angular displacement}}{\text{total time}} = \dfrac{\theta_f - \theta_i}{t_2 - t_1}$

$\theta_f = 2\,(3)^2 + 3 = 21$ rad

$\theta_i = 2\,(0) + 3 = 3$ rad.

So, $\omega_{avg} = \dfrac{21-3}{3} = 6$ rad/sec

(d) Angular Acceleration ~ :

Average Angular Acceleration :

Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Ex.2 A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5.0 m/s to 6.0 m/s in 2.0s, find the angular acceleration.

Sol. The tangential acceleration is given by

$$a_t = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1}$$

($\because$ Here speed increases uniformly $a_t = \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$)

$$= \frac{6.0 - 5.0}{2.0}\ \text{m/s}^2 = 0.5\ \text{m/s}^2$$

The angular acceleration is $\alpha = a_t/r$

$$= \frac{0.5\,\text{m/s}^2}{20\,\text{cm}} = 2.5\ \text{rad/s}^2$$