

## 9. Values of Trigonometric Functions at Multiples and Submultiple of an Angles

### Exercise 9.1

#### 1. Question

Prove the following identities:

$$\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \tan x$$

**Answer**

**To prove:**  $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \tan x$

**Proof:**

Take LHS:

$$\text{Let } I = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$$

**Identities used:**

$$\cos 2x = 1 - 2 \sin^2 x$$

$$= 2 \cos^2 x - 1$$

Therefore,

$$= \sqrt{\frac{1 - (1 - 2 \sin^2 x)}{1 + (2 \cos^2 x - 1)}}$$

$$= \sqrt{\frac{1 - 1 + 2 \sin^2 x}{1 + 2 \cos^2 x - 1}}$$

$$= \sqrt{\frac{2 \sin^2 x}{2 \cos^2 x}}$$

$$= \sqrt{\frac{\sin^2 x}{\cos^2 x}}$$

$$= \sqrt{\tan^2 x}$$

$$\left\{ \because \frac{\sin x}{\cos x} = \tan x \right\}$$

$$= \tan x$$

$$= \text{RHS}$$

**Hence Proved**

#### 2. Question

Prove the following identities:

$$\frac{\sin 2x}{1 - \cos 2x} = \cot x$$

**Answer**

**To prove:**  $\frac{\sin 2x}{1 - \cos 2x} = \cot x$

Proof:

Take LHS:

$$\frac{\sin 2x}{1 - \cos 2x}$$

**Identities used:**

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

Therefore,

$$= \frac{2 \sin x \cos x}{1 - (1 - 2 \sin^2 x)}$$

$$= \frac{2 \sin x \cos x}{1 - 1 + 2 \sin^2 x}$$

$$= \frac{2 \sin x \cos x}{2 \sin^2 x}$$

$$= \frac{\cos x}{\sin x}$$

$$\left\{ \because \frac{\cos x}{\sin x} = \cot x \right\}$$

$$= \cot x$$

$$= \text{RHS}$$

**Hence Proved**

### 3. Question

Prove the following identities:

$$\frac{\sin 2x}{1 + \cos 2x} = \tan x$$

**Answer**

**To prove:**  $\frac{\sin 2x}{1 + \cos 2x} = \tan x$

Proof:

Take LHS:

$$\frac{\sin 2x}{1 + \cos 2x}$$

**Identities used:**

$$\cos 2x = 2 \cos^2 x - 1$$

$$\sin 2x = 2 \sin x \cos x$$

Therefore,

$$= \frac{2 \sin x \cos x}{1 + (2 \cos^2 x - 1)}$$

$$= \frac{2 \sin x \cos x}{1 + 2 \cos^2 x - 1}$$

$$= \frac{2 \sin x \cos x}{2 \cos^2 x}$$

$$= \frac{\sin x}{\cos x}$$

$$\left\{ \because \frac{\sin x}{\cos x} = \tan x \right\}$$

$$= \tan x$$

$$= \text{RHS}$$

**Hence Proved**

#### 4. Question

Prove the following identities:

$$\sqrt{2 + \sqrt{2 + 2 \cos 4x}} = 2 \cos x, 0 < x < \frac{\pi}{4}$$

**Answer**

$$\text{To prove: } \sqrt{2 + \sqrt{2 + 2 \cos 4x}} = 2 \cos x$$

Proof:

Take LHS:

$$\sqrt{2 + \sqrt{2 + 2 \cos 4x}}$$

$$= \sqrt{2 + \sqrt{2 + 2(2 \cos^2 2x - 1)}}$$

$$\{\because \cos 2x = 2 \cos^2 x - 1 \Rightarrow \cos 4x = 2 \cos^2 2x - 1\}$$

$$= \sqrt{2 + \sqrt{2 + 4 \cos^2 2x - 2}}$$

$$= \sqrt{2 + \sqrt{4 \cos^2 2x}}$$

$$= \sqrt{2 + 2 \cos 2x}$$

$$= \sqrt{2 + 2(2 \cos^2 x - 1)}$$

$$\{\because \cos 2x = 2 \cos^2 x - 1\}$$

$$= \sqrt{2 + 4 \cos^2 x - 2}$$

$$= \sqrt{4 \cos^2 x}$$

$$= 2 \cos x$$

$$= \text{RHS}$$

**Hence Proved**

### 5. Question

Prove the following identities:

$$\frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$$

**Answer**

**To prove:**  $\frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x} = \tan x$

Proof:

Take LHS

$$\frac{1 - \cos 2x + \sin 2x}{1 + \cos 2x + \sin 2x}$$

**Identities used:**

$$\cos 2x = 2 \cos^2 x - 1$$

$$= 1 - 2 \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

Therefore,

$$= \frac{1 - (1 - 2 \sin^2 x) + 2 \sin x \cos x}{1 + (2 \cos^2 x - 1) + 2 \sin x \cos x}$$

$$= \frac{1 - 1 + 2 \sin^2 x + 2 \sin x \cos x}{1 + 2 \cos^2 x - 1 + 2 \sin x \cos x}$$

$$= \frac{2 \sin^2 x + 2 \sin x \cos x}{2 \cos^2 x + 2 \sin x \cos x}$$

$$= \frac{2 \sin x (\sin x + \cos x)}{2 \cos x (\cos x + \sin x)}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$\left\{ \because \frac{\sin x}{\cos x} = \tan x \right\}$$

$$= \text{RHS}$$

**Hence Proved**

### 6. Question

Prove the following identities:

$$\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$$

**Answer**

**To prove:**  $\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$

Proof:

Take LHS:

$$\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x}$$

**Identities used:**

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

Therefore,

$$= \frac{\sin x + 2 \sin x \cos x}{1 + \cos x + (2 \cos^2 x - 1)}$$

$$= \frac{\sin x + 2 \sin x \cos x}{1 + \cos x + 2 \cos^2 x - 1}$$

$$= \frac{\sin x + 2 \sin x \cos x}{\cos x + 2 \cos^2 x}$$

$$= \frac{\sin x (1 + 2 \cos x)}{\cos x (1 + 2 \cos x)}$$

$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$\left\{ \because \frac{\sin x}{\cos x} = \tan x \right\}$$

$$= \text{RHS}$$

**Hence Proved**

## 7. Question

Prove the following identities:

$$\frac{\cos 2x}{1 + \sin 2x} = \tan \left( \frac{\pi}{4} - x \right)$$

**Answer**

$$\text{To prove: } \frac{\cos 2x}{1 + \sin 2x} = \tan \left( \frac{\pi}{4} - x \right)$$

Proof:

Take LHS:

$$\frac{\cos 2x}{1 + \sin 2x}$$

**Identities used:**

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin 2x = 2 \sin x \cos x$$

Therefore,

$$= \frac{\cos^2 x - \sin^2 x}{1 + 2 \sin x \cos x}$$

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{\sin^2 x + \cos^2 x + 2 \sin x \cos x}$$

$$\{\because a^2 - b^2 = (a - b)(a + b) \text{ \& } \sin^2 x + \cos^2 x = 1\}$$

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)^2}$$

$$\{\because a^2 + b^2 + 2ab = (a + b)^2\}$$

$$= \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\sin x + \cos x)(\sin x + \cos x)}$$

$$= \frac{(\cos x - \sin x)}{(\sin x + \cos x)}$$

Multiplying numerator and denominator by  $\frac{1}{\sqrt{2}}$ :

$$= \frac{\frac{1}{\sqrt{2}}(\cos x - \sin x)}{\frac{1}{\sqrt{2}}(\sin x + \cos x)}$$

$$= \frac{\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}{\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)}$$

$$= \frac{\left(\sin \frac{\pi}{4} \cos x - \cos \frac{\pi}{4} \sin x\right)}{\left(\sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x\right)}$$

$$\left\{\because \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}\right\}$$

$$= \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)}$$

$$\{\because \sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B\}$$

$$= \tan\left(\frac{\pi}{4} - x\right)$$

$$\left\{\because \frac{\sin x}{\cos x} = \tan x\right\}$$

$$= \text{RHS}$$

**Hence Proved**

## 8. Question

Prove the following identities:

$$\frac{\cos x}{1 - \sin x} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

**Answer**

$$\text{To prove: } \frac{\cos x}{1 - \sin x} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

Proof:

Take LHS:

$$\frac{\cos x}{1 - \sin x}$$

**Identities used:**

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\Rightarrow \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\Rightarrow \sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

Therefore,

$$\begin{aligned} &= \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{1 - 2 \sin \frac{x}{2} \cos \frac{x}{2}} \\ &= \frac{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}} \end{aligned}$$

$$\{\because a^2 - b^2 = (a - b)(a + b) \text{ \& } \sin^2 x + \cos^2 x = 1\}$$

$$= \frac{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)}{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2}$$

$$\{\because a^2 + b^2 + 2ab = (a + b)^2\}$$

$$\begin{aligned} &= \frac{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)}{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)} \\ &= \frac{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)}{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)} \\ &= \frac{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)}{\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)} \end{aligned}$$

Multiplying numerator and denominator by  $\frac{1}{\sqrt{2}}$ :

$$\begin{aligned} &= \frac{\frac{1}{\sqrt{2}}\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)}{\frac{1}{\sqrt{2}}\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)} \\ &= \frac{\left(\frac{1}{\sqrt{2}} \cos \frac{x}{2} + \frac{1}{\sqrt{2}} \sin \frac{x}{2}\right)}{\left(\frac{1}{\sqrt{2}} \sin \frac{x}{2} - \frac{1}{\sqrt{2}} \cos \frac{x}{2}\right)} \end{aligned}$$

$$= \frac{\left(\sin \frac{\pi}{4} \cos \frac{x}{2} + \cos \frac{\pi}{4} \sin \frac{x}{2}\right)}{\left(\sin \frac{\pi}{4} \sin \frac{x}{2} - \cos \frac{\pi}{4} \cos \frac{x}{2}\right)}$$

$$\left\{ \because \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}} \right\}$$

$$= \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)}$$

$$\{ \because \sin(A - B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B \}$$

$$= \tan\left(\frac{\pi}{4} - x\right)$$

$$\left\{ \because \frac{\sin x}{\cos x} = \tan x \right\}$$

$$= \text{RHS}$$

**Hence Proved**

## 9. Question

Prove the following identities:

$$\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} = 2$$

**Answer**

$$\text{To prove: } \cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8} = 2$$

Proof:

Take LHS:

$$\cos^2 \frac{\pi}{8} + \cos^2 \frac{3\pi}{8} + \cos^2 \frac{5\pi}{8} + \cos^2 \frac{7\pi}{8}$$

**Identities used:**

$$\cos 2x = 2 \cos^2 x - 1$$

$$\Rightarrow 2 \cos^2 x = 1 + \cos 2x$$

$$\Rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}$$

Therefore,

$$= \frac{1 + \cos \frac{2\pi}{8}}{2} + \frac{1 + \cos \frac{6\pi}{8}}{2} + \frac{1 + \cos \frac{10\pi}{8}}{2} + \frac{1 + \cos \frac{14\pi}{8}}{2}$$

$$= \frac{1 + \cos \frac{2\pi}{8}}{2} + \frac{1 + \cos\left(\pi - \frac{2\pi}{8}\right)}{2} + \frac{1 + \cos\left(\pi + \frac{2\pi}{8}\right)}{2} + \frac{1 + \cos\left(2\pi - \frac{2\pi}{8}\right)}{2}$$

$$\left\{ \because \pi - \frac{2\pi}{8} = \frac{6\pi}{8}; \pi + \frac{2\pi}{8} = \frac{10\pi}{8}; 2\pi - \frac{2\pi}{8} = \frac{14\pi}{8} \right\}$$



$$= \frac{1 + \cos \frac{2\pi}{8}}{2} + \frac{1 - \cos \frac{2\pi}{8}}{2} + \frac{1 - \cos \frac{2\pi}{8}}{2} + \frac{1 + \cos \frac{2\pi}{8}}{2}$$

$$\{\because \cos(\pi - \theta) = -\cos \theta, \cos(\pi + \theta) = -\cos \theta \text{ \& } \cos(2\pi - \theta) = \cos \theta\}$$

$$= 2 \times \frac{1 + \cos \frac{2\pi}{8}}{2} + 2 \times \frac{1 - \cos \frac{2\pi}{8}}{2}$$

$$= 1 + \cos \frac{2\pi}{8} + 1 - \cos \frac{2\pi}{8}$$

$$= 2$$

$$= \text{RHS}$$

**Hence Proved**

## 10. Question

Prove the following identities:

$$\sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8} = 2$$

**Answer**

$$\text{To prove: } \sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8} = 2$$

Proof:

Take LHS:

$$\sin^2 \frac{\pi}{8} + \sin^2 \frac{3\pi}{8} + \sin^2 \frac{5\pi}{8} + \sin^2 \frac{7\pi}{8}$$

**Identities used:**

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\Rightarrow 2 \sin^2 x = 1 - \cos 2x$$

$$\Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

Therefore,

$$= \frac{1 - \cos \frac{2\pi}{8}}{2} + \frac{1 - \cos \frac{6\pi}{8}}{2} + \frac{1 - \cos \frac{10\pi}{8}}{2} + \frac{1 - \cos \frac{14\pi}{8}}{2}$$

$$= \frac{1 - \cos \frac{2\pi}{8}}{2} + \frac{1 - \cos \left( \pi - \frac{2\pi}{8} \right)}{2} + \frac{1 - \cos \left( \pi + \frac{2\pi}{8} \right)}{2} + \frac{1 - \cos \left( 2\pi - \frac{2\pi}{8} \right)}{2}$$

$$\left\{ \because \pi - \frac{2\pi}{8} = \frac{6\pi}{8}; \pi + \frac{2\pi}{8} = \frac{10\pi}{8}; 2\pi - \frac{2\pi}{8} = \frac{14\pi}{8} \right\}$$

$$= \frac{1 - \cos \frac{2\pi}{8}}{2} + \frac{1 - \left( -\cos \frac{2\pi}{8} \right)}{2} + \frac{1 - \left( -\cos \frac{2\pi}{8} \right)}{2} + \frac{1 - \cos \frac{2\pi}{8}}{2}$$

$$\{\because \cos(\pi - \theta) = -\cos \theta,$$

$$\cos(\pi + \theta) = -\cos \theta \text{ \& }$$

$$\cos(2\pi - \theta) = \cos \theta\}$$

$$\begin{aligned}
&= \frac{1 - \cos \frac{2\pi}{8}}{2} + \frac{1 + \cos \frac{2\pi}{8}}{2} + \frac{1 + \cos \frac{2\pi}{8}}{2} + \frac{1 - \cos \frac{2\pi}{8}}{2} \\
&= 2 \times \frac{1 - \cos \frac{2\pi}{8}}{2} + 2 \times \frac{1 + \cos \frac{2\pi}{8}}{2} \\
&= 1 - \cos \frac{2\pi}{8} + 1 + \cos \frac{2\pi}{8} \\
&= 2 \\
&= \text{RHS}
\end{aligned}$$

**Hence Proved**

### 11. Question

Prove the following identities:

$$(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = 4 \cos^2 \left( \frac{\alpha - \beta}{2} \right)$$

**Answer**

$$\text{To prove: } (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = 4 \cos^2 \frac{\alpha - \beta}{2}$$

Proof:

Take LHS:

$$\begin{aligned}
&(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\
&= \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta + \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta \\
&= 2 + 2 \cos \alpha \cos \beta + 2 \sin \alpha \sin \beta \\
&= 2(1 + \cos \alpha \cos \beta + \sin \alpha \sin \beta) \\
&= 2(1 + \cos(\alpha - \beta)) \\
&\{\because \cos(A - B) = \cos A \cos B + \sin A \sin B\} \\
&= 2 \left( 1 + 2 \cos^2 \frac{\alpha - \beta}{2} - 1 \right) \\
&\{\because \cos 2x = 2 \cos^2 x - 1\} \\
&= 2 \left( 2 \cos^2 \frac{\alpha - \beta}{2} \right) \\
&= 4 \cos^2 \frac{\alpha - \beta}{2} \\
&= \text{RHS}
\end{aligned}$$

**Hence Proved**

### 12. Question

Prove the following identities:

$$\sin^2 \left( \frac{\pi}{8} + \frac{x}{2} \right) - \sin^2 \left( \frac{\pi}{8} - \frac{x}{2} \right) = \frac{1}{\sqrt{2}} \sin x$$

**Answer**

**To prove:**  $\sin^2\left(\frac{\pi}{8} + \frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{x}{2}\right) = \frac{1}{\sqrt{2}} \sin x$

Proof:

Take LHS:

$$\sin^2\left(\frac{\pi}{8} + \frac{x}{2}\right) - \sin^2\left(\frac{\pi}{8} - \frac{x}{2}\right)$$

**Identities used:**

$$\sin^2 A - \sin^2 B = \sin(A + B) \sin(A - B)$$

Therefore,

$$= \sin\left(\frac{\pi}{8} + \frac{x}{2} + \frac{\pi}{8} - \frac{x}{2}\right) \sin\left(\frac{\pi}{8} + \frac{x}{2} - \left(\frac{\pi}{8} - \frac{x}{2}\right)\right)$$

$$= \sin\left(\frac{\pi}{8} + \frac{\pi}{8}\right) \sin\left(\frac{\pi}{8} + \frac{x}{2} - \frac{\pi}{8} + \frac{x}{2}\right)$$

$$= \sin \frac{\pi}{4} \sin x$$

$$= \frac{1}{\sqrt{2}} \sin x$$

= RHS

**Hence Proved**

### 13. Question

Prove the following identities:

$$1 + \cos^2 2x = 2(\cos^4 x + \sin^4 x)$$

**Answer**

**To prove:**  $1 + \cos^2 2x = 2(\cos^4 x + \sin^4 x)$

Proof:

Take LHS:

$$1 + \cos^2 2x$$

$$= [(\cos^2 x + \sin^2 x)]^2 + [(\cos^2 x - \sin^2 x)]^2$$

$$\{\because \cos 2x = \cos^2 x - \sin^2 x \text{ \& } \cos^2 x + \sin^2 x = 1\}$$

$$= (\cos^4 x + \sin^4 x + 2 \cos^2 x \sin^2 x) + (\cos^4 x + \sin^4 x - 2 \cos^2 x \sin^2 x)$$

$$= \cos^4 x + \sin^4 x + \cos^4 x + \sin^4 x$$

$$= 2 \cos^4 x + 2 \sin^4 x$$

$$= 2(\cos^4 x + \sin^4 x)$$

= RHS

### 14. Question

Prove the following identities:

$$\cos^3 2x + 3 \cos 2x = 4(\cos^6 x - \sin^6 x)$$

**Answer**

**To prove:**  $\cos^3 2x + 3 \cos 2x = 4(\cos^6 x - \sin^6 x)$

Proof:

Take RHS:

$$\begin{aligned} & 4(\cos^6 x - \sin^6 x) \\ &= 4((\cos^2 x)^3 - (\sin^2 x)^3) \\ &= 4(\cos^2 x - \sin^2 x)(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x) \\ &= 4(\cos^2 x - \sin^2 x)(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x) \\ &\{\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)\} \\ &= 4 \cos 2x(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x + \cos^2 x \sin^2 x - \cos^2 x \sin^2 x) \\ &\{\because \cos 2x = \cos^2 x - \sin^2 x\} \\ &= 4 \cos 2x(\cos^4 x + \sin^4 x + 2 \cos^2 x \sin^2 x - \cos^2 x \sin^2 x) \\ &= 4 \cos 2x\{(\cos^2 x)^2 + (\sin^2 x)^2 + 2 \cos^2 x \sin^2 x - \cos^2 x \sin^2 x\} \\ &\{\because a^2 + b^2 + 2ab = (a + b)^2\} \\ &= 4 \cos 2x\{(\cos^2 x + \sin^2 x)^2 - \cos^2 x \sin^2 x\} \\ &\{\because \cos^2 x + \sin^2 x = 1\} \\ &= 4 \cos 2x\{(1)^2 - \frac{1}{4}(4 \cos^2 x \sin^2 x)\} \\ &= 4 \cos 2x\{(1)^2 - \frac{1}{4}(2 \cos x \sin x)^2\} \\ &\{\because \sin 2x = 2 \sin x \cos x\} = 4 \cos 2x\left\{(1)^2 - \frac{1}{4}(\sin 2x)^2\right\} \\ &= 4 \cos 2x\left(1 - \frac{1}{4}\sin^2 2x\right) \\ &\{\because \sin^2 x = 1 - \cos^2 x\} \\ &= 4 \cos 2x\left(1 - \frac{1}{4}(1 - \cos^2 2x)\right) \\ &= 4 \cos 2x\left(1 - \frac{1}{4} + \frac{1}{4}\cos^2 2x\right) \\ &= 4 \cos 2x\left(\frac{3}{4} + \frac{1}{4}\cos^2 2x\right) \\ &= 4\left(\frac{3}{4}\cos 2x + \frac{1}{4}\cos^3 2x\right) \\ &= 3 \cos 2x + \cos^3 2x \\ &= \text{LHS} \end{aligned}$$

**Hence Proved**

### 15. Question

Prove the following identities:

$$(\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0$$

**Answer****To prove:**  $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x = 0$ 

Proof:

Take LHS:

$$(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x$$

$$= (\sin 3x)(\sin x) + \sin^2 x + (\cos 3x)(\cos x) - \cos^2 x$$

$$= [(\sin 3x)(\sin x) + (\cos 3x)(\cos x)] + (\sin^2 x - \cos^2 x)$$

$$= [(\sin 3x)(\sin x) + (\cos 3x)(\cos x)] - (\cos^2 x - \sin^2 x)$$

$$= \cos(3x - x) - \cos 2x$$

$$\{\because \cos 2x = \cos^2 x - \sin^2 x \text{ \&}$$

$$\cos A \cos B + \sin A \sin B = \cos(A - B)\}$$

$$= \cos 2x - \cos 2x$$

$$= 0$$

$$= \text{RHS}$$

**Hence Proved****16. Question**

Prove the following identities:

$$\cos^2\left(\frac{\pi}{4} - x\right) - \sin^2\left(\frac{\pi}{4} - x\right) = \sin 2x$$

**Answer**

$$\textbf{To prove: } \cos^2\left(\frac{\pi}{4} - x\right) - \sin^2\left(\frac{\pi}{4} - x\right) = \sin 2x$$

Proof:

Take LHS:

$$\cos^2\left(\frac{\pi}{4} - x\right) - \sin^2\left(\frac{\pi}{4} - x\right)$$

**Identities used:**

$$\cos^2 A - \sin^2 A = \cos 2A$$

Therefore,

$$= \cos 2\left(\frac{\pi}{4} - x\right)$$

$$= \cos\left(\frac{\pi}{2} - 2x\right)$$

$$= \sin 2x$$

$$\left\{\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta\right\}$$

$$= \text{RHS}$$

**Hence Proved****17. Question**

Prove the following identities:

$$\cos 4x = 1 - 8 \cos^2 x + 8 \cos^4 x$$

**Answer**

**To prove:**  $\cos 4x = 1 - 8 \cos^2 x + 8 \cos^4 x$

Proof:

Take LHS:

$$\cos 4x$$

**Identities used:**

$$\cos 2x = 2 \cos^2 x - 1$$

Therefore,

$$= 2 \cos^2 2x - 1$$

$$= 2(2 \cos^2 2x - 1)^2 - 1$$

$$= 2\{(2 \cos^2 2x)^2 + 1^2 - 2 \times 2 \cos^2 x\} - 1$$

$$= 2(4 \cos^4 2x + 1 - 4 \cos^2 x) - 1$$

$$= 8 \cos^4 2x + 2 - 8 \cos^2 x - 1$$

$$= 8 \cos^4 2x + 1 - 8 \cos^2 x$$

$$= \text{RHS}$$

**Hence Proved**

### 18. Question

Prove the following identities:

$$\sin 4x = 4 \sin x \cos^3 x - 4 \cos x \sin^3 x$$

**Answer**

**To prove:**  $\sin 4x = 4 \sin x \cos^3 x - 4 \cos x \sin^3 x$

Proof:

Take LHS:

$$\sin 4x$$

**Identities used:**

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

Therefore,

$$= 2 \sin 2x \cos 2x$$

$$= 2 (2 \sin x \cos x) (\cos^2 x - \sin^2 x)$$

$$= 4 \sin x \cos x (\cos^2 x - \sin^2 x)$$

$$= 4 \sin x \cos^3 x - 4 \sin^3 x \cos x$$

$$= \text{RHS}$$

**Hence Proved**

### 19. Question

Prove the following identities:

$$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13$$

#### Answer

**To prove:**  $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13$

Proof:

Take LHS:

$$3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$$

#### Identities used:

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

Therefore,

$$= 3\{(\sin x - \cos x)^2\}^2 + 6\{(\sin x)^2 + (\cos x)^2 + 2\sin x \cos x\} + 4\{(\sin^2 x)^3 + (\cos^2 x)^3\}$$

$$= 3\{(\sin x)^2 + (\cos x)^2 - 2\sin x \cos x\}^2 + 6(\sin^2 x + \cos^2 x + 2\sin x \cos x) + 4\{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)\}$$

$$= 3(1 - 2\sin x \cos x)^2 + 6(1 + 2\sin x \cos x) + 4\{(1)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x)\}$$

$$\{\because \sin^2 x + \cos^2 x = 1\}$$

$$= 3\{1^2 + (2\sin x \cos x)^2 - 4\sin x \cos x\} + 6(1 + 2\sin x \cos x) + 4\{(\sin^2 x)^2 + (\cos^2 x)^2 + 2\sin^2 x \cos^2 x - 3\sin^2 x \cos^2 x\}$$

$$= 3\{1 + 4\sin^2 x \cos^2 x - 4\sin x \cos x\} + 6(1 + 2\sin x \cos x) + 4\{(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x\}$$

$$= 3 + 12\sin^2 x \cos^2 x - 12\sin x \cos x + 6 + 12\sin x \cos x + 4\{1^2 - 3\sin^2 x \cos^2 x\}$$

$$= 9 + 12\sin^2 x \cos^2 x + 4(1 - 3\sin^2 x \cos^2 x)$$

$$= 9 + 12\sin^2 x \cos^2 x + 4 - 12\sin^2 x \cos^2 x$$

$$= 13$$

$$= \text{RHS}$$

**Hence Proved**

### 20. Question

Prove the following identities:

$$2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0$$

#### Answer

**To prove:**  $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1 = 0$

**Proof:**

Take LHS:

$$2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1$$

**Identities used:**

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

Therefore,

$$= 2\{(\sin^2 x)^3 + (\cos^2 x)^3\} - 3\{(\sin^2 x)^2 + (\cos^2 x)^2\} + 1$$

$$= 2\{(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x \cos^2 x) - 3\{(\sin^2 x)^2 + (\cos^2 x)^2 + 2\sin^2 x \cos^2 x - 2\sin^2 x \cos^2 x\} + 1$$

$$= 2\{(1)(\sin^4 x + \cos^4 x + 2\sin^2 x \cos^2 x - 3\sin^2 x \cos^2 x) - 3\{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x\} + 1$$

$$\{\because \sin^2 x + \cos^2 x = 1\}$$

$$= 2\{(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x\} - 3\{(1)^2 - 2\sin^2 x \cos^2 x\} + 1$$

$$= 2\{(1)^2 - 3\sin^2 x \cos^2 x\} - 3(1 - 2\sin^2 x \cos^2 x) + 1$$

$$= 2(1 - 3\sin^2 x \cos^2 x) - 3 + 6\sin^2 x \cos^2 x + 1$$

$$= 2 - 6\sin^2 x \cos^2 x - 2 + 6\sin^2 x \cos^2 x$$

$$= 0$$

$$= \text{RHS}$$

**Hence Proved**

## 21. Question

Prove the following identities:

$$\cos^6 x - \sin^6 x = \cos 2x \left(1 - \frac{1}{4}\sin^2 2x\right)$$

**Answer**

$$\text{To prove: } \cos^6 x - \sin^6 x = \cos 2x \left(1 - \frac{1}{4}\sin^2 2x\right)$$

Proof:

Take LHS:

$$\cos^6 x - \sin^6 x$$

**Identities used:**

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$$

Therefore,

$$= (\cos^2 x)^3 - (\sin^2 x)^3$$

$$= (\cos^2 x - \sin^2 x)(\cos^4 x + \sin^4 x + \cos^2 x \sin^2 x)$$

$$\{\because \cos 2x = \cos^2 x - \sin^2 x\}$$

$$= \cos 2x((\cos^2 x)^2 + (\sin^2 x)^2 + 2\cos^2 x \sin^2 x - \cos^2 x \sin^2 x)$$

$$= \cos 2x\left((\cos^2 x + \sin^2 x)^2 - \frac{1}{4} \times 4\cos^2 x \sin^2 x\right)$$

$$\{\because \sin^2 x + \cos^2 x = 1\}$$



$$= \cos 2x \left( (1)^2 - \frac{1}{4} \times (2 \cos x \sin x)^2 \right)$$

$$\{\because \sin 2x = 2 \sin x \cos x\}$$

$$= \cos 2x \left( 1 - \frac{1}{4} \times (\sin 2x)^2 \right)$$

$$= \cos 2x \left( 1 - \frac{1}{4} \sin^2 2x \right)$$

$$= \text{RHS}$$

**Hence Proved**

## 22. Question

Prove the following identities:

$$\tan \left( \frac{\pi}{4} + x \right) + \tan \left( \frac{\pi}{4} - x \right) = 2 \sec 2x$$

**Answer**

$$\text{To prove: } \tan \left( \frac{\pi}{4} + x \right) + \tan \left( \frac{\pi}{4} - x \right) = 2 \sec 2x$$

Proof:

Take LHS:

$$\tan \left( \frac{\pi}{4} + x \right) + \tan \left( \frac{\pi}{4} - x \right)$$

**Identities used:**

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Therefore,

$$= \frac{\tan \frac{\pi}{4} + \tan x}{1 - \tan \frac{\pi}{4} \tan x} + \frac{\tan \frac{\pi}{4} - \tan x}{1 + \tan \frac{\pi}{4} \tan x}$$

$$\left\{ \because \tan \frac{\pi}{4} = 1 \right\}$$

$$= \frac{1 + \tan x}{1 - \tan x} + \frac{1 - \tan x}{1 + \tan x}$$

$$= \frac{(1 + \tan x)^2 + (1 - \tan x)^2}{(1 - \tan x)(1 + \tan x)}$$

$$\{\because (a - b)(a + b) = a^2 - b^2;$$

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ \&}$$

$$(a - b)^2 = a^2 + b^2 - 2ab\}$$

$$= \frac{1^2 + \tan^2 x + 2 \tan x + 1^2 + \tan^2 x - 2 \tan x}{1^2 - \tan^2 x}$$

$$= \frac{1 + \tan^2 x + 1 + \tan^2 x}{1 - \tan^2 x}$$

$$= \frac{2(1 + \tan^2 x)}{1 - \tan^2 x}$$

$$\left\{ \because \tan x = \frac{\sin x}{\cos x} \right\}$$

$$= \frac{2 \left( 1 + \left( \frac{\sin x}{\cos x} \right)^2 \right)}{1 - \left( \frac{\sin x}{\cos x} \right)^2}$$

$$= \frac{2 \left( 1 + \frac{\sin^2 x}{\cos^2 x} \right)}{1 - \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{2 \left( \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \right)}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$$

$$\{ \because \cos^2 x + \sin^2 x = 1 \text{ \& } \cos 2x = \cos^2 x - \sin^2 x \}$$

$$= \frac{2 \left( \frac{1}{\cos^2 x} \right)}{\frac{\cos 2x}{\cos^2 x}}$$

$$= \frac{2}{\cos 2x}$$

$$= 2 \sec 2x$$

$$\left\{ \because \frac{1}{\cos 2x} = \sec 2x \right\}$$

$$= \text{RHS}$$

**Hence Proved**

### 23. Question

Prove the following identities:

$$\cot^2 x - \tan^2 x = 4 \cot 2x \operatorname{cosec} 2x$$

**Answer**

$$\textbf{To prove: } \cot^2 x - \tan^2 x = 4 \cot 2x \operatorname{cosec} 2x$$

Proof:

Take LHS:

$$\cot^2 x - \tan^2 x$$

**Identities used:**

$$a^2 - b^2 = (a - b)(a + b)$$

Therefore,

$$= (\cot x - \tan x)(\cot x + \tan x)$$

$$\left\{ \because \tan x = \frac{1}{\cot x} \right\}$$

$$= \left( \cot x - \frac{1}{\cot x} \right) \left( \cot x + \frac{1}{\cot x} \right)$$

$$= \left( \frac{\cot^2 x - 1}{\cot x} \right) \left( \frac{\cot^2 x + 1}{\cot x} \right)$$

$$= 2 \left( \frac{\cot^2 x - 1}{2 \cot x} \right) \left( \frac{\cot^2 x + 1}{\cot x} \right)$$

$$\{\because \cot^2 x + 1 = \operatorname{cosec}^2 x\}$$

$$= 2 \left( \frac{\cot^2 x - 1}{2 \cot x} \right) \left( \frac{\operatorname{cosec}^2 x}{\cot x} \right)$$

$$= 2(\cot 2x) \left( \frac{1}{\frac{\sin^2 x}{\cos x}} \right)$$

$$\left\{ \begin{array}{l} \because \cot 2x = \frac{\cot^2 x - 1}{2 \cot x}; \\ \operatorname{cosec} x = \frac{1}{\sin x}; \\ \cot x = \frac{\cos x}{\sin x} \end{array} \right\}$$

$$= 2(\cot 2x) \left( \frac{1}{\sin x \cos x} \right)$$

$$= 2(\cot 2x) \left( \frac{2}{2 \cos x \sin x} \right)$$

$$= \frac{4 \cot 2x}{\sin 2x}$$

$$\{\because \sin 2x = 2 \sin x \cos x\}$$

$$= 4 \cot 2x \operatorname{cosec} 2x$$

$$\left\{ \because \operatorname{cosec} x = \frac{1}{\sin x} \right\}$$

$$= \text{RHS}$$

**Hence Proved**

## 24. Question

Prove the following identities:

$$\cos 4x - \cos 4\alpha = 8(\cos x - \cos \alpha)(\cos x + \cos \alpha)(\cos x - \sin \alpha)(\cos x + \sin \alpha)$$

**Answer**

**To prove:**  $\cos 4x - \cos 4\alpha = 8(\cos x - \cos \alpha)(\cos x + \cos \alpha)(\cos x - \sin \alpha)(\cos x + \sin \alpha)$

Proof:

Take LHS:

$$\cos 4x - \cos 4\alpha$$

$$\{\because \cos 2\theta = 2 \cos^2 \theta - 1\}$$

$$= 2 \cos^2 2x - 1 - (2 \cos^2 2\alpha - 1)$$

$$= 2 \cos^2 2x - 1 - 2 \cos^2 2\alpha + 1$$

$$= 2 \cos^2 2x - 2 \cos^2 2\alpha$$

$$= 2(\cos^2 2x - \cos^2 2\alpha)$$

$$\begin{aligned}
& \{\because (a - b)(a + b) = a^2 - b^2\} \\
& = 2(\cos 2x - \cos 2\alpha)(\cos 2x + \cos 2\alpha) \\
& \{\because \cos 2\theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta\} \\
& = 2\{2 \cos^2 x - 1 - (2 \cos^2 \alpha - 1)\}(2 \cos^2 x - 1 + 1 - 2 \sin^2 \alpha) \\
& = 2\{2 \cos^2 x - 1 - 2 \cos^2 \alpha + 1\}(2 \cos^2 x - 2 \sin^2 \alpha) \\
& = 2 \times 2\{2 \cos^2 x - 2 \cos^2 \alpha\}(\cos^2 x - \sin^2 \alpha) \\
& = 4 \times 2\{\cos^2 x - \cos^2 \alpha\}(\cos^2 x - \sin^2 \alpha) \\
& = 8(\cos x - \cos \alpha)(\cos x + \cos \alpha)(\cos x - \sin \alpha)(\cos x + \sin \alpha) \\
& = \text{RHS}
\end{aligned}$$

**Hence Proved**

## 25. Question

Prove the following identities:

$$\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$$

**Answer**

**To prove:**  $\sin 3x + \sin 2x - \sin x = 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$

Proof:

Take LHS:

$$\sin 3x + \sin 2x - \sin x$$

**Identities used:**

$$\sin 2x = 2 \sin x \cos x$$

$$\sin A - \sin B = 2 \sin \frac{A - B}{2} \cos \frac{A + B}{2}$$

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}$$

Therefore,

$$= 2 \sin \frac{3x}{2} \cos \frac{3x}{2} + 2 \sin \frac{2x - x}{2} \cos \frac{2x + x}{2}$$

$$= 2 \sin \frac{3x}{2} \cos \frac{3x}{2} + 2 \sin \frac{x}{2} \cos \frac{3x}{2}$$

$$= 2 \cos \frac{3x}{2} \left( \sin \frac{3x}{2} + \sin \frac{x}{2} \right)$$

$$= 2 \cos \frac{3x}{2} \left( 2 \sin \frac{\frac{3x}{2} + \frac{x}{2}}{2} \cos \frac{\frac{3x}{2} - \frac{x}{2}}{2} \right)$$

$$= 2 \cos \frac{3x}{2} \left( 2 \sin \frac{\frac{4x}{2}}{2} \cos \frac{\frac{2x}{2}}{2} \right)$$

$$= 2 \cos \frac{3x}{2} \left( 2 \sin \frac{2x}{2} \cos \frac{x}{2} \right)$$

$$= 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2}$$

= RHS

**Hence Proved**

## 26. Question

Prove that:  $\tan 82 \frac{1}{2}^\circ = (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1) = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

**Answer**

**To prove:**  $\tan 82 \frac{1}{2}^\circ = (\sqrt{3} + \sqrt{2})(\sqrt{2} + 1) = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$

Proof:

**Identities used:**

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Therefore,

$$\tan 15^\circ = \tan (45^\circ - 30^\circ)$$

$$\Rightarrow \tan 15^\circ = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 30^\circ \tan 45^\circ}$$

$$\Rightarrow \tan 15^\circ = \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$\left\{ \because \tan 45^\circ = 1 \text{ \& } \tan 30^\circ = \frac{1}{\sqrt{3}} \right\}$$

$$\Rightarrow \tan 15^\circ = \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$$

$$\Rightarrow \tan 15^\circ = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

On rationalising:

$$\Rightarrow \tan 15^\circ = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$\Rightarrow \tan 15^\circ = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3})^2 - 1}$$

$$\{ \because (a - b)(a + b) = a^2 - b^2 \}$$

$$\Rightarrow \tan 15^\circ = \frac{3 + 1 - 2\sqrt{3}}{3 - 1}$$

$$\Rightarrow \tan 15^\circ = \frac{4 - 2\sqrt{3}}{2}$$

$$\Rightarrow \tan 15^\circ = \frac{2(2 - \sqrt{3})}{2}$$

$$\Rightarrow \tan 15^\circ = 2 - \sqrt{3}$$

$$\Rightarrow \cot 15^\circ = \frac{1}{2 - \sqrt{3}}$$

$$\left\{ \because \cot x = \frac{1}{\tan x} \right\}$$

On rationalising

$$\Rightarrow \cot 15^\circ = \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$\Rightarrow \cot 15^\circ = \frac{2 + \sqrt{3}}{(2)^2 - (\sqrt{3})^2}$$

$$\{ \because (a - b)(a + b) = a^2 - b^2 \}$$

$$\Rightarrow \cot 15^\circ = \frac{2 + \sqrt{3}}{4 - 3}$$

$$\Rightarrow \cot 15^\circ = 2 + \sqrt{3}$$

$$\text{Let } 2\theta = 15^\circ$$

$$\Rightarrow \cot 2\theta = 2 + \sqrt{3}$$

We know,

$$\cot 2\theta = \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$\Rightarrow \frac{\cot^2 \theta - 1}{2 \cot \theta} = 2 + \sqrt{3}$$

$$\Rightarrow \cot^2 \theta - 1 = 2(2 + \sqrt{3}) \cot \theta$$

$$\Rightarrow \cot^2 \theta - 2(2 + \sqrt{3}) \cot \theta - 1 = 0$$

**Formula used:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ for } ax^2 + bx + c = 0$$

$$\Rightarrow \cot \theta = \frac{-[-2(2 + \sqrt{3})] \pm \sqrt{[-2(2 + \sqrt{3})]^2 - 4(1)(-1)}}{2(1)}$$

$$\Rightarrow \cot \theta = \frac{2(2 + \sqrt{3}) \pm \sqrt{4(4 + 3 + 4\sqrt{3}) + 4}}{2}$$

$$\{ \because (a + b)^2 = a^2 + b^2 + 2ab \}$$

$$\Rightarrow \cot \theta = \frac{2(2 + \sqrt{3}) \pm 2\sqrt{7 + 4\sqrt{3} + 1}}{2}$$

$$\Rightarrow \cot \theta = (2 + \sqrt{3}) \pm \sqrt{8 + 4\sqrt{3}}$$

$\cot \theta < 0$  as  $\theta$  is in 1<sup>st</sup> quadrant.

So,

$$\cot \theta = (2 + \sqrt{3}) + \sqrt{8 + 4\sqrt{3}}$$

$$\Rightarrow \cot \theta = (2 + \sqrt{3}) + \sqrt{(\sqrt{6})^2 + (\sqrt{2})^2 + 2 \cdot (\sqrt{6})(\sqrt{2})}$$

$$\{\because (a + b)^2 = a^2 + b^2 + 2ab\}$$

$$\Rightarrow \cot \theta = (2 + \sqrt{3}) + \sqrt{(\sqrt{6} + \sqrt{2})^2}$$

$$\Rightarrow \cot \theta = (2 + \sqrt{3}) + (\sqrt{6} + \sqrt{2})$$

$$\text{As, } 2\theta = 15^\circ \Rightarrow \theta = \frac{15^\circ}{2} = 7\frac{1}{2}^\circ$$

$$\Rightarrow \cot 7\frac{1}{2}^\circ = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$$

$$\{\because 4 = \sqrt{2}\}$$

$$\Rightarrow \tan\left(90^\circ - 7\frac{1}{2}^\circ\right) = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$$

$$\{\because \cot \theta = \tan(90^\circ - \theta)\}$$

$$\Rightarrow \tan 82\frac{1}{2}^\circ = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$$

**Hence Proved**

## 27. Question

Prove that:  $\cot \frac{\pi}{8} = \sqrt{2} + 1$

**Answer**

**To prove:**  $\cot \frac{\pi}{8} = \sqrt{2} + 1$

Proof:

Take LHS:

$$\text{Let } 2\theta = 45^\circ$$

We know,

$$\cot 2\theta = \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$\Rightarrow \cot 45^\circ = \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$\{\because \cot 45^\circ = 1\}$$

$$\Rightarrow 1 = \frac{\cot^2 \theta - 1}{2 \cot \theta}$$

$$\Rightarrow 2 \cot \theta = \cot^2 \theta - 1$$

$$\Rightarrow \cot^2 \theta - 2 \cot \theta - 1 = 0$$

**Formula used:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ for } ax^2 + bx + c = 0$$

$$\Rightarrow \cot \theta = \frac{-[-2] \pm \sqrt{[-2]^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1}$$

$$\Rightarrow \cot \theta = \frac{2 \pm \sqrt{4 + 4}}{2}$$

$$\Rightarrow \cot \theta = \frac{2 \pm 2\sqrt{2}}{2}$$

$$\Rightarrow \cot \theta = 1 \pm \sqrt{2}$$

$\cot \theta < 0$  as  $\theta$  is in 1<sup>st</sup> quadrant.

So,

$$\cot \theta = 1 + \sqrt{2}$$

$$\text{As, } 2\theta = 45^\circ \Rightarrow \theta = \frac{45^\circ}{2} = \frac{\pi}{8}$$

$$\Rightarrow \cot \frac{\pi}{8} = 1 + \sqrt{2}$$

**Hence Proved**

## 28 A. Question

If  $\cos x = -\frac{3}{5}$  and  $x$  lies in the III<sup>rd</sup> quadrant, find the values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$  and  $\sin 2x$ .

**Answer**

**Given:**

$$\cos x = -\frac{3}{5} \text{ and } x \text{ lies in 3<sup>rd</sup> quadrant} \Rightarrow x \in \left(\pi, \frac{3\pi}{2}\right)$$

**To find:** Values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$ ,  $\sin 2x$

We know,

$$\cos 2x = 2 \cos^2 x - 1$$

$$\Rightarrow \cos x = 2 \cos^2 \frac{x}{2} - 1$$

$$\Rightarrow -\frac{3}{5} = 2 \cos^2 \frac{x}{2} - 1$$

$$\left\{ \because \cos x = -\frac{3}{5} \right\}$$

$$\Rightarrow 2 \cos^2 \frac{x}{2} = -\frac{3}{5} + 1$$

$$\Rightarrow 2 \cos^2 \frac{x}{2} = \frac{2}{5}$$



$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{5}$$

$$\Rightarrow \cos \frac{x}{2} = \pm \frac{1}{\sqrt{5}}$$

Since,

$$x \in \left( \pi, \frac{3\pi}{2} \right) \Rightarrow \frac{x}{2} \in \left( \frac{\pi}{2}, \frac{3\pi}{4} \right)$$

$$\Rightarrow \cos \frac{x}{2} \text{ will be negative in third quadrant}$$

So,

$$\cos x = -\frac{1}{\sqrt{5}}$$

We know,

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\Rightarrow \cos x = 1 - 2 \sin^2 \frac{x}{2}$$

$$\left\{ \because \cos x = -\frac{3}{5} \right\}$$

$$\Rightarrow -\frac{3}{5} = 1 - 2 \sin^2 \frac{x}{2}$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{3}{5} + 1$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{8}{5}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{4}{5}$$

$$\Rightarrow \sin \frac{x}{2} = \pm \frac{2}{\sqrt{5}}$$

Since,

$$x \in \left( \pi, \frac{3\pi}{2} \right) \Rightarrow \frac{x}{2} \in \left( \frac{\pi}{2}, \frac{3\pi}{4} \right)$$

$$\Rightarrow \sin \frac{x}{2} \text{ will be positive in second quadrant}$$

So,

$$\Rightarrow \sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

We know,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left( -\frac{3}{5} \right)^2$$

$$\left\{ \because \cos x = -\frac{3}{5} \right\}$$

$$\Rightarrow \sin^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \sin^2 x = \frac{25-9}{25}$$

$$\Rightarrow \sin^2 x = \frac{16}{25}$$

$$\Rightarrow \sin x = \pm \frac{4}{5}$$

Since,

$$x \in \left( \pi, \frac{3\pi}{2} \right)$$

$\Rightarrow \sin x$  will be negative in third quadrant

So,

$$\Rightarrow \sin x = -\frac{4}{5}$$

Now,

$$\sin 2x = 2(\sin x)(\cos x)$$

$$\left\{ \because \cos x = -\frac{3}{5} \text{ \& } \sin x = -\frac{4}{5} \right\}$$

$$\Rightarrow \sin 2x = 2 \times -\frac{4}{5} \times -\frac{3}{5}$$

$$\Rightarrow \sin 2x = \frac{24}{25}$$

Hence, values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$ ,  $\sin 2x$  are  $-\frac{1}{\sqrt{5}}$ ,  $\frac{2}{\sqrt{5}}$  and  $\frac{24}{25}$

## 28 B. Question

If  $\cos x = -\frac{3}{5}$  and  $x$  lies in the II<sup>nd</sup> quadrant, find the values of  $\sin 2x$  and  $\sin \frac{x}{2}$ .

**Answer**

**Given:**

$$\cos x = -\frac{3}{5} \text{ and } x \text{ lies in 2<sup>nd</sup> quadrant} \Rightarrow x \in \left( \frac{\pi}{2}, \pi \right)$$

**To find:** Values of  $\sin \frac{x}{2}$ ,  $\sin 2x$

We know,

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\Rightarrow \cos x = 1 - 2 \sin^2 \frac{x}{2}$$

$$\left\{ \because \cos x = -\frac{3}{5} \right\}$$

$$\Rightarrow -\frac{3}{5} = 1 - 2 \sin^2 \frac{x}{2}$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{3}{5} + 1$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{8}{5}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{4}{5}$$

$$\Rightarrow \sin \frac{x}{2} = \pm \frac{2}{\sqrt{5}}$$

Since,

$$x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{3\pi}{2}\right)$$

$$\Rightarrow \sin \frac{x}{2} \text{ will be positive in first quadrant}$$

So,

$$\Rightarrow \sin \frac{x}{2} = \frac{2}{\sqrt{5}}$$

We know,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(-\frac{3}{5}\right)^2$$

$$\left\{ \because \cos x = -\frac{3}{5} \right\}$$

$$\Rightarrow \sin^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \sin^2 x = \frac{25 - 9}{25}$$

$$\Rightarrow \sin^2 x = \frac{16}{25}$$

$$\Rightarrow \sin x = \pm \frac{4}{5}$$

Since,

$$x \in \left(\frac{\pi}{2}, \pi\right)$$

$$\Rightarrow \sin x \text{ will be positive in second quadrant}$$

So,

$$\Rightarrow \sin x = \frac{4}{5}$$

Now,

$$\sin 2x = 2(\sin x)(\cos x)$$

$$\left\{ \because \cos x = -\frac{3}{5} \text{ \& } \sin x = \frac{4}{5} \right\}$$

$$\Rightarrow \sin 2x = 2 \times \frac{4}{5} \times -\frac{3}{5}$$

$$\Rightarrow \sin 2x = -\frac{24}{25}$$

Hence, values of  $\sin \frac{x}{2}$ ,  $\sin 2x$  are  $\frac{2}{\sqrt{5}}$  and  $-\frac{24}{25}$

## 29. Question

If  $\sin x = \frac{\sqrt{5}}{3}$  and  $x$  lies in II<sup>nd</sup> quadrant, find the values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$  and  $\tan \frac{x}{2}$ .

## Answer

**Given:**

$$\sin x = \frac{\sqrt{5}}{3} \text{ and } x \text{ lies in 2<sup>nd</sup> quadrant} \Rightarrow x \in \left(\frac{\pi}{2}, \pi\right)$$

**To find:** Values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$ ,  $\tan \frac{x}{2}$

We know,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{\sqrt{5}}{3}\right)^2$$

$$\left\{ \because \sin x = \frac{\sqrt{5}}{3} \right\}$$

$$\Rightarrow \cos^2 x = 1 - \frac{5}{9}$$

$$\Rightarrow \cos^2 x = \frac{9-5}{9}$$

$$\Rightarrow \cos^2 x = \frac{4}{9}$$

$$\Rightarrow \cos x = \pm \frac{2}{3}$$

Since,

$$x \in \left(\frac{\pi}{2}, \pi\right)$$

$\Rightarrow \cos x$  will be negative in second quadrant

So,

$$\Rightarrow \cos x = -\frac{2}{3}$$

We know,

$$\cos 2x = 2 \cos^2 x - 1$$

$$\Rightarrow \cos x = 2 \cos^2 \frac{x}{2} - 1$$

$$\Rightarrow -\frac{2}{3} = 2 \cos^2 \frac{x}{2} - 1$$

$$\left\{ \because \cos x = -\frac{2}{3} \right\}$$

$$\Rightarrow 2 \cos^2 \frac{x}{2} = -\frac{2}{3} + 1$$

$$\Rightarrow 2 \cos^2 \frac{x}{2} = \frac{-2 + 3}{3}$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{1}{6}$$

$$\Rightarrow \cos \frac{x}{2} = \pm \frac{1}{\sqrt{6}}$$

Since,

$$x \in \left( \frac{\pi}{2}, \pi \right) \Rightarrow \frac{x}{2} \in \left( \frac{\pi}{4}, \frac{\pi}{2} \right)$$

$$\Rightarrow \cos \frac{x}{2} \text{ will be positive in first quadrant}$$

So,

$$\cos \frac{x}{2} = \frac{1}{\sqrt{6}}$$

We know,

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\Rightarrow \cos x = 1 - 2 \sin^2 \frac{x}{2}$$

$$\left\{ \because \cos x = -\frac{2}{3} \right\}$$

$$\Rightarrow -\frac{2}{3} = 1 - 2 \sin^2 \frac{x}{2}$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{2}{3} + 1$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{2 + 3}{3}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{5}{6}$$

$$\Rightarrow \sin \frac{x}{2} = \pm \sqrt{\frac{5}{6}}$$

Since,

$$x \in \left( \frac{\pi}{2}, \pi \right) \Rightarrow \frac{x}{2} \in \left( \frac{\pi}{4}, \frac{\pi}{2} \right)$$

$$\Rightarrow \sin \frac{x}{2} \text{ will be positive in first quadrant}$$

So,

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{5}{6}}$$

We know,

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}$$

$$\Rightarrow \tan \frac{x}{2} = \frac{\sqrt{\frac{5}{6}}}{\frac{1}{\sqrt{6}}}$$

$$\Rightarrow \tan \frac{x}{2} = \sqrt{5}$$

Hence, values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$ ,  $\tan \frac{x}{2}$  are  $\frac{1}{\sqrt{6}}$ ,  $\sqrt{\frac{5}{6}}$  and  $\sqrt{5}$

### 30 A. Question

$0 \leq x \leq \pi$  and  $x$  lies in the II<sup>nd</sup> quadrant such that  $\sin x = \frac{1}{4}$ . Find the values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$  and  $\tan \frac{x}{2}$ .

**Answer**

**Given:**

$$\sin x = \frac{1}{4} \text{ and } x \text{ lies in 2<sup>nd</sup> quadrant} \Rightarrow x \in \left(\frac{\pi}{2}, \pi\right)$$

**To find:** Values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$ ,  $\tan \frac{x}{2}$

We know,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{1}{4}\right)^2$$

$$\left\{ \because \sin x = \frac{1}{4} \right\}$$

$$\Rightarrow \cos^2 x = 1 - \frac{1}{16}$$

$$\Rightarrow \cos^2 x = \frac{16-1}{16}$$

$$\Rightarrow \cos^2 x = \frac{15}{16}$$

$$\Rightarrow \cos x = \pm \frac{\sqrt{15}}{4}$$

Since,

$$x \in \left(\frac{\pi}{2}, \pi\right)$$

$\Rightarrow \cos x$  will be negative in second quadrant

So,

$$\Rightarrow \cos x = -\frac{\sqrt{15}}{4}$$

We know,

$$\cos 2x = 2 \cos^2 x - 1$$

$$\Rightarrow \cos x = 2 \cos^2 \frac{x}{2} - 1$$

$$\Rightarrow -\frac{\sqrt{15}}{4} = 2 \cos^2 \frac{x}{2} - 1$$

$$\left\{ \because \cos x = -\frac{\sqrt{15}}{4} \right\}$$

$$\Rightarrow 2 \cos^2 \frac{x}{2} = -\frac{\sqrt{15}}{4} + 1$$

$$\Rightarrow 2 \cos^2 \frac{x}{2} = \frac{-\sqrt{15} + 4}{4}$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{-\sqrt{15} + 4}{8}$$

$$\Rightarrow \cos \frac{x}{2} = \pm \sqrt{\frac{-\sqrt{15} + 4}{8}}$$

Since,

$$x \in \left( \frac{\pi}{2}, \pi \right) \Rightarrow \frac{x}{2} \in \left( \frac{\pi}{4}, \frac{\pi}{2} \right)$$

$$\Rightarrow \cos \frac{x}{2} \text{ will be positive in first quadrant}$$

So,

$$\cos \frac{x}{2} = \sqrt{\frac{-\sqrt{15} + 4}{8}}$$

We know,

$$\cos 2x = 1 - 2 \sin^2 x$$

$$\Rightarrow \cos x = 1 - 2 \sin^2 \frac{x}{2}$$

$$\left\{ \because \cos x = -\frac{\sqrt{15}}{4} \right\}$$

$$\Rightarrow -\frac{\sqrt{15}}{4} = 1 - 2 \sin^2 \frac{x}{2}$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{\sqrt{15}}{4} + 1$$

$$\Rightarrow 2 \sin^2 \frac{x}{2} = \frac{\sqrt{15} + 4}{4}$$

$$\Rightarrow \sin^2 \frac{x}{2} = \frac{\sqrt{15} + 4}{8}$$

$$\Rightarrow \sin \frac{x}{2} = \pm \sqrt{\frac{\sqrt{15} + 4}{8}}$$

Since,

$$x \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$\Rightarrow \sin \frac{x}{2}$  will be positive in first quadrant

So,

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{\sqrt{15} + 4}{8}}$$

We know,

$$\tan \frac{x}{2} = \frac{\sqrt{\frac{\sqrt{15} + 4}{8}}}{\sqrt{\frac{-\sqrt{15} + 4}{8}}}$$

$$\Rightarrow \tan \frac{x}{2} = \sqrt{\frac{\sqrt{15} + 4}{8} \times \frac{8}{-\sqrt{15} + 4}}$$

$$\Rightarrow \tan \frac{x}{2} = \sqrt{\frac{\sqrt{15} + 4}{-\sqrt{15} + 4}}$$

On rationalising:

$$\Rightarrow \tan \frac{x}{2} = \sqrt{\frac{4 + \sqrt{15}}{4 - \sqrt{15}} \times \frac{4 + \sqrt{15}}{4 + \sqrt{15}}}$$

$$\Rightarrow \tan \frac{x}{2} = \sqrt{\frac{(4 + \sqrt{15})^2}{4^2 - (\sqrt{15})^2}}$$

$$\{\because (a + b)(a - b) = a^2 - b^2\}$$

$$\Rightarrow \tan \frac{x}{2} = \sqrt{\frac{(4 + \sqrt{15})^2}{16 - 15}}$$

$$\Rightarrow \tan \frac{x}{2} = \sqrt{\frac{(4 + \sqrt{15})^2}{1}}$$

$$\Rightarrow \tan \frac{x}{2} = 4 + \sqrt{15}$$

Hence, values of  $\cos \frac{x}{2}$ ,  $\sin \frac{x}{2}$ ,  $\tan \frac{x}{2}$  are  $\sqrt{\frac{-\sqrt{15} + 4}{8}}$ ,  $\sqrt{\frac{\sqrt{15} + 4}{8}}$  and  $4 + \sqrt{15}$

### 30 B. Question



If  $\cos x = \frac{4}{5}$  and  $x$  is acute, find  $\tan 2x$ .

**Answer**

**Given:**

$$\cos x = \frac{4}{5} \text{ and } x \text{ is acute} \Rightarrow x \in \left(0, \frac{\pi}{2}\right)$$

**To find:** Value of  $\tan 2x$

We know,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin^2 x = 1 - \left(\frac{4}{5}\right)^2$$

$$\left\{ \because \cos x = \frac{4}{5} \right\}$$

$$\Rightarrow \sin^2 x = 1 - \frac{16}{25}$$

$$\Rightarrow \sin^2 x = \frac{25 - 16}{25}$$

$$\Rightarrow \sin^2 x = \frac{9}{25}$$

$$\Rightarrow \sin x = \pm \frac{3}{5}$$

Since,

$$x \in \left(0, \frac{\pi}{2}\right)$$

$\Rightarrow \sin x$  will be negative in first quadrant

So,

$$\Rightarrow \sin x = \frac{3}{5}$$

Now,

$$\tan x = \frac{\sin x}{\cos x}$$

$$\Rightarrow \tan x = \frac{\frac{3}{5}}{\frac{4}{5}}$$

$$\Rightarrow \tan x = \frac{3}{4}$$

We know,

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\Rightarrow \tan 2x = \frac{2\left(\frac{3}{4}\right)}{1 - \left(\frac{3}{4}\right)^2}$$

$$\Rightarrow \tan 2x = \frac{\frac{3}{2}}{1 - \frac{9}{16}}$$

$$\Rightarrow \tan 2x = \frac{\frac{3}{2}}{\frac{16 - 9}{16}}$$

$$\Rightarrow \tan 2x = \frac{\frac{3}{2}}{\frac{7}{16}}$$

$$\Rightarrow \tan 2x = \frac{3}{2} \times \frac{16}{7}$$

$$\Rightarrow \tan 2x = \frac{24}{7}$$

Hence, value of  $\tan 2x = \frac{24}{7}$

### 30 C. Question

If  $\sin x = \frac{4}{5}$  and  $0 < x < \frac{\pi}{2}$ , find the value of  $\sin 4x$ .

**Answer**

**Given:**

$$\sin x = \frac{4}{5} \text{ and } x \in \left(0, \frac{\pi}{2}\right)$$

**To find:** Values of  $\sin 4x$

We know,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{4}{5}\right)^2$$

$$\left\{ \because \sin x = \frac{4}{5} \right\}$$

$$\Rightarrow \cos^2 x = 1 - \frac{16}{25}$$

$$\Rightarrow \cos^2 x = \frac{25 - 16}{25}$$

$$\Rightarrow \cos^2 x = \frac{9}{25}$$

$$\Rightarrow \cos x = \pm \frac{3}{5}$$

Since,

$$x \in \left(0, \frac{\pi}{2}\right)$$

$\Rightarrow \cos x$  will be negative in first quadrant

So,

$$\Rightarrow \cos x = \frac{3}{5}$$

We know,

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1$$

Therefore,

$$\sin 4x = 2 \sin 2x \cos 2x$$

$$\Rightarrow \sin 4x = 2 (2 \sin x \cos x) (2 \cos^2 x - 1)$$

$$\left\{ \because \sin x = \frac{4}{5} \text{ \& } \cos x = \frac{3}{5} \right\}$$

$$\Rightarrow \sin 4x = 2 \left( 2 \times \frac{4}{5} \times \frac{3}{5} \right) \left( 2 \left( \frac{4}{5} \right)^2 - 1 \right)$$

$$\Rightarrow \sin 4x = 2 \left( \frac{24}{25} \right) \left( 2 \times \frac{16}{25} - 1 \right)$$

$$\Rightarrow \sin 4x = \frac{48}{25} \left( \frac{32}{25} - 1 \right)$$

$$\Rightarrow \sin 4x = \frac{48}{25} \left( \frac{32 - 25}{25} \right)$$

$$\Rightarrow \sin 4x = \frac{48}{25} \left( \frac{7}{25} \right)$$

$$\Rightarrow \sin 4x = \frac{336}{625}$$

Hence, value of  $\sin 4x = \frac{336}{625}$

### 31. Question

If  $\tan x = \frac{b}{a}$ , then find the value of  $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$ .

**Answer**

$$\text{Given: } \tan x = \frac{b}{a}$$

$$\text{To find: } \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$$

On taking LCM:

$$\begin{aligned}
&= \frac{(\sqrt{a+b})^2 + (\sqrt{a-b})^2}{\sqrt{a+b}\sqrt{a-b}} \\
&= \frac{a+b+a-b}{\sqrt{a+b}\sqrt{a-b}} \\
&= \frac{2a}{\sqrt{a+b}\sqrt{a-b}}
\end{aligned}$$

Dividing numerator and denominator by a:

$$\begin{aligned}
&= \frac{\frac{2a}{a}}{\frac{\sqrt{a+b}\sqrt{a-b}}{a}} \\
&= \frac{2}{\sqrt{\frac{a+b}{a}}\sqrt{\frac{a-b}{a}}} \\
&= \frac{2}{\sqrt{1+\frac{b}{a}}\sqrt{1-\frac{b}{a}}} \\
&= \frac{2}{\sqrt{1+\tan x}\sqrt{1-\tan x}} \\
&\left\{ \because \tan x = \frac{b}{a} \right\} \\
&= \frac{2}{\sqrt{(1+\tan x)(1-\tan x)}} \\
&\{ \because (a+b)(a-b) = a^2 - b^2 \} \\
&= \frac{2}{\sqrt{1-\tan^2 x}}
\end{aligned}$$

### 32. Question

If  $\tan A = \frac{1}{7}$  and  $\tan B = \frac{1}{3}$ , show that  $\cos 2A = \sin 4B$

**Answer**

**Given:**  $\tan A = \frac{1}{7}$  &  $\tan B = \frac{1}{3}$

**To prove:**  $\cos 2A = \sin 4B$

We know,

$$\begin{aligned}
\tan 2B &= \frac{2 \tan B}{1 - \tan^2 B} \\
\Rightarrow \tan 2B &= \frac{2\left(\frac{1}{3}\right)}{1 - \left(\frac{1}{3}\right)^2} \\
\Rightarrow \tan 2B &= \frac{\frac{2}{3}}{1 - \frac{1}{9}}
\end{aligned}$$

$$\Rightarrow \tan 2B = \frac{\frac{2}{3}}{\frac{9-1}{9}}$$

$$\Rightarrow \tan 2B = \frac{\frac{2}{3}}{\frac{8}{9}}$$

$$\Rightarrow \tan 2B = \frac{3}{4}$$

Take LHS:

$\cos 2A$

$$= \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$\left\{ \because \tan A = \frac{1}{7} \right\}$$

$$= \frac{1 - \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2}$$

$$= \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}}$$

$$= \frac{\frac{49-1}{49}}{\frac{49+1}{49}}$$

$$= \frac{\frac{48}{49}}{\frac{50}{49}}$$

$$= \frac{48}{50}$$

$$= \frac{24}{25}$$

Now,

Take RHS:

$\sin 4B$

$$= \frac{2 \tan 2B}{1 + \tan^2 2B}$$

$$\left\{ \because \tan 2B = \frac{3}{4} \right\}$$

$$= \frac{2 \left(\frac{3}{4}\right)}{1 + \left(\frac{3}{4}\right)^2}$$

$$= \frac{\frac{3}{2}}{1 + \frac{9}{16}}$$

$$= \frac{\frac{3}{2}}{\frac{16+9}{16}}$$

$$= \frac{\frac{3}{2}}{\frac{25}{16}}$$

$$= \frac{24}{25}$$

Clearly, LHS = RHS =  $\frac{24}{25}$

**Hence Proved**

### 33. Question

Prove that:

$$\cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ = \frac{\sin 68^\circ}{16 \cos 83^\circ}$$

**Answer**

$$\text{To prove: } \cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ = \frac{\sin 68^\circ}{16 \cos 83^\circ}$$

Proof:

Take LHS:

$$\cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ$$

Multiplying and Dividing  $2^4 \sin 7^\circ$

$$= \frac{2^4 \sin 7^\circ \cos 7^\circ \cos 14^\circ \cos 28^\circ \cos 56^\circ}{2^4 \sin 7^\circ}$$

$$= \frac{2^3 (2 \sin 7^\circ \cos 7^\circ) \cos 14^\circ \cos 28^\circ \cos 56^\circ}{2^4 \sin 7^\circ}$$

{ $\because \sin 2x = 2 \sin x \cos x$ }

$$= \frac{2^3 (\sin 14^\circ) \cos 14^\circ \cos 28^\circ \cos 56^\circ}{2^4 \sin 7^\circ}$$

$$= \frac{2^2 (2 \sin 14^\circ \cos 14^\circ) \cos 28^\circ \cos 56^\circ}{2^4 \sin 7^\circ}$$

$$= \frac{2^2 (\sin 28^\circ) \cos 28^\circ \cos 56^\circ}{2^4 \sin 7^\circ}$$

$$= \frac{2^1 (2 \sin 28^\circ \cos 28^\circ) \cos 56^\circ}{2^4 \sin 7^\circ}$$

$$= \frac{2^1 (\sin 56^\circ) \cos 56^\circ}{2^4 \sin 7^\circ}$$

$$= \frac{2 \sin 56^\circ \cos 56^\circ}{2^4 \sin 7^\circ}$$

$$= \frac{\sin 112^\circ}{2^4 \sin 7^\circ}$$

We know,

$$\sin(180^\circ - \theta) = \sin \theta$$

$$\sin(90^\circ - \theta) = \cos \theta$$

Now,

$$= \frac{\sin(180^\circ - 112^\circ)}{2^4 \cos(90^\circ - 7^\circ)}$$

$$= \frac{\sin 68^\circ}{16 \cos 83^\circ}$$

$$= \text{RHS}$$

**Hence Proved**

### 34. Question

Prove that:

$$\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} = \frac{1}{16}$$

**Answer**

$$\text{To prove: } \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15} = \frac{1}{16}$$

Proof:

Take LHS:

$$\cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15}$$

Multiplying and Dividing by  $2^4 \sin \frac{2\pi}{15}$ :

$$= \frac{2^4 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$= \frac{2^3 \left( 2 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \right) \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$\{\because \sin 2x = 2 \sin x \cos x\}$$

$$= \frac{2^3 \sin \frac{4\pi}{15} \cos \frac{4\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$= \frac{2^2 \left( 2 \sin \frac{4\pi}{15} \cos \frac{4\pi}{15} \right) \cos \frac{8\pi}{15} \cos \frac{16\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$= \frac{2^2 \sin \frac{8\pi}{15} \cos \frac{8\pi}{15} \cos \frac{16\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$= \frac{2 \left( 2 \sin \frac{8\pi}{15} \cos \frac{8\pi}{15} \right) \cos \frac{16\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$= \frac{2 \sin \frac{16\pi}{15} \cos \frac{16\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$= \frac{\sin \frac{32\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$= \frac{\sin \left( 2\pi + \frac{2\pi}{15} \right)}{2^4 \sin \frac{2\pi}{15}}$$

$$\left\{ \because 2\pi + \frac{2\pi}{15} = \frac{30\pi + 2\pi}{15} = \frac{32\pi}{15} \right\}$$

$$= \frac{\sin \frac{2\pi}{15}}{2^4 \sin \frac{2\pi}{15}}$$

$$\{ \because \sin (2\pi + \theta) = \sin \theta \}$$

$$= \frac{1}{2^4}$$

$$= \frac{1}{16}$$

$$= \text{RHS}$$

**Hence Proved**

### 35. Question

Prove that:

$$\cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} = \frac{-1}{16}$$

**Answer**

$$\text{To prove: } \cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5} = \frac{-1}{16}$$

Proof:

Take LHS:

$$\cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5}$$

Multiplying and Dividing  $2^4 \sin \frac{\pi}{5}$ :

$$= \frac{2^4 \sin \frac{\pi}{5} \cos \frac{\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$= \frac{2^3 \left( 2 \sin \frac{\pi}{5} \cos \frac{\pi}{5} \right) \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$\{ \because \sin 2x = 2 \sin x \cos x \}$$



$$= \frac{2^3 \sin \frac{2\pi}{5} \cos \frac{2\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$= \frac{2^2 \left( 2 \sin \frac{2\pi}{5} \cos \frac{2\pi}{5} \right) \cos \frac{4\pi}{5} \cos \frac{8\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$= \frac{2^2 \sin \frac{4\pi}{5} \cos \frac{4\pi}{5} \cos \frac{8\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$= \frac{2 \left( 2 \sin \frac{4\pi}{5} \cos \frac{4\pi}{5} \right) \cos \frac{8\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$= \frac{2 \sin \frac{8\pi}{5} \cos \frac{8\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$= \frac{\sin \frac{16\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$= \frac{\sin \left( 3\pi + \frac{\pi}{5} \right)}{2^4 \sin \frac{\pi}{5}}$$

$$\left\{ \because 3\pi + \frac{\pi}{5} = \frac{15\pi + \pi}{5} = \frac{16\pi}{5} \right\}$$

$$= - \frac{\sin \frac{\pi}{5}}{2^4 \sin \frac{\pi}{5}}$$

$$\{ \because \sin (3\pi + \theta) = - \sin \theta \}$$

$$= - \frac{1}{2^4}$$

$$= - \frac{1}{16}$$

$$= \text{RHS}$$

**Hence Proved**

### 36. Question

Prove that:

$$\cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65}$$

$$\cos \frac{32\pi}{65} = \frac{1}{64}$$

**Answer**

$$\text{To prove: } \cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} = \frac{1}{64}$$

Proof:

Take LHS:

$$\cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$$

Multiplying and Dividing  $2^6 \sin \frac{\pi}{65}$ :

$$= \frac{2^6 \sin \frac{\pi}{65} \cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2^5 \left( 2 \sin \frac{\pi}{65} \cos \frac{\pi}{65} \right) \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

{ $\because \sin 2x = 2 \sin x \cos x$ }

$$= \frac{2^5 \sin \frac{2\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2^4 \left( 2 \sin \frac{2\pi}{65} \cos \frac{2\pi}{65} \right) \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2^4 \sin \frac{4\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2^3 \left( 2 \sin \frac{4\pi}{65} \cos \frac{4\pi}{65} \right) \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2^3 \sin \frac{8\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2^2 \left( 2 \sin \frac{8\pi}{65} \cos \frac{8\pi}{65} \right) \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2^2 \sin \frac{16\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2 \left( 2 \sin \frac{16\pi}{65} \cos \frac{16\pi}{65} \right) \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{2 \sin \frac{32\pi}{65} \cos \frac{32\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{\sin \frac{64\pi}{65}}{2^6 \sin \frac{\pi}{65}}$$

$$= \frac{\sin \left( \pi - \frac{\pi}{65} \right)}{2^6 \sin \frac{\pi}{65}}$$

$$\left\{ \because \pi - \frac{\pi}{65} = \frac{65\pi - \pi}{65} = \frac{64\pi}{65} \right\}$$

$$= \frac{\sin \frac{\pi}{65}}{2^5 \sin \frac{\pi}{65}}$$

$$\{ \because \sin (\pi - \theta) = \sin \theta \}$$

$$= \frac{1}{2^5}$$

$$= \frac{1}{64}$$

$$= \text{RHS}$$

**Hence Proved**

### 37. Question

$$\text{If } 2 \tan \alpha = 3 \tan \beta, \text{ prove that } \tan (\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}.$$

**Answer**

$$\textbf{Given: } 2 \tan \alpha = 3 \tan \beta$$

$$\textbf{To prove: } \tan (\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$$

Proof:

Take LHS:

$$\tan \alpha - \tan \beta$$

$$= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$= \frac{\frac{3}{2} \tan \beta - \tan \beta}{1 + \frac{3}{2} \tan \beta \tan \beta}$$

$$\left\{ \because 2 \tan \alpha = 3 \tan \beta \Rightarrow \tan \alpha = \frac{3}{2} \tan \beta \right\}$$

$$= \frac{\tan \beta \left( \frac{3}{2} - 1 \right)}{1 + \frac{3}{2} \tan^2 \beta}$$

$$= \frac{\frac{1}{2} \tan \beta}{1 + \frac{3}{2} \tan^2 \beta}$$

$$= \frac{\frac{1}{2} \frac{\sin \beta}{\cos \beta}}{1 + \frac{3}{2} \cdot \left( \frac{\sin \beta}{\cos \beta} \right)^2}$$

$$\left\{ \because \tan \beta = \frac{\sin \beta}{\cos \beta} \right\}$$

$$\begin{aligned}
&= \frac{\frac{\sin \beta}{2 \cos \beta}}{1 + \frac{3 \sin^2 \beta}{2 \cos^2 \beta}} \\
&= \frac{\frac{\sin \beta}{2 \cos \beta}}{\frac{2 \cos^2 \beta + 3 \sin^2 \beta}{2 \cos^2 \beta}} \\
&= \frac{2 \cos^2 \beta \sin \beta}{2 \cos \beta (2 \cos^2 \beta + 3 \sin^2 \beta)} \\
&= \frac{2 \cos \beta \sin \beta}{2(2 \cos^2 \beta + 3 \sin^2 \beta)} \\
&= \frac{\sin 2 \beta}{2(2 \cos^2 \beta) + 3(2 \sin^2 \beta)}
\end{aligned}$$

$$\{\because \sin 2x = 2(\sin x)(\cos x)\}$$

$$= \frac{\sin 2 \beta}{2(1 + \cos 2\beta) + 3(1 - \cos 2\beta)}$$

$$\{\because 2 \cos^2 x = 1 + \cos 2x \text{ \& } 2 \sin^2 x = 1 - \cos 2x\}$$

$$\begin{aligned}
&= \frac{\sin 2 \beta}{2 + 2 \cos 2\beta + 3 - 3 \cos 2\beta} \\
&= \frac{\sin 2 \beta}{5 - \cos 2\beta}
\end{aligned}$$

= RHS

**Hence Proved**

### 38 A. Question

If  $\sin \alpha + \sin \beta = a$  and  $\cos \alpha + \cos \beta = b$ , prove that

$$\sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$$

**Answer**

**Given:**  $\sin \alpha + \sin \beta = a$  &  $\cos \alpha + \cos \beta = b$

$$\text{To prove: } \sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$$

Proof:

$$\sin \alpha + \sin \beta = a \dots\dots\dots(3)$$

$$\cos \alpha + \cos \beta = b \dots\dots\dots(4)$$

Dividing equation 3 and 4:

$$\begin{aligned}
\Rightarrow \frac{(\sin \alpha + \sin \beta)}{(\cos \alpha + \cos \beta)} &= \frac{a}{b} \\
\Rightarrow \frac{2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}}{2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}} &= \frac{a}{b}
\end{aligned}$$

$$\Rightarrow \frac{\sin \frac{\alpha + \beta}{2}}{\cos \frac{\alpha + \beta}{2}} = \frac{a}{b}$$

$$\Rightarrow \tan \frac{\alpha + \beta}{2} = \frac{a}{b}$$

We know,

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

Therefore,

$$\sin(\alpha + \beta) = \frac{2 \tan \frac{\alpha + \beta}{2}}{1 + \tan^2 \frac{\alpha + \beta}{2}}$$

$$\Rightarrow \sin(\alpha + \beta) = \frac{2 \left( \frac{a}{b} \right)}{1 + \left( \frac{a}{b} \right)^2}$$

$$\Rightarrow \sin(\alpha + \beta) = \frac{\frac{2a}{b}}{1 + \frac{a^2}{b^2}}$$

$$\Rightarrow \sin(\alpha + \beta) = \frac{\frac{2a}{b}}{\frac{b^2 + a^2}{b^2}}$$

$$\Rightarrow \sin(\alpha + \beta) = \frac{\frac{2a}{b} \cdot \frac{1}{1}}{\frac{b^2 + a^2}{b}}$$

$$\Rightarrow \sin(\alpha + \beta) = \frac{2ab}{a^2 + b^2}$$

**Hence Proved**

### 38 B. Question

If  $\sin \alpha + \sin \beta = a$  and  $\cos \alpha + \cos \beta = b$ , prove that

$$\cos(\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$$

**Answer**

**Given:**  $\sin \alpha + \sin \beta = a$  &  $\cos \alpha + \cos \beta = b$

**To prove:**  $\cos(\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$

Proof:

$$\sin \alpha + \sin \beta = a$$

Squaring both sides, we get

$$(\sin \alpha + \sin \beta)^2 = a^2$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta = a^2 \dots\dots(1)$$

$$\cos \alpha + \cos \beta = b$$

Squaring both sides, we get

$$(\cos \alpha + \cos \beta)^2 = a^2$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = b^2 \dots\dots\dots(2)$$

Adding equation 1 and 2, we get

$$\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = a^2 + b^2$$

$$\Rightarrow \sin^2 \alpha + \cos^2 \alpha + \sin^2 \beta + \cos^2 \beta + 2 \sin \alpha \sin \beta + 2 \cos \alpha \cos \beta = a^2 + b^2$$

$$\Rightarrow 1 + 1 + 2 \sin \alpha \sin \beta + 2 \cos \alpha \cos \beta = a^2 + b^2$$

$$\{\because \sin^2 x + \cos^2 x = 1\}$$

$$\Rightarrow 2 + 2 \sin \alpha \sin \beta + 2 \cos \alpha \cos \beta = a^2 + b^2$$

$$\Rightarrow 2(\sin \alpha \sin \beta + \cos \alpha \cos \beta) = a^2 + b^2 - 2$$

$$\Rightarrow (\sin \alpha \sin \beta + \cos \alpha \cos \beta) = \frac{a^2 + b^2 - 2}{2}$$

We know,

$$\sin A \sin B + \cos A \cos B = \cos (A - B)$$

Therefore,

$$\Rightarrow \cos(\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$$

**Hence Proved**

### 39. Question

$$\text{If } 2 \tan \frac{\alpha}{2} = \tan \frac{\beta}{2}, \text{ prove that } \cos \alpha = \frac{3 + 5 \cos \beta}{5 + 3 \cos \beta}.$$

**Answer**

$$\textbf{Given: } 2 \tan \frac{\alpha}{2} = \tan \frac{\beta}{2}$$

$$\textbf{To prove: } \cos \alpha = \frac{3 + 5 \cos \beta}{5 + 3 \cos \beta}$$

Proof:

Take LHS:

$$\cos \alpha$$

$$= \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}$$

$$\left\{ \because \tan \frac{\alpha}{2} = \frac{1}{2} \tan \frac{\beta}{2} \right\}$$

$$= \frac{1 - \left( \frac{1}{2} \tan \frac{\beta}{2} \right)^2}{1 + \left( \frac{1}{2} \tan \frac{\beta}{2} \right)^2}$$

$$= \frac{1 - \frac{1}{4} \tan^2 \frac{\beta}{2}}{1 + \frac{1}{4} \tan^2 \frac{\beta}{2}}$$

$$= \frac{\frac{4 - \tan^2 \frac{\beta}{2}}{4}}{\frac{4 + \tan^2 \frac{\beta}{2}}{4}}$$

$$= \frac{4 - \tan^2 \frac{\beta}{2}}{4 + \tan^2 \frac{\beta}{2}}$$

Now, Take RHS:

$$\frac{3 + 5 \cos \beta}{5 + 3 \cos \beta}$$

$$= \frac{3 + 5 \left( \frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}{5 + 3 \left( \frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}$$

$$\left\{ \because \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right\}$$

$$= \frac{3 \left( 1 + \tan^2 \frac{\beta}{2} \right) + 5 \left( 1 - \tan^2 \frac{\beta}{2} \right)}{5 \left( 1 + \tan^2 \frac{\beta}{2} \right) + 3 \left( 1 - \tan^2 \frac{\beta}{2} \right)}$$

$$= \frac{3 + 3 \tan^2 \frac{\beta}{2} + 5 - 5 \tan^2 \frac{\beta}{2}}{5 + 5 \tan^2 \frac{\beta}{2} + 3 - 3 \tan^2 \frac{\beta}{2}}$$

$$= \frac{8 - 2 \tan^2 \frac{\beta}{2}}{8 + 2 \tan^2 \frac{\beta}{2}}$$

$$= \frac{2 \left( 4 - \tan^2 \frac{\beta}{2} \right)}{2 \left( 4 + \tan^2 \frac{\beta}{2} \right)}$$

$$= \frac{4 - \tan^2 \frac{\beta}{2}}{4 + \tan^2 \frac{\beta}{2}}$$

$$\left\{ \because \cos \alpha = \frac{4 - \tan^2 \frac{\beta}{2}}{4 + \tan^2 \frac{\beta}{2}} \right\}$$

$$= \cos \alpha$$

**Hence Proved**

#### 40. Question

If  $\cos x = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$ , prove that  $\tan \frac{x}{2} = \pm \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$

**Answer**

$$\text{Given: } \cos x = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$$

$$\text{To prove: } \tan \frac{x}{2} = \pm \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$$

$$\cos x = \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}$$

We know,

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{\frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} + \frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}}}{1 + \left( \frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}} \right) \left( \frac{1 - \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\beta}{2}} \right)}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{\frac{(1 - \tan^2 \frac{\alpha}{2})(1 + \tan^2 \frac{\beta}{2}) + (1 - \tan^2 \frac{\beta}{2})(1 + \tan^2 \frac{\alpha}{2})}{(1 + \tan^2 \frac{\alpha}{2})(1 + \tan^2 \frac{\beta}{2})}}{\frac{(1 + \tan^2 \frac{\alpha}{2})(1 + \tan^2 \frac{\beta}{2}) + (1 - \tan^2 \frac{\alpha}{2})(1 - \tan^2 \frac{\beta}{2})}{(1 + \tan^2 \frac{\alpha}{2})(1 + \tan^2 \frac{\beta}{2})}}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{(1 - \tan^2 \frac{\alpha}{2})(1 + \tan^2 \frac{\beta}{2}) + (1 - \tan^2 \frac{\beta}{2})(1 + \tan^2 \frac{\alpha}{2})}{(1 + \tan^2 \frac{\alpha}{2})(1 + \tan^2 \frac{\beta}{2}) + (1 - \tan^2 \frac{\alpha}{2})(1 - \tan^2 \frac{\beta}{2})}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{1 - \tan^2 \frac{\alpha}{2} + \tan^2 \frac{\beta}{2} - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2} + 1 - \tan^2 \frac{\beta}{2} + \tan^2 \frac{\alpha}{2} - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\alpha}{2} + \tan^2 \frac{\beta}{2} + \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2} + 1 - \tan^2 \frac{\alpha}{2} - \tan^2 \frac{\beta}{2} + \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{2 - 2 \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}{2 + 2 \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{2 \left( 1 - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2} \right)}{2 \left( 1 + \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2} \right)}$$



$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{1 - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}{1 + \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}$$

Applying componendo and dividendo, we get

$$\Rightarrow \frac{\left(1 - \tan^2 \frac{x}{2}\right) + \left(1 + \tan^2 \frac{x}{2}\right)}{\left(1 - \tan^2 \frac{x}{2}\right) - \left(1 + \tan^2 \frac{x}{2}\right)} = \frac{\left(1 - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}\right) + \left(1 + \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}\right)}{\left(1 - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}\right) - \left(1 + \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}\right)}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{x}{2} + 1 + \tan^2 \frac{x}{2}}{1 - \tan^2 \frac{x}{2} - 1 - \tan^2 \frac{x}{2}} = \frac{1 - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2} + 1 + \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}{1 - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2} - 1 - \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}$$

$$\Rightarrow \frac{2}{-2 \tan^2 \frac{x}{2}} = \frac{2}{-2 \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}$$

$$\Rightarrow \frac{1}{-\tan^2 \frac{x}{2}} = \frac{1}{-\tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}$$

Taking reciprocal both sides:

$$\Rightarrow -\tan^2 \frac{x}{2} = -\tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}$$

$$\Rightarrow \tan^2 \frac{x}{2} = \tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}$$

$$\Rightarrow \tan \frac{x}{2} = \pm \sqrt{\tan^2 \frac{\alpha}{2} \tan^2 \frac{\beta}{2}}$$

$$\Rightarrow \tan \frac{x}{2} = \pm \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$$

**Hence Proved**

#### 41. Question

If  $\sec(x + \alpha) + \sec(x - \alpha) = 2 \sec x$ , prove that  $\cos x = \pm \sqrt{2} \cos \frac{\alpha}{2}$

**Answer**

**Given:**  $\sec(x + \alpha) + \sec(x - \alpha) = 2 \sec x$

**To prove:**  $\cos x = \pm \sqrt{2} \cos \frac{\alpha}{2}$

$$\sec(x + \alpha) + \sec(x - \alpha) = 2 \sec x$$

$$\Rightarrow \frac{1}{\cos(x + \alpha)} + \frac{1}{\cos(x - \alpha)} = \frac{2}{\cos x}$$

$$\left\{ \because \sec x = \frac{1}{\cos x} \right\}$$

$$\Rightarrow \frac{\cos(x - \alpha) + \cos(x + \alpha)}{\cos(x + \alpha) \cos(x - \alpha)} = \frac{2}{\cos x}$$

$$\left\{ \because \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \right\}$$

$$\Rightarrow \frac{2 \cos\left(\frac{x + \alpha + x - \alpha}{2}\right) \cos\left(\frac{x + \alpha - x + \alpha}{2}\right)}{\cos(x + \alpha) \cos(x - \alpha)} = \frac{2}{\cos x}$$

$$\Rightarrow \frac{2 \cos\left(\frac{2x}{2}\right) \cos\left(\frac{2\alpha}{2}\right)}{2 \cos(x + \alpha) \cos(x - \alpha)} = \frac{1}{\cos x}$$

$$\{\because 2 \cos A \cos B = \cos(A + B) + \cos(A - B)\}$$

$$\Rightarrow \frac{2 \cos x \cos \alpha}{\cos(x + \alpha + x - \alpha) + \cos(x + \alpha - x + \alpha)} = \frac{1}{\cos x}$$

$$\Rightarrow \frac{2 \cos x \cos \alpha}{\cos 2x + \cos 2\alpha} = \frac{1}{\cos x}$$

$$\Rightarrow 2 \cos^2 x \cos \alpha = \cos 2x + \cos 2\alpha$$

$$\Rightarrow 2 \cos^2 x \cos \alpha = 2 \cos^2 x - 1 + \cos 2\alpha$$

$$\{\because \cos 2x = 2 \cos^2 x - 1\}$$

$$\Rightarrow 2 \cos^2 x \cos \alpha - 2 \cos^2 x = \cos 2\alpha - 1$$

$$\Rightarrow 2 \cos^2 x (\cos \alpha - 1) = 2 \cos^2 \alpha - 1 - 1$$

$$\{\because \cos 2x = 2 \cos^2 x - 1\}$$

$$\Rightarrow 2 \cos^2 x = \frac{2 \cos^2 \alpha - 2}{\cos \alpha - 1}$$

$$\Rightarrow 2 \cos^2 x = \frac{2(\cos^2 \alpha - 1)}{\cos \alpha - 1}$$

$$\Rightarrow \cos^2 x = \frac{(\cos \alpha - 1)(\cos \alpha + 1)}{\cos \alpha - 1}$$

$$\Rightarrow \cos^2 x = \cos \alpha + 1$$

$$\Rightarrow \cos^2 x = 2 \cos^2 \frac{\alpha}{2} - 1 + 1$$

$$\left\{ \because \cos x = 2 \cos^2 \frac{x}{2} - 1 \right\}$$

$$\Rightarrow \cos^2 x = 2 \cos^2 \frac{\alpha}{2}$$

$$\Rightarrow \cos x = \pm \sqrt{2 \cos^2 \frac{\alpha}{2}}$$

$$\Rightarrow \cos x = \pm \sqrt{2} \cos \frac{\alpha}{2}$$

**Hence Proved**

#### 42. Question

If  $\cos \alpha + \cos \beta = \frac{1}{3}$  and  $\sin \alpha + \sin \beta = \frac{1}{4}$ , prove that  $\cos \frac{\alpha - \beta}{2} = \pm \frac{5}{24}$ .

**Answer**

**Given:**  $\cos \alpha + \cos \beta = \frac{1}{3}$  &  $\sin \alpha + \sin \beta = \frac{1}{4}$

**To prove:**  $\cos \frac{\alpha - \beta}{2} = \pm \frac{5}{24}$

$$\sin \alpha + \sin \beta = \frac{1}{4}$$

Squaring both sides, we get

$$\Rightarrow (\sin \alpha + \sin \beta)^2 = \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta = \frac{1}{16} \dots \dots \dots (1)$$

$$\cos \alpha + \cos \beta = \frac{1}{3}$$

Squaring both sides, we get

$$\Rightarrow (\cos \alpha + \cos \beta)^2 = \left(\frac{1}{3}\right)^2$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = \frac{1}{9} \dots \dots \dots (2)$$

Adding equation (1) and (2), we get

$$\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = \frac{1}{16} + \frac{1}{9}$$

$$\Rightarrow \sin^2 \alpha + \cos^2 \alpha + \sin^2 \beta + \cos^2 \beta + 2 \sin \alpha \sin \beta + 2 \cos \alpha \cos \beta = \frac{16 + 9}{(16)(9)}$$

$$\Rightarrow 1 + 1 + 2(\sin \alpha \sin \beta + \cos \alpha \cos \beta) = \frac{25}{144}$$

We know,

$$\sin A \sin B + \cos A \cos B = \cos (A - B)$$

Therefore,

$$\Rightarrow 2 + 2(\cos(\alpha - \beta)) = \frac{25}{144}$$

$$\Rightarrow 2(\cos(\alpha - \beta)) = \frac{25}{144} - 2$$

$$\Rightarrow 2 \cos(\alpha - \beta) = \frac{25 - 288}{144}$$

$$\Rightarrow \cos(\alpha - \beta) = -\frac{253}{288}$$

$$\left\{ \because \cos x = 2 \cos^2 \frac{x}{2} - 1 \right\}$$

$$\Rightarrow 2 \cos^2 \frac{(\alpha - \beta)}{2} - 1 = -\frac{253}{288}$$

$$\Rightarrow 2 \cos^2 \frac{(\alpha - \beta)}{2} = 1 - \frac{253}{288}$$

$$\Rightarrow 2 \cos^2 \frac{(\alpha - \beta)}{2} = \frac{288 - 253}{288}$$

$$\Rightarrow 2 \cos^2 \frac{(\alpha - \beta)}{2} = \frac{25}{288}$$

$$\Rightarrow \cos^2 \frac{(\alpha - \beta)}{2} = \frac{25}{576}$$

$$\Rightarrow \cos \frac{\alpha - \beta}{2} = \pm \sqrt{\frac{25}{576}}$$

$$\Rightarrow \cos \frac{\alpha - \beta}{2} = \pm \frac{5}{24}$$

**Hence Proved**

#### 43. Question

If  $\sin \alpha = \frac{4}{5}$  and  $\cos \beta = \frac{5}{13}$ , prove that  $\cos \frac{\alpha - \beta}{2} = \frac{8}{\sqrt{65}}$ .

**Answer**

**Given:**  $\sin \alpha = \frac{4}{5}$  &  $\cos \beta = \frac{5}{13}$

**To prove:**  $\cos \frac{\alpha - \beta}{2} = \frac{8}{\sqrt{65}}$

Proof:

We know,

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\Rightarrow \cos^2 \alpha = 1 - \sin^2 \alpha$$

$$\Rightarrow \cos \alpha = \sqrt{1 - \sin^2 \alpha}$$

$$\Rightarrow \cos \alpha = \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

$$\Rightarrow \cos \alpha = \sqrt{1 - \frac{16}{25}}$$

$$\Rightarrow \cos \alpha = \sqrt{\frac{9}{25}}$$

$$\Rightarrow \cos \alpha = \frac{3}{5}$$

Similarly,

$$\sin^2 \beta + \cos^2 \beta = 1$$

$$\Rightarrow \sin^2 \beta = 1 - \cos^2 \beta$$

$$\Rightarrow \sin \beta = \sqrt{1 - \cos^2 \beta}$$

$$\Rightarrow \sin \beta = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \sin \beta = \sqrt{1 - \frac{25}{169}}$$

$$\Rightarrow \sin \beta = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \sin \beta = \frac{12}{13}$$

**Identity used:**

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\Rightarrow \cos(\alpha - \beta) = \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$$

$$\Rightarrow 2 \cos^2\left(\frac{\alpha - \beta}{2}\right) - 1 = \frac{15}{65} + \frac{48}{65}$$

$$\Rightarrow 2 \cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{63}{65} + 1$$

$$\Rightarrow 2 \cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{63 + 65}{65}$$

$$\Rightarrow 2 \cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{128}{65}$$

$$\Rightarrow \cos^2\left(\frac{\alpha - \beta}{2}\right) = \frac{64}{65}$$

$$\Rightarrow \cos\left(\frac{\alpha - \beta}{2}\right) = \sqrt{\frac{64}{65}}$$

$$\Rightarrow \cos\left(\frac{\alpha - \beta}{2}\right) = \frac{8}{\sqrt{65}}$$

**Hence Proved**

#### 44 A. Question

If  $a \cos 2x + b \sin 2x = c$  has  $\alpha$  and  $\beta$  as its roots, then prove that

$$\tan \alpha + \tan \beta = \frac{2b}{a + c}$$

**Answer**

**Given:**  $a \cos 2x + b \sin 2x = c$

**To prove:**  $\tan \alpha + \tan \beta = \frac{2b}{a + c}$

We know,

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

Therefore,

$$a \cos 2x + b \sin 2x = c$$

$$\Rightarrow a\left(\frac{1 - \tan^2 x}{1 + \tan^2 x}\right) + b\left(\frac{2 \tan x}{1 + \tan^2 x}\right) = c$$

$$\Rightarrow \frac{a(1 - \tan^2 x)}{1 + \tan^2 x} + \frac{2b \tan x}{1 + \tan^2 x} = c$$

$$\Rightarrow \frac{a(1 - \tan^2 x) + 2b \tan x}{1 + \tan^2 x} = c$$

$$\Rightarrow a(1 - \tan^2 x) + 2b \tan x = c(1 + \tan^2 x)$$

$$\Rightarrow 2b \tan x + a - a \tan^2 x = c + c \tan^2 x$$

$$\Rightarrow 2b \tan x + a - a \tan^2 x - c - c \tan^2 x = 0$$

$$\Rightarrow (-a - c) \tan^2 x + 2b \tan x + a - c = 0$$

We know,

If m and n are roots of the equation  $ax^2 + bx + c = 0$

then,

$$\text{Sum of the roots}(m+n), = -\frac{b}{a}$$

Therefore,

If  $\tan \alpha$  and  $\tan \beta$  are the roots of the equation

$$(-a - c) \tan^2 x + 2b \tan x + a - c = 0$$

then,

$$\tan \alpha + \tan \beta = \frac{-2b}{-a - c}$$

$$\Rightarrow \tan \alpha + \tan \beta = \frac{-2b}{-(a + c)}$$

$$\Rightarrow \tan \alpha + \tan \beta = \frac{2b}{a + c}$$

**Hence Proved**

#### 44 B. Question

If  $a \cos 2x + b \sin 2x = c$  has  $\alpha$  and  $\beta$  as its roots, then prove that

$$\tan \alpha \tan \beta = \frac{c - a}{c + a}$$

**Answer**

**Given:**  $a \cos 2x + b \sin 2x = c$

**To prove:**  $\tan \alpha \tan \beta = \frac{c - a}{c + a}$

We know,

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

Therefore,

$$a \cos 2x + b \sin 2x = c$$

$$\Rightarrow a \left( \frac{1 - \tan^2 x}{1 + \tan^2 x} \right) + b \left( \frac{2 \tan x}{1 + \tan^2 x} \right) = c$$

$$\Rightarrow \frac{a(1 - \tan^2 x)}{1 + \tan^2 x} + \frac{2b \tan x}{1 + \tan^2 x} = c$$

$$\Rightarrow \frac{a(1 - \tan^2 x) + 2b \tan x}{1 + \tan^2 x} = c$$

$$\Rightarrow a(1 - \tan^2 x) + 2b \tan x = c(1 + \tan^2 x)$$

$$\Rightarrow 2b \tan x + a - a \tan^2 x = c + c \tan^2 x$$

$$\Rightarrow 2b \tan x + a - a \tan^2 x - c - c \tan^2 x = 0$$

$$\Rightarrow (-a - c) \tan^2 x + 2b \tan x + a - c = 0$$

We know,

If  $m$  and  $n$  are roots of the equation  $ax^2 + bx + c = 0$

then,

$$\text{Product of the roots}(mn), = \frac{c}{a}$$

Therefore,

If  $\tan \alpha$  and  $\tan \beta$  are the roots of the equation

$$(-a - c) \tan^2 x + 2b \tan x + a - c = 0$$

then,

$$\tan \alpha \tan \beta = \frac{a - c}{-a - c}$$

$$\Rightarrow \tan \alpha \tan \beta = \frac{-(c - a)}{-(c + a)}$$

$$\Rightarrow \tan \alpha \tan \beta = \frac{c - a}{c + a}$$

**Hence Proved**

#### 44 C. Question

If  $a \cos 2x + b \sin 2x = c$  has  $\alpha$  and  $\beta$  as its roots, then prove that

$$\tan(\alpha + \beta) = \frac{b}{a}$$

**Answer**

$$\text{To prove: } \tan(\alpha + \beta) = \frac{b}{a}$$

We know,

$$\tan(x + y) = \frac{\tan x + \tan y}{1 + \tan x \tan y}$$

Therefore,

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 + \tan \alpha \tan \beta}$$

From previous question:

$$\tan \alpha + \tan \beta = \frac{2b}{a+c} \text{ \& } \tan \alpha \tan \beta = \frac{c-a}{c+a}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{\frac{2b}{a+c}}{1 + \frac{c-a}{c+a}}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{\frac{2b}{a+c}}{\frac{c+a+c-a}{c+a}}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{2b}{2c}$$

$$\Rightarrow \tan(\alpha + \beta) = \frac{b}{c}$$

**Hence Proved**

#### 45. Question

If  $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$ , then prove that  $\cos 2\alpha + \cos 2\beta = -2 \cos (\alpha + \beta)$ .

**Answer**

**Given:**  $\cos \alpha + \cos \beta = \sin \alpha + \sin \beta = 0$

**To prove:**  $\cos 2\alpha + \cos 2\beta = -2 \cos (\alpha + \beta)$

Proof:

$$\cos \alpha + \cos \beta = 0$$

Squaring both sides:

$$\Rightarrow (\cos \alpha + \cos \beta)^2 = (0)^2$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta = 0 \dots\dots(1)$$

$$\sin \alpha + \sin \beta = 0$$

Squaring both sides:

$$\Rightarrow (\sin \alpha + \sin \beta)^2 = (0)^2$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta = 0 \dots\dots\dots(2)$$

Subtracting equation (1) from (2), we get

$$\cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta - (\sin^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta) = 0$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + 2 \cos \alpha \cos \beta - \sin^2 \alpha - \sin^2 \beta - 2 \sin \alpha \sin \beta = 0$$

$$\Rightarrow \cos^2 \alpha - \sin^2 \alpha + \cos^2 \beta - \sin^2 \beta + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = 0$$

$$\{\because \cos^2 x - \sin^2 x = 2x \text{ \& }$$

$$\cos A \cos B - \sin A \sin B = \cos(A + B)\}$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + 2 \cos (\alpha + \beta) = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta = -2 \cos (\alpha + \beta)$$

**Hence Proved**

#### Exercise 9.2



## 1. Question

Prove that:

$$\sin 5x = 5 \sin x - 20 \sin^3 x + 16 \sin^5 x$$

### Answer

LHS is

$$\sin 5x = \sin(3x+2x)$$

But we know,

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \dots (i)$$

$$\Rightarrow \sin 5x = \sin 3x \cos 2x + \cos 3x \sin 2x$$

$$\Rightarrow \sin 5x = \sin (2x+x) \cos 2x + \cos (2x+x) \sin 2x \dots (ii)$$

And

$$\cos (x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \dots (iii)$$

Now substituting equation (i) and (iii) in equation (ii), we get

$$\Rightarrow \sin 5x = (\sin 2x \cos x + \cos 2x \sin x) \cos 2x + (\cos 2x \cos x - \sin 2x \sin x) \sin 2x$$

$$\Rightarrow \sin 5x = \sin 2x \cos 2x \cos x + \cos^2 2x \sin x + (\sin 2x \cos 2x \cos x - \sin^2 2x \sin x)$$

$$\Rightarrow \sin 5x = 2\sin 2x \cos 2x \cos x + \cos^2 2x \sin x - \sin^2 2x \sin x \dots (iv)$$

$$\text{Now } \sin 2x = 2\sin x \cos x \dots (v)$$

$$\text{And } \cos 2x = \cos^2 x - \sin^2 x \dots (vi)$$

Substituting equation (v) and (vi) in equation (iv), we get

$$\Rightarrow \sin 5x = 2(2\sin x \cos x)(\cos^2 x - \sin^2 x)\cos x + (\cos^2 x - \sin^2 x)^2 \sin x - (2\sin x \cos x)^2 \sin x$$

$$\Rightarrow \sin 5x = 4(\sin x \cos^2 x)([1 - \sin^2 x] - \sin^2 x) + ([1 - \sin^2 x] - \sin^2 x)^2 \sin x - (4\sin^2 x \cos^2 x)\sin x \quad (\text{as } \cos^2 x + \sin^2 x = 1 \\ \Rightarrow \cos^2 x = 1 - \sin^2 x)$$

$$\Rightarrow \sin 5x = 4(\sin x [1 - \sin^2 x])(1 - 2\sin^2 x) + (1 - 2\sin^2 x)^2 \sin x - 4\sin^3 x [1 - \sin^2 x]$$

$$\Rightarrow \sin 5x = 4\sin x(1 - \sin^2 x)(1 - 2\sin^2 x) + (1 - 4\sin^2 x + 4\sin^4 x)\sin x - 4\sin^3 x + 4\sin^5 x$$

$$\Rightarrow \sin 5x = (4\sin x - 4\sin^3 x)(1 - 2\sin^2 x) + \sin x - 4\sin^3 x + 4\sin^5 x - 4\sin^3 x + 4\sin^5 x$$

$$\Rightarrow \sin 5x = 4\sin x - 8\sin^3 x - 4\sin^3 x + 8\sin^5 x + \sin x - 8\sin^3 x + 8\sin^5 x$$

$$\Rightarrow \sin 5x = 5\sin x - 20\sin^3 x + 16\sin^5 x$$

Hence LHS = RHS

[Hence proved]

## 2. Question

Prove that:

$$4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\cos 10^\circ + \sin 20^\circ)$$

### Answer

We know that

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = \cos 30^\circ \Rightarrow \sin (3 \times 20^\circ) = \cos (3 \times 10^\circ)$$

$$\Rightarrow 3\sin 20^\circ - 4\sin^3 20^\circ = 4\cos^3 10^\circ - 3\cos 10^\circ$$

(as  $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$  and  $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$ )

$$\Rightarrow 4(\cos^3 10^\circ + \sin^3 20^\circ) = 3(\sin 20^\circ + \cos 10^\circ)$$

LHS=RHS

Hence proved

### 3. Question

Prove that:

$$\cos^3 x \sin 3x + \sin^3 x \cos 3x = \frac{3}{4} \sin 4x$$

### Answer

We know that,

$$\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$$

$$\Rightarrow 4\cos^3 \theta = \cos 3\theta + 3\cos \theta$$

$$\Rightarrow \cos^3 \theta = \frac{\cos 3\theta + 3\cos \theta}{4} \dots (i)$$

And similarly

$$\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$$

$$\Rightarrow 4\sin^3 \theta = 3\sin \theta - \sin 3\theta$$

$$\Rightarrow \sin^3 \theta = \frac{3\sin \theta - \sin 3\theta}{4} \dots (ii)$$

Now,

$$\text{LHS} = \cos^3 x \sin 3x + \sin^3 x \cos 3x$$

Substituting the values from equation (i) and (ii), we get

$$\Rightarrow \left( \frac{\cos 3x + 3\cos x}{4} \right) \sin 3x + \left( \frac{3\sin x - \sin 3x}{4} \right) \cos 3x$$

$$\Rightarrow \frac{1}{4} (\sin 3x \cos 3x + 3\sin 3x \cos x + 3\sin x \cos 3x - \sin 3x \cos 3x)$$

$$\Rightarrow \frac{1}{4} (3[\sin 3x \cos x + \sin x \cos 3x] + 0)$$

$$\Rightarrow \frac{1}{4} (3\sin(3x + x))$$

(as  $\sin(x+y) = \sin x \cos y + \cos x \sin y$ )

$$\Rightarrow \frac{3}{4} \sin 4x$$

=RHS

Hence Proved

### 4. Question

Prove that:

$$\tan x \tan\left(x + \frac{\pi}{3}\right) + \tan x \tan\left(\frac{\pi}{3} - x\right) + \tan\left(x + \frac{\pi}{3}\right) \tan\left(x - \frac{\pi}{3}\right) = -3$$

**Answer**

$$\text{LHS} = \tan x \tan\left(x + \frac{\pi}{3}\right) + \tan x \tan\left(\frac{\pi}{3} - x\right) + \tan\left(x + \frac{\pi}{3}\right) \tan\left(x - \frac{\pi}{3}\right)$$

$$\Rightarrow \tan x \left( \frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \tan \frac{\pi}{3}} \right) + \tan x \left( \frac{\tan \frac{\pi}{3} - \tan x}{1 + \tan x \tan \frac{\pi}{3}} \right) + \left( \frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \tan \frac{\pi}{3}} \right) \left( \frac{\tan \frac{\pi}{3} - \tan x}{1 + \tan x \tan \frac{\pi}{3}} \right)$$

$$\left( \because \tan(A + B) = \left( \frac{\tan A + \tan B}{1 - \tan A \tan B} \right) \text{ and } \tan(A - B) = \left( \frac{\tan A - \tan B}{1 + \tan A \tan B} \right) \right)$$

$$\Rightarrow \tan x \left( \frac{\tan x + \sqrt{3}}{1 - \tan x (\sqrt{3})} \right) + \tan x \left( \frac{\sqrt{3} - \tan x}{1 + \tan x (\sqrt{3})} \right) + \left( \frac{\tan x + \sqrt{3}}{1 - \tan x (\sqrt{3})} \right) \left( \frac{\sqrt{3} - \tan x}{1 + \tan x (\sqrt{3})} \right)$$

$$\left( \text{as } \tan \frac{\pi}{3} = \sqrt{3} \right)$$

$$= \left( \frac{(1 + \tan x (\sqrt{3})) \tan x (\tan x + \sqrt{3}) + (1 - \tan x (\sqrt{3})) \tan x (\sqrt{3} - \tan x) + (\tan x + \sqrt{3})(\sqrt{3} - \tan x)}{(1 - \tan x (\sqrt{3})) (1 + \tan x (\sqrt{3}))} \right)$$

$$= \left( \frac{(1 + \sqrt{3} \tan x) \tan x (\tan x + \sqrt{3}) + (1 - \sqrt{3} \tan x) \tan x (\sqrt{3} - \tan x) + (\tan^2 x - (\sqrt{3})^2)}{(1 - (\sqrt{3} \tan x)^2)} \right)$$

$$= \left( \frac{(\tan x + \sqrt{3} \tan^2 x)(\tan x + \sqrt{3}) + (\tan x - \sqrt{3} \tan^2 x)(\sqrt{3} - \tan x) + (\tan^2 x - 3)}{(1 - (\sqrt{3} \tan x)^2)} \right)$$

$$= \left( \frac{(\tan^2 x + \sqrt{3} \tan x + \sqrt{3} \tan^3 x + 3 \tan^2 x) + (\sqrt{3} \tan x - 3 \tan^2 x - \tan^2 x + \sqrt{3} \tan^3 x) + (\tan^2 x - 3)}{(1 - (\sqrt{3} \tan x)^2)} \right)$$

$$= \left( \frac{(0 + 2\sqrt{3} \tan x + 2\sqrt{3} \tan^3 x + 3 \tan^2 x) + (\tan^2 x - 3)}{(1 - (\sqrt{3} \tan x)^2)} \right)$$

$$= \left( \frac{2\sqrt{3} \tan x + 2\sqrt{3} \tan^3 x + 4 \tan^2 x - 3}{(1 - 3 \tan^2 x)} \right)$$

$$\neq -3$$

Hence LHS  $\neq$  RHS

## 5. Question

Prove that:

$$\tan x + \tan\left(\frac{\pi}{3} + x\right) - \tan\left(\frac{\pi}{3} - x\right) = 3 \tan 3x$$

**Answer**

$$\text{LHS} = \tan x + \tan\left(\frac{\pi}{3} + x\right) - \tan\left(\frac{\pi}{3} - x\right)$$

$$\Rightarrow \tan x + \left(\frac{\tan \frac{\pi}{3} + \tan x}{1 - \tan x \tan \frac{\pi}{3}}\right) - \left(\frac{\tan \frac{\pi}{3} - \tan x}{1 + \tan x \tan \frac{\pi}{3}}\right)$$

$$\left(\because \tan(A+B) = \left(\frac{\tan A + \tan B}{1 - \tan A \tan B}\right) \text{ and } \tan(A-B) = \left(\frac{\tan A - \tan B}{1 + \tan A \tan B}\right)\right)$$

$$\Rightarrow \tan x + \left(\frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \tan x}\right) - \left(\frac{\sqrt{3} - \tan x}{1 + \sqrt{3} \tan x}\right)$$

$$\Rightarrow \tan x + \left(\frac{(1 + \sqrt{3} \tan x)(\sqrt{3} + \tan x) - (1 - \sqrt{3} \tan x)(\sqrt{3} - \tan x)}{(1 - \tan x(\sqrt{3}))(1 + \tan x(\sqrt{3}))}\right)$$

$$\Rightarrow$$

$$= \tan x$$

$$+ \left(\frac{(\sqrt{3} + 3 \tan x + \tan x + \sqrt{3} \tan^2 x) - (\sqrt{3} - 3 \tan x - \tan x + \sqrt{3} \tan^2 x)}{(1 - 3 \tan^2 x)}\right)$$

$$\Rightarrow \tan x + \left(\frac{(0 + 6 \tan x + 2 \tan x + 0)}{(1 - 3 \tan^2 x)}\right)$$

$$\Rightarrow \tan x + \left(\frac{8 \tan x}{(1 - 3 \tan^2 x)}\right)$$

$$\Rightarrow \left(\frac{\tan x (1 - 3 \tan^2 x) + 8 \tan x}{(1 - 3 \tan^2 x)}\right)$$

$$\Rightarrow \left(\frac{(\tan x - 3 \tan^3 x) + 8 \tan x}{(1 - 3 \tan^2 x)}\right)$$

$$\Rightarrow \left(\frac{9 \tan x - 3 \tan^3 x}{(1 - 3 \tan^2 x)}\right)$$

$$\Rightarrow 3 \left(\frac{3 \tan x - \tan^3 x}{(1 - 3 \tan^2 x)}\right)$$

$$\Rightarrow 3 \tan 3x = \text{RHS}$$

$$\left(\text{as } \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}\right)$$

Hence proved

**6. Question**

Prove that:

$$\cot x + \cot\left(\frac{\pi}{3} + x\right) - \cot\left(\frac{\pi}{3} - x\right) = 3 \cot 3x$$

**Answer**

$$\text{LHS} = \cot x + \cot\left(\frac{\pi}{3} + x\right) - \cot\left(\frac{\pi}{3} - x\right)$$

$$\Rightarrow \frac{1}{\tan x} + \frac{1}{\tan\left(\frac{\pi}{3} + x\right)} - \frac{1}{\tan\left(\frac{\pi}{3} - x\right)}$$

$$\Rightarrow \frac{1}{\tan x} + \left( \frac{1 - \tan x \tan \frac{\pi}{3}}{\tan \frac{\pi}{3} + \tan x} \right) - \left( \frac{1 + \tan x \tan \frac{\pi}{3}}{\tan \frac{\pi}{3} - \tan x} \right)$$

$$\left( \because \tan(A + B) = \left( \frac{\tan A + \tan B}{1 - \tan A \tan B} \right) \text{ and } \tan(A - B) = \left( \frac{\tan A - \tan B}{1 + \tan A \tan B} \right) \right)$$

$$\Rightarrow \frac{1}{\tan x} + \left( \frac{1 - \sqrt{3} \tan x}{\sqrt{3} + \tan x} \right) - \left( \frac{1 + \sqrt{3} \tan x}{\sqrt{3} - \tan x} \right)$$

$$\Rightarrow \frac{1}{\tan x} + \left( \frac{(1 - \sqrt{3} \tan x)(\sqrt{3} - \tan x) - (1 + \sqrt{3} \tan x)(\sqrt{3} + \tan x)}{(\sqrt{3} + \tan x)(\sqrt{3} - \tan x)} \right)$$

$$\Rightarrow \frac{1}{\tan x} + \left( \frac{(\sqrt{3} - \tan x - 3 \tan x + \sqrt{3} \tan^2 x) - (\sqrt{3} + 3 \tan x + \tan x + \sqrt{3} \tan^2 x)}{(3 - \tan^2 x)} \right)$$

$$\Rightarrow \frac{1}{\tan x} + \left( \frac{(0 - 4 \tan x - 4 \tan x + 0)}{(3 - \tan^2 x)} \right)$$

$$\Rightarrow \frac{1}{\tan x} - \left( \frac{8 \tan x}{(3 - \tan^2 x)} \right)$$

$$\Rightarrow \left( \frac{(3 - \tan^2 x) - 8 \tan^2 x}{\tan x (3 - \tan^2 x)} \right)$$

$$\Rightarrow \left( \frac{3 - 9 \tan^2 x}{(3 \tan x - \tan^3 x)} \right)$$

$$\Rightarrow 3 \left( \frac{1 - 3 \tan^2 x}{(3 \tan x - \tan^3 x)} \right)$$

$$\Rightarrow 3 \times \frac{1}{\tan 3x}$$

$$\left( \text{as } \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \right)$$

$$\Rightarrow 3 \cot 3x = \text{RHS}$$

Hence proved

## 7. Question

Prove that:

$$\cot x + \cot\left(\frac{\pi}{3} + x\right) + \cot\left(\frac{2\pi}{3} + x\right) = 3 \cot 3x$$

**Answer**

$$\text{LHS} = \cot x + \cot\left(\frac{\pi}{3} + x\right) + \cot\left(\frac{2\pi}{3} + x\right)$$

We know,

$$\cot\left(\frac{2\pi}{3} + x\right) = \cot\left(\pi - \left(\frac{\pi}{3} - x\right)\right) = -\cot\left(\frac{\pi}{3} - x\right) \text{ (as } -\cot \theta = \cot(180^\circ - \theta)\text{)}$$

Hence the above LHS becomes

$$= \cot x + \cot\left(\frac{\pi}{3} + x\right) - \cot\left(\frac{\pi}{3} - x\right)$$

$$\Rightarrow \frac{1}{\tan x} + \frac{1}{\tan\left(\frac{\pi}{3} + x\right)} - \frac{1}{\tan\left(\frac{\pi}{3} - x\right)}$$

$$\Rightarrow \frac{1}{\tan x} + \left(\frac{1 - \tan x \tan \frac{\pi}{3}}{\tan \frac{\pi}{3} + \tan x}\right) - \left(\frac{1 + \tan x \tan \frac{\pi}{3}}{\tan \frac{\pi}{3} - \tan x}\right)$$

$$\left(\because \tan(A + B) = \left(\frac{\tan A + \tan B}{1 - \tan A \tan B}\right) \text{ and } \tan(A - B) = \left(\frac{\tan A - \tan B}{1 + \tan A \tan B}\right)\right)$$

$$\Rightarrow \frac{1}{\tan x} + \left(\frac{1 - \sqrt{3} \tan x}{\sqrt{3} + \tan x}\right) - \left(\frac{1 + \sqrt{3} \tan x}{\sqrt{3} - \tan x}\right)$$

$$\Rightarrow \frac{1}{\tan x} + \left(\frac{(1 - \sqrt{3} \tan x)(\sqrt{3} - \tan x) - (1 + \sqrt{3} \tan x)(\sqrt{3} + \tan x)}{(\sqrt{3} + \tan x)(\sqrt{3} - \tan x)}\right)$$

$$\Rightarrow$$

$$= \frac{1}{\tan x}$$

$$+ \left(\frac{(\sqrt{3} - \tan x - 3 \tan x + \sqrt{3} \tan^2 x) - (\sqrt{3} + 3 \tan x + \tan x + \sqrt{3} \tan^2 x)}{(3 - \tan^2 x)}\right)$$

$$\Rightarrow \frac{1}{\tan x} + \left(\frac{(0 - 4 \tan x - 4 \tan x + 0)}{(3 - \tan^2 x)}\right)$$

$$\Rightarrow \frac{1}{\tan x} - \left(\frac{8 \tan x}{(3 - \tan^2 x)}\right)$$

$$\Rightarrow \left(\frac{(3 - \tan^2 x) - 8 \tan^2 x}{\tan x (3 - \tan^2 x)}\right)$$

$$\Rightarrow \left(\frac{3 - 9 \tan^2 x}{(3 \tan x - \tan^3 x)}\right)$$

$$\Rightarrow 3 \left(\frac{1 - 3 \tan^2 x}{(3 \tan x - \tan^3 x)}\right)$$

$$\Rightarrow 3 \times \frac{1}{\tan 3x}$$

$$\left(\text{as } \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}\right)$$

$$\Rightarrow 3 \cot 3x = \text{RHS}$$

Hence proved

## 8. Question

Prove that:

$$\sin 5x = 5 \cos^4 x \sin x - 10 \cos^2 x \sin^3 x + \sin^5 x$$

### Answer

LHS is

$$\sin 5x = \sin(3x+2x)$$

But we know,

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \dots (i)$$

$$\Rightarrow \sin 5x = \sin 3x \cos 2x + \cos 3x \sin 2x$$

$$\Rightarrow \sin 5x = \sin (2x+x) \cos 2x + \cos (2x+x) \sin 2x \dots (ii)$$

And

$$\cos (x+y) = \cos(x)\cos(y) - \sin(x)\sin(y) \dots (iii)$$

Now substituting equation (i) and (iii) in equation (ii), we get

$$\Rightarrow \sin 5x = (\sin 2x \cos x + \cos 2x \sin x)(\cos 2x) + (\cos 2x \cos x - \sin 2x \sin x)(\sin 2x) \dots (iv)$$

$$\text{Now } \sin 2x = 2 \sin x \cos x \dots (v)$$

$$\text{And } \cos 2x = \cos^2 x - \sin^2 x \dots (vi)$$

Substituting equation (v) and (vi) in equation (iv), we get

$$\Rightarrow \sin 5x = [(2 \sin x \cos x) \cos x + (\cos^2 x - \sin^2 x) \sin x](\cos^2 x - \sin^2 x) + [(\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x](2 \sin x \cos x)$$

$$\Rightarrow \sin 5x = [2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x](\cos^2 x - \sin^2 x) + [\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x](2 \sin x \cos x)$$

$$\Rightarrow \sin 5x = \cos^2 x [3 \sin x \cos^2 x - \sin^3 x] - \sin^2 x [3 \sin x \cos^2 x - \sin^3 x] + 2 \sin x \cos^4 x - 2 \sin^3 x \cos^2 x - 4 \sin^3 x \cos^2 x$$

$$\Rightarrow \sin 5x = 3 \sin x \cos^4 x - \sin^3 x \cos^2 x - 3 \sin^3 x \cos^2 x - \sin^5 x + 2 \sin x \cos^4 x - 2 \sin^3 x \cos^2 x - 4 \sin^3 x \cos^2 x$$

$$\Rightarrow \sin 5x = 5 \sin x \cos^4 x - 10 \sin^3 x \cos^2 x + \sin^5 x$$

Hence LHS = RHS

[Hence proved]

### 9. Question

Prove that:

$$\sin^3 x + \sin^3 \left( \frac{2\pi}{3} + x \right) + \sin^3 \left( \frac{4\pi}{3} + x \right) = -\frac{3}{4} \sin 3x$$

### Answer

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\Rightarrow 4 \sin^3 \theta = 3 \sin \theta - \sin 3\theta$$

$$\Rightarrow \sin^3 \theta = \frac{3 \sin \theta - \sin 3\theta}{4} \dots (i)$$

Now,

$$\text{LHS} = \sin^3 x + \sin^3 \left( \frac{2\pi}{3} + x \right) + \sin^3 \left( \frac{4\pi}{3} + x \right)$$

Substituting equation (i) in above LHS, we get

$$= \frac{3 \sin x - \sin 3x}{4} + \frac{3 \sin\left(\frac{2\pi}{3} + x\right) - \sin 3\left(\frac{2\pi}{3} + x\right)}{4} \\ + \frac{3 \sin\left(\frac{4\pi}{3} + x\right) - \sin 3\left(\frac{4\pi}{3} + x\right)}{4} \dots \text{(ii)}$$

We know,

$$\sin\left(\frac{2\pi}{3} + x\right) = \sin\left(\pi - \left(\frac{\pi}{3} - x\right)\right) = \sin\left(\frac{\pi}{3} - x\right) \dots \dots \text{(iii)} \text{ (as } \sin \theta = \sin(180^\circ - \theta)\text{)}$$

Similarly,

$$\sin\left(\frac{4\pi}{3} + x\right) = \sin\left(\pi + \left(\frac{\pi}{3} - x\right)\right) = -\sin\left(\frac{\pi}{3} - x\right) \dots \dots \text{(iv)} \text{ (as } -\sin \theta = \sin(180^\circ + \theta)\text{)}$$

Substituting the equation (iii) and (iv) in equation (ii), we get

$$= \frac{3 \sin x - \sin 3x}{4} + \frac{3 \sin\left\{\pi - \left(\frac{\pi}{3} - x\right)\right\} - \sin(2\pi + 3x)}{4} \\ + \frac{3 \sin\left\{\pi + \left(\frac{\pi}{3} - x\right)\right\} - \sin(4\pi + 3x)}{4} \\ = \frac{1}{4} \left[ 3 \sin x - \sin 3x + 3 \sin\left\{\pi - \left(\frac{\pi}{3} - x\right)\right\} - \sin(2\pi + 3x) + 3 \sin\left\{\pi + \left(\frac{\pi}{3} - x\right)\right\} \right. \\ \left. - \sin(4\pi + 3x) \right] \\ = \frac{1}{4} \left[ 3 \sin x - \sin 3x + 3 \sin\left(\frac{\pi}{3} - x\right) - \sin(3x) - 3 \sin\left(\frac{\pi}{3} + x\right) - \sin(3x) \right] \\ = \frac{1}{4} \left[ 3 \sin x - 3 \sin 3x + 3 \left\{ \sin\left(\frac{\pi}{3} - x\right) - 3 \sin\left(\frac{\pi}{3} + x\right) \right\} \right] \\ = \frac{1}{4} \left[ 3 \sin x - 3 \sin 3x + 3 \left\{ \sin\left(\frac{\pi}{3} - x\right) - 3 \sin\left(\frac{\pi}{3} + x\right) \right\} \right]$$

We know,

$$\left[ \because \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2} \right]$$

Substituting this in the above equation, we get

$$= \frac{1}{4} \left[ 3 \sin x - 3 \sin 3x + 3 \left\{ 2 \cos\left(\frac{\frac{\pi}{3} - x + \frac{\pi}{3} + x}{2}\right) \sin\left(\frac{\frac{\pi}{3} - x - \frac{\pi}{3} - x}{2}\right) \right\} \right] \\ = \frac{3}{4} \left[ \sin x - \sin 3x + 2 \left\{ \cos\left(\frac{\pi}{3}\right) \sin(-x) \right\} \right] \\ = \frac{3}{4} \left[ \sin x - \sin 3x - 2 \left\{ \frac{1}{2} \sin x \right\} \right] \\ = -\frac{3}{4} \sin 3x = \text{RHS}$$

Hence proved

## 10. Question

Prove that:



$$\left| \sin x \sin \left( \frac{\pi}{3} - x \right) \sin \left( \frac{\pi}{3} + x \right) \right| \leq \frac{1}{4} \text{ For all values of } x.$$

### Answer

We know

$$\sin (A+B)\sin (A-B)=\sin^2A-\sin^2B$$

So the above LHS becomes,

$$\left| \sin x \sin \left( \frac{\pi}{3} - x \right) \sin \left( \frac{\pi}{3} + x \right) \right|$$

$$\Rightarrow \left| \sin x \left\{ \sin^2 \frac{\pi}{3} - \sin^2 x \right\} \right|$$

$$\Rightarrow \left| \sin x \left\{ \left( \frac{\sqrt{3}}{2} \right)^2 - \sin^2 x \right\} \right|$$

$$\Rightarrow \left| \sin x \left\{ \frac{3}{4} - \sin^2 x \right\} \right|$$

$$\Rightarrow \frac{1}{4} |3 \sin x - 4 \sin^3 x|$$

$$\text{But } 3 \sin x - 4 \sin^3 x = \sin 3x$$

$$\Rightarrow \frac{1}{4} |\sin 3x|$$

But  $|\sin \theta| \leq 1$  for all values of  $x$

$$\text{Hence LHS} \leq \frac{1}{4}$$

$$\text{Therefore } \left| \sin x \sin \left( \frac{\pi}{3} - x \right) \sin \left( \frac{\pi}{3} + x \right) \right| \leq \frac{1}{4} \text{ For all values of } x$$

### 11. Question

Prove that:

$$\left| \cos x \cos \left( \frac{\pi}{3} - x \right) \cos \left( \frac{\pi}{3} + x \right) \right| \leq \frac{1}{4} \text{ for all values of } x$$

### Answer

We know

$$\cos (A+B)\cos (A-B)=\cos^2A-\sin^2B$$

So the above LHS becomes,

$$\left| \cos x \cos \left( \frac{\pi}{3} - x \right) \cos \left( \frac{\pi}{3} + x \right) \right|$$

$$\Rightarrow \left| \cos x \left\{ \cos^2 \frac{\pi}{3} - \sin^2 x \right\} \right|$$

$$\Rightarrow \left| \cos x \left\{ \left( \frac{1}{2} \right)^2 - \sin^2 x \right\} \right|$$

$$\Rightarrow \left| \cos x \left\{ \frac{1}{4} - (1 - \cos^2 x) \right\} \right|$$

$$\Rightarrow \frac{1}{4} |\cos x - 4 \cos x + 4 \cos^3 x|$$

$$\Rightarrow \frac{1}{4} |4 \cos^3 x - 3 \cos x|$$

$$\text{But } 4 \cos^3 x - 3 \cos x = \cos 3x$$

$$\Rightarrow \frac{1}{4} |\cos 3x|$$

But  $|\cos \theta| \leq 1$  for all values of  $x$

$$\text{Hence LHS} \leq \frac{1}{4}$$

Therefore  $\left| \cos x \cos \left( \frac{\pi}{3} - x \right) \cos \left( \frac{\pi}{3} + x \right) \right| \leq \frac{1}{4}$  For all values of  $x$

### Exercise 9.3

#### 1. Question

Prove that:

$$\sin^2 \frac{2\pi}{5} - \sin^2 \frac{\pi}{3} = \frac{\sqrt{5} - 1}{8}$$

**Answer**

$$\begin{aligned} \text{LHS} &= \sin^2 \frac{2\pi}{5} - \sin^2 \frac{\pi}{3} \\ &= \sin^2 \left( \frac{\pi}{2} - \frac{\pi}{10} \right) - \sin^2 \frac{\pi}{3} \end{aligned}$$

$$\text{But } \sin (90^\circ - \theta) = \cos \theta$$

Then the above equation becomes,

$$= \cos^2 \left( \frac{\pi}{10} \right) - \left( \frac{\sqrt{3}}{2} \right)^2$$

$$\text{And } \therefore \cos \frac{\pi}{10} = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

Hence the above equation becomes,

$$= \left( \frac{\sqrt{10+2\sqrt{5}}}{4} \right)^2 - \frac{3}{4}$$

$$= \frac{10+2\sqrt{5}}{16} - \frac{3}{4}$$

$$= \frac{10+2\sqrt{5}-12}{16}$$

$$= \frac{2\sqrt{5}-2}{16}$$

$$= \frac{\sqrt{5}-1}{8} = \text{RHS}$$

Hence proved

## 2. Question

Prove that:

$$\sin^2 24^\circ - \sin^2 6^\circ = \frac{\sqrt{5}-1}{8}$$

### Answer

$$\text{LHS} = \sin^2 24^\circ - \sin^2 6^\circ$$

$$\text{But } \sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$$

Then the above equation becomes,

$$= \sin(24^\circ + 6^\circ) \sin(24^\circ - 6^\circ)$$

$$= \sin(30^\circ) \sin(18^\circ)$$

$$\text{And } \therefore \sin(18^\circ) = \frac{\sqrt{5}-1}{4}$$

Hence the above equation becomes,

$$= \frac{1}{2} \times \frac{\sqrt{5}-1}{4}$$

$$\frac{\sqrt{5}-1}{8} = \text{RHS}$$

Hence proved

## 3. Question

Prove that:

$$\sin^2 42^\circ - \cos^2 78^\circ = \frac{\sqrt{5}+1}{8}$$

### Answer

$$\text{LHS} = \sin^2 42^\circ - \cos^2 78^\circ$$

$$\Rightarrow \sin^2(90^\circ - 48^\circ) - \cos^2(90^\circ - 12^\circ)$$

$$= \cos^2 48^\circ - \sin^2 12^\circ (\because \sin(90 - \theta) = \cos \theta \text{ and } \cos(90 - \theta) = \sin \theta)$$

$$\text{But } \cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$$

Then the above equation becomes,

$$= \cos(48^\circ + 12^\circ) \cos(48^\circ - 12^\circ)$$

$$= \cos(60^\circ) \cos(36^\circ)$$

$$\text{And } \therefore \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$

Hence the above equation becomes,

$$= \frac{1}{2} \times \frac{\sqrt{5}+1}{4}$$

$$= \frac{\sqrt{5}+1}{8} = \text{RHS}$$

Hence proved

#### 4. Question

Prove that:

$$\cos 78^\circ \cos 42^\circ \cos 36^\circ = \frac{1}{8}$$

#### Answer

$$\text{LHS} = \cos 78^\circ \cos 42^\circ \cos 36^\circ$$

Multiply and divide by 2, we get

$$= \frac{1}{2} (2 \cos 78^\circ \cos 42^\circ \cos 36^\circ)$$

$$\text{But } 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

Then the above equation becomes,

$$= \frac{1}{2} (\cos(78^\circ + 42^\circ) + \cos(78^\circ - 42^\circ)) \times \cos 36^\circ$$

$$= \frac{1}{2} (\cos 120^\circ + \cos 36^\circ) \cos 36^\circ$$

$$= \frac{1}{2} (\cos(180^\circ - 60^\circ) + \cos 36^\circ) \cos 36^\circ$$

$$\text{But } \cos(180^\circ - \theta) = -\cos \theta$$

So the above equation becomes,

$$= \frac{1}{2} (-\cos(60^\circ) + \cos 36^\circ) \cos 36^\circ$$

$$\text{And } \therefore \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$

Hence the above equation becomes,

$$= \frac{1}{2} \left( -\frac{1}{2} + \frac{\sqrt{5}+1}{4} \right) \left( \frac{\sqrt{5}+1}{4} \right)$$

$$= \frac{1}{2} \left( \frac{\sqrt{5}+1-2}{4} \right) \left( \frac{\sqrt{5}+1}{4} \right)$$

$$= \frac{1}{2} \left( \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{5}+1}{4} \right)$$

$$= \frac{1}{2} \left( \frac{(\sqrt{5})^2 - 1^2}{16} \right)$$

$$= \frac{1}{2} \left( \frac{5-1}{16} \right)$$

$$= \frac{1}{2} \left( \frac{4}{16} \right)$$

$$= \frac{1}{8} = \text{RHS}$$

Hence proved

## 5. Question

Prove that:

$$\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{16}$$

## Answer

$$\text{LHS} = \cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{7\pi}{15}$$

Multiply and divide by  $2 \sin \frac{\pi}{15}$ , we get

$$= \frac{\left(2 \sin \frac{\pi}{15} \cos \frac{\pi}{15}\right) \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{7\pi}{15}}{2 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$= \frac{\left(\sin \frac{2\pi}{15}\right) \cos \frac{2\pi}{15} \cos \frac{4\pi}{15} \cos \frac{7\pi}{15}}{2 \sin \frac{\pi}{15}}$$

Multiply and divide by 2, we get

$$= \frac{\left(2 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15}\right) \cos \frac{4\pi}{15} \cos \frac{7\pi}{15}}{2 \times 2 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$= \frac{\left(\sin \frac{4\pi}{15}\right) \cos \frac{4\pi}{15} \cos \frac{7\pi}{15}}{4 \sin \frac{\pi}{15}}$$

Multiply and divide by 2, we get

$$= \frac{\left(2 \sin \frac{4\pi}{15} \cos \frac{4\pi}{15}\right) \cos \frac{7\pi}{15}}{2 \times 4 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$= \frac{\left(\sin \frac{8\pi}{15}\right) \cos \frac{7\pi}{15}}{8 \sin \frac{\pi}{15}}$$

Multiply and divide by 2, we get

$$= \frac{\left(2 \sin \frac{8\pi}{15} \cos \frac{7\pi}{15}\right)}{2 \times 8 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos B = \sin (A+B) + \sin(A-B)$ , so the above equation becomes,

$$= \frac{\sin\left(\frac{8\pi}{15} + \frac{7\pi}{15}\right) + \sin\left(\frac{8\pi}{15} - \frac{7\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{\sin(\pi) + \sin\left(\frac{\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{0 + \sin\left(\frac{\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{\sin\left(\frac{\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{1}{16} = \text{RHS}$$

Hence proved

## 6. Question

Prove that:

$$\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ = \frac{1}{16}$$

## Answer

$$\text{LHS} = \cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ$$

By regrouping the LHS and multiplying and dividing by 4 we get,

$$= \frac{1}{4} (2 \cos 66^\circ \cos 6^\circ) (2 \cos 78^\circ \cos 42^\circ)$$

$$\text{But } 2 \cos A \cos B = \cos (A+B) + \cos (A-B)$$

Then the above equation becomes,

$$= \frac{1}{4} (\cos(66^\circ + 6^\circ) + \cos(66^\circ - 6^\circ)) (\cos(78^\circ + 42^\circ) + \cos(78^\circ - 42^\circ))$$

$$= \frac{1}{4} (\cos(72^\circ) + \cos(60^\circ)) (\cos(120^\circ) + \cos(36^\circ))$$

$$= \frac{1}{4} (\cos(90^\circ - 18^\circ) + \cos(60^\circ)) (\cos(180^\circ - 60^\circ) + \cos(36^\circ))$$

$$\text{But } \cos(90^\circ - \theta) = \sin \theta \text{ and } \cos(180^\circ - \theta) = -\cos(\theta).$$

Then the above equation becomes,

$$= \frac{1}{4} (\sin(18^\circ) + \cos(60^\circ)) (-\cos(60^\circ) + \cos(36^\circ))$$

$$\text{Now, } \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$

$$\sin(18^\circ) = \frac{\sqrt{5}-1}{4}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Substituting the corresponding values, we get

$$\begin{aligned}
 &= \frac{1}{4} \left( \frac{\sqrt{5}-1}{4} + \frac{1}{2} \right) \left( -\frac{1}{2} + \frac{\sqrt{5}+1}{4} \right) \\
 &= \frac{1}{4} \left( \frac{\sqrt{5}-1+2}{4} \right) \left( \frac{\sqrt{5}+1-2}{4} \right) \\
 &= \frac{1}{4} \left( \frac{\sqrt{5}+1}{4} \right) \left( \frac{\sqrt{5}-1}{4} \right) \\
 &= \frac{1}{4} \left( \frac{(\sqrt{5})^2 - 1^2}{4 \times 4} \right) \\
 &= \frac{1}{4} \left( \frac{4}{16} \right) \\
 &= \frac{1}{16} = \text{RHS}
 \end{aligned}$$

Hence proved

## 7. Question

Prove that:

$$\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$$

## Answer

$$\text{LHS} = \sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ$$

By regrouping the LHS and multiplying and dividing by 4 we get,

$$= \frac{1}{4} (2 \sin 66^\circ \sin 6^\circ) (2 \sin 78^\circ \sin 42^\circ)$$

$$\text{But } 2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Then the above equation becomes,

$$\begin{aligned}
 &= \frac{1}{4} (\cos(66^\circ - 6^\circ) - \cos(66^\circ + 6^\circ)) (\cos(78^\circ - 42^\circ) - \cos(78^\circ + 42^\circ)) \\
 &= \frac{1}{4} (\cos(60^\circ) - \cos(72^\circ)) (\cos(36^\circ) - \cos(120^\circ)) \\
 &= \frac{1}{4} (\cos(60^\circ) - \cos(90^\circ - 18^\circ)) (\cos(36^\circ) - \cos(180^\circ - 60^\circ))
 \end{aligned}$$

$$\text{But } \cos(90^\circ - \theta) = \sin \theta \text{ and } \cos(180^\circ - \theta) = -\cos(\theta).$$

Then the above equation becomes,

$$= \frac{1}{4} (\cos(60^\circ) - \sin(18^\circ)) (\cos(36^\circ) + \cos(60^\circ))$$

$$\text{Now, } \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$

$$\sin(18^\circ) = \frac{\sqrt{5}-1}{4}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Substituting the corresponding values, we get

$$= \frac{1}{4} \left( \frac{1}{2} - \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{5}+1}{4} + \frac{1}{2} \right)$$

$$= \frac{1}{4} \left( \frac{2 - \sqrt{5} + 1}{4} \right) \left( \frac{\sqrt{5} + 1 + 2}{4} \right)$$

$$= \frac{1}{4} \left( \frac{3 - \sqrt{5}}{4} \right) \left( \frac{3 + \sqrt{5}}{4} \right)$$

$$= \frac{1}{4} \left( \frac{3^2 - (\sqrt{5})^2}{4 \times 4} \right)$$

$$= \frac{1}{4} \left( \frac{9 - 5}{16} \right)$$

$$= \frac{1}{16} = \text{RHS}$$

Hence proved

### 8. Question

Prove that:

$$\cos 36^\circ \cos 42^\circ \cos 60^\circ \cos 78^\circ = \frac{1}{16}$$

### Answer

$$\text{LHS} = \cos 36^\circ \cos 42^\circ \cos 60^\circ \cos 78^\circ$$

By regrouping the LHS and multiplying and dividing by 2 we get,

$$= \frac{1}{2} \cos 36^\circ \cos 60^\circ (2 \cos 78^\circ \cos 42^\circ)$$

$$\text{But } 2 \cos A \cos B = \cos (A+B) + \cos (A-B)$$

Then the above equation becomes,

$$= \frac{1}{2} \cos 36^\circ \cos 60^\circ (\cos(78^\circ + 42^\circ) + \cos(78^\circ - 42^\circ))$$

$$= \frac{1}{2} \cos 36^\circ \cos 60^\circ (\cos(120^\circ) + \cos(36^\circ))$$

$$= \frac{1}{2} \cos 36^\circ \cos 60^\circ (\cos(180^\circ - 60^\circ) + \cos(36^\circ))$$

$$\text{But } \cos(90^\circ - \theta) = \sin \theta \text{ and } \cos(180^\circ - \theta) = -\cos(\theta).$$

Then the above equation becomes,

$$= \frac{1}{2} \cos 36^\circ \cos 60^\circ (-\cos(60^\circ) + \cos(36^\circ))$$

$$\text{Now, } \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$



$$\cos(60^\circ) = \frac{1}{2}$$

Substituting the corresponding values, we get

$$\begin{aligned} &= \frac{1}{2} \left( \frac{\sqrt{5}+1}{4} \right) \left( \frac{1}{2} \right) \left( -\frac{1}{2} + \frac{\sqrt{5}+1}{4} \right) \\ &= \left( \frac{\sqrt{5}+1}{16} \right) \left( \frac{\sqrt{5}+1-2}{4} \right) \\ &= \left( \frac{(\sqrt{5})^2 - 1^2}{16 \times 4} \right) \\ &= \left( \frac{5-1}{64} \right) \\ &= \frac{1}{16} = \text{RHS} \end{aligned}$$

Hence proved

### 9. Question

Prove that:

$$\sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5} = \frac{5}{16}$$

### Answer

$$\text{LHS} = \sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5}$$

This can be rewritten as,

$$= \sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \left( \pi - \frac{2\pi}{5} \right) \sin \left( \pi - \frac{\pi}{5} \right)$$

But  $\sin(\pi - \theta) = \sin \theta$  so the above equation becomes,

$$= \sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \left( \frac{2\pi}{5} \right) \sin \left( \frac{\pi}{5} \right)$$

$$= \sin^2 \frac{\pi}{5} \sin^2 \frac{2\pi}{5}$$

This can be rewritten as,

$$= \sin^2 \frac{\pi}{5} \sin^2 \left( \frac{\pi}{2} - \frac{\pi}{10} \right)$$

But  $\sin(90^\circ - \theta) = \cos \theta$

Then the above equation becomes,

$$= \sin^2 \frac{\pi}{5} \cos^2 \left( \frac{\pi}{10} \right)$$

Now,

$$\therefore \cos \frac{\pi}{10} = \frac{\sqrt{10+2\sqrt{5}}}{4}, \sin \frac{\pi}{5} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

Hence the above equation becomes,

$$\begin{aligned}
&= \left( \frac{\sqrt{10 - 2\sqrt{5}}}{4} \right)^2 \left( \frac{\sqrt{10 + 2\sqrt{5}}}{4} \right)^2 \\
&= \left( \frac{10 - 2\sqrt{5}}{16} \right) \left( \frac{10 + 2\sqrt{5}}{16} \right) \\
&= \left( \frac{(10)^2 - (2\sqrt{5})^2}{16 \times 16} \right) \\
&= \left( \frac{100 - 20}{16 \times 16} \right) \\
&= \left( \frac{80}{16 \times 16} \right) \\
&= \frac{5}{16} = \text{RHS}
\end{aligned}$$

Hence proved

### 10. Question

Prove that:

$$\cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15} = \frac{1}{128}$$

**Answer**

$$\text{LHS} = \cos \frac{\pi}{15} \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}$$

Multiply and divide by  $2 \sin \frac{\pi}{15}$ , we get

$$= \frac{\left( 2 \sin \frac{\pi}{15} \cos \frac{\pi}{15} \right) \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}}{2 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$= \frac{\left( \sin \frac{2\pi}{15} \right) \cos \frac{2\pi}{15} \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}}{2 \sin \frac{\pi}{15}}$$

Multiply and divide by 2, we get

$$= \frac{\left( 2 \sin \frac{2\pi}{15} \cos \frac{2\pi}{15} \right) \cos \frac{3\pi}{15} \cos \frac{4\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}}{2 \times 2 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$= \frac{\left( \sin \frac{4\pi}{15} \right) \cos \frac{4\pi}{15} \cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}}{4 \sin \frac{\pi}{15}}$$

Multiply and divide by 2, we get

$$= \frac{\left(2 \sin \frac{4\pi}{15} \cos \frac{4\pi}{15}\right) \cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}}{2 \times 4 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$= \frac{\left(\sin \frac{8\pi}{15}\right) \cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15} \cos \frac{7\pi}{15}}{8 \sin \frac{\pi}{15}}$$

Multiply and divide by 2, we get

$$= \frac{\left(2 \sin \frac{8\pi}{15} \cos \frac{7\pi}{15}\right) \cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15}}{2 \times 8 \sin \frac{\pi}{15}}$$

But  $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$ , so the above equation becomes,

$$= \frac{\left(\sin\left(\frac{8\pi}{15} + \frac{7\pi}{15}\right) + \sin\left(\frac{8\pi}{15} - \frac{7\pi}{15}\right)\right) \left(\cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{\left(\sin(\pi) + \sin\left(\frac{\pi}{15}\right)\right) \left(\cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{\left(0 + \sin\left(\frac{\pi}{15}\right)\right) \left(\cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

$$= \frac{\sin\left(\frac{\pi}{15}\right) \left(\cos \frac{3\pi}{15} \cos \frac{5\pi}{15} \cos \frac{6\pi}{15}\right)}{16 \sin \frac{\pi}{15}}$$

Multiply and divide by  $2 \sin \frac{3\pi}{15}$ , we get

$$= \frac{\left(2 \sin \frac{3\pi}{15} \cos \frac{3\pi}{15}\right) \cos \frac{5\pi}{15} \cos \frac{6\pi}{15}}{16 \times 2 \sin \frac{3\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$= \frac{\left(\sin \frac{6\pi}{15}\right) \cos \frac{5\pi}{15} \cos \frac{6\pi}{15}}{32 \sin \frac{3\pi}{15}}$$

Multiply and divide by 2, we get

$$= \frac{\left(2 \sin \frac{6\pi}{15} \cos \frac{6\pi}{15}\right) \cos \frac{5\pi}{15}}{2 \times 32 \sin \frac{3\pi}{15}}$$

But  $2 \sin A \cos A = \sin 2A$

Then the above equation becomes,

$$\begin{aligned}
&= \frac{\left(\sin \frac{12\pi}{15}\right) \cos \frac{5\pi}{15}}{64 \sin \frac{3\pi}{15}} \\
&= \frac{\left(\sin \left(\pi - \frac{3\pi}{15}\right)\right) \left(\cos \frac{5\pi}{15}\right)}{64 \sin \frac{3\pi}{15}} \\
&= \frac{\left(\sin \left(\frac{3\pi}{15}\right)\right) \left(\cos \frac{5\pi}{15}\right)}{64 \sin \frac{3\pi}{15}} \quad (\because \sin(\pi - \theta) = \sin \theta) \\
&= \frac{\cos \frac{\pi}{3}}{64} \\
&= \frac{\frac{1}{2}}{64} \\
&= \frac{1}{128} = \text{RHS}
\end{aligned}$$

Hence proved

## Very Short Answer

### 1. Question

If  $\cos 4x = 1 + k \sin^2 x \cos^2 x$ , then write the value of k.

### Answer

Given equation is

$$\cos 4x = 1 + k \sin^2 x \cos^2 x$$

Now consider the LHS of the equation,

$$\cos 4x = 2\cos^2 2x - 1$$

$$[\text{Formula for } \cos 2x = 2\cos^2 x - 1]$$

$$= 2[2\cos^2 x - 1]^2 - 1$$

$$= 2[(2\cos^2 x)^2 - 2 \times (2 \cos^2 x) \times (1) + (1)^2] - 1$$

$$[\text{Applying } (a-b)^2 = a^2 - 2ab + b^2 \text{ formula}]$$

$$= 2[4\cos^4 x - 4\cos^2 x + 1] - 1$$

$$= 8 \cos^4 x - 8\cos^2 x + 2 - 1$$

$$= 8\cos^2 x (\cos^2 x - 1) + 1$$

$$= 8\cos^2 x (-\sin^2 x) + 1$$

$$= -8\cos^2 x \sin^2 x + 1$$

$$\text{Now as per the LHS } \cos 4x = -8\cos^2 x \sin^2 x + 1 \text{ ----- (1)}$$

Comparing LHS with the RHS,

$$\cos 4x = 1 - 8\cos^2 x \sin^2 x = 1 + k \sin^2 x \cos^2 x$$

by comparing we get  $k = -8$

## 2. Question

If  $\tan \frac{x}{2} = \frac{m}{n}$ , then write the value of  $m \sin x + n \cos x$ .

### Answer

Given,

$$\tan \frac{x}{2} = \frac{m}{n}$$

We need to find the value of  $m \sin x + n \cos x$

Now consider

$$m \sin x + n \cos x = m \left[ \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right] + n \left[ \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right]$$

[ using the formulas  $\sin 2x$  &  $\cos 2x$  in terms of  $\tan x$

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \text{ and } \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}]$$

$$= m \left[ \frac{2 \left( \frac{m}{n} \right)}{1 + \left( \frac{m}{n} \right)^2} \right] + n \left[ \frac{1 - \left( \frac{m}{n} \right)^2}{1 + \left( \frac{m}{n} \right)^2} \right]$$

[Substituting  $\tan \frac{x}{2} = \frac{m}{n}$ ]

$$= m \left[ \frac{2 \left( \frac{m}{n} \right)}{\frac{n^2 + m^2}{n^2}} \right] + n \left[ \frac{\frac{n^2 - m^2}{n^2}}{\frac{n^2 + m^2}{n^2}} \right]$$

$$= m \left[ \frac{2mn}{n^2 + m^2} \right] + n \left[ \frac{n^2 - m^2}{n^2 + m^2} \right]$$

$$= \left[ \frac{2m^2n}{n^2 + m^2} \right] + \left[ \frac{n^3 - m^2n}{n^2 + m^2} \right]$$

$$= \left[ \frac{2m^2n + n^3 - m^2n}{n^2 + m^2} \right]$$

$$= \left[ \frac{m^2n + n^3}{n^2 + m^2} \right]$$

$$= \left[ \frac{n(m^2 + n^2)}{m^2 + n^2} \right]$$

$$= n$$

Hence the value of  $m \sin x + n \cos x = n$ .

## 3. Question

If  $\frac{\pi}{2} < x < \frac{3\pi}{2}$ , then write the value of  $\sqrt{\frac{1 + \cos 2x}{2}}$ .

### Answer

Given  $\frac{\pi}{2} < x < \frac{3\pi}{2}$  then the value of

$$\begin{aligned}
\sqrt{\frac{1 + \cos 2x}{2}} &= \sqrt{\frac{1 + (\cos^2 x - \sin^2 x)}{2}} \\
&= \sqrt{\frac{\cos^2 x + (1 - \sin^2 x)}{2}} \\
&= \sqrt{\frac{\cos^2 x + \cos^2 x}{2}} \\
&= \sqrt{\frac{2\cos^2 x}{2}} \\
&= \sqrt{\cos^2 x} \\
&= \pm \cos x
\end{aligned}$$

Hence

$$\sqrt{\frac{1 + \cos 2x}{2}} = \pm \cos x$$

But as given,  $\frac{\pi}{2} < x < \frac{3\pi}{2}$

This states that,  $90^\circ < x < 270^\circ$ , which means  $x$  lies between 2<sup>nd</sup> and 3<sup>rd</sup> quadrants.

In the 2<sup>nd</sup> and 3<sup>rd</sup> quadrants, the cosine function is negative, so the value of

$$\sqrt{\frac{1 + \cos 2x}{2}} = -\cos x$$

#### 4. Question

If  $\frac{\pi}{2} < x < \pi$ , then write the value of  $\sqrt{2 + \sqrt{2 + 2\cos 2x}}$  in the simplest form.

#### Answer

Given,  $\frac{\pi}{2} < x < \pi$

To find the value of  $\sqrt{2 + \sqrt{2 + 2\cos 2x}}$

$$= \sqrt{2 + \sqrt{2(1 + \cos 2x)}}$$

[using the formula  $\cos 2x = 2\cos^2 x - 1$ ]

$$= \sqrt{2 + \sqrt{2(1 + 1 - 2\cos^2 x - 1)}}$$

$$= \sqrt{2 + \sqrt{2(2\cos^2 x)}}$$

$$= \sqrt{2 + \sqrt{4\cos^2 x}}$$

[using the formula  $\cos 2x = 2\cos^2 x - 1$ , here  $2x = \theta$  so  $x = \frac{\theta}{2}$ ]

$$= \sqrt{2 + 2\cos x}$$

$$= \sqrt{2 + 2 \left[ 2 \cos^2 \left( \frac{x}{2} \right) - 1 \right]}$$

$$= \sqrt{2 + 4 \cos^2 \left( \frac{x}{2} \right) - 2}$$

$$= \sqrt{4 \cos^2 \left( \frac{x}{2} \right)}$$

$$= \pm 2 \cos \left( \frac{x}{2} \right)$$

As given,  $\frac{\pi}{2} < x < \pi$  now by dividing the whole inequation with 2 we get,  $\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ .

This clearly state that  $\frac{x}{2}$  lies in the 1<sup>st</sup> quadrant and between 45° and 90°.

$$\text{So } \sqrt{2 + \sqrt{2 + 2 \cos 2x}} = 2 \cos \left( \frac{x}{2} \right)$$

## 5. Question

If  $\frac{\pi}{2} < x < \pi$ , then write the value of  $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ .

## Answer

Given, for  $\frac{\pi}{2} < x < \pi$  the value of  $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$

Consider,

$$\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \sqrt{\frac{1 - (\cos^2 x - \sin^2 x)}{1 + (\cos^2 x - \sin^2 x)}}$$

[by using the formula  $\cos 2x = \cos^2 x - \sin^2 x$ ]

$$= \sqrt{\frac{(1 - \cos^2 x) + \sin^2 x}{(1 - \sin^2 x) + \cos^2 x}}$$

$$= \sqrt{\frac{\sin^2 x + \sin^2 x}{\cos^2 x + \cos^2 x}}$$

[by using the formula  $\cos^2 x + \sin^2 x = 1$ ]

$$= \sqrt{\frac{2 \sin^2 x}{2 \cos^2 x}}$$

$$= \sqrt{\tan^2 x}$$

$$= \pm \tan x$$

As already mentioned in the question,  $\frac{\pi}{2} < x < \pi$ , x is in the 2nd quadrant, where tangent function is negative.

$$\text{Therefore, } \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = -\tan x$$

## 6. Question

If  $\pi < x < \frac{3\pi}{2}$ , then write the value of  $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ .

**Answer**

Given, for  $\pi < x < \frac{3\pi}{2}$  the value of  $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$

Consider,

$$\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \sqrt{\frac{1 - (\cos^2 x - \sin^2 x)}{1 + (\cos^2 x - \sin^2 x)}}$$

[by using the formula  $\cos 2x = \cos^2 x - \sin^2 x$ ]

$$= \sqrt{\frac{(1 - \cos^2 x) + \sin^2 x}{(1 - \sin^2 x) + \cos^2 x}}$$

$$= \sqrt{\frac{\sin^2 x + \sin^2 x}{\cos^2 x + \cos^2 x}}$$

[by using the formula  $\cos^2 x + \sin^2 x = 1$ ]

$$= \sqrt{\frac{2 \sin^2 x}{2 \cos^2 x}}$$

$$= \sqrt{\tan^2 x}$$

$$= \pm \tan x$$

As already mentioned in the question,  $\pi < x < \frac{3\pi}{2}$ ,  $x$  is in the 3<sup>rd</sup> quadrant, where tangent function is positive.

$$\text{Therefore, } \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} = \tan x$$

## 7. Question

In a right-angled triangle ABC, write the value of  $\sin^2 A + \sin^2 B + \sin^2 C$ .

**Answer**

Given, triangle ABC is right angle.

So, let  $\angle B = 90^\circ$

Then as per the property of angles in a triangle

$$\angle A + \angle B + \angle C = 180^\circ$$

As  $\angle B = 90^\circ$

$$\angle A + 90^\circ + \angle C = 180^\circ$$

$$\text{Then } \angle A + \angle C = 180^\circ - 90^\circ = 90^\circ$$

Now, consider  $\sin^2 A + \sin^2 B + \sin^2 C$

As  $\angle B = 90^\circ$

$$\sin^2 A + \sin^2 B + \sin^2 C = \sin^2 A + \sin^2(90^\circ) + \sin^2 C$$

$$= \sin^2 A + 1 + \sin^2 C$$



From before, we know that  $\angle A + \angle C = 90^\circ$  ;  $\angle C = 90^\circ - \angle A$

$$\sin^2 A + \sin^2 B + \sin^2 C = \sin^2 A + 1 + \sin^2(90^\circ - A)$$

$$= \sin^2 A + \cos^2(A) + 1$$

[by using the identity  $\cos x = \sin(90^\circ - x)$ ]

$$\sin^2 A + \sin^2 B + \sin^2 C = (\sin^2 A + \cos^2 A) + 1$$

$$= 1 + 1$$

$$= 2$$

[by using the identity  $\sin^2 \theta + \cos^2 \theta = 1$ ]

Therefore,  $\sin^2 A + \sin^2 B + \sin^2 C = 2$ .

### 8. Question

Write the value of  $\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ$ .

### Answer

Given to find the value for,

$$\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ$$

In the above expression consider  $\cos 76^\circ \cos 16^\circ$

[By using the trigonometric sum formula, we can say that,

$$\cos(C+D) + \cos(C-D) = 2 \cos C \cos D]$$

Now multiply and divide this with 2, we get

$$\frac{2 \times (\cos 76^\circ \cos 16^\circ)}{2} = \frac{\cos(76^\circ + 16^\circ) + \cos(76^\circ - 16^\circ)}{2}$$

$$\frac{\cos 92^\circ + \cos 60^\circ}{2}$$

Consider the full expression,

$$\begin{aligned} & \cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ \\ &= \cos^2 76^\circ + \cos^2 16^\circ - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right) \\ &= \cos^2 76^\circ + \cos^2 16^\circ - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right) \end{aligned}$$

Multiplying and dividing the terms  $\cos^2 76^\circ + \cos^2 16^\circ$  with 2

$$\begin{aligned} &= \frac{2\cos^2 76^\circ}{2} + \frac{2\cos^2 16^\circ}{2} - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right) \\ &= \frac{1}{2} [\cos 2(76) + 1] + \frac{1}{2} [\cos 2(16) + 1] - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right) \end{aligned}$$

[ by using the formula,  $\cos 2\theta = 2\cos^2 \theta - 1 \Leftrightarrow 2\cos^2 \theta = \cos 2\theta + 1$  ]

$$= \frac{1}{2} [2 + (\cos 152^\circ + \cos 32^\circ)] - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right)$$

[ by using the formula,  $\cos A + \cos B = 2 \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$  ]

$$= 1 + \frac{1}{2} \left[ 2 \cos \left( \frac{152^\circ + 32^\circ}{2} \right) \cos \left( \frac{152^\circ - 32^\circ}{2} \right) \right] - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right)$$

$$\begin{aligned}
&= 1 + \frac{1}{2} \left[ 2 \cos\left(\frac{184^\circ}{2}\right) \cos\left(\frac{120^\circ}{2}\right) \right] - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right) \\
&= 1 + \frac{1}{2} [ 2 \cos(92^\circ) \cos(60^\circ) ] - \left( \frac{\cos 92^\circ + \cos 60^\circ}{2} \right) \\
&= 1 + \frac{\cos 92^\circ}{2} - \frac{\cos 92^\circ}{2} - \frac{1}{2} \\
&= 1 - \frac{1}{4} = \frac{3}{4}
\end{aligned}$$

Hence,  $\cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ = \frac{3}{4}$

### 9. Question

If  $\frac{\pi}{4} < x < \frac{\pi}{2}$ , then write the value of  $\sqrt{1 - \sin 2x}$ .

### Answer

Given,  $\frac{\pi}{4} < x < \frac{\pi}{2}$

We should find the value for  $\sqrt{1 - \sin 2x}$

$$\sqrt{1 - \sin 2x} = \sqrt{(\sin^2 x + \cos^2 x) - 2 \sin x \cos x}$$

[by using the formulae,  $\sin^2 \theta + \cos^2 \theta = 1$  and  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$\sqrt{1 - \sin 2x} = \sqrt{(\sin^2 x - \cos^2 x)^2}$$

$$= \sqrt{(\sin x - \cos x)^2}$$

$$= \pm(\sin x - \cos x)$$

As already mentioned in the question,  $\frac{\pi}{4} < x < \frac{\pi}{2}$ , so  $x$  lies in the 1<sup>st</sup> quadrant and both sine and cosine functions are positive.

Therefore,  $\sqrt{1 - \sin 2x} = \sin x + \cos x$

### 10. Question

Write the value of  $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7}$ .

### Answer

Given expression is  $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7}$

[by using  $\sin 2\theta = 2 \sin \theta \cos \theta \Leftrightarrow \cos \theta = \frac{\sin 2\theta}{2 \sin \theta}$ ]

$$\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = \left( \frac{\sin 2\left(\frac{\pi}{7}\right)}{2 \sin\left(\frac{\pi}{7}\right)} \right) \left( \frac{\sin 2\left(\frac{2\pi}{7}\right)}{2 \sin\left(\frac{2\pi}{7}\right)} \right) \left( \frac{\sin 2\left(\frac{4\pi}{7}\right)}{2 \sin\left(\frac{4\pi}{7}\right)} \right)$$

$$= \left( \frac{\sin 2\left(\frac{\pi}{7}\right)}{2 \sin\left(\frac{\pi}{7}\right)} \right) \left( \frac{\sin 2\left(\frac{2\pi}{7}\right)}{2 \sin\left(\frac{2\pi}{7}\right)} \right) \left( \frac{\sin 2\left(\frac{4\pi}{7}\right)}{2 \sin\left(\frac{4\pi}{7}\right)} \right)$$

$$= \left( \frac{\sin\left(\frac{2\pi}{7}\right)}{2 \sin\left(\frac{\pi}{7}\right)} \right) \left( \frac{\sin\left(\frac{4\pi}{7}\right)}{2 \sin\left(\frac{2\pi}{7}\right)} \right) \left( \frac{\sin\left(\frac{8\pi}{7}\right)}{2 \sin\left(\frac{4\pi}{7}\right)} \right)$$

$$= \left( \frac{\sin\left(\frac{8\pi}{7}\right)}{2^3 \sin\left(\frac{\pi}{7}\right)} \right) = \left( \frac{\sin\left(\pi + \frac{\pi}{7}\right)}{2^3 \sin\left(\frac{\pi}{7}\right)} \right) = \left( \frac{-\sin\left(\frac{\pi}{7}\right)}{2^3 \sin\left(\frac{\pi}{7}\right)} \right)$$

$$= -\frac{1}{8}$$

Hence  $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = -\frac{1}{8}$

### 11. Question

If  $A = \frac{1 - \cos B}{\sin B}$ , then find the value of  $\tan 2A$ .

### Answer

Given,  $\tan A = \frac{1 - \cos B}{\sin B}$

To find the value for  $\tan 2A$ ,

Consider

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

[ by using the formula for  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$  ]

$$\tan 2A = \frac{2 \left( \frac{1 - \cos B}{\sin B} \right)}{1 - \left( \frac{1 - \cos B}{\sin B} \right)^2}$$

[by substituting the value of  $\tan A$  as given in the problem]

$$\begin{aligned} \tan 2A &= \frac{2 \left( \frac{1 - \cos B}{\sin B} \right)}{\frac{\sin^2 B - (1 - \cos B)^2}{\sin^2 B}} \\ &= \frac{2(1 - \cos B) \sin B}{\sin^2 B - (1 - \cos B)^2} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos^2 B) - (1 - \cos B)^2} \\ &= \frac{2(1 - \cos B) \sin B}{(1 + \cos B)(1 - \cos B) - (1 - \cos B)^2} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos B)[1 + \cos B - 1 + \cos B]} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos B)2 \cos B} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos B)2 \cos B} \\ &= \frac{\sin B}{\cos B} \end{aligned}$$

$$= \tan B$$

Therefore,  $\tan 2A = \tan B$

### 12. Question

If  $\sin x + \cos x = a$ , find the value of  $\sin^6 x + \cos^6 x$ .

**Answer**

Given,  $\sin x + \cos x = a$

We need to find the value of the expression,

$$\begin{aligned}\sin^6 x + \cos^6 x &= (\sin^2 x)^3 + (\cos^2 x)^3 \\&= (\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) \\& \text{[ by using the formula } a^3 + b^3 = (a+b)^3 - 3ab(a+b) \text{]} \\&= (1)^3 - 3 \sin^2 x \cos^2 x (1)\end{aligned}$$

[ by using the formula  $\sin^2 x + \cos^2 x = 1$  ]

$$\begin{aligned}&= 1 - 3 \left\{ \frac{(\sin x + \cos x)^2 - \sin^2 x - \cos^2 x}{2} \right\}^2 \\&= 1 - 3 \left\{ \frac{a^2 - (\sin^2 x + \cos^2 x)}{2} \right\}^2\end{aligned}$$

[ by using the formula  $\sin^2 x + \cos^2 x = 1$  ]

$$\begin{aligned}&= 1 - 3 \left\{ \frac{a^2 - 1}{2} \right\}^2 \\&= 1 - \frac{3}{4} (a^2 - 1)^2 \\&= \frac{4 - 3(a^2 - 1)^2}{4} \\&= \frac{1}{4} \{ 4 - 3(a^2 - 1)^2 \}\end{aligned}$$

$$\text{Hence } \sin^6 x + \cos^6 x = \frac{1}{4} \{ 4 - 3(a^2 - 1)^2 \}$$

**13. Question**

If  $\sin x + \cos x = a$ , find the value of  $|\sin x - \cos x|$

**Answer**

Given,  $\sin x + \cos x = a$

To find the value of  $|\sin x - \cos x|$

Consider square of  $|\sin x - \cos x|$

$$\begin{aligned}|\sin x - \cos x|^2 &= |\sin x|^2 + |\cos x|^2 - 2|\sin x| |\cos x| \\& \text{[using the formula } (a + b)^2 = a^2 + b^2 + 2ab \text{]} \\|\sin x - \cos x|^2 &= |\sin x|^2 + |\cos x|^2 - 2|\sin x| |\cos x| \\&= (\sin^2 x + \cos^2 x) - [(\sin x + \cos x)^2 - \sin^2 x - \cos^2 x] \\&= (\sin^2 x + \cos^2 x) - [a^2 - (\sin^2 x + \cos^2 x)]\end{aligned}$$

[using the formula  $\sin^2 x + \cos^2 x = 1$ ]

$$\begin{aligned}&= 1 - a^2 + 1 \\&= 2 - a^2\end{aligned}$$

$$|\sin x - \cos x|^2 = 2 - a^2$$

Taking square root on both sides.

$$\sqrt{|\sin x - \cos x|^2} = \sqrt{2 - a^2}$$

$$\text{Hence } |\sin x - \cos x| = \sqrt{2 - a^2}$$

## MCQ

### 1. Question

Mark the Correct alternative in the following:

$8 \sin \frac{x}{8} \cos \frac{x}{2} \cos \frac{x}{4} \cos \frac{x}{8}$  is equal to

A.  $8 \cos x$

B.  $\cos x$

C.  $8 \sin x$

D.  $\sin x$

### Answer

Given expression,  $8 \sin \frac{x}{8} \cos \frac{x}{2} \cos \frac{x}{4} \cos \frac{x}{8}$

$$4 \left( 2 \sin \frac{x}{8} \cos \frac{x}{8} \right) \cos \frac{x}{2} \cos \frac{x}{4}$$

[by rearranging terms]

$$4 \left( \sin \frac{2x}{8} \right) \cos \frac{x}{2} \cos \frac{x}{4}$$

[using the formula  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$= 4 \left( \sin \frac{x}{4} \right) \cos \frac{x}{2} \cos \frac{x}{4}$$

$$= 2 \left( 2 \sin \frac{x}{4} \cos \frac{x}{4} \right) \cos \frac{x}{2}$$

$$= 2 \left( \sin \frac{2x}{4} \right) \cos \frac{x}{2}$$

$$= \left( 2 \sin \frac{x}{2} \cos \frac{x}{2} \right)$$

$$= \sin x$$

$$\text{Hence } 8 \sin \frac{x}{8} \cos \frac{x}{2} \cos \frac{x}{4} \cos \frac{x}{8} = \sin x$$

### 2. Question

Mark the Correct alternative in the following:

$\frac{\sec 8A - 1}{\sec 4A - 1}$  is equal to

A.  $\frac{\tan 2A}{\tan 8A}$

B.  $\frac{\tan 8A}{\tan 2A}$

C.  $\frac{\cot 8A}{\cot 2A}$

D. None of these

### Answer

Given expression is  $\frac{\sec 8A - 1}{\sec 4A - 1}$

$$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\frac{1}{\cos 8A} - 1}{\frac{1}{\cos 4A} - 1}$$

[using  $\sec \theta = \frac{1}{\cos \theta}$ ]

$$= \frac{\frac{1 - \cos 8A}{\cos 8A}}{\frac{1 - \cos 4A}{\cos 4A}}$$

$$= \frac{\cos 4A (1 - \cos 8A)}{\cos 8A (1 - \cos 4A)}$$

$$= \frac{\cos 4A \{1 - (1 - 2\sin^2 4A)\}}{\cos 8A \{1 - (1 - 2\sin^2 2A)\}}$$

[using  $\cos 2\theta = 1 - 2\sin^2 \theta$ ]

$$= \frac{\cos 4A (2\sin^2 4A)}{\cos 8A (2\sin^2 2A)}$$

$$= \frac{\sin 4A (2 \sin 4A \cos 4A)}{\cos 8A (2\sin^2 2A)}$$

[using  $\sin 2\theta = 2\sin \theta \cos \theta$ ]

$$= \frac{2 \sin 2A \cos 2A (\sin 8A)}{\cos 8A (2\sin^2 2A)}$$

$$= \frac{\cos 2A (\sin 8A)}{\cos 8A (\sin 2A)}$$

$$= \frac{\left(\frac{\sin 8A}{\cos 8A}\right)}{\left(\frac{\sin 2A}{\cos 2A}\right)}$$

[using  $\tan \theta = \frac{\sin \theta}{\cos \theta}$ ]

$$\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$$

### 3. Question

Mark the Correct alternative in the following:

The value of  $\cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$  is

A.  $\frac{1}{8}$

B.  $\frac{1}{16}$

C.  $\frac{1}{32}$

D. None of these

**Answer**

Given expression,  $\cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65}$

Multiply and divide the expression with  $2 \sin \frac{\pi}{65}$

$$= \frac{1}{2 \sin \frac{\pi}{65}} \left\{ \left( 2 \sin \frac{\pi}{65} \cos \frac{\pi}{65} \right) \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

[using the formula  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$= \frac{1}{2 \sin \frac{\pi}{65}} \left\{ \sin \frac{2\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

Multiply and divide the expression with 2

$$= \frac{1}{2^2 \sin \frac{\pi}{65}} \left\{ \left( 2 \sin \frac{2\pi}{65} \cos \frac{2\pi}{65} \right) \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

[using the formula  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$= \frac{1}{2^2 \sin \frac{\pi}{65}} \left\{ \sin \frac{4\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

Multiply and divide the expression with 2

$$= \frac{1}{2^3 \sin \frac{\pi}{65}} \left\{ \left( 2 \sin \frac{4\pi}{65} \cos \frac{4\pi}{65} \right) \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

[using the formula  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$= \frac{1}{2^3 \sin \frac{\pi}{65}} \left\{ \sin \frac{8\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

Multiply and divide the expression with 2

$$= \frac{1}{2^4 \sin \frac{\pi}{65}} \left\{ \left( 2 \sin \frac{8\pi}{65} \cos \frac{8\pi}{65} \right) \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

[using the formula  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$= \frac{1}{2^4 \sin \frac{\pi}{65}} \left\{ \sin \frac{16\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} \right\}$$

Multiply and divide the expression with 2

$$= \frac{1}{2^5 \sin \frac{\pi}{65}} \left\{ \left( 2 \sin \frac{16\pi}{65} \cos \frac{16\pi}{65} \right) \cos \frac{32\pi}{65} \right\}$$

[using the formula  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$= \frac{1}{2^5 \sin \frac{\pi}{65}} \left\{ \sin \frac{32\pi}{65} \cos \frac{32\pi}{65} \right\}$$

Multiply and divide the expression with 2

$$= \frac{1}{2^6 \sin \frac{\pi}{65}} \left\{ 2 \sin \frac{32\pi}{65} \cos \frac{32\pi}{65} \right\}$$

$$= \frac{1}{2^6 \sin \frac{\pi}{65}} \left\{ \sin \frac{64\pi}{65} \right\}$$

$$= \frac{1}{2^6 \sin \frac{\pi}{65}} \left\{ \sin \left( \pi - \frac{\pi}{65} \right) \right\}$$

$$= \frac{1}{2^6 \sin \frac{\pi}{65}} \left\{ \sin \frac{\pi}{65} \right\}$$

$$= \frac{1}{2^6} = \frac{1}{64}$$

$$\text{As } \cos \frac{\pi}{65} \cos \frac{2\pi}{65} \cos \frac{4\pi}{65} \cos \frac{8\pi}{65} \cos \frac{16\pi}{65} \cos \frac{32\pi}{65} = \frac{1}{64}$$

Hence answer is option D.

#### 4. Question

Mark the Correct alternative in the following:

If  $\cos 2x + 2 \cos x = 1$  then,  $(2 - \cos^2 x) \sin^2 x$  is equal to

A. 1

B. -1

C.  $-\sqrt{5}$

D.  $\sqrt{5}$

#### Answer

Given  $\cos 2x + 2 \cos x = 1$ , we need to find the expression,

$$(2 - \cos^2 x) \sin^2 x$$

$$\text{Consider } \cos 2x + 2 \cos x = 1$$

$$2\cos^2 x - 1 + 2 \cos x - 1 = 0$$

$$2\cos^2 x + 2\cos x - 2 = 0$$

$$\cos^2 x + \cos x = 1 \text{ ----- (1)}$$

Now consider the expression

$$(2 - \cos^2 x) \sin^2 x = (2 - \cos^2 x)(1 - \cos^2 x)$$

$$= \{2 - (1 - \cos x)\} \{1 - (1 - \cos x)\}$$

$$[\text{from equation (1) } \cos^2 x = 1 - \cos x]$$

$$= (1 + \cos x) (\cos x)$$

$$= \cos x + \cos^2 x$$

$$[\text{from equation (1) } \cos^2 x + \cos x = 1]$$



$$= 1$$

Hence  $(2 - \cos^2 x) \sin^2 x = 1$ , so option A is the answer.

### 5. Question

Mark the Correct alternative in the following:

For all real values of  $x$ ,  $\cot x - 2 \cot 2x$  is equal to

- A.  $\tan 2x$
- B.  $\tan x$
- C.  $-\cot 3x$
- D. None of these

### Answer

Given expression is  $\cot x - 2 \cot 2x$  for all real values of  $x$

$$\text{Consider } \cot x - 2 \cot 2x = \left( \frac{1}{\tan x} \right) - 2 \left( \frac{1 - \tan^2 x}{2 \tan x} \right)$$

$$[ \text{using } \cot x = \left( \frac{1}{\tan x} \right) \text{ and } \cot 2x = \left( \frac{1 - \tan^2 x}{2 \tan x} \right) ]$$

$$= \frac{1 - 1 + \tan^2 x}{\tan x}$$

$$= \frac{\tan^2 x}{\tan x}$$

$$= \tan x$$

Therefore  $\cot x - 2 \cot 2x = \tan x$ .

Option B is the answer.

### 6. Question

Mark the Correct alternative in the following:

The value of  $2 \tan \frac{\pi}{10} + 3 \sec \frac{\pi}{10} - 4 \cos \frac{\pi}{10}$  is

- A. 0
- B.  $\sqrt{5}$
- C. 1
- D. None of these

### Answer

Given expression is  $2 \tan \frac{\pi}{10} + 3 \sec \frac{\pi}{10} - 4 \cos \frac{\pi}{10}$

Now

$$2 \tan \frac{\pi}{10} + 3 \sec \frac{\pi}{10} - 4 \cos \frac{\pi}{10} = 2 \left( \frac{\sin \frac{\pi}{10}}{\cos \frac{\pi}{10}} \right) + 3 \left( \frac{1}{\cos \frac{\pi}{10}} \right) - 4 \cos \frac{\pi}{10}$$

$$= \frac{2 \sin \frac{\pi}{10} + 3 - 4 \cos^2 \frac{\pi}{10}}{\cos \frac{\pi}{10}}$$

Multiplying and dividing the whole expression with  $\cos \frac{\pi}{10}$

$$= \frac{\cos \frac{\pi}{10} \left( 2 \sin \frac{\pi}{10} + 3 - 4 \cos^2 \frac{\pi}{10} \right)}{\cos \frac{\pi}{10} \cos \frac{\pi}{10}}$$

$$= \frac{\left( 2 \sin \frac{\pi}{10} \cos \frac{\pi}{10} + 3 \cos \frac{\pi}{10} - 4 \cos^3 \frac{\pi}{10} \right)}{\cos^2 \frac{\pi}{10}}$$

[using  $\sin 2x = 2 \sin x \cos x$  formula]

$$= \frac{\sin \frac{2\pi}{10} - \left( 4 \cos^3 \frac{\pi}{10} - 3 \cos \frac{\pi}{10} \right)}{\cos^2 \frac{\pi}{10}}$$

[using  $\cos 3x = 4\cos^3 x - 3 \cos x$  formula]

$$= \frac{\sin \frac{2\pi}{10} - \cos \frac{3\pi}{10}}{\cos^2 \frac{\pi}{10}} = \frac{\sin \frac{2\pi}{10} - \sin \left( \frac{\pi}{2} - \frac{2\pi}{10} \right)}{\cos^2 \frac{\pi}{10}}$$

$$= \frac{\sin \frac{2\pi}{10} - \sin \left( \frac{\pi}{2} - \frac{3\pi}{10} \right)}{\cos^2 \frac{\pi}{10}}$$

[using  $\cos x = \sin \left( \frac{\pi}{2} - x \right)$ ]

$$= \frac{\sin \frac{2\pi}{10} - \sin \left( \frac{2\pi}{10} \right)}{\cos^2 \frac{\pi}{10}}$$

$$= 0$$

$$\text{Therefore } 2 \tan \frac{\pi}{10} + 3 \sec \frac{\pi}{10} - 4 \cos \frac{\pi}{10} = 0$$

The answer is option A.

## 7. Question

Mark the Correct alternative in the following:

If in a  $\Delta ABC$ ,  $\tan A + \tan B + \tan C = 0$ , then  $\cot A \cot B \cot C =$ -

A. 6

B. 1

C.  $\frac{1}{6}$

D. None of these

## Answer

Given ABC is a triangle, so  $\angle A + \angle B + \angle C = 180^\circ$

Now applying tan on both sides

$$\tan (A+B +C) = \tan (180^\circ)$$

$$\tan (A + B + C) = 0 \text{ ----- (1)}$$

$$\text{Also given } \tan A + \tan B + \tan C = 0 \text{ ----- (2)}$$

As per the formula of  $\tan (A+B+C)$

$$\tan(A + B + C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

$$\text{Now, } \tan(A + B + C) = \frac{0 - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

[from equation (1) ]

$$0 = \frac{-\tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

[from equation (2) ]

By cross multiplying

$$-\tan A \tan B \tan C = 0$$

$$\tan A \tan B \tan C = 0$$

$$\text{therefore } \frac{1}{\tan A \tan B \tan C} = 0$$

$$\text{Hence } \cot A \cot B \cot C = 0$$

The answer is option D.

### 8. Question

Mark the Correct alternative in the following:

$$\text{If } \cos x = \frac{1}{2} \left( a + \frac{1}{a} \right), \text{ and } \cos 3x = \lambda \left( a^3 + \frac{1}{a^3} \right), \text{ then } \lambda =$$

A.  $\frac{1}{4}$

B.  $\frac{1}{2}$

C. 1

D. None of these

### Answer

$$\text{Given } \cos x = \frac{1}{2} \left( a + \frac{1}{a} \right) \text{ and } \cos 3x = \lambda \left( a^3 + \frac{1}{a^3} \right)$$

$$\text{Consider the equation } \cos 3x = \lambda \left( a^3 + \frac{1}{a^3} \right)$$

Now take the LHS of the equation,

$$\cos 3x = 4\cos^3 x - 3\cos x$$

$$[\text{using the formula for } \cos 3x = 4\cos^3 x - 3\cos x]$$

$$\text{From the question we know, } \cos x = \frac{1}{2} \left( a + \frac{1}{a} \right)$$

Substituting the known  $\cos x$  values in the  $\cos 3x$  expansion,

$$\cos 3x = 4 \left[ \frac{1}{2} \left( a + \frac{1}{a} \right) \right]^3 - 3 \left[ \frac{1}{2} \left( a + \frac{1}{a} \right) \right]$$

$$= 4 \left[ \frac{1}{8} \left( a^3 + \frac{1}{a^3} + 3a \frac{1}{a} \left( a + \frac{1}{a} \right) \right) \right] - 3 \left[ \frac{1}{2} \left( a + \frac{1}{a} \right) \right]$$

$$\begin{aligned}
&= 4 \left[ \frac{1}{8} \left( a^3 + \frac{1}{a^3} \right) + \frac{3}{8} \left( a + \frac{1}{a} \right) \right] - 3 \left[ \frac{1}{2} \left( a + \frac{1}{a} \right) \right] \\
&= 4 \left[ \frac{1}{8} \left( a^3 + \frac{1}{a^3} \right) \right] + \frac{3 \times 4}{8} \left( a + \frac{1}{a} \right) - 3 \left[ \frac{1}{2} \left( a + \frac{1}{a} \right) \right] \\
&= 4 \left[ \frac{1}{8} \left( a^3 + \frac{1}{a^3} \right) \right] + \frac{3}{2} \left( a + \frac{1}{a} \right) - \frac{3}{2} \left( a + \frac{1}{a} \right) \\
&= 4 \left[ \frac{1}{8} \left( a^3 + \frac{1}{a^3} \right) \right]
\end{aligned}$$

$$\cos 3x = \frac{1}{2} \left( a^3 + \frac{1}{a^3} \right) \text{----- (1)}$$

If we compare the RHS of the  $\cos 3x$  equation with the now derived equation (1) we get,

$$\lambda \left( a^3 + \frac{1}{a^3} \right) = \frac{1}{2} \left( a^3 + \frac{1}{a^3} \right)$$

From the here we can clearly say that  $\lambda = \frac{1}{2}$

Hence the answer is option B.

### 9. Question

Mark the Correct alternative in the following:

If  $2 \tan \alpha = 3 \tan \beta$ , then  $\tan (\alpha - \beta) =$

A.  $\frac{\sin 2\beta}{5 - \cos 2\beta}$

B.  $\frac{\cos 2\beta}{5 - \cos 2\beta}$

C.  $\frac{\sin 2\beta}{5 + \cos 2\beta}$

D. None of these

### Answer

Given,  $2 \tan \alpha = 3 \tan \beta$

From here we get,  $\tan \alpha = \frac{3}{2} \tan \beta$  ----- (1)

Now consider  $\tan (\alpha - \beta)$ ,

The expansion of  $\tan (\alpha - \beta)$  is given by

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

As we already know the value of  $\tan \alpha$  from equation (1), we have,

$$\tan(\alpha - \beta) = \frac{\left( \frac{3}{2} \tan \beta \right) - \tan \beta}{1 + \left( \frac{3}{2} \tan \beta \right) \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\left( \frac{3 \tan \beta - 2 \tan \beta}{2} \right)}{\left( \frac{2 + 3 \tan^2 \beta}{2} \right)}$$

$$= \frac{\tan \beta}{2 + 3 \tan^2 \beta}$$

[ by using  $\tan \theta = \frac{\sin \theta}{\cos \theta}$  ]

$$= \frac{\left( \frac{\sin \beta}{\cos \beta} \right)}{2 + 3 \left( \frac{\sin \beta}{\cos \beta} \right)^3}$$

$$= \frac{\sin \beta \cos \beta}{2 \cos^2 \beta + 3 \sin^2 \beta}$$

$$= \frac{\sin \beta \cos \beta}{2 \cos^2 \beta + 3(1 - \cos^2 \beta)}$$

$$= \frac{\sin \beta \cos \beta}{2 \cos^2 \beta + 3 - 3 \cos^2 \beta}$$

$$= \frac{\sin \beta \cos \beta}{3 - \cos^2 \beta}$$

Multiplying and dividing the equation with 2

$$= \frac{2 \sin \beta \cos \beta}{2(3 - \cos^2 \beta)}$$

[using  $\sin 2\theta = 2 \sin \theta \cos \theta$ ]

$$= \frac{\sin 2\beta}{6 - 2 \cos^2 \beta}$$

In the denominator adding and subtracting 1

$$= \frac{\sin 2\beta}{6 - 2 \cos^2 \beta + 1 - 1}$$

$$= \frac{\sin 2\beta}{(6 - 1) - (2 \cos^2 \beta - 1)}$$

[using  $\cos 2\theta = 2 \cos^2 \theta - 1$ ]

$$\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$$

Hence, in the question the answer matches with option A.

### 10. Question

Mark the Correct alternative in the following:

If  $\tan \alpha = \frac{1 - \cos \beta}{\sin \beta}$ , then

A.  $\tan 3 \alpha = \tan 2 \beta$  ok

B.  $\tan 2 \alpha = \tan \beta$

C.  $\tan 2 \alpha = \tan \alpha$

D. None of these

### Answer

Given,  $\tan A = \frac{1 - \cos B}{\sin B}$

As there are 2 option in terms of  $\tan 2A$ , let us consider  $\tan 2A$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

[by using the formula for  $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$ ]

$$\tan 2A = \frac{2 \left( \frac{1 - \cos B}{\sin B} \right)}{1 - \left( \frac{1 - \cos B}{\sin B} \right)^2}$$

[by substituting the value of  $\tan A$  as given in the problem]

$$\begin{aligned} \tan 2A &= \frac{2 \left( \frac{1 - \cos B}{\sin B} \right)}{\frac{\sin^2 B - (1 - \cos B)^2}{\sin^2 B}} \\ &= \frac{2(1 - \cos B) \sin B}{\sin^2 B - (1 - \cos B)^2} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos^2 B) - (1 - \cos B)^2} \\ &= \frac{2(1 - \cos B) \sin B}{(1 + \cos B)(1 - \cos B) - (1 - \cos B)^2} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos B)[1 + \cos B - 1 + \cos B]} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos B)2 \cos B} \\ &= \frac{2(1 - \cos B) \sin B}{(1 - \cos B)2 \cos B} \\ &= \frac{\sin B}{\cos B} \\ &= \tan B \end{aligned}$$

Therefore,  $\tan 2A = \tan B$

Hence the option B is the correct answer.

### 11. Question

Mark the Correct alternative in the following:

If  $\sin \alpha + \sin \beta = a$  and  $\cos \alpha - \cos \beta = b$ , then  $\tan \frac{\alpha - \beta}{2} =$

A.  $-\frac{a}{b}$

B.  $-\frac{b}{a}$

C.  $\sqrt{a^2 + b^2}$

D. None of these

### Answer

Given,  $\sin \alpha + \sin \beta = a$  and  $\cos \alpha - \cos \beta = b$ , then the value of

$$\tan \frac{\alpha - \beta}{2}$$

Consider  $\sin \alpha + \sin \beta = a$

As per the expansion of  $\sin A + \sin B = 2 \sin \left( \frac{A+B}{2} \right) \cos \left( \frac{A-B}{2} \right)$

$$\text{Now, } \sin \alpha + \sin \beta = 2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right) = a \text{ ----- (1)}$$

Similarly,  $\cos \alpha - \cos \beta = b$

As per the expansion of  $\cos A - \cos B = -2 \sin \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right)$

$$\text{Now } \cos \alpha - \cos \beta = -2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right) = b \text{ ----- (2)}$$

By dividing equation (1) with (2) we get,

$$\frac{\sin \alpha + \sin \beta}{\cos \alpha - \cos \beta} = \frac{2 \sin \left( \frac{\alpha + \beta}{2} \right) \cos \left( \frac{\alpha - \beta}{2} \right)}{-2 \sin \left( \frac{\alpha + \beta}{2} \right) \sin \left( \frac{\alpha - \beta}{2} \right)} = \frac{a}{b}$$

$$= -\frac{\cos \left( \frac{\alpha - \beta}{2} \right)}{\sin \left( \frac{\alpha - \beta}{2} \right)} = \frac{a}{b}$$

$$= -\cot \left( \frac{\alpha - \beta}{2} \right) = \frac{a}{b}$$

$$[\text{As } \tan \theta = \frac{1}{\cot \theta}]$$

$$= \tan \left( \frac{\alpha - \beta}{2} \right) = -\frac{b}{a}$$

Therefore the answer is option B.

## 12. Question

Mark the Correct alternative in the following:

The value of  $\left( \cot \frac{x}{2} - \tan \frac{x}{2} \right)^2 (1 - 2 \tan x \cot 2x)$  is

- A. 1
- B. 2
- C. 3
- D. 4

## Answer

Given to find the value of  $\left( \cot \frac{x}{2} - \tan \frac{x}{2} \right)^2 (1 - 2 \tan x \cot 2x)$

We will solve the expression in two parts,

Now solving 1<sup>st</sup> term

$$\left( \cot \frac{x}{2} - \tan \frac{x}{2} \right)^2 = \left( \frac{1}{\tan \frac{x}{2}} - \tan \frac{x}{2} \right)^2$$

$$= \left( \frac{1}{\tan \frac{x}{2}} - \tan \frac{x}{2} \right)^2$$

$$= \left( \frac{1 - \tan^2 \frac{x}{2}}{\tan \frac{x}{2}} \right)^2$$

If we multiply and divide the term by 2, we get,

$$= \left( \frac{2 \left( 1 - \tan^2 \frac{x}{2} \right)}{2 \tan \frac{x}{2}} \right)^2$$

$$= 2^2 \left( \frac{1 - \tan^2 \frac{x}{2}}{2 \tan \frac{x}{2}} \right)^2$$

[using the formula for  $\cot 2x = \frac{1 - \tan^2 x}{2 \tan x}$  and  $\cot x = \frac{1}{\tan x}$  ]

$$= 2^2 \left( \frac{1}{\tan x} \right)^2$$

$$\left( \cot \frac{x}{2} - \tan \frac{x}{2} \right)^2 = \frac{4}{\tan^2 x} \text{ ----- (1)}$$

Solving the 2<sup>nd</sup> term

$$(1 - 2 \tan x \cot 2x) = 1 - 2 \tan x \left( \frac{1 - \tan^2 x}{2 \tan x} \right)$$

[using the formula for  $\cot 2x = \frac{1 - \tan^2 x}{2 \tan x}$ ]

$$1 - 2 \tan x \cot 2x = 1 - (1 - \tan^2 x)$$

$$= 1 - 1 + \tan^2 x$$

$$1 - 2 \tan x \cot 2x = \tan^2 x \text{ ----- (2)}$$

Now by combining (1) and (2) we get,

$$\left( \cot \frac{x}{2} - \tan \frac{x}{2} \right)^2 (1 - 2 \tan x \cot 2x) = \left( \frac{4}{\tan^2 x} \right) (\tan^2 x)$$

$$\left( \cot \frac{x}{2} - \tan \frac{x}{2} \right)^2 (1 - 2 \tan x \cot 2x) = 4$$

Hence the answer is option D.

### 13. Question

Mark the Correct alternative in the following:

The value of  $\tan x \sin \left( \frac{\pi}{2} + x \right) \cos \left( \frac{\pi}{2} - x \right)$  is

- A. 1
- B. -1
- C.  $\frac{1}{2} \sin 2x$



D. None of these

### Answer

Given to find the value of the expression  $\tan x \sin\left(\frac{\pi}{2} + x\right) \cos\left(\frac{\pi}{2} - x\right)$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x \text{ (as sine is positive in 2<sup>nd</sup> quadrant)}$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \text{ (as cosine is positive in 1<sup>st</sup> quadrant)}$$

$$\tan x \sin\left(\frac{\pi}{2} + x\right) \cos\left(\frac{\pi}{2} - x\right) = \tan x \cos x \sin x$$

$$= \frac{\sin x}{\cos x} \cos x \sin x$$

$$= \sin^2 x$$

$$\text{There for } \tan x \sin\left(\frac{\pi}{2} + x\right) \cos\left(\frac{\pi}{2} - x\right) = \sin^2 x$$

Hence the answer is option D.

### 14. Question

Mark the Correct alternative in the following:

The value of  $\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{4\pi}{9}\right)$  is

A. 1

B. 2

C. 4

D. None of these

### Answer

Given to find the value of  $\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{4\pi}{9}\right)$

The angles can be modified as  $\frac{7\pi}{18} = \frac{\pi}{2} - \frac{\pi}{9}$  and  $\frac{4\pi}{9} = \frac{\pi}{2} - \frac{\pi}{18}$

$$\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{4\pi}{9}\right)$$

$$= \sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{9}\right) + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{18}\right)$$

Using the identity  $\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$ , we have

$$= \sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \cos^2\left(\frac{\pi}{9}\right) + \cos^2\left(\frac{\pi}{18}\right)$$

$$= \left[\sin^2\left(\frac{\pi}{18}\right) + \cos^2\left(\frac{\pi}{18}\right)\right] + \left[\sin^2\left(\frac{\pi}{9}\right) + \cos^2\left(\frac{\pi}{9}\right)\right]$$

[using the identity  $\cos^2\theta + \sin^2\theta = 1$ ]

$$= 1 + 1 = 2$$

$$\sin^2\left(\frac{\pi}{18}\right) + \sin^2\left(\frac{\pi}{9}\right) + \sin^2\left(\frac{7\pi}{18}\right) + \sin^2\left(\frac{4\pi}{9}\right) = 2$$

Hence the answer is option B.

### 15. Question

Mark the Correct alternative in the following:

If  $5 \sin \alpha = 3 \sin (\alpha + 2 \beta) \neq 0$ , then  $\tan (\alpha + \beta)$  is equal to

- A.  $2 \tan \beta$
- B.  $3 \tan \beta$
- C.  $4 \tan \beta$
- D.  $6 \tan \beta$

**Answer**

Given  $5 \sin \alpha = 3 \sin (\alpha + 2 \beta) \neq 0$ , then the value of  $\tan (\alpha + \beta)$  is

Consider the given equation,

$$5 \sin \alpha = 3 \sin (\alpha + 2 \beta)$$

$$\frac{\sin(\alpha + 2 \beta)}{\sin \alpha} = \frac{5}{3}$$

By applying componendo and dividendo  $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

We get

$$\frac{\sin(\alpha + 2 \beta) + \sin \alpha}{\sin(\alpha + 2 \beta) - \sin \alpha} = \frac{5 + 3}{5 - 3}$$

[ using  $\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$  and  $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$  sum of angles ]

$$\frac{2 \sin\left(\frac{\alpha + 2\beta + \alpha}{2}\right) \cos\left(\frac{\alpha + 2\beta - \alpha}{2}\right)}{2 \cos\left(\frac{\alpha + 2\beta + \alpha}{2}\right) \sin\left(\frac{\alpha + 2\beta - \alpha}{2}\right)} = \frac{8}{2}$$

$$\frac{2 \sin\left(\frac{2(\alpha + \beta)}{2}\right) \cos\left(\frac{2\beta}{2}\right)}{2 \cos\left(\frac{2(\alpha + \beta)}{2}\right) \sin\left(\frac{2\beta}{2}\right)} = 4$$

$$\frac{2 \sin(\alpha + \beta) \cos(\beta)}{2 \cos(\alpha + \beta) \sin(\beta)} = 4$$

$$\frac{\left[\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}\right]}{\left[\frac{\sin \beta}{\cos \beta}\right]} = 4$$

$$\frac{\tan(\alpha + \beta)}{\tan \beta} = 4$$

This clearly shows,  $\tan (\alpha + \beta) = 4 \tan \beta$

Hence the answer is option C.

**16. Question**

Mark the Correct alternative in the following:

The value of  $2 \cos x - \cos 3x - \cos 5x - 16 \cos^3 x \sin^2 x$  is

- A. 2
- B. 1
- C. 0

D. -1

### Answer

Given expression is  $2 \cos x - \cos 3x - \cos 5x - 16 \cos^3 x \sin^2 x$

Consider the expression

$$2 \cos x - \cos 3x - \cos 5x - 16 \cos^3 x \sin^2 x$$

$$= 2 \cos x - (\cos 5x + \cos 3x) - 16 \cos^3 x \sin^2 x$$

$$[\text{using the sum of angles } \cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)]$$

$$= 2 \cos x - \left[ 2 \cos\left(\frac{5x+3x}{2}\right) \cos\left(\frac{5x-3x}{2}\right) \right] - 16 \cos^3 x \sin^2 x$$

$$= 2 \cos x - [2 \cos 4x \cos x] - 16 \cos^3 x \sin^2 x$$

$$= 2 \cos x (1 - \cos 4x) - 16 \cos^3 x \sin^2 x$$

$$[\text{using the property } \cos 2\theta = 1 - 2 \sin^2 \theta]$$

$$= 2 \cos x [1 - (1 - 2 \sin^2 2x)] - 16 \cos^3 x \sin^2 x$$

$$= 2 \cos x [2 \sin^2 2x] - 16 \cos^3 x \sin^2 x$$

$$= 4 \cos x [2 \sin x \cos x]^2 - 16 \cos^3 x \sin^2 x$$

$$[\text{using } \sin 2\theta = 2 \sin \theta \cos \theta]$$

$$= 4 \times 4 (\cos x \sin^2 x \cos^2 x) - 16 \cos^3 x \sin^2 x$$

$$= 16 \cos^3 x \sin^2 x - 16 \cos^3 x \sin^2 x$$

$$= 0$$

$$\text{Hence } \cos x - \cos 3x - \cos 5x - 16 \cos^3 x \sin^2 x = 0$$

The answer is option C.

### 17. Question

Mark the Correct alternative in the following:

If  $A = 2 \sin^2 x - \cos 2x$ , then A lies in the interval

A.  $[-1, 3]$

B.  $[1, 2]$

C.  $[-2, 4]$

D. None of these

### Answer

$$\text{Given } A = 2 \sin^2 x - \cos 2x$$

$$[\text{using } \cos 2x = 1 - 2 \sin^2 x]$$

$$\text{so } A = 2 \sin^2 x - \cos 2x = 2 \sin^2 x - [1 - 2 \sin^2 x]$$

$$= 2 \sin^2 x - 1 + 2 \sin^2 x$$

$$= 4 \sin^2 x - 1$$

$$\text{Now } A = 2 \sin^2 x - \cos 2x = 4 \sin^2 x - 1$$

As we know  $\sin x$  lies between -1 and 1

$$-1 \leq \sin x \leq 1$$

$$0 \leq \sin^2 x \leq 1$$

Multiplying the inequality by 4

$$0 \leq 4 \sin^2 x \leq 4$$

Subtracting 1 from the inequality

$$-1 \leq (4 \sin^2 x - 1) \leq 3$$

From the above inequation, we can say that

$$A = (4 \sin^2 x - 1) \text{ belongs to the closed interval } [-1, 3]$$

Hence the answer is A.

### 18. Question

Mark the Correct alternative in the following:

The value of  $\frac{\cos 3x}{2 \cos 2x - 1}$  is equal to

A.  $\cos x$

B.  $\sin x$

C.  $\tan x$

D. None of these

### Answer

Given expression is  $\frac{\cos 3x}{2 \cos 2x - 1}$

Consider

$$\frac{\cos 3x}{2 \cos 2x - 1} = \frac{4 \cos^3 x - 3 \cos x}{2 [2 \cos^2 x - 1] - 1}$$

[using the formulae  $\cos 3x = 4 \cos^3 x - 3 \cos x$  and

$$\cos 2x = 2 \cos^2 x - 1]$$

$$\frac{\cos 3x}{2 \cos 2x - 1} = \frac{\cos x (4 \cos^2 x - 3)}{4 \cos^2 x - 2 - 1}$$

$$= \frac{\cos x (4 \cos^2 x - 3)}{4 \cos^2 x - 3}$$

$$= \cos x$$

Therefore  $\frac{\cos 3x}{2 \cos 2x - 1} = \cos x$

Hence the answer is option A.

### 19. Question

Mark the Correct alternative in the following:

If  $\tan (\pi/4 + x) + \tan (\pi/4 - x) = \lambda \sec 2x$ , then

A. 3

B. 4

C. 1

D. 2

### Answer

Given equation is

$$\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = \lambda \sec 2x$$

Let us consider LHS

$$\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = \left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right) + \left(\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right)$$

[ using the formulae  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$  and  $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$  ]

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right) + \left(\frac{1 - \tan x}{1 + \tan x}\right)$$

[ the value of  $\tan 45^\circ = 1$  ]

$$= \frac{(1 + \tan x)^2 + (1 - \tan x)^2}{(1 + \tan x)(1 - \tan x)}$$

$$= \frac{(1 + \tan^2 x + 2 \tan x) + (1 + \tan^2 x - 2 \tan x)}{(1 + \tan x)(1 - \tan x)}$$

$$= \frac{2(1 + \tan^2 x)}{(1 - \tan^2 x)}$$

$$= \frac{2\left(1 + \frac{\sin^2 x}{\cos^2 x}\right)}{\left(1 - \frac{\sin^2 x}{\cos^2 x}\right)}$$

$$= \frac{2\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)}{\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x}\right)}$$

[using the formulae  $\cos 2x = \cos^2 x - \sin^2 x$  and  $\cos^2 x + \sin^2 x = 1$ ]

$$= \frac{2(1)}{(\cos 2x)}$$

$$= 2 \sec 2x$$

Now comparing with the LHS with RHS

$$\tan\left(\frac{\pi}{4} + x\right) + \tan\left(\frac{\pi}{4} - x\right) = 2 \sec 2x = \lambda \sec 2x$$

From here we can clearly say that the answer is option D.

### 20. Question

Mark the Correct alternative in the following:

The value of  $\cos^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right)$  is

A.  $\frac{1}{2} \cos 2x$

B. 0

C.  $-\frac{1}{2}\cos 2x$

D.  $\frac{1}{2}$

**Answer**

Given expression is  $\cos^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right)$

[using the identity  $\sin^2 x + \cos^2 x = 1$ ]

$$\begin{aligned}\cos^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right) &= 1 - \sin^2\left(\frac{\pi}{6} + x\right) - \sin^2\left(\frac{\pi}{6} - x\right) \\ &= 1 - \left[\sin^2\left(\frac{\pi}{6} + x\right) + \sin^2\left(\frac{\pi}{6} - x\right)\right]\end{aligned}$$

[using the formula  $a^2 + b^2 = (a + b)^2 - 2ab$ ]

$$= 1 - \left[\left(\sin\left(\frac{\pi}{6} + x\right) + \sin\left(\frac{\pi}{6} - x\right)\right)^2 - 2\sin\left(\frac{\pi}{6} + x\right)\sin\left(\frac{\pi}{6} - x\right)\right]$$

[ using the sum of angle formula  $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$  ]

$$= 1 - \left[\left(2\sin\left(\frac{\frac{\pi}{6} + x + \frac{\pi}{6} - x}{2}\right)\cos\left(\frac{\frac{\pi}{6} + x - \frac{\pi}{6} - x}{2}\right)\right)^2 - 2\sin\left(\frac{\pi}{6} + x\right)\sin\left(\frac{\pi}{6} - x\right)\right]$$

$$= 1 - \left[\left(2\sin\left(\frac{\pi}{6}\right)\cos(x)\right)^2 + \left(-2\sin\left(\frac{\pi}{6} + x\right)\sin\left(\frac{\pi}{6} - x\right)\right)\right]$$

[Using the identity  $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$  ]

$$= 1 - \left[\left(2\left(\frac{1}{2}\right)\cos(x)\right)^2 + \left(\cos\left(\frac{\pi}{6} + x + \frac{\pi}{6} - x\right) - \cos\left(\frac{\pi}{6} + x - \frac{\pi}{6} + x\right)\right)\right]$$

$$= 1 - \left[\cos^2 x + \cos\frac{\pi}{3} - \cos 2x\right]$$

$$= 1 - \cos^2 x - \frac{1}{2} + \cos 2x$$

[multiplying and dividing the term  $\cos^2 x$  with 2]

$$= 1 - \frac{2\cos^2 x}{2} - \frac{1}{2} + \cos 2x$$

$$= \frac{1}{2} - \frac{2\cos^2 x}{2} + \cos 2x$$

$$= \cos 2x - \left(\frac{2\cos^2 x - 1}{2}\right)$$

[using the  $\cos 2\theta = 2\cos^2 \theta - 1$ ]

$$= \cos 2x - \frac{1}{2}\cos 2x$$

$$= \frac{1}{2}\cos 2x$$

Hence the answer is option A.

### 21. Question

Mark the Correct alternative in the following:

$\frac{\sin 3x}{1 + 2 \cos 2x}$  is equal to

- A.  $\cos x$
- B.  $\sin x$
- C.  $-\cos x$
- D.  $\sin x$

### Answer

Given expression  $\frac{\sin 3x}{1 + 2 \cos 2x}$

$$\frac{\sin 3x}{1 + 2 \cos 2x} = \frac{3 \sin x - 4 \sin^3 x}{1 + 2 (1 - 2 \sin^2 x)}$$

[Using the formulae  $\sin 3x = 3 \sin x - 4 \sin^3 x$  and  $\cos 2x = 1 - 2 \sin^2 x$ ]

$$\begin{aligned} &= \frac{3 \sin x - 4 \sin^3 x}{1 + 2 - 4 \sin^2 x} \\ &= \frac{\sin x (3 - 4 \sin^2 x)}{3 - 4 \sin^2 x} \end{aligned}$$

$$= \sin x$$

$$\frac{\sin 3x}{1 + 2 \cos 2x} = \sin x$$

Hence the answer is option B.

### 22. Question

Mark the Correct alternative in the following:

The value of  $2 \sin^2 B + 4 \cos (A + B) \sin A \sin B + \cos 2 (A + B)$  is

- A. 0
- B.  $\cos 3 A$
- C.  $\cos 2 A$
- D. None of these

### Answer

Given expression is

$$2 \sin^2 B + 4 \cos (A + B) \sin A \sin B + \cos 2 (A + B)$$

[ using the  $\cos (A+B) = \cos A \cos B - \sin A \sin B$  ]

$$= 2 \sin^2 B + 4 \sin A \sin B [\cos A \cos B - \sin A \sin B] + \cos 2 (A + B)$$

$$= 2 \sin^2 B + 4 \sin A \sin B \cos A \cos B - 4 \sin A \sin B \sin A \sin B + \cos 2 (A + B)$$

$$= 2 \sin^2 B + (2 \sin A \cos A) (2 \sin B \cos B) - 4 \sin^2 A \sin^2 B + \cos 2 (A + B)$$

[ using  $\sin 2A = 2 \sin A \cos A$  ]

$$\begin{aligned}
&= 2 \sin^2 B + \sin 2A \sin 2B - 4 \sin^2 A \sin^2 B + \cos (2A + 2B) \\
&= 2 \sin^2 B (1 - 2 \sin^2 A) + \sin 2A \sin 2B + (\cos 2A \cos 2B - \sin 2A \sin 2B) \\
&[ \text{using } \cos (A+B) = \cos A \cos B - \sin A \sin B ] \\
&= 2 \sin^2 B (1 - 2 \sin^2 A) + \sin 2A \sin 2B + \cos 2A \cos 2B - \sin 2A \sin 2B \\
&[ \text{using } \cos 2A = 1 - 2 \sin^2 x ] \\
&= 2 \sin^2 B \cos 2A + \cos 2A \cos 2B \\
&= \cos 2A (2 \sin^2 B + \cos 2B) \\
&[ \text{using } \cos 2A = \cos^2 x - \sin^2 x ] \\
&= \cos 2A (2 \sin^2 B + \cos^2 B - \sin^2 B) \\
&= \cos 2A (\sin^2 B + \cos^2 B) \\
&[ \text{using the identity } \sin^2 x + \cos^2 x = 1 ] \\
&= \cos 2A (1) \\
&= \cos 2A
\end{aligned}$$

Hence

$$2 \sin^2 B + 4 \cos (A + B) \sin A \sin B + \cos 2 (A + B) = \cos 2A$$

The answer is option C.

### 23. Question

Mark the Correct alternative in the following:

The value of  $\frac{2(\sin 2x + 2 \cos^2 x - 1)}{\cos x - \sin x - \cos 3x + \sin 3x}$  is

- A.  $\cos x$
- B.  $\sec x$
- C.  $\operatorname{cosec} x$
- D.  $\sin x$

### Answer

Given expression is  $\frac{2(\sin 2x + 2 \cos^2 x - 1)}{\cos x - \sin x - \cos 3x + \sin 3x}$

$$\frac{2(\sin 2x + 2 \cos^2 x - 1)}{\cos x - \sin x - \cos 3x + \sin 3x}$$

[ using  $\cos 2A = \cos^2 x - \sin^2 x$  ]

$$= \frac{2(\sin 2x + \cos 2x)}{\cos x - \sin x - \cos 3x + \sin 3x}$$

$$= \frac{2(\sin 2x + \cos 2x)}{(\sin 3x - \sin x) - (\cos 3x - \cos x)}$$

[ using  $\sin A - \sin B = \cos \frac{A+B}{2} \sin \frac{A-B}{2}$  and  $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$  ]

$$= \frac{2(\sin 2x + \cos 2x)}{\left(2 \cos \frac{3x+x}{2} \sin \frac{3x-x}{2}\right) - \left(-2 \sin \frac{3x+x}{2} \sin \frac{3x-x}{2}\right)}$$



$$= \frac{2 (\sin 2x + \cos 2x)}{2 \cos 2x \sin x + 2 \sin 2x \sin x}$$

$$= \frac{2 (\sin 2x + \cos 2x)}{2 \sin x (\cos 2x + \sin 2x)}$$

$$= \frac{1}{\sin x}$$

$$= \operatorname{cosec} x$$

$$\text{Therefore } \frac{2 (\sin 2x + 2 \cos^2 x - 1)}{\cos x - \sin x - \cos 3x + \sin 3x} = \operatorname{cosec} x$$

Answer is option C.

#### 24. Question

Mark the Correct alternative in the following:

$2(1 - 2 \sin^2 7x) \sin 3x$  is equal to

A.  $\sin 17x - \sin 11x$

B.  $\sin 11x - \sin 17x$

C.  $\cos 17x - \cos 11x$

D.  $\cos 17x + \cos 11x$

#### Answer

Given expression is  $2(1 - 2 \sin^2 7x) \sin 3x$

$$2(1 - 2 \sin^2 7x) \sin 3x = 2 \cos 2(7x) \sin 3x$$

$$[ \text{using } \cos 2A = 1 - 2\sin^2 A ]$$

$$= 2 \cos 14x \sin 3x$$

$$[ \text{using the sum of angles formula } \sin A - \sin B = 2 \cos \left( \frac{A+B}{2} \right) \sin \left( \frac{A-B}{2} \right) ]$$

$$= 2 \cos \left( \frac{17x + 11x}{2} \right) \sin \left( \frac{17x - 11x}{2} \right)$$

$$= \sin (17x) - \sin (11x)$$

$$\text{Therefore } 2(1 - 2 \sin^2 7x) \sin 3x = \sin (17x) - \sin (11x)$$

The answer is option A.

#### 25. Question

Mark the Correct alternative in the following:

$$\text{If } \alpha \text{ and } \beta \text{ are acute angles satisfying } \cos 2\alpha = \frac{3 \cos 2\beta - 1}{3 - \cos 2\beta}, \text{ then } \tan \alpha =$$

A.  $\sqrt{2} \tan \beta$

B.  $\frac{1}{\sqrt{2}} \tan \beta$

C.  $\sqrt{2} \cot \beta$

D.  $\frac{1}{\sqrt{2}} \cot \beta$

### Answer

Given for  $\alpha < 90^\circ$  and  $\beta < 90^\circ$ ,  $\cos 2\alpha = \frac{3 \cos 2\beta - 1}{3 - \cos 2\beta}$

Then  $\tan \alpha$  is given by

Consider

$$\frac{\cos 2\alpha}{1} = \frac{2 \cos 2\beta - 1}{3 - \cos 2\beta}$$

[using componendo and dividend principle, if  $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$ ]

$$\frac{\cos 2\alpha + 1}{\cos 2\alpha - 1} = \frac{(3 \cos 2\beta - 1) + (3 - \cos 2\beta)}{(3 \cos 2\beta - 1) - (3 - \cos 2\beta)}$$

$$\frac{(1 - 2 \sin^2 \alpha) + 1}{(2 \cos^2 \alpha - 1) - 1} = \frac{(2 \cos 2\beta + 2)}{(4 \cos 2\beta - 4)}$$

[ using  $\cos 2x = 1 - 2 \sin^2 x = 2 \cos^2 x - 1$  ]

$$\frac{2(1 - \sin^2 \alpha)}{-2(1 - \cos^2 \alpha)} = \frac{2(\cos 2\beta + 1)}{4(\cos 2\beta - 1)}$$

[ using  $\cos 2x = \cos^2 x - \sin^2 x$  ]

$$-\frac{\cos^2 \alpha}{\sin^2 \alpha} = \frac{(\cos^2 \beta - \sin^2 \beta + 1)}{2(\cos^2 \beta - \sin^2 \beta - 1)}$$

$$-\frac{\cos^2 \alpha}{\sin^2 \alpha} = \frac{(\cos^2 \beta + 1 - \sin^2 \beta)}{-2(1 - \cos^2 \alpha + \sin^2 x)}$$

[ using  $\cos^2 x + \sin^2 x = 1$  ]

$$-\frac{\cos^2 \alpha}{\sin^2 \alpha} = -\frac{2(\cos^2 \beta)}{4(\sin^2 \beta)}$$

$$\frac{1}{\tan^2 \alpha} = \frac{1}{2 \tan^2 \beta}$$

$$\tan^2 \alpha = 2 \tan^2 \beta$$

applying square root on both sides

$$\sqrt{\tan^2 \alpha} = \sqrt{2 \tan^2 \beta}$$

$$\tan \alpha = \sqrt{2} \tan \beta$$

Hence the answer is option A.

### 26. Question

Mark the Correct alternative in the following:

$$\text{If } \tan \frac{x}{2} = \sqrt{\frac{1-e}{1+e}} \tan \frac{\alpha}{2}, \text{ then } \cos \alpha =$$

A.  $1 - e \cos (\cos x + e)$

B.  $\frac{1+e \cos x}{\cos x - e}$

C.  $\frac{1-e \cos x}{\cos x - e}$

D.  $\frac{\cos x - e}{1 - e \cos x}$

**Answer**

Given  $\tan \frac{x}{2} = \sqrt{\frac{1-e}{1+e}} \tan \frac{\alpha}{2}$ , then  $\cos \alpha$  is

Let

$$\tan \frac{\alpha}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{x}{2}$$

By using the expansion of  $\cos 2x$  in terms of  $\tan x$

$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

We get,

$$\begin{aligned} \cos \alpha &= \frac{1 - \left( \sqrt{\frac{1+e}{1-e}} \tan \frac{x}{2} \right)^2}{1 + \left( \sqrt{\frac{1+e}{1-e}} \tan \frac{x}{2} \right)^2} \\ &= \frac{1 - \left( \frac{1+e}{1-e} \tan^2 \frac{x}{2} \right)}{1 + \left( \frac{1+e}{1-e} \tan^2 \frac{x}{2} \right)} \\ &= \frac{1 - e - \left[ (1+e) \tan^2 \frac{x}{2} \right]}{1 - e + \left[ (1+e) \tan^2 \frac{x}{2} \right]} \\ &= \frac{1 - e - \tan^2 \frac{x}{2} - e \tan^2 \frac{x}{2}}{1 - e + \tan^2 \frac{x}{2} + e \tan^2 \frac{x}{2}} \\ &= \frac{1 - \tan^2 \frac{x}{2} - e - e \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2} - e + e \tan^2 \frac{x}{2}} \\ &= \frac{\left( 1 - \tan^2 \frac{x}{2} \right) - e \left( 1 + \tan^2 \frac{x}{2} \right)}{\left( 1 + \tan^2 \frac{x}{2} \right) - e \left( 1 - \tan^2 \frac{x}{2} \right)} \end{aligned}$$

Dividing the numerator and denominator by  $1 + \tan^2 \frac{x}{2}$

$$\begin{aligned} &= \frac{\left( \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) - e \left( \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)}{\left( \frac{1 + \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) - e \left( \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} \end{aligned}$$

[using the formula for  $\cos 2x$  in terms of  $\tan x$   $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$ ]

$$= \frac{\cos x - e}{1 - e \cos x}$$

Hence the answer is option D.

## 27. Question

Mark the Correct alternative in the following:

If  $(2^n + 1)x = \pi$ , then  $2^n \cos x \cos 2x \cos 2^2 x \dots \cos 2^{n-1} x =$

- A. -1
- B. 1
- C. 1/2
- D. None of these

## Answer

Given  $(2^n - 1)x = \pi$

Then evaluate the expression

$$2^n \cos x \cos 2x \cos 2^2 x \dots \cos 2^{n-1} x$$

by taking a 2 from  $2^n$  and multiplying and dividing by  $\sin x$ , we get

$$= \frac{2^{n-1}}{\sin x} (2 \sin x \cos x) \cos 2x \cos 2^2 x \dots \cos 2^{n-1} x$$

[by using the formula  $\sin 2x = 2 \sin x \cos x$ ]

$$= \frac{2^{n-1}}{\sin x} (\sin 2x) \cos 2x \cos 2^2 x \dots \cos 2^{n-1} x$$

Now borrowing another 2 from  $2^{n-1}$

$$= \frac{2^{n-2}}{\sin x} (2 \sin 2x \cos 2x) \cos 2^2 x \dots \cos 2^{n-1} x$$

$$= \frac{2^{n-2}}{\sin x} (\sin 4x) \cos 4x \dots \cos 2^{n-1} x$$

These iterations repeat till we reach the last term

$$= \frac{2^{n-(n-1)}}{\sin x} \sin 2^{n-1} x \cos 2^{n-1} x$$

$$= \frac{2 \sin 2^{n-1} x \cos 2^{n-1} x}{\sin x}$$

$$= \frac{\sin 2^n x}{\sin x}$$

As already given that

$$2^n x + x = 180^\circ$$

$$2^n x = 180^\circ - x$$

So substituting the same in the above solution

$$2^n \cos x \cos 2x \cos 2^2 x \dots \cos 2^{n-1} x = \frac{\sin(\pi - x)}{\sin x} = \frac{\sin x}{\sin x} = 1$$

So the answer is option B.

### 28. Question

Mark the Correct alternative in the following:

If  $\tan x = t$  then  $\tan 2x + \sec 2x$  is equal to

A.  $\frac{1+t}{1-t}$

B.  $\frac{1-t}{1+t}$

C.  $\frac{2t}{1-t}$

D.  $\frac{2t}{1+t}$

### Answer

Given  $\tan x = t$

then  $\tan 2x + \sec 2x =$

[ using the formulae for  $\tan 2x$  and  $\sec 2x$  in terms of  $\tan x$ ,

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} \text{ and } \sec 2x = \frac{1 + \tan^2 x}{1 - \tan^2 x} ]$$

Now

$$\begin{aligned} \tan 2x + \sec 2x &= \frac{2 \tan x}{1 - \tan^2 x} + \frac{1 + \tan^2 x}{1 - \tan^2 x} \\ &= \frac{2 \tan x + 1 + \tan^2 x}{1 - \tan^2 x} \\ &= \frac{(1 + \tan x)^2}{(1 + \tan x)(1 - \tan x)} \\ &= \frac{(1 + \tan x)}{(1 - \tan x)} \end{aligned}$$

As already given  $\tan x = t$

$$\tan 2x + \sec 2x = \frac{1 + t}{1 - t}$$

Hence the answer is option A.

### 29. Question

Mark the Correct alternative in the following:

The value of  $\cos^4 x + \sin^4 x - 6 \cos^2 x \sin^2 x$  is

A.  $\cos 2x$

B.  $\sin 2x$

C.  $\cos 4x$

D. None of these

### Answer

Given expression is  $\cos^4 x + \sin^4 x - 6 \cos^2 x \sin^2 x$

$$= [(\cos^2 x)^2 + (\sin^2 x)^2 - 2 \cos^2 x \sin^2 x] - 4 \cos^2 x \sin^2 x$$

[ using the formula  $a^2 + b^2 = (a+b)^2 - 2ab$  ]

$$= (\cos^2 x - \sin^2 x)^2 - 4 \cos^2 x \sin^2 x$$

[ using the formula  $\cos 2x = \cos^2 x - \sin^2 x$  ]

$$= (\cos 2x)^2 - (2 \sin x \cos x)^2$$

[ using the formula  $\sin 2x = 2 \sin x \cos x$  ]

$$= (\cos 2x)^2 - (\sin 2x)^2$$

[ using the formula  $\cos 2x = \cos^2 x - \sin^2 x$  ]

$$= \cos 4x$$

Therefore  $\cos^4 x + \sin^4 x - 6 \cos^2 x \sin^2 x = \cos 4x$

The answer is option A.

### 30. Question

Mark the Correct alternative in the following:

The value of  $\cos(36^\circ - A) \cos(36^\circ + A) + \cos(54^\circ - A) \cos(54^\circ + A)$  is

- A.  $\cos 2A$
- B.  $\sin 2A$
- C.  $\cos A$
- D. 0

### Answer

Given expression

$$\cos(36^\circ - A) \cos(36^\circ + A) + \cos(54^\circ - A) \cos(54^\circ + A)$$

In the above expression angle  $\cos(54^\circ + A) = \sin[90^\circ - (54^\circ + A)]$

And  $\cos(54^\circ - A) = \sin[90^\circ - (54^\circ - A)]$

[using  $\cos \theta = \sin(90^\circ - \theta)$  ]

Now substituting the same in the expression

$$= \cos(36^\circ - A) \cos(36^\circ + A) + \sin[90^\circ - (54^\circ - A)] \sin[90^\circ - (54^\circ + A)]$$

$$= \cos(36^\circ - A) \cos(36^\circ + A) + \sin(36^\circ + A) \sin(36^\circ - A)$$

$$= \cos(36^\circ + A) \cos(36^\circ - A) + \sin(36^\circ + A) \sin(36^\circ - A)$$

[using  $\cos(A-B) = \cos A \cos B + \sin A \sin B$  ]

$$= \cos[(36^\circ + A) - (36^\circ - A)]$$

$$= \cos(2A)$$

Therefore the answer is option A.

### 31. Question

Mark the Correct alternative in the following:

The value of  $\tan x \tan\left(\frac{\pi}{3} - x\right) \tan\left(\frac{\pi}{3} + x\right)$  is

- A.  $\cot 3x$
- B.  $2 \cot 3x$
- C.  $\tan 3x$
- D.  $3 \tan 3x$

**Answer**

Given expression is  $\tan x \tan\left(\frac{\pi}{3} - x\right) \tan\left(\frac{\pi}{3} + x\right)$

[using  $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$  and  $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$ ]

Then

$$\begin{aligned} & \tan x \tan\left(\frac{\pi}{3} - x\right) \tan\left(\frac{\pi}{3} + x\right) \\ &= \tan x \left( \frac{\tan \frac{\pi}{3} - \tan x}{1 + \tan \frac{\pi}{3} \tan x} \right) \left( \frac{\tan \frac{\pi}{3} + \tan x}{1 - \tan \frac{\pi}{3} \tan x} \right) \\ &= \tan x \left( \frac{\sqrt{3} - \tan x}{1 + \sqrt{3} \tan x} \right) \left( \frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \tan x} \right) \end{aligned}$$

[ using  $a^2 - b^2 = (a-b)(a+b)$ ]

$$\begin{aligned} &= \tan x \left( \frac{(\sqrt{3})^2 - \tan^2 x}{1 - (\sqrt{3})^2 \tan^2 x} \right) \\ &= \tan x \left( \frac{3 - \tan^2 x}{1 - 3 \tan^2 x} \right) \\ &= \left( \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \right) \end{aligned}$$

[using  $\tan 3x = \left( \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \right)$  formula ]

$$= \tan 3x$$

$$\text{Therefore } \tan x \tan\left(\frac{\pi}{3} - x\right) \tan\left(\frac{\pi}{3} + x\right) = \tan 3x$$

The answer is option C.

### 32. Question

Mark the Correct alternative in the following:

The value  $\tan x + \tan\left(\frac{\pi}{3} + x\right) + \tan\left(\frac{2\pi}{3} + x\right)$  of is

- A.  $3 \tan 3x$
- B.  $\tan 3x$
- C.  $3 \cot 3x$
- D.  $\cot 3x$

**Answer**

$$\text{Given } \tan x + \tan\left(\frac{\pi}{3} + x\right) + \tan\left(\frac{2\pi}{3} + x\right)$$

$$[\text{using } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}]$$

Then

$$\begin{aligned} & \tan x + \tan\left(\frac{\pi}{3} + x\right) + \tan\left(\frac{2\pi}{3} + x\right) \\ &= \tan x + \left(\frac{\tan \frac{\pi}{3} + \tan x}{1 - \tan \frac{\pi}{3} \tan x}\right) + \left(\frac{\tan \frac{2\pi}{3} + \tan x}{1 - \tan \frac{2\pi}{3} \tan x}\right) \\ &= \tan x + \left(\frac{\sqrt{3} + \tan x}{1 - \sqrt{3} \tan x}\right) + \left(\frac{-\sqrt{3} + \tan x}{1 - (-\sqrt{3}) \tan x}\right) \\ &= \tan x + \left(\frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x}\right) + \left(\frac{\tan x - \sqrt{3}}{1 + \sqrt{3} \tan x}\right) \\ &= \tan x + \frac{(\tan x + \sqrt{3})(1 + \sqrt{3} \tan x) + (\tan x - \sqrt{3})(1 - \sqrt{3} \tan x)}{(1 - \sqrt{3} \tan x)(1 + \sqrt{3} \tan x)} \end{aligned}$$

$$[\text{using } a^2 - b^2 = (a-b)(a+b)]$$

$$\begin{aligned} &= \tan x \\ &+ \frac{(\tan x + \sqrt{3} \tan^2 x + \sqrt{3} + 3 \tan x) + (\tan x - \sqrt{3} \tan^2 x - \sqrt{3} + 3 \tan x)}{1 - 3 \tan^2 x} \\ &= \frac{\tan x (1 - 3 \tan^2 x) + 8 \tan x}{1 - 3 \tan^2 x} \\ &= \frac{\tan x - 3 \tan^3 x + 8 \tan x}{1 - 3 \tan^2 x} \\ &= \frac{9 \tan x - 3 \tan^3 x}{1 - 3 \tan^2 x} \\ &= \frac{3(3 \tan x - \tan^3 x)}{1 - 3 \tan^2 x} \end{aligned}$$

$$[\text{using } \tan 3x = \left(\frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}\right) \text{ formula}]$$

$$= 3 \tan 3x$$

$$\text{Therefore } \tan x + \tan\left(\frac{\pi}{3} + x\right) + \tan\left(\frac{2\pi}{3} + x\right) = 3 \tan 3x$$

The answer is option A.

### 33. Question

Mark the Correct alternative in the following:

$$\text{The value of is } \frac{\sin 5\alpha - \sin 3\alpha}{\cos 5\alpha + 2 \cos 4\alpha + \cos 3\alpha}$$

- A.  $\cot \alpha/2$
- B.  $\cot \alpha$
- C.  $\tan \alpha/2$
- D. None of these



## Answer

Given

$$\frac{\sin 5\alpha - \sin 3\alpha}{\cos 5\alpha + 2 \cos 4\alpha + \cos 3\alpha}$$

[Using  $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$$\cos A + \cos B = 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) ]$$

$$= \frac{2 \cos\left(\frac{5\alpha + 3\alpha}{2}\right) \sin\left(\frac{5\alpha - 3\alpha}{2}\right)}{2 \cos\left(\frac{5\alpha + 3\alpha}{2}\right) \cos\left(\frac{5\alpha - 3\alpha}{2}\right) + 2 \cos 4\alpha}$$

$$= \frac{2 \cos 4\alpha \sin \alpha}{2 \cos 4\alpha \cos \alpha + 2 \cos 4\alpha}$$

$$= \frac{2 \cos 4\alpha \sin \alpha}{2 \cos 4\alpha (\cos \alpha + 1)}$$

$$= \frac{\sin \alpha}{(\cos \alpha + 1)}$$

[ using  $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$  and  $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$  ]

$$= \frac{\frac{2 \tan \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}}{\left(\frac{1 - \tan^2 \frac{\alpha}{2}}{1 + \tan^2 \frac{\alpha}{2}}\right) + 1}$$

$$= \frac{2 \tan \frac{\alpha}{2}}{1 - \tan^2 \frac{\alpha}{2} + 1 + \tan^2 \frac{\alpha}{2}}$$

$$= \frac{2 \tan \frac{\alpha}{2} x}{2}$$

$$= \tan \frac{\alpha}{2}$$

$$\text{Therefore } \frac{\sin 5\alpha - \sin 3\alpha}{\cos 5\alpha + 2 \cos 4\alpha + \cos 3\alpha} = \tan \frac{\alpha}{2}$$

Answer is option C.

## 34. Question

Mark the Correct alternative in the following:

$\frac{\sin 5x}{\sin x}$  is equal to

A.  $16 \cos^4 x - 12 \cos^2 x + 1$

B.  $16 \cos^4 x + 12 \cos^2 x + 1$

C.  $16 \cos^4 x - 12 \cos^2 x - 1$

D.  $16 \cos^4 x + 12 \cos^2 x - 1$

## Answer

Given  $\frac{\sin 5x}{\sin x}$

Let  $5x = 3x + 2x$

Then

$$\frac{\sin 5x}{\sin x} = \frac{\sin (3x + 2x)}{\sin x}$$

[using  $\sin (A+B) = \sin A \cos B + \cos A \sin B$ ]

$$= \frac{\sin 3x \cos 2x + \cos 3x \sin 2x}{\sin x}$$

[using the formulae :

$$\sin 3x = 3\sin x - 4\sin^3x$$

$$\cos 3x = 4\cos^3x - 3\cos x$$

$$\cos 2x = 2\cos^2x - 1$$

$$\sin 2x = 2\sin x \cos x]$$

$$= \frac{(3\sin x - 4\sin^3x)(2\cos^2x - 1) + (4\cos^3x - 3\cos x)(2\sin x \cos x)}{\sin x}$$

$$= \frac{\sin x (3 - 4\sin^2x)(2\cos^2x - 1) + \sin x (4\cos^3x - 3\cos x)(2\cos x)}{\sin x}$$

$$= \frac{\sin x [(3 - 4\sin^2x)(2\cos^2x - 1) + (4\cos^3x - 3\cos x)(2\cos x)]}{\sin x}$$

$$= (3 - 4\sin^2x)(2\cos^2x - 1) + (4\cos^3x - 3\cos x)(2\cos x)$$

$$= (6\cos^2x - 3 - 8\sin^2x \cos^2x + 4\sin^2x) + (8\cos^4x - 6\cos^2x)$$

[using  $\sin^2x + \cos^2x = 1$ ]

$$= -3 - 8(1 - \cos^2x) \cos^2x + 4(1 - \cos^2x) + 8\cos^4x$$

$$= -3 - 8\cos^2x + 8\cos^4x + 4 - 4\cos^2x + 8\cos^4x$$

$$= 16\cos^4x - 12\cos^2x + 1$$

Therefore the answer is option A.

### 35. Question

Mark the Correct alternative in the following:

If  $n = 1, 2, 3, \dots$ , then  $\cos \alpha \cos 2\alpha \cos 4\alpha \dots \cos 2^{n-1}\alpha$  is equal to

A.  $\frac{\sin 2^n \alpha}{2^n \sin \alpha}$

B.  $\frac{\sin 2^n \alpha}{2^n \sin 2^{n-1}\alpha}$

C.  $\frac{\sin 4^{n-1}\alpha}{4^{n-1}\sin \alpha}$

D.  $\frac{\sin 2^n \alpha}{2^n \sin \alpha}$

### Answer

Given expression

$$\cos \alpha \cos 2 \alpha \cos 4 \alpha \dots \cos 2^{n-1} \alpha$$

multiplying and dividing the expression by  $2 \sin \alpha$ , we get,

$$= \frac{1}{2 \sin \alpha} (2 \sin \alpha \cos \alpha) \cos 2 \alpha \cos 4 \alpha \dots \dots \dots \cos 2^{n-1} \alpha$$

[using  $\sin 2x = 2 \sin x \cos x$ ]

$$= \frac{1}{2 \sin \alpha} (\sin 2 \alpha) \cos 2 \alpha \cos 4 \alpha \dots \dots \dots \cos 2^{n-1} \alpha$$

Now multiplying and dividing the expression with 2.

$$= \frac{1}{2^2 \sin \alpha} (2 \sin 2 \alpha \cos 2 \alpha) \cos 4 \alpha \dots \dots \dots \cos 2^{n-1} \alpha$$

$$= \frac{1}{2^2 \sin \alpha} (\sin 4 \alpha) \cos 4 \alpha \dots \dots \dots \cos 2^{n-1} \alpha$$

Continuing this process for  $n-1$  times we will get

$$= \frac{1}{2^{n-1} \sin \alpha} \sin 2^{n-1} \alpha \cos 2^{n-1} \alpha$$

Now repeating for the last time,

$$= \frac{1}{(2^{n-1} \times 2) \sin \alpha} (2 \sin 2^{n-1} \alpha \cos 2^{n-1} \alpha)$$

$$= \frac{1}{2^n \sin \alpha} (\sin 2^n \alpha)$$

This proves that

$$\cos \alpha \cos 2 \alpha \cos 4 \alpha \dots \dots \dots \cos 2^{n-1} \alpha = \frac{\sin 2^n \alpha}{2^n \sin \alpha}$$

Hence the answer is option D.

### 36. Question

Mark the Correct alternative in the following:

If  $\tan x = \frac{a}{b}$ , then  $b \cos 2x + a \sin 2x$  is equal to

A. a

B. b

C.  $\frac{a}{b}$

D.  $\frac{b}{a}$

### Answer

Given  $\tan x = \frac{a}{b}$

The value of the expression  $b \cos 2x + a \sin 2x$

Now consider  $b \cos 2x + a \sin 2x$

[ by using  $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$  and  $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$  ]

$$b \cos 2x + a \sin 2x = b \left( \frac{1 - \tan^2 x}{1 + \tan^2 x} \right) + a \left( \frac{2 \tan x}{1 + \tan^2 x} \right)$$

As already given  $\tan x = \frac{a}{b}$

Then

$$\begin{aligned} b \cos 2x + a \sin 2x &= b \left( \frac{1 - \left(\frac{a}{b}\right)^2}{1 + \left(\frac{a}{b}\right)^2} \right) + a \left( \frac{2 \frac{a}{b}}{1 + \left(\frac{a}{b}\right)^2} \right) \\ &= b \left( \frac{\frac{b^2 - a^2}{b^2}}{\frac{b^2 + a^2}{b^2}} \right) + a \left( \frac{2 \frac{a}{b}}{\frac{b^2 + a^2}{b^2}} \right) \\ &= b \left( \frac{b^2 - a^2}{b^2 + a^2} \right) + a \left( \frac{2ab}{b^2 + a^2} \right) \\ &= \left( \frac{b^3 - a^2b}{b^2 + a^2} \right) + \left( \frac{2a^2b}{b^2 + a^2} \right) \\ &= \left( \frac{b^3 - a^2b + 2a^2b}{b^2 + a^2} \right) \\ &= \left( \frac{b^3 + a^2b}{b^2 + a^2} \right) \\ &= \frac{b(b^2 + a^2)}{b^2 + a^2} \\ &= b \end{aligned}$$

Hence  $b \cos 2x + a \sin 2x = b$ .

The answer is option B.

### 37. Question

Mark the Correct alternative in the following:

If  $\tan \alpha = \frac{1}{7}$ ,  $\tan \beta = \frac{1}{3}$ , then  $\cos 2\alpha$  is equal to

- A.  $\sin 2\beta$
- B.  $\sin 4\beta$
- C.  $\sin 3\beta$
- D.  $\cos 2\beta$

### Answer

Given  $\tan \alpha = \frac{1}{7}$  and  $\tan \beta = \frac{1}{3}$

Now to find the value of  $\cos 2\alpha$

[By using  $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$ ]

$$\cos 2\alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

[as  $\tan \alpha = \frac{1}{7}$  is given]

$$\cos 2\alpha = \frac{1 - \left(\frac{1}{7}\right)^2}{1 + \left(\frac{1}{7}\right)^2}$$

$$= \frac{49 - 1}{49 + 1}$$

$$= \frac{48}{50} = \frac{24}{25}$$

$$\text{Hence } \cos 2\alpha = \frac{24}{25}$$

The same value is obtained for  $\sin 4\beta$ .

[By  $\sin 2x = 2 \sin x \cos x$ ]

$$\sin 4\alpha = 2 \sin 2\alpha \cos 2\alpha$$

$$[\text{using } \sin 2x = \frac{2 \tan x}{1 + \tan^2 x} \text{ and } \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}]$$

We have

$$\sin 4\beta = 2 \left( \frac{2 \tan \beta}{1 + \tan^2 \beta} \right) \left( \frac{1 - \tan^2 \beta}{1 + \tan^2 \beta} \right)$$

$$\text{As } \tan \beta = \frac{1}{3}$$

$$\sin 4\beta = 2 \left( \frac{2 \left(\frac{1}{3}\right)}{1 + \left(\frac{1}{3}\right)^2} \right) \left( \frac{1 - \left(\frac{1}{3}\right)^2}{1 + \left(\frac{1}{3}\right)^2} \right)$$

$$= 2 \left( \frac{6}{9 + 1} \right) \left( \frac{9 - 1}{9 + 1} \right)$$

$$= 2 \left( \frac{48}{100} \right) = \frac{48}{50} = \frac{24}{25}$$

As the value of  $\cos 2\alpha$  and  $\sin 4\alpha$  are the same, the answer is option B.

### 38. Question

Mark the Correct alternative in the following:

The value of  $\cos^2 48^\circ - \sin^2 12^\circ$  is

A.  $\frac{\sqrt{5} + 1}{8}$

B.  $\frac{\sqrt{5} - 1}{8}$

C.  $\frac{\sqrt{5} + 1}{5}$  D.

$$\frac{\sqrt{5} + 1}{2\sqrt{2}}$$

**Answer**

Given

$$\cos^2 48^\circ - \sin^2 12^\circ$$

[by using the formula  $\cos 2x = 2\cos^2 x - 1$  and  $\cos 2x = 1 - 2\sin^2 x$ ]

$$\begin{aligned}\cos^2 48^\circ - \sin^2 12^\circ &= \left( \frac{\cos(2 \times 48^\circ) + 1}{2} \right) - \left( \frac{1 - \cos(2 \times 12^\circ)}{2} \right) \\&= \left( \frac{\cos(96^\circ) + 1}{2} \right) - \left( \frac{1 - \cos(24^\circ)}{2} \right) \\&= \left( \frac{\cos(96^\circ) + 1 - 1 + \cos(24^\circ)}{2} \right) \\&= \left( \frac{\cos(96^\circ) + \cos(24^\circ)}{2} \right)\end{aligned}$$

[by using the formula  $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$ ]

$$\begin{aligned}&= \frac{1}{2} \left[ 2\cos\left(\frac{96^\circ + 24^\circ}{2}\right)\cos\left(\frac{96^\circ - 24^\circ}{2}\right) \right] \\&= \cos\left(\frac{120^\circ}{2}\right)\cos\left(\frac{72^\circ}{2}\right) \\&= \cos(60^\circ)\cos(36^\circ) \\&= \frac{1}{2} \left( \frac{1 + \sqrt{5}}{4} \right) \\&= \frac{1 + \sqrt{5}}{8}\end{aligned}$$

$$\text{Therefore } \cos^2 48^\circ - \sin^2 12^\circ = \frac{1 + \sqrt{5}}{8}$$

Hence the answer is option A.