Statistics-I

Ex. 5.1

Answer 1.

Money (in Rs.)	Class marks x_i	Number of students f_i	f_ix_i
0-10	5	5	25
10-20	15	7	105
20-30	25	5	125
30-40	35	2	70
40-50	45	6	270
Total	....	$\Sigma f_i = 25$	$\Sigma f_ix_i = 595$

Here, $\Sigma f_ix_i = 595, \Sigma f_i = 25$

The formula to calculate the mean using the Direct method is,

$$\bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} \qquad \left(\bar{x} \text{ is the ariithmetic mean}\right)$$

$$= \frac{595}{25}$$

$$= 23.80$$

The mean of money collected by a student is Rs. 23.80

Answer 2.

Age (in years)	Class marks x_i	Number of People f_i	$f_i x_i$
7-11	9	5	45
11-15	13	9	117
15-19	17	13	221
19-23	21	21	441
23-27	25	16	400
27-31	29	15	435
31-35	33	12	396
35-39	37	9	333
Total	...	$\Sigma f_i = 100$	$\Sigma f_i x_i = 2388$

Here, $\Sigma f_i x_i = 2388$,

$\Sigma f_i = 100$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{2388}{100} = 23.88$$

The mean age of persons suffering from *Asthama* is 3.88 years.

Answer 3.

Let A be the assumed mean.

A is taken as the class mark of the middle class.

Hence, let us take 40 as the assumed mean.

Then A = 40 and deviation $d_i = x_i - A = x_i - 40$.

Diameter (in mm)	Class marks x_i	Deviations $d_i = x_i - A$ $d_i = x_i - 40$	Number of Screws f_i	$f_i x_i$
33 - 35	34	-6	10	-60
36 - 38	37	-3	19	-57
39 - 41	40 = A	0	23	0
42 - 44	43	3	21	63
45 - 47	46	6	27	162
Total	...	...	$\sum f_i = 100$	$\sum f_i x_i = 108$

Here, $\sum f_i d_i = 108$, $\sum f_i = 100$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

$$= \frac{108}{100} = 1.08$$

$$\bar{x} = A + \bar{d}$$

$$= 40 + 1.08$$

$$= 41.08$$

The mean diameter of the screw heads is 41.08 mm.

Answer 4.

50 is the class mark of the middle class.

Hence, let us take 50 as the assumed mean.

Then A = 50 and deviation $d_i = x_i - A = x_i - 50$.

Marks	Class marks x_i	Deviations $D_i = x_i - A$ $D_i = x_i - 50$	Number of students f_i	$f_i d_i$
0-10	5	-45	3	-135
10-20	15	-35	5	-175
20-30	25	-25	7	-175
30-40	35	-15	10	-150
40-50	45	-5	12	-60
50-60	55	5	15	75
60-70	65	15	12	180
70-80	75	25	6	150
80-90	85	35	2	70
90-100	95	45	8	360
Total	...	...	$\Sigma f_i = 80$	$\Sigma f_i d_i = 140$

Here, $\Sigma f_i d_i = 140, \Sigma f_i = 80.$

$$\bar{d} = \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= \frac{140}{80} = 1.75$$

$$\bar{x} = A + \bar{d}$$

$$= 50 + 1.75 = 51.75$$

The mean marks scored by a student is 51.75

Answer 5.

Shift of Origin method is the same as the Assumed Mean method.

57 is the class mark of the middle class.

Hence, let us take 57 as the assumed mean.

Here, we take A = 57.

Deviation $d_i = x_i - A = x_i - 57$.

Milk (in litres)	Class marks x_i	Deviations $d_i=x_i-A$ $d_i=x_i-57$	Number of cows f_i	f_id_i
24-30	27	-30	1	-30
30-36	33	-24	3	-72
36-42	39	-18	8	-144
42-48	45	-12	5	-60
48-54	51	-6	5	-30
54-60	57=A	0	5	0
60-66	63	6	8	48
66-72	69	12	4	48
72-78	75	18	6	108
78-84	81	24	2	48
84-90	87	30	3	90
Total	...	...	$\Sigma f_i = 50$	$\Sigma f_id_i = 6$

Here, $\Sigma f_id_i = 6, \Sigma f_i = 50.$

$$\bar{d} = \frac{\Sigma fidi}{\Sigma fi} = \frac{6}{50} = 0.12$$

$$\bar{x} = A + \bar{d}$$

$$= 57 + 0.12$$

$$= 57.12$$

The average (mean) amount of milk given by a cow per week is 57.12 litres.

Answer 6.

Here, we take A =22.5 and h (width of the class) = 5.

Number of saplings	Class Marks x_i	$d_i = x_i - A = x_i - 22.5$	$u_i = \frac{d_i}{h} = \frac{d_i}{5}$	Number of societies f_i	$f_i u_i$
10-15	12.5	-10	-2	2	-4
15-20	17.5	-5	-1	7	-7
20-25	22.5	0	0	9	0
25-30	27.5	5	1	8	8
30-35	32.5	10	2	6	12
35-40	37.5	15	3	4	12
Total	....	...	....	$\Sigma f_i = 36$	$\Sigma f_i u_i = 21$

Here, $\Sigma f_i u_i = 21, \Sigma f_i = 36, h = 5$

$$\bar{u} = \frac{\Sigma f_i u_i}{\Sigma f_i} = \frac{21}{36} = \frac{7}{12}$$

$$\bar{x} = A + h\bar{u}$$

$$= 22.5 + 5x\frac{7}{12}$$

$$= 22.5 + 2.92 (\text{Approx.})$$

$$= 25.42$$

The mean number of saplings planted by the housing societies is 25.42.

Answer 7.

Solution: Here, we take A = 25, h(Width of the class) = 4

Age (in years)	Class marks x_i	$D_i=x_i-A$ $=x_i-25$	$u_i = \frac{d_i}{h} = \frac{d_i}{4}$	Number of people f_i	f_iu_i
7-11	9	-16	-4	5	-20
11-15	13	-12	-3	9	-27
15-19	17	-8	-2	13	-26
19-23	21	-4	-1	21	-21
23-27	25=A	0	0	16	0
27-31	29	4	1	15	15
31-35	33	8	2	12	24
35-39	37	12	3	9	27
Total	...	...	...	$\Sigma f_i = 100$	$\Sigma f_iu_i = -28$

Here, $\Sigma f_iu_i = -28, \Sigma f_i = 100, h = 4$

$$\bar{u} = \frac{\Sigma f_iu_i}{\Sigma f_i} = \frac{-28}{100} = -0.28$$

$$\bar{x} = A + h\bar{u}$$

$$= 25 + 4(-0.28)$$

$$= 25 - 1.12 = 23.88$$

The mean age of persons suffering from Asthama is 23.88 years.

Answer 8.

Here, we take A = 40, h (Width of the class) = 3.

Diameter (in mm)	Class marks x_i	$D_i=x_i-A$ $=x_i-40$	$u_i=\frac{d_i}{h}=\frac{d_i}{3}$	Number of Screws f_i	f_iu_i
33-35	34	-6	-2	10	-20
36-38	37	-3	-1	19	-19
39-41	40=A	0	0	23	0
42-44	43	3	1	21	21
45-47	46	6	2	27	54
Total	...	...	....	$\Sigma f_i = 100$	$\Sigma f_iu_i = 36$

Here, $\Sigma f_iu_i = 36, \Sigma f_i = 100, h = 3$

$$\bar{u}=\frac{\Sigma f_iu_i}{\Sigma f_i}=\frac{36}{100}=0.36$$

$$\bar{x}=A+h\bar{u}$$

$$=40+3(0.36)=40+1.08=41.08$$

The mean diameter of the screw heads is 41.08 mm.

Answer 9.

We take 22.5 as the assumed mean

A = 22.5 and Deviaton $d_i = x_i - A = x_i - 22.5$

Number of saplings	Class marks x_i	Deviations $d_i=x_i-A$ $d_i=x_i-22.5$	Number of societies f_i	f_id_i
10-15	12.5	-10	2	-20
15-20	17.5	-5	7	-35
20-25	22.5	0	9	0
25-30	27.5	5	8	40
30-35	32.5	10	6	60
35-40	37.5	15	4	60
Total	...	....	$\Sigma f_i = 36$	$\Sigma f_id_i = 105$

Here, $\Sigma f_id_i = 105, \Sigma f_i = 36.$

$$\bar{d} = \frac{\Sigma f_id_i}{\Sigma f_i} = \frac{105}{36} = \frac{35}{12} = 2.92$$

$$\bar{x} = A + \bar{d}$$
$$= 22.5 + 2.92$$
$$= 25.42$$

The mean number of saplings planted by the housing societies is 25.42.

Answer 10.

Let A =55, h(Width of the class) = 1

Marks	Class Marks x_i	$D_i=x_i-A$ $=x_i-55$	$u_i = \frac{d_i}{h} = \frac{d_i}{10}$	Number of students f_i	f_iu_i
0-10	5	-50	-5	3	-15
10-20	15	-40	-4	5	-20
20-30	25	-30	-3	7	-21
30-40	35	-20	-2	10	-20
40-50	45	-10	-1	12	-12
50-60	55	0	0	15	0
60-70	65	10	1	12	12
70-80	75	20	2	6	12
80-90	85	30	3	2	6
90-100	95	40	4	8	32
Total	....	....	.....	$\Sigma fi = 80$	$\Sigma f_iu_i = -26$

Here, $\sum f_i u_i = -26, \sum f_i = 80$

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-26}{80} = -0.325$$

$$\bar{x} = A + h\bar{u}$$

$$= 55 + 10(-0.325)$$

$$= 55 - 3.25$$

$$= 51.75$$

A The mean marks scored by a student is 51.75

Ex. 5.2

Answer 1.

Size of farm (in acres)	No. of farms (f)	C.f (less than type)
5-15	7	7
15-25	12	(7+12)=19
25-35	17	(19+17)=36→c.f.
35-45 Median class	25→f	(36+25)=61
45-55	31	(61+31)=92
55-65	5	(92+5)=97
65-75	3	(97+3)=100

Here, N = 100, $\frac{N}{2}=\frac{100}{2}=50$.

The cumulative frequency which is just greater than 50 is 61.

∴ The corresponding class 35 - 45 is the median class.

Hence, L = 35, f = 25, c.f. = 36, h = 10

Where,

L: lower boundary of median class

f: frequency of median class

c.f.: cumulative frequency less than type of the preceding median class.

h: width of median class

$$\text{Median} = L + \left(\frac{N}{2} - c.f\right)\frac{h}{f} \quad \ldots(\text{Formula})$$

$$= 35 + (50\text{-}36) \times \frac{10}{25}$$

$$= 35 + 14 \times \frac{10}{25}$$

$$= 35 + 5.6 = 40.6$$

= The median size of the farm is 40.60 acres.

Answer 2.

Profit (In Rs.)	Number of shops (f)	c.f. (less than type)
500-1000	8	8
1000-1500	18	26
1500-2000	27	53 →c.f
2000-2500 Median class	21→f	74
2500-3000	20	94
3000-3500	18	112
3500-4000	8	120

According to the given data,

$N = 120, \ \frac{N}{2} = \frac{120}{2} = 60.$

The cumulative frequency just greater than 60 is 74.

∴The corresponding class 2000-2500 is the median class.

Hence,

L = 2000, f = 21, c.f. = 53, h = 500

Where,

L: lower boundary of median class

f: frequency of median class

c.f.: cumulative frequency less than type of the preceding median class.

h: width of median class

$$\text{Median} = L + \left(\frac{N}{2} - C.f\right)\frac{h}{f} \quad \ldots(\text{Formula})$$

$$= 2000 + (60 - 53) \times \frac{500}{21}$$

$$= 2000 + 166.67$$

$$= 2166.67 \approx 2167$$

The median profit is Rs. 2167.

Answer 3.

Monthly expenditure (In Rs.)	Number of households (f)	c.f (less than type)
150-225	65	65
225-300	171	236→c.f
300-375 Median class	196→f	432
375-450	75	507
450-525	53	560
525-600	26	586
600 and above	14	600

Here, $N = 600, \frac{N}{2} = 300.$

The cumulative frequency just greater than 300 is 432.
∴ The corresponding class 300-375 is the median class.
∴ L = 300, f = 196, c.f = 236, h = 75

$$\text{Median} = L + \left(\frac{N}{2} - \text{c.f.}\right)\frac{h}{f}$$

$$= 300 + (300 - 236) \times \frac{75}{196}$$

$$= 300 + 64 \times \frac{75}{196}$$

$$= 300 + 24.49 = 324.49$$

The median expenditure on electricity per month is Rs. 324.49

Answer 4.

Age (in years)	Number of patients (f)	c.f (less than type)
10-20	60	60
20-30	42	102 → c.f.
30-40 Median class	55 → f	157
40-50	70	227
50-60	53	280
60-70	20	300

Here, N = 300,

$\therefore \frac{N}{2} = 150.$

The cumulative frequency just greater than 150 is 157.
∴ The corresponding class 30-40 is the median class.
L = 30, f = 55, c.f. = 102, h = 10

$$\text{Median} = L + \left(\frac{N}{2} - \text{c.f.}\right)\frac{h}{f} \quad \ldots(\text{Formula})$$

$$= 30 + (150 - 102) \times \frac{10}{55}$$

$$= 30 + 48 \times \frac{10}{55}$$

$$= 30 + 8.73$$

$$= 38.73$$

The median age of a patient is 38.73 years.

Ex. 5.3

Answer 1.

Weight (in grams)	Number of Packets Frequency (f)
200-201	$12 \rightarrow f_1$
201-202 modal class	$26 \rightarrow f_m$
202-203	$20 \rightarrow f2$
203-204	9
204-205	2
205-206	1

Here, the maximum frequency $f_m = 26$

So, the corresponding class 201-202 is the modal class.

$$\text{Mode} = L + \left[\frac{f_m - f_1}{2f_m - f_1 - f_2}\right]h$$

Where,

L : Lower boundary of the class = 201

f_m: Maximum frequency = 26

f_1: Frequency of pre modal class = 12

f_2: Frequency of post modal class = 20

h: Width of the class = 1

$\therefore$ 'Modal weight' of coffe in a packet is,

$$= 201 + \left[\frac{26 - 12}{2(26) - 12 - 20}\right] \times 1 \ldots \text{(Substituting the values)}$$

$$= 201 + \frac{14}{52 - 32} = 201 + \frac{14}{20}$$

$$= 201 + 0.70 = 201.70$$

The modal weight of coffee in a packet is 201.70 grams.

Answer 2.

Here, the class intervals are of inclusive type.

But, for the calculation of mode, let us first convert the class intervals into exclusive or continuous type.

To do this:

Find the difference between the lower limit and the upper limit of the class intervals, which is = 0.1 here.

Divide 0.1 by 2

$\therefore 0.1 \div 2 = 0.05$

Subtract 0.05 from the lower limit of every class.

Add 0.05 to the upper limit of every class.

After doing this, frequency distribution will be as follows:

Haemoglobin% (mg/100 ml)	Class boundaries	Number of persons Frequency (f)
13.1-14	13.05-14.05	$8 \rightarrow f_1$
14.1-15	14.05-15.05 Modal Class	$12 \rightarrow f_m$
15.1-16	15.05-16.05	$10 \rightarrow f_2$
16.1-17	16.05-17.05	6
17.1-18	17.05-18.05	4

Here, the maximum frequency $f_m = 12$

$\therefore$ The corresponding class boundary 14.05-15.05 is the Modal class.

$L = 14.05, f_m = 12, f_1 = 8, f_2 = 10, h = 1$

∴ Value of Haemoglobin % in the blood of a person is,

$$\text{Mode} = L + \left[\frac{fm - f_1}{2f_m - f_1 - f_2}\right]h$$

(Substituting the values),

$$\text{Mode} = 14.05 + \left[\frac{12 - 8}{2(12) - 8 - 10}\right] \times 1$$

$$= 14.05 + \frac{4}{24 - 18} = 14.05 + \frac{4}{6}$$

$$= 14.05 + 0.67 = 14.72$$

The modal value of Haemoglobin in blood is 14.72 mg/100 ml.

Answer 3.

Bowling speed (km/hr)	Number of players Frequency (f)
85-100	$9 \rightarrow f_1$
100-115 Modal class	$11 \rightarrow f_m$
115-130	$8 \rightarrow f_2$
130-145	5

Here, the maximum frequency $f_m = 11$.

∴ The corresponding class 100-115 is the Modal class.

$L = 100, f_m = 11, f_1 = 9, f_2 = 8, h = 15$

∴The modal bowling speed of a player is,

$$\text{Mode} = L + \left[\frac{f_m - f_1}{2f_m - f_1 - f_2}\right]h$$

$$= 100 + \left[\frac{11-9}{2(11)-9-8}\right]\times 15 \ldots \text{(Substituting the values)}$$

$$= 100 + \frac{2}{22-17}\times 15$$

$$= 100 + \frac{2}{5}\times 15$$

$$= 100 + 6 = 106$$

The modal bowling speed of a player is 106 km/hr

Answer 4.

Here, the class intervals are of inclusive type.

But, for the calculation of mode let us first convert the class intervals into exclusive or continuous type.

After doing this, the frequency distribution will be as follows:

Body weight (in grams)	Class boundaries	Number of fish Frequency (f)
15-15.9	14.95-15.95	2
16-16.9	15.95-16.95	$4 \rightarrow f_1$
17-17.9	16.95-17.95 Modal class	$8 \rightarrow f_m$
18-18.9	17.95-18.95	$6 \rightarrow f_2$
19-19.9	18.95-19.95	6
20-20.9	19.95-20.95	4

Here the maximum frequency f_m= 8, So, the corresponding class boundary 16.95-17.95 is the modal class.

L = 16.95, f_m = 8,f_1= 4,f_2 = 6, h = 1.

$$\text{Mode} = L + \left[\frac{f_m - f_1}{2f_m - f_1 - f_2}\right]h$$

∴ Substituting the values,

$$= 16.95 + \left[\frac{8-4}{2(8)-4-6}\right]\times 1$$

$$= 16.95 + \frac{4}{16-10}$$

$$= 16.95 + \frac{4}{6}$$

$$= 16.95 + 0.67 = 17.62$$

The modal body weight of a fish in the pond is 17.62 g.

Ex. 5.4

Answer 1.

Let the value of the median of the data set be x.
Then the relationship between the mean, median and mode of the data set is such that,
Mean - Mode = 3(Mean-Median)
$\therefore$ 54.6 - 54 = 3(54.6 - x)

$$\therefore \frac{0.6}{3} = 54.6 - x$$

$\therefore$ x = 54.6 - 0.2 = 54.4
The value of the median is 54.4.

Answer 2.

Let the mean be x.
Then Mean - Mode = 3(Mean - Median)
$\therefore$ x - 95.5 = 3(x - 95.75)
$\therefore$ x - 95.5 = 3x - 287.25
$\therefore$ 3x - x = 287.25 - 95.5
$\therefore$ 2x = 191.75
$\therefore$ x = 95.875
The value of the mean is 95.875.

Answer 3.

Let the mode be x.
Then Mean - Mode = 3(Mean - Median)
$\therefore$ 101 - x = 3(101 - 100)
$\therefore$ 101 - x = 3
$\therefore$ x = 101 - 3
$\therefore$ x = 98
The value of the mode is 98.