CHAPTER

1 INDICES AND SURDS

☐ Before we proceed to exponents (Indices) and surds, it is proper to learn about Real numbers.

☐ **Number System**

Natural Numbers : These are the numbers (1, 2, 3, ...etc) that are used for counting. In other words, all positive integers are natural numbers.

The least natural number is 1 but there is no largest natural number.

The set of natural number is denoted by N.

Thus, N = {1, 2, 3,......}

☐ **Whole Numbers :** The set of numbers that includes all natural numbers and the number zero are called whole numbers.

The set of whole numbers is denoted by W.

Thus, W = {0, 1, 2, 3, ...}

Note : Whole numbers are also called as "Non-negative Integers".

Integers : All the natural numbers, zero, and the the negatives of natural numbers are called integers.

I = {......, -3, -2, -1, 0, 1, 2, 3, ...}

(i) Set of negative integers = {-1, -2, -3, ...}

(ii) Set of non-negative integers = {0, 1, 2, 3...}

(iii) Set of positive integers = {1, 2, 3 ....}

(iv) Set of non-positive integers = {0, 1, 2, 3.....}

Note: '0' is definately a non-negative integer as well as a non-positive integer.

☐ **Rational numbers** : The numbers which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers and their set is denoted by Q.

e.g. $\frac{1}{4}, \frac{2}{5}, -\frac{3}{7}, 6\left(as\ 6 = \frac{6}{1}\right)$etc. are rational numbers.

The set of rational numbers encloses the set of integers and fractions.

Representation of Rational Numbers as Decimals : The decimal form of a rational number is either terminating or non-terminating.

E.g.: $\frac{17}{4} = 4.25,\ \frac{21}{5} = 4.2 \rightarrow$ terminating (or finite) decimal.

$\frac{16}{3} = 5.\bar{3},\ \frac{2}{3} = 0.\bar{6} \rightarrow$ Non-terminating (or Recurring) decimal.

Note: If the denominator of a rational number has no prime factors other than 2 or 5, then and only then it is expressible as a terminating decimal.

☐ **Irrational numbers** : The numbers which when expressed in decimal form are neither terminating nor repeating decimals are called " Irrational numbers".

e.g. $\sqrt{2}, \sqrt{3}, \sqrt{50}, \sqrt{7}, \pi$ etc

Note: The exact value of π is not $\frac{22}{7}$, as $\frac{22}{7}$ is rational while π is irrational. $\frac{22}{7}$ is the approximate value of π. Similarly 3.14 is not an exact value of π.

☐ **Real numbers** : All rational and irrational numbers together form the set of real numbers, denoted by R.

Thus, every natural number, every whole number, every integer, every rational number and every irrational number is a real number.

Note :

(i) The sum (or difference) of a rational and an irrational number is irrational.

e.g. $(4+\sqrt{3}),(2-\sqrt{5}),\left(\frac{3}{2}-\sqrt{2}\right),7+\pi$ etc. are all irrational.

(ii) The product of a rational and an irrational number is irrational, e.g. $4\sqrt{3}$, $-2\sqrt{5}$ etc. are all irrational.

☐ **Even and Odd numbers** : Integers divisible by 2 are called even numbers, while those which are not divisible by 2 are known as odd integers.

Thus, ..., -6, -4, -2, 0, 2, 4, 6, ...etc. are even integers.

And, ..., -5, -3, -1, 1, 3, 5, ...etc are odd integers.

☐ **Prime numbers** : A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and itself.

E.g.: 2, 3, 5, 7, 11, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, etc.

→ 2 is the only even number which is prime.

☐ **Composite numbers** : Composite numbers are the numbers greater than 1 which are not prime.

e.g. 4, 6, 9, 14, 15, etc.

Note: 1 is neither prime nor composite. There are 25 prime numbers between numbers 1 & 100.

Test for Prime Numbers : Let x be a given number and let k be an integer very near to $\sqrt{x}$ s.t. $k > \sqrt{x}$. If x is not divisible by any prime number less than k, then x is prime, otherwise, it is not prime.

E.g.: Check whether 571 is prime or not ?

clearly, $24 > \sqrt{571}$

So, we divide 571 by each prime number less than 24 which are 2, 3, 5, 7, 9, 11, 17. 19. and 23.

we find that 571 is not divisible by any of them. So, 571 is a prime number.

☐ **Co-Prime Numbers** : Two numbers are co-prime, if their H.C.F (Highest common factor) is 1.

E.g. (2, 3), (3, 13), (5, 7) etc are co-prime numbers.

☐ **Perfect Numbers** : If the sum of divisors of a number excluding N itself is equal to N, then N is called a perfect number.

E.g.: 6, 28, 496, 8128 etc.

For 6, divisors are 1, 2 and 3.

6 : 1 + 2 + 3 = 6

28 : 1 + 2 + 4 + 7 + 14 = 28

Note: The sum of the reciprocals of the divisors of a perfect number including that of its own is always equal to 2.

E.g. For 6, divisors are 1, 2 and 3.

$$\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=\frac{6+3+2+1}{6}=\frac{12}{6}=2$$

- **Simplification** : In simplification an expression, we must remove the brackets strictly in the order ( ), { }, [] and then we must apply the operations : - Of, Divison, Multiplication, Addition and S for Subtraction.
- **Remember** : - 'BODMAS' where B stands for bracket, O for of ; D for division; M for multiplication, A for addition and S for Substraction strictly in this order.
  Note : 'Of' means multiplication.
- **Division Algorithm** : - Dividend = (Divisor × Quotient) + Remainder

2. **Moduls or Absolute value** :
The absolute value of a real number X is denoted by the symbol $|x|$ and is defined as -

$$|x|=\begin{cases} x, \text{ if } x>0 \\ -x, \text{ if } x<0 \\ 0, \text{ if } x=0 \end{cases}$$

E.g.: $|5| = 5$, $|-5| = -(-5) = 5$

Note: In multiplication and division, when both the numbers carry similar sign, we get positive sign in the result, otherwise we get negative sign in the result i.e.

$(+)\times(+) = +$
$(+)\times(-) = -$
$(-)\times(+) = -$
$(-)\times(-) = +$
$(+)\div(+) = +$
$(+)\div(-) = -$
$(-)\div(+) = -$
$(-)\div(-) = +$

Important terms :

1. **Identity element of Addition** : '0' (zero) is called identity element of addition as Addition of 'O' in any number does not affect that number.
e.g. $x + 0 = x$ ($x \in Q$)
2. **Identity element of Multiplication** : '1' is called identity element of multiplication as multiplication of '1' in any number does not affect that number.
e.g. $x \times 1 = x$
3. **Inverse Element of Addition/ Negative element of Addition/ Additive Inverse :**
The number is called "Additive inverse" of a certain number, when it is added to the certain number and result becomes '0' (zero).

E.g. (i) $x + (-x) = 0$
Here $(-x)$ is Additive inverse of x.
(ii) (9) + (-9) is Additive inverse of '9'

4. **Inverse Element of Multiplication/Reciprocal Element/ Multiplicative Inverse** :
The number is called "Multiplicative inverse" of a certain number, when the product of number and multiplicative inverse is1.

E.g. $x\times\frac{1}{x}=1$

Here, $\frac{1}{x}$ is multiplicative inverse of 'x'

- **Indices and Surds :**

→ Let n be a positive integer and a be a real number, then :

$$a^n = \underbrace{a\times a\times a\times......\times a}_{\text{(n factors)}}$$

a^n is called "n^{th} power of a" or "a raised to the power n" where, a is called the base and n is called index or exponent of the power a^n.

E.g. 3^2 = square of 3, 3^3 = cube of 3 etc.

☐ **Laws of Indices** :

1. $a^m \times a^n = a^{m+n}$ where $a \neq 0$ and $(m, n) \in I$
2. $a^m \times a^n \times a^p$ ....= $a^{m+n+P+.....}$
3. $\dfrac{a^m}{a^n} = \begin{cases} a^{m-n} & if \quad m > n \\ \dfrac{1}{a^{n-m}} & if \quad n > m \\ 1 & if \quad m = n \end{cases}$
4. $(a^m)^n = a^{mn} = (a^n)^m$
5. $a^{m^n} = a^{m \times m \times ....n \text{ times}} \neq (a^m)^n$
6. $(ab)^n = a^n b^n$
7. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, \quad b \neq 0$
8. $(-a)^n = \left[\begin{array}{l} a^n, \text{ when n is even} \\ -a^n, \text{ when n is odd} \end{array}\right.$

   Remark : These rules are also true when n is negative or fraction.

9. $a^{-n} = a^{(-1)n} = \left(a^{-1}\right)^n = \left(\dfrac{1}{a}\right)^n$

   $= \dfrac{1}{a} \times \dfrac{1}{a} \times \dfrac{1}{a}$......n times
10. $a^{p/q} = a^{1/q \times p} = (a^{1/q})^p$ p is positive integer, $q \neq 0$

    $= a^{1/q} \times a^{1/q} \times$ ....p times

☐ If the index of a power is unit (i.e. 1) then the value of the power is equal to its base, i.e.

$a^1 = a, 0^1 = 0$

◆ $a^m = a^n \Rightarrow m = n$ when $a \neq 0$,1

◆ $a^m = b^m \Rightarrow a = b$

◆ **Surd :** If a is rational and n is a positive integer and $a^{1/n} = \sqrt[n]{a}$ is **irrational,** then $\sqrt[n]{a}$ is called a "surd of order n" or "nth root of a"

For the surd $\sqrt[a]{n}$, n is called the surd-index or the order of the surd and 'a' is called the radicand. The synbol '$\sqrt{\ }$' is called the surd sign or radical.

E.g. $\sqrt{5}$ is a surd of order 2 or square root of 5

$\sqrt[3]{6}$ is a surd of order 3 or cube root of 6.

$\sqrt{6+\sqrt{5}}$ is not a surd as $6+\sqrt{5}$ is not a rational number.

Note: Every surd is an irrational number but every irrational number is not a surd.

→ In the surd $a\sqrt[n]{b}$, a and b are called factors of the surd.

(i) A surd which has unity as its rational factor (i.e. a = 1) is called "pure surd".

E.g.: $\sqrt[4]{3}, \sqrt{2}, \sqrt[3]{3}$ etc.

(ii) A surd which has a rational factor other than unity, the other irrational, is called "mixed surd". e.g. $3\sqrt{5}, 2\sqrt{7}, 5\sqrt[3]{7}$

→ If $\sqrt[n]{a}$ is a surd it implies :

(i) a is a rational number.

(ii) $\sqrt[n]{a}$ is an irrational number.

→ **Quadratic surd** : A surd of order 2 (i.e. $\sqrt{a}$) is called a quadratic surd.

E.g.: $\sqrt{2} = 2^{1/2}$ is a quadratic surd but $\sqrt{4} = 4^{1/2}$ is not a quadratic surd because $\sqrt{4} = 2$ is a rational number. Therefore $\sqrt{4}$ is not a surd.

→ **Cubic Surd :** A surd of order 3(i.e. $\sqrt[3]{a}$) is called a cubic surd.

E.g: $\sqrt[3]{9}$ is a cubic surd but $\sqrt[3]{27}$ is not a surd because $\sqrt[3]{27}=3$ is a rational number.

→ **Quartic or Biquadratic surd :** A surd of order $4\left(i.e.\ \sqrt[4]{a}\right)$ is called a quartic surd.

E.g.: $\sqrt[4]{3}$ is a quartic surd but $\sqrt[4]{81}$ is not a quartic surd.

Note : Each surd can be represented on the number line.

Important Formulae Based on Surds :

(i) $\sqrt[n]{a^n}=a$

(ii) $\sqrt[n]{ab}=\sqrt[n]{a}.\sqrt[n]{b}$

(iii) $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ and $\frac{k\sqrt[n]{a}}{l\sqrt[n]{b}}=\frac{k}{l}\sqrt[n]{\frac{a}{b}}$

(iv) $\sqrt[m]{\sqrt[n]{a}}=\sqrt[mn]{a}=\sqrt[n]{\sqrt[m]{a}}$

(v) $\left(\sqrt[n]{a}\right)^m=(a)^{m/n}=\left(a^m\right)^{1/n}=\sqrt[n]{a^m}$

(vi) $\sqrt{a}\times\sqrt{a}=a$

(vii) $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

and $k.\sqrt[n]{a}\times l.\sqrt[m]{b}=kl.\sqrt[n]{a}.\sqrt[m]{b}=kl.\sqrt[mn]{a^m b^n}$

(viii) $\sqrt{a^2b}=a\sqrt{b}$

(ix) $(\sqrt{a}+\sqrt{b})^2=a+b+2\sqrt{ab}$

(x) $(\sqrt{a}-\sqrt{b})^2=a+b-2\sqrt{ab}$

(xi) $(\sqrt{a}+\sqrt{b})\times(\sqrt{a}-\sqrt{b})$ = a − b, where a and b are positive rational numbers.

→ A surd is in its simplest form if :

(i) There is no factor which has n^th^ power of a rational number, under the radical sign whose index is n,

(ii) There is no fracrtion under the radical sign, and

(iii) The index of the surd is the smallest possible.

E.g. The surd $\sqrt[4]{3\times5^4}$ is not in its simplest form since the number under the radical sign has factor 5^4 s.t. its index is equal to the order of the surd. Its simplest form :

$$\sqrt[4]{3\times5^4}=\sqrt[4]{3}.\sqrt[4]{5^4}=\left(\sqrt[4]{3}\right)(5)=5.\left(\sqrt[4]{3}\right)$$

☐ **Similar or like Surds :** Surds having same irrational factors are called "similar or like surd" .

E.g.: $3\sqrt{3},\ 7\sqrt{3},\ \frac{2}{5}\sqrt{3},\ \sqrt{3}$ *etc.* are similar surds

☐ **Unlike Surds :** Surds having no-common irrational factors are called "unlike surds".

E.g.: $3\sqrt{3},\ 5\sqrt{2},\ 6\sqrt{7}$, *etc.* are unlike surds.

→ **Comparison of Surds :** (i) If two surds are of the same order, then the one whose radicand is larger, is the larger of the two.

E.g.: $\sqrt[3]{19}>\sqrt[3]{15},\ \sqrt{7}>\sqrt{5},\ \sqrt[5]{9}>\sqrt[5]{7}$ etc.

(ii) If two surds are of **distinct order,** we change them into the surds of the same order.

This order is the L.C.M. of the orders of the given surds.

E.g.: Which is larger $\sqrt{2}$ *or* $\sqrt[3]{3}$?

Sol. Given surds are of order 2 & 3 respectively whose L.C.M. is 6.

Convert each into a surd of order 6, as shown below :

$$\sqrt{2}=2^{1/2}=2^{\frac{1}{2}\times\frac{3}{3}}=2^{3/6}=(2^3)^{1/6}=(8)^{1/6}=\sqrt[6]{8}$$

$$\sqrt[3]{3}=3^{1/3}=3^{\frac{1}{3}\times\frac{2}{2}}=3^{2/6}=(9)^{1/6}=\sqrt[6]{9}$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$, so, $\sqrt[3]{3} > \sqrt{2}$

→ **Rationalisation of Surds** : If the product of two surds is rational, then each of them is called the (R.F.) rationalising factor of the other.

E.g.: $5\sqrt{7}\times\sqrt{7} = 5\sqrt{7\times7} = 5\times7 = 35.$

$\therefore \sqrt{7}$ is a rationalising factor of $5\sqrt{7}$.

→ Rationalising factor (R.F.) of the surd $\sqrt[n]{a}$ is $a^{1-\frac{1}{n}}$.

→ R.F. of the surd $\sqrt{a}\pm\sqrt{b}$ is $\sqrt{a}\mp\sqrt{b}$.

E.g. $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$

$=(\sqrt{3})^2-(\sqrt{2})^2=3-2=1$

$\therefore(\sqrt{3}-\sqrt{2})$ is a R.F. of $\sqrt{3}+\sqrt{2}$

Note: The R.F. of a given surd is not unique.

Some Useful Results :

(i) *if* $y=\sqrt{x+\sqrt{x+\sqrt{x+........\infty}}}$

and $x=n(n+1)$ then $y=(n+1)$

E.g.: $y=\sqrt{12+\sqrt{12+\sqrt{12+........\infty}}}$

we have $x = 12 = 3\times4 = n(n+1)$

$\therefore\ y=4$

(ii) if $y=\sqrt{x-\sqrt{x-\sqrt{x-........\infty}}}$ and $x = n(n+1)$
then y = n

E.g.: $y=\sqrt{12-\sqrt{12-\sqrt{12-........\infty}}}$

we have, $x = 12 = 3\times4 = n(n+1)$

$\therefore\ y=3.$

→ **Square-root of an Irrational number :**

As we know that, $(a+b)^2=(a^2+b^2)+2ab$

$\therefore(\sqrt{2}+\sqrt{3})^2 = \underbrace{5}_{(a^2+b^2)} + \underbrace{2\sqrt{6}}_{(2ab)}$

$\therefore 5+2\sqrt{6} = 5+\underbrace{2\sqrt{2}\sqrt{3}}_{(2ab)}$ $\therefore a=\sqrt{2}\ \&\ b=\sqrt{3}$

$\&\ a^2+b^2=5$

$\therefore\ 5+2\sqrt{6}=(\sqrt{2}+\sqrt{3})^2 \Rightarrow a=\sqrt{5+2\sqrt{6}}=\sqrt{2}+\sqrt{3}$

Examples : (i) $\sqrt{7-4\sqrt{3}}=\sqrt{7-2\times\underset{\substack{\downarrow\\ a}}{2}\times\underset{\substack{\downarrow\\ b}}{\sqrt{3}}}$

& $a^2+b^2=2^2+\sqrt{3}^2=7$

$=\sqrt{(2-\sqrt{3})^2}=(2-\sqrt{3})$

(ii) $\sqrt{5-\underbrace{3\sqrt{7}}_{2ab}}$ for making it 2ab

$\sqrt{\frac{(5-3\sqrt{7})\times2}{2}}=\sqrt{\frac{10-2\times3\times\sqrt{7}}{2}}$

$=\frac{1}{\sqrt{2}}\sqrt{(10-2\times3\times\sqrt{7})}$ $\left[\begin{array}{l}\therefore a=3\ \&\ b=\sqrt{7}\\ \because a^2+b^2=10\end{array}\right]$

$=\frac{1}{\sqrt{2}}\sqrt{(3-\sqrt{7})^2}=\frac{3-\sqrt{7}}{\sqrt{2}}$

Note :- In these type of questions there is maximum possibility of having answer '2'.
For e.g. Q 5,9m & 10 (you can observe it)

Useful Result

(i) If $x=\frac{4\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$\frac{x+2\sqrt{a}}{x-2\sqrt{a}}+\frac{x+2\sqrt{b}}{x-2\sqrt{b}}=2$

(ii) if $= x\ \frac{2\sqrt{ab}}{\sqrt{a}+\sqrt{b}}$

$\frac{x+\sqrt{a}}{x-\sqrt{a}}+\frac{x+\sqrt{b}}{x-\sqrt{b}}=2$

Exercise

LEVEL - 1

1. $2\sqrt{5}+4\sqrt{5}-3\sqrt{5}$ is equal to :

(a) $6\sqrt{5}$
(b) $3\sqrt{5}$
(c) $2\sqrt{5}$
(d) $\sqrt{5}$

2. If $x=\sqrt{11+\sqrt{8+\sqrt{289}}}$, then x is equal to :

(a) 4
(b) 2
(c) -2
(d) 8

3. $2^{(-2)^2}$ is equal to :

(a) -8
(b) $\frac{1}{16}$
(c) 16
(d) $\frac{1}{8}$

4. If $\sqrt{x}=12.3/123$, the value of x is :

(a) 0.1
(b) 1
(c) 10
(d) 0.01

5. The value of $\sqrt{\frac{3+\sqrt{8}}{3-\sqrt{8}}}$ is :

(a) $3-\sqrt{8}$
(b) $3+2\sqrt{2}$
(c) $3+2\sqrt{8}$
(d) 3

6. The value of $(32)^{0.16}\times(32)^{0.04}$ is :

(a) 2
(b) $\frac{1}{2}$
(c) 4
(d) $\frac{1}{4}$

7. The value of

$$\left(\frac{16}{81}\right)^{-3/4}\times\left(\frac{49}{9}\right)^{3/2}\times\left(\frac{343}{216}\right)^{2/3}$$ is:

(a) $\frac{16087}{208}$
(b) $\frac{16087}{280}$
(c) $\frac{16087}{288}$
(d) None of these

8. $\sqrt{20}+\sqrt{12}$ is equal to :

(a) $\sqrt{32}$
(b) $2\sqrt{5}+2\sqrt{3}$
(c) $5\sqrt{2}$
(d) $\sqrt{240}$

9. The value of $\frac{4^{10+n}.16^{3n-4}}{4^{7n}}$ is :

(a) 4
(b) 8
(c) 20
(d) 16

10. $\frac{5^{n+3}-6\times 5^{n+1}}{9\times 5^{n}-5^{n}\times 2^{2}}$ is equal to :

(a) 5
(b) 19
(c) 25
(d) 95

11. The value of x in $3^{1/9}.3^{2/9}.3^{3/9}=(9)^{x}$ is :

(a) $\frac{2}{3}$
(b) 1
(c) $\frac{1}{3}$
(d) 2

12. If $x=81$, then $4\sqrt{x}-\sqrt[4]{x}$ is equal to :

(a) 0
(b) -33
(c) 40
(d) 33

13. The least prime number is :

(a) 0
(b) 1
(c) 2
(d) 3

14. Which of the following is a correct statement :

(a) Sum of two irrational numbers is always irrational.
(b) Sum of rational and an irrational number is always rational.
(c) Square of an irrational number is always rational.
(d) Sum of two rational numbers can never be an integer.

15. If n is a natural number, then $\sqrt{n}$ is:

(a) always a natural number
(b) always a rational number
(c) always an irrational number
(d) Sometimes a natural number and sometimes an irrational number.

16. $\left(\frac{243}{32}\right)^{-3/5}$ is equal to :

(a) $\frac{8}{27}$ (b) $\frac{27}{8}$
(c) $\frac{4}{9}$ (d) $\frac{9}{4}$

17. Which is largest amongst $\sqrt[4]{4}, \sqrt[3]{3}$, and $\sqrt{2}$?

(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$
(c) $\sqrt[4]{4}$
(d) can't be determined

18. The value of $\sqrt{\frac{1.21 \times .9}{1.1 \times .11}}$ is :

(a) 2 (b) 3
(c) 9 (d) 11

19. $\left(\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}+\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}\right)$ is equal to :

(a) $2\sqrt{7}-3\sqrt{5}$ (b) $2\sqrt{7}+3\sqrt{5}$
(c) 12 (d) 2

20. $\sqrt{217+\sqrt{48+\sqrt{256}}} = ?$

(a) 14 (b) 16
(c) 14.4 (d) 15

21. $\sqrt{1.21}-\sqrt{.01}$ is equal to :

(a) 0.99 (b) 1
(c) $\sqrt{1.2}$ (d) .82

22. If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$, then $\frac{a}{b}$ is:

(a) 0.016 (b) 0.16
(c) 1 (d) 16

23. $\sqrt{\frac{.081 \times .484}{.0064 \times 6.25}} = ?$

(a) 9 (b) 0.9
(c) 99 (d) 0.99

24. The value of $\frac{3}{\sqrt{0.09}}$ is :

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$
(c) 1 (d) 10

25. Which one is smallest ?

$\sqrt{3}, \sqrt[3]{2}, \sqrt{2}$ and $\sqrt[3]{4}$.

(a) $\sqrt{3}$ (b) $\sqrt[3]{4}$
(c) $\sqrt[3]{2}$ (d) $\sqrt{2}$

26. If $\sqrt{15} = 3.88$, then $\sqrt{5/3}$ is :

(a) 1.68 (b) 1.59
(c) 1.43 (d) 1.29

27. If $\frac{(21)^{5.36}}{(21)^{3.47}} = (21)^x$, then value of x is :

(a) 8.83 (b) 1.54
(c) 9.32 (d) 1.89

28. $\sqrt{\frac{0.289}{0.00121}}+\frac{\sqrt{24}+\sqrt{216}}{\sqrt{96}} = ?$

(a) $16\frac{5}{11}$ (b) $17\frac{5}{11}$
(c) $15\frac{5}{11}$
(d) None of these

29. If $\sqrt{3^n} = 81$, then n = ?

(a) 2 (b) 4
(c) 6 (d) 8

30. The value of $\sqrt[3]{\sqrt{0.000064}}$ is :

(a) 0.02 (b) 0.2
(c) 2.0
(d) None of these

31. $\dfrac{\sqrt[7]{\sqrt[5]{\left(21^7\right)^5}}}{\sqrt[5]{\sqrt[3]{\left(7^5\right)^3}}} = ?$

(a) 3 (b) 5
(c) 7 (d) 9

32. $(-3)^{(-2)^{(-2)^{(-1/4)}}} = ?$

(a) 3 (b) 3^{16}
(c) $\dfrac{1}{3}$ (d) $\dfrac{1}{9}$

33. If $\sqrt[5]{2x-7} - 3 = 0$, the value of x is :

(a) 8 (b) 250
(c) 120 (d) 125

34. The value of expression

$$\left\{\frac{x^b}{x^c}\right\}^{(b+c-a)} \left\{\frac{x^c}{x^a}\right\}^{(c+a-b)} \left\{\frac{x^a}{x^b}\right\}^{(a+b-c)} :$$

(a) x^{a+b+c} (b) 1
(c) $x^{ab+bc+ca}$ (d) x^{abc}

35. $\dfrac{1}{1+x^{(b-a)}+x^{(c-a)}} + \dfrac{1}{1+x^{(a-b)}+x^{(c-b)}} + \dfrac{1}{1+x^{(b-c)}+x^{(a-c)}} = ?$

(a) 0 (b) x^{a-b-c}
(c) 1 (d) x^{a+b+c}

36. $\dfrac{1}{1+a^{x-y}} + \dfrac{1}{1+a^{y-x}} = ?$

(a) 0 (b) 1
(c) $\dfrac{1}{2}$ (d) a^{x+y+c}

37. $\left(\left(\left(3^{1-\frac{1}{2}}\right)^{1-\frac{1}{3}}\right)^{1-\frac{1}{4}}\right)^{\cdots\cdots n \text{ terms}}$

(a) $(3)^{\frac{1}{n+1}}$ (b) 3^n
(c) $3^{\frac{n}{n+1}}$ (d) $3^{2\log n}$

38. $\dfrac{\sqrt{x+2}+\sqrt{x-2}}{\sqrt{x+2}-\sqrt{x-2}} = \dfrac{3}{2}$, then the value of $6x$ will be :

(a) $\dfrac{13}{6}$ (b) $\dfrac{6}{13}$
(c) 13 (d) 6

39. If $a^{1/3} = 11$, then the value of $a^2 - 331a$ will be :

(a) 1331331 (b) 1331000
(c) 1334331 (d) 1330030

40. If $\sqrt{\dfrac{x}{y}} + \sqrt{\dfrac{y}{x}} = \dfrac{10}{3}$ and $x + y = 10$, the value of xy will be :

(a) 36 (b) 24
(c) 16 (d) 9

41. $2^{73} - 2^{72} - 2^{71}$ is the same as :

(a) 2^{69} (b) 2^{70}
(c) 2^{71} (d) 2^{72}

42. $\sqrt{x}\sqrt{y} = \sqrt{xy}$ is true only when

(a) $x > 0$, $y > 0$
(b) $x > 0$ and $y < 0$
(c) $x < 0$ and $y > 0$
(d) All of these

43. If $\sqrt{15625} = 125$, find

$\sqrt{156.25} + \sqrt{1.5625} - \sqrt{0.015625}$:

(a) 13.625 (b) 13.6
(c) 12.5 (d) 2.375

44. If a,b are rationals and $a\sqrt{2}+b\sqrt{3}=\sqrt{98}+\sqrt{108}-\sqrt{48}-\sqrt{72}$,then the values of a, b are respectively :

(a) 2, 3 (b) 1, 2

(c) 1, 3 (d) 2, 1

45. Find the value of

$$3+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}+3}+\frac{1}{\sqrt{3}-3}:$$

(a) $\frac{3}{2\left(\sqrt{3}+3\right)}$ (b) $2\sqrt{3}$

(c) 6 (d) 3

46. Find the value of $\sqrt{40+\sqrt{9\sqrt{81}}}$ is :

(a) 9 (b) 11

(c) 7 (d) $\sqrt{111}$

47. The value of $\frac{\sqrt{72}\times\sqrt{363}\times\sqrt{175}}{\sqrt{32}\times\sqrt{147}\times\sqrt{252}}$ is :

(a) $\frac{55}{28}$ (b) $\frac{55}{42}$

(c) $\frac{45}{56}$ (d) $\frac{45}{28}$

48. If $\sqrt{4^n}=1024$, then the value of n is:

(a) 5 (b) 8

(c) 10 (d) 12

49. $\left(x^{b+c}\right)^{b-c}\cdot\left(x^{c+a}\right)^{(c-a)}\cdot\left(x^{a+b}\right)^{(a-b)}=?$

(a) 0 (b) 1

(c) x (d) $x^{a^2+b^2+c^2}$

50. If $5\sqrt{5}\times5^{3}\div5^{-3/2}=5^{a+2}$, then a = ?

(a) 4 (b) 5

(c) 6 (d) 8

51. If $\left(\frac{3}{5}\right)^{3}\left(\frac{3}{5}\right)^{-6}=\left(\frac{3}{5}\right)^{2x-1}$ then x is equal to :

(a) - 2 (b) 2

(c) - 1 (d) 1

52. For what value(s) of a $x+\frac{1}{4}\sqrt{x}+a^2$ a perfect square ?

(a) $\pm\frac{1}{18}$ (b) $\pm\frac{1}{8}$

(c) $-\frac{1}{5}$ (d) $\frac{1}{4}$

53. If $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ and $y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ then $(x+y)$ equals :

(a) 8 (b) 16

(c) $2\sqrt{15}$ (d) $2\left(\sqrt{5}+\sqrt{3}\right)$

54. Which of the following numbers is the least $(0.5)^2, \sqrt{0.49}, \sqrt[3]{0.008}, 0.23$

(a) $(0.5)^2$ (b) $\sqrt{0.49}$

(c) $\sqrt[3]{0.008}$ (d) 0.23

55. $\left[\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right]$ simplifies to :

(a) $2\sqrt{6}$ (b) $4\sqrt{6}$

(c) $2\sqrt{3}$ (d) $3\sqrt{2}$

56. By how much does $\left(\sqrt{12}+\sqrt{18}\right)$ exceed $\left(2\sqrt{3}+2\sqrt{2}\right)$?

(a) 2 (b) $\sqrt{3}$

(c) $\sqrt{2}$ (d) 3

57. If $2\sqrt{x} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, the value of x is

(a) 6 (b) 30

(c) $\sqrt{15}$ (d) 15

58. If $\sqrt{3} = 1.732$, the value of

$$\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$$ is :

(a) 4.899 (b) 2.551

(c) 1.414 (d) 1.732

59. The value of $\frac{(81)^{3.6}\times(9)^{2.7}}{(81)^{4.2}\times(3)}$ is :

(a) 3 (b) 6

(c) 9 (d) 8.2

60. The largest among the numbers $(0.1)^2$, $\sqrt{0.0121}$, 0.12 and $\sqrt{0.0004}$ is:

(a) $(0.1)^2$ (b) $\sqrt{0.0121}$

(c) 0.12 (d) $\sqrt{0.0004}$

LEVEL - 2

1. If $3^x - 3^{x-1} = 18$, then the value of x^2 is:

(a) 9 (b) 16

(c) 3 (d) 8

2. If $a^x = b^y = c^z$ and $b^2 = ac$, then y is equal to :

(a) $\frac{xz}{x+z}$ (b) $\frac{2xz}{x+z}$

(c) $\frac{2xz}{x-z}$ (d) $\frac{xz}{2(z-x)}$

3. If $a^x = b$, $b^y = c$ and $c^z = a$, then xyz equals :

(a) abc (b) $\frac{1}{abc}$

(c) 1

(d) None of these

4. The square root of $3+\sqrt{5}$ is :

(a) $\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{5}}{2}-\frac{1}{\sqrt{2}}$ (d) $\sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}$

5. The square root of $7+2\sqrt{10}$ is :

(a) $\sqrt{5}+\sqrt{2}$ (b) $\sqrt{3}+\sqrt{5}$

(c) $\sqrt{6}+\sqrt{1}$ (d) $2+\sqrt{5}$

6. $$\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}} = ?$$

(a) 0 (b) 1

(c) 5 (d) 1/3

7. $\sqrt{20+\sqrt{20+\sqrt{20+\ldots\ldots.\infty}}}$ = ?

(a) 2
(b) 3
(c) - 4
(d) 5

8. $\sqrt{12+\sqrt{12+\sqrt{12+\ldots\ldots.\infty}}}$ = ?

(a) 3
(b) 4
(c) -3
(d) - 4

9. If $\sqrt{1+\frac{55}{729}}=1+\frac{x}{27}$, then the value of x is :

(a) 1
(b) 3
(c) 5
(d) 7

10. If $x^2 = 4224$, then :

(a) $64 < x < 65$
(b) $210 < x < 215$
(c) $24 < x < 25$
(d) $21 < x < 22$

11. If $x=\sqrt{6+\sqrt{6+\sqrt{6+\ldots\ldots\ldots\infty}}}$, then

(a) 2.5
(b) 3
(c) 6
(d) 8

12. If $2^{x+1} + 2^{x+3} = 2560$, then x is equal to :

(a) 12
(b) 11
(c) 8
(d) 6

13. If $a=\sqrt{8}-\sqrt{7}$, $b=\sqrt{7}-\sqrt{6}$, $c=\sqrt{6}-\sqrt{5}$, then:

(a) a > b > c
(b) a < b < c
(c) b < a < c
(d) a < c < b

14. $\sqrt[3]{(4.032)^2-(3.968)^2}$ is equal to :

(a) 1
(b) 0.8
(c) 4/5
(d) 1.6

15. Among $\left(\sqrt{19}-\sqrt{17}\right)$, $\left(\sqrt{13}-\sqrt{11}\right)$, $\left(\sqrt{7}-\sqrt{5}\right)$ the greatest expression is :

(a) $\sqrt{19}-\sqrt{17}$
(b) $\sqrt{13}-\sqrt{11}$
(c) $\sqrt{7}-\sqrt{5}$
(d) $\sqrt{5}-\sqrt{3}$

16. The value of the expression $\sqrt{4+\sqrt{15}}+\sqrt{4-\sqrt{15}}-\sqrt{12-4\sqrt{5}}$ is :

(a) natural number
(b) a negative number
(c) a non-negative number
(d) an irrational number

17. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$, then b is equal to :

(a) - 6
(b) 6
(c) - 11
(d) 11

18. $\sqrt{6\sqrt{6\sqrt{6\ldots\ldots\ldots\infty}}}$ = ?

(a) 6
(b) - 6
(c) 4
(d) - 4

19. $\sqrt{6\sqrt{6\sqrt{6\sqrt{6\sqrt{6}}}}}$ = ?

(a) $6^{11/32}$
(b) $6^{13/32}$
(c) $6^{31/32}$
(d) $6^{26/32}$

20. $\sqrt{20-\sqrt{20-\sqrt{20-\ldots.\infty}}}$ =?

(a) 4
(b) 5
(c) 3
(d) - 4

21. If $\sqrt{5}=2.236$ and $\sqrt{10}=3.162$, then the value of

$$\frac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$$ is :

(a) 4.398
(b) 5.398
(c) 4.938
(d) 5.938

22. If $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots\ldots+\frac{1}{\sqrt{8}+\sqrt{9}}$ = ?

(a) 1
(b) 3
(c) 2
(d) 2

23. The simplified value of

$$\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}+\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}:$$

(a) $2\sqrt{6}$ (b) 0

(c) 1 (d) $\sqrt{6}$

24. The value of

$$\left(28+10\sqrt{3}\right)^{\frac{1}{2}}-\left(7-4\times\sqrt{3}\right)^{-\frac{1}{2}}:$$

(a) - 3 (b) $2\sqrt{3}$

(c) 3 (d) 0

25. Simplified value of $\dfrac{\sqrt{4-\sqrt{7}}}{\sqrt{8+3\sqrt{7}}-2\sqrt{2}}$ is:

(a) $\left(\sqrt{7}+1\right)$ (b) 1

(c) 2 (d) $\frac{1}{2}$

26. Find the value of $\sqrt{72+\sqrt{72+\sqrt{72+\ldots\infty}}}\div$

$\sqrt{12-\sqrt{12-\sqrt{12-\ldots\ldots\infty}}}$

(a) 3 (b) 2

(c) 9/4 (d) 8/3

27. If $p=a^x$, $q=a^y$, $(p^y q^x)^z=a^2$, then xyz =?

(a) 0 (b) p^2q^2

(c) $\dfrac{1}{pq}$ (d) 1

28. Which one of the following is the largest ?

(a) $8\sqrt{2}$ (b) $2\sqrt{5}$

(c) $6\sqrt{3}$ (d) $3\sqrt{7}$

29. If $2^x=3^y=6^{-z}$, then $\left(\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}\right)$ is equal to :

(a) 1 (b) 0

(c) $-\dfrac{1}{2}$ (d) $\dfrac{3}{2}$

30. If $\sqrt{3}=1.732$, the value of

$$\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}\text{ is :}$$

(a) $\frac{1}{2}$ (b) $\sqrt{3}$

(c) 1 (d) $\dfrac{1}{\sqrt{3}}$

31. If $x=\dfrac{1}{2+\sqrt{3}}, y=\dfrac{1}{2-\sqrt{3}}$, then the value of $\dfrac{1}{x+1}+\dfrac{1}{y+1}$ is :

(a) 4.899 (b) 2.551

(c) 1.414 (d) 1.732

32. Given that $\sqrt[3]{3^x}=5^{1/4}$ and $\sqrt[4]{5^y}=\sqrt{3}$, then find the value of $2xy$:

(a) 2 (b) 3

(c) - 3 (d) 5

33. If $\dfrac{\left(x-\sqrt{24}\right)\left(\sqrt{75}+\sqrt{50}\right)}{\left(\sqrt{75}-\sqrt{50}\right)}=1$, then the value of x is :

(a) $\sqrt{5}$ (b) 3

(c) $2\sqrt{5}$ (d) $3\sqrt{5}$

34. Evaluate

$$\sqrt{20}+\sqrt{12}+\sqrt[3]{729}-\frac{4}{\sqrt{5}-\sqrt{3}}-81:$$

(a) $\sqrt{2}$ (b) $\sqrt{3}$

(c) 0 (d) $2\sqrt{2}$

35. $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$ simplifies to :

(a) $\sqrt{5}+\sqrt{6}$ (b) $2\sqrt{5}+\sqrt{6}$

(c) $\sqrt{5}-\sqrt{6}$ (d) $2\sqrt{5}-3\sqrt{6}$

36. If $a=\frac{1}{2-\sqrt{3}}+\frac{1}{3-\sqrt{8}}+\frac{1}{4-\sqrt{15}}$

Then we have :

(a) $a = 9$

(b) $a < 18$ but $a \neq 9$

(c) $a > 18$ (d) $a = 18$

37. The value of

$$\left[\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots\ldots\right.$$

$$\left.+\frac{1}{\sqrt{120}+\sqrt{121}}\right] \text{ is :}$$

(a) $\sqrt{120}$ (b) $12\sqrt{2}$

(c) 8 (d) 10

38. $\sqrt{7\sqrt{3}+12}=?$

(a) $\sqrt[4]{3}\left(2+\sqrt{3}\right)$ (b) $\sqrt[3]{3}\left(2+\sqrt{3}\right)$

(c) $\sqrt{3}\left(2+\sqrt{3}\right)$ (d) $\sqrt{3}\left(2-\sqrt{3}\right)$

39. If $\frac{9^n.3^2.3^n-(27)^n}{3^{3m}.2^3}=\frac{1}{27}$, then the value of (m - n) is :

(a) 1 (b) 2

(c) $\sqrt{3}$ (d) $\sqrt{2/3}$

40. $\sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}}$ is equal to:

(a) $\left(\sqrt{2}+\sqrt{3}-\sqrt{5}\right)$

(b) $\left(\sqrt{3}+\sqrt{5}-\sqrt{2}\right)$

(c) $\left(\sqrt{2}+\sqrt{5}-\sqrt{3}\right)$

(d) $\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)$

41. If $\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$, then the value of a is

(a) $\frac{11}{3}$ (b) $-\frac{4}{3}$

(c) $\frac{4}{3}$ (d) $\frac{-4\sqrt{7}}{3}$

42. $\left[\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\right]$ is simplified to :

(a) 0 (b) 1

(c) $\sqrt{3}$ (d) $\sqrt{6}$

43. If $x=\frac{\sqrt{3}}{2}$, then $\frac{\sqrt{1+x}}{1+\sqrt{1+x}}+\frac{\sqrt{1-x}}{1-\sqrt{1-x}}$ is eual to :

(a) 1 (b) $2/\sqrt{3}$

(c) $2-\sqrt{3}$ (d) 2

44. If $\sqrt{2}$ = 1.414, the square root of $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ is nearest to :

(a) 0.172 (b) 0.414

(c) 0.586 (d) 1.414

45. The value of $\sqrt{\frac{\left(\sqrt{12}-\sqrt{8}\right)\left(\sqrt{3}+\sqrt{2}\right)}{5+\sqrt{24}}}$ is:

(a) $\sqrt{6}-\sqrt{2}$ (b) $\sqrt{6}+\sqrt{2}$

(c) $\sqrt{6}-2$ (d) $2-\sqrt{6}$

46. Arrange the following in descending order : $\sqrt[3]{4},\sqrt{2},\sqrt[6]{3},\sqrt[4]{5}$:

(a) $\sqrt[3]{4}>\sqrt[4]{5}>\sqrt{2}>\sqrt[6]{3}$

(b) $\sqrt[4]{5}>\sqrt[3]{4}>\sqrt[6]{3}>\sqrt{2}$

(c) $\sqrt{2}>\sqrt[6]{3}>\sqrt[3]{4}>\sqrt[4]{5}$

(d) $\sqrt[3]{4}>\sqrt[4]{5}>\sqrt[6]{3}>\sqrt{2}$

47. The greatest number among $\sqrt{5}, \sqrt[3]{4}, \sqrt[5]{2}, \sqrt[7]{3}$ is :

(a) $\sqrt[3]{4}$ (b) $\sqrt[7]{3}$

(c) $\sqrt{5}$ (d) $\sqrt[5]{2}$

48. $\dfrac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}}$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

49. The largest among the numbers $\sqrt{7}-\sqrt{5}, \sqrt{5}-\sqrt{3}, \sqrt{9}-\sqrt{7}, \sqrt{11}-\sqrt{9}$ is:

(a) $\sqrt{7}-\sqrt{5}$ (b) $\sqrt{5}-\sqrt{3}$

(c) $\sqrt{9}-\sqrt{7}$ (d) $\sqrt{11}-\sqrt{9}$

50. If $\sqrt{7\sqrt{7\sqrt{7\sqrt{7......}}}} = (343)^{y-1}$, then y is equal to :

(a) $\frac{2}{3}$ (b) 1

(c) $\frac{4}{3}$ (d) $\frac{3}{4}$

51. $\dfrac{1}{\sqrt{1}+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{4}}+.....$

$......+\dfrac{1}{\sqrt{99}+\sqrt{100}}$ is equal to :

(a) $10-\sqrt{99}$ (b) $\sqrt{2}-10$

(c) 7 (d) 9

52. If $x=\dfrac{\sqrt{5}-2}{\sqrt{5}+2}$, then $x^4 + x^4$ is :

(a) a surd

(b) a rational number but not an integer

(c) an integer

(d) an irrational number but not a surd

53. If $x+\dfrac{1}{x}=3$, the value of $x^5+\dfrac{1}{x^5}$ is :

(a) 123 (b) 126

(c) 113 (d) 129

54. If $a=\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$ and $b=\dfrac{2-\sqrt{3}}{2+\sqrt{3}}$, then the value of (a^2+b^2+ab) is :

(a) 185 (b) 195

(c) 200 (d) 175

55. If $a=\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}, b=\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$, then the value of $\dfrac{a^2}{b}+\dfrac{b^2}{a}$ is :

(a) 900 (b) 970

(c) 1030 (d) 630

56. If $x=2+\sqrt{3}$, then the value of $\sqrt{x}+\dfrac{1}{\sqrt{x}}$ is :

(a) $\sqrt{3}$ (b) $\sqrt{6}$

(c) $2\sqrt{6}$ (d) 6

57. If $x=\dfrac{\sqrt{3}}{2}$, then the value of $\sqrt{1+x}+\sqrt{1-x}$ is :

(a) $\frac{1}{\sqrt{3}}$ (b) $2\sqrt{3}$

(c) $\sqrt{3}$ (d) 2

58. $\sqrt{6+\sqrt{6+\sqrt{6+.......}}}$ is equal to :

(a) 2 (b) 5

(c) 4 (d) 3

LEVEL - 3

1. The value of $\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}$ is :
(a) 20 (b) 4
(c) 6 (d) 14

2. If $a=\frac{\sqrt{5}+1}{\sqrt{5}-1}, b=\frac{\sqrt{5}-1}{\sqrt{5}+1}$ and $\sqrt{5}$ = 2.236, then the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ is :
(a) 0.75 (b) 0.6
(c) 1.33 (d) 1.66

3. If $x=\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-........\infty}}}}$, then x equals :
(a) 3 (b) $\sqrt{13}$
(c) $\left(\frac{\sqrt{3}-1}{2}\right)$ (d) $\left(\frac{\sqrt{3}+1}{2}\right)$

4. Find the value of x in
$\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{3x}}}}=x$:
(a) 1 (b) 3
(c) 6 (d) 12

5. If $x=\frac{4\sqrt{15}}{\sqrt{5}+\sqrt{3}}$, the value of $\frac{x+\sqrt{20}}{x-\sqrt{20}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$ is :
(a) 1 (b) $\sqrt{3}$
(c) 2 (d) $\sqrt{5}$

6. If $x=$ $x=5-\sqrt{21}$, then the value of $\frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}}$ is :
(a) $\frac{1}{\sqrt{2}}\left(7-\sqrt{3}\right)$
(b) $\frac{1}{\sqrt{2}}\left(\sqrt{3}-\sqrt{7}\right)$
(c) $\frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$
(d) $\frac{1}{\sqrt{2}}\left(\sqrt{7}+\sqrt{3}\right)$

7. If $a=\sqrt{3+\sqrt{3+\sqrt{3+........\ \infty}}}$, then which of the following is true ?
(a) a = 3 (b) 3 < a < 4
(c) a > 3 (d) 2 < a < 3

8. $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1}$ is equal to :
(a) $\sqrt[3]{2}+1$ (b) $\sqrt[3]{2}-1$
(c) $-\sqrt[3]{2}-1$ (d) $1-\sqrt[3]{2}$

9. If $x=\frac{2\sqrt{24}}{\sqrt{3}+\sqrt{2}}$, then the value of $\frac{x+\sqrt{8}}{x-\sqrt{8}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$ is :
(a) 1 (b) 2
(c) -1 (d) - 2

10. If $x=\frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}}$, then the value of $\frac{x+\sqrt{2}}{x-\sqrt{2}}+\frac{x+\sqrt{3}}{x-\sqrt{3}}$ is :
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\sqrt{6}$ (d) 2

Hints and Solutions :

LEVEL - 1

1.(b) $2\sqrt{5}+4\sqrt{5}-3\sqrt{5}$

$= \sqrt{5}(2+4-3)$

$= 3\sqrt{5}$

2.(a) $x=\sqrt{11+\sqrt{8+\sqrt{289}}}$

$=\sqrt{11+\sqrt{8+17}}$

$=\sqrt{11+\sqrt{25}}$

$=\sqrt{11+5}=\sqrt{16}=4$

3.(c) $2^{(-2)^2}=2^4=16$

4.(d) $\sqrt{x}=\frac{12.3}{123}=0.1$

$\Rightarrow x=(0.1)^2=0.01$

5.(b) $\sqrt{\frac{3+\sqrt{8}}{3-\sqrt{8}}}=\sqrt{\frac{3+\sqrt{8}}{3-\sqrt{8}}\times\frac{(3+\sqrt{8})}{(3+\sqrt{8})}}$

$=\sqrt{\frac{(3+\sqrt{8})^2}{9-8}}=3+\sqrt{8}$

$=3+2\sqrt{2}$

6.(a) $(32)^{0.16}\times(32)^{0.04}=(32)^{0.20}=(32)^{1/5}$

$=(2^5)^{1/5}=2$

7.(c) Given Exp. =

$$\left[\left(\frac{2}{3}\right)^4\right]^{-3/4}\times\left[\left(\frac{7}{3}\right)^2\right]^{3/2}\times\left[\left(\frac{7}{6}\right)^3\right]^{2/3}$$

$=\left(\frac{2}{3}\right)^{-3}\times\left(\frac{7}{3}\right)^3\times\left(\frac{7}{6}\right)^2$

$=\left(\frac{3}{2}\right)^3\times\left(\frac{7}{3}\right)^3\times\left(\frac{7}{6}\right)^2$

$=\frac{27}{8}\times\frac{343}{27}\times\frac{49}{36}$

$=\frac{16087}{288}$

8.(b) $\sqrt{20}+\sqrt{12}=\sqrt{2\times2\times5}+\sqrt{2\times2\times3}$

$=2\sqrt{5}+2\sqrt{3}$

9.(d) Given Exp. $=\frac{4^{10+n}.4^{2(3n-4)}}{4^{7n}}$

$=4^{(10+n+6n-8-7n)}$

$=4^{(10-8)}=4^2=16$

10.(b) Given Exp. $=\frac{5^n\times5^3-6\times5^n\times5^1}{5^n(9-2^2)}$

$\frac{5^n(5^3-5\times6)}{5^n(9-4)}$

$\frac{125-30}{5}=\frac{95}{5}$

$=19$

or

$\frac{5^n\times5^1(5^2-6)}{5^n(9-4)}$

$\frac{5\times19}{5}$

$=19$

11.(c) $3^{1/9}.3^{2/9}.3^{3/9}=(9)^x$

$\Rightarrow 3^{\frac{1}{9}+\frac{2}{9}+\frac{3}{9}}=(3^2)^x\Rightarrow 3^{6/9}=(3^2)^x$

$\Rightarrow 3^{2/3}=(3^2)^x\Rightarrow(3^2)^{1/3}=(3^2)^x\Rightarrow x=\frac{1}{3}$

12.(d) $4\sqrt{x}-\sqrt[4]{x}=4\sqrt{81}-\sqrt[4]{81}$

$=4\times9-(3^4)^{1/4}$

$=36-3=33$

13.(c) The least prime number is 2.

14.(b) Clearly (b) is true.

15.(d) Clearly (d) is true.

16.(a) $\left(\frac{243}{32}\right)^{-3/5} = \left(\left(\frac{3}{2}\right)^{5}\right)^{-3/5} = \left(\frac{3}{2}\right)^{5\times\left(\frac{-3}{5}\right)}$

$= \left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^{3} = \frac{8}{27}$

17.(b) Given surds are of order 4, 3 & 2 whose L.C.M. is 12.

$\sqrt[4]{4} = 4^{1/4} = 4^{\frac{1}{4}\times\frac{3}{3}} = 4^{3/12} = \left(4^{3}\right)^{1/12}$

$= (64)^{1/12} = \sqrt[12]{64}$

$\sqrt[3]{3} = 3^{1/3} = 3^{\frac{1}{3}\times\frac{4}{4}} = 3^{4/12} = \left(3^{4}\right)^{1/12}$

$= (81)^{1/12} = \sqrt[12]{81}$

$= \sqrt{2} = 2^{\frac{1}{2}} = 2^{\frac{1}{2}\times\frac{6}{6}} = 2^{6/12}$

$= \left(2^{6}\right)^{1/12} = (64)^{1/12} = \sqrt[12]{64}$

Clearly, $\sqrt[12]{81}$ is largest, i.e. $\sqrt[3]{3}$ is $\frac{9^{99-8.9} - 9^{8.9}}{9^{8.9}}$ largest.

Alternatively :

L.C.M. of surds 4,3 and 2 = 12

$\left(\sqrt[4]{4}\right)^{12} = 4^{\frac{1}{4}\times 12} = 4^{3} = 64$

$\left(\sqrt[3]{3}\right)^{12} = 3^{\frac{1}{3}\times 12} = 3^{4} = 81$

$\left(\sqrt{2}\right)^{12} = 2^{\frac{1}{2}\times 12} = 2^{6} = 64$

Clearly, $\sqrt[3]{3}$ is largest.

18.(b) Given Exp. $= \sqrt{\frac{121\times 9}{11\times 11}} = \sqrt{9} = 3$

19.(c) Given Exp.=

$$\frac{\left(\sqrt{7}+\sqrt{5}\right)^{2}+\left(\sqrt{7}-\sqrt{5}\right)^{2}}{\left(\sqrt{7}\right)^{2}-\left(\sqrt{5}\right)^{2}} = \frac{2\left[\left(\sqrt{7}\right)^{2}+\left(\sqrt{5}\right)^{2}\right]}{7-5}$$

$= \frac{2\times 12}{2} = 12$

20.(d)

$\sqrt{217+\sqrt{48+\sqrt{256}}} = \sqrt{217+\sqrt{48+16}}$

$= \sqrt{217+\sqrt{64}} = \sqrt{217+8} = \sqrt{225} = 15$

21.(b) $\sqrt{1.21}-\sqrt{0.01} = \sqrt{\frac{121}{100}} - \sqrt{\frac{1}{100}} =$

$\frac{\sqrt{121}}{\sqrt{100}} - \frac{1}{\sqrt{100}}$

$= \frac{11}{10} - \frac{1}{10} = \frac{10}{10} = 1$

22.(a) $\sqrt{0.04\times 0.4\times a} = 0.4\times 0.04\times\sqrt{b}$

$\Rightarrow \frac{\sqrt{a}}{\sqrt{b}} = \frac{0.4\times 0.04}{\sqrt{0.04\times 0.4}} = \sqrt{0.4\times 0.04}$

$\Rightarrow \frac{a}{b} = 0.4\times 0.04 = 0.016$

23.(d) $\sqrt{\frac{81\times 484}{64\times 625}} = \frac{9\times 22}{8\times 25} = \frac{99}{100} = 0.99$

24.(d) $\frac{3}{\sqrt{0.09}} = \frac{3}{\sqrt{\frac{9}{100}}} = \frac{3}{\frac{3}{10}} = 10$

25.(c) $\sqrt{3} = 3^{1/2}, \sqrt[3]{2} = 2^{1/3}$ $\sqrt{2} = 2^{1/2}$ and

$\sqrt[3]{4} = 4^{1/3}$

L.C.M. of 2, 3, 2, 3 = 6

$\sqrt{3} = 3^{\frac{1}{2}\times\frac{3}{3}} = \sqrt[6]{27}$

Similarly, $\sqrt[3]{2} = 2^{\frac{1}{3}\times\frac{2}{2}} = \sqrt[6]{4}$

$\sqrt{2} = 2^{\frac{1}{2}\times\frac{3}{3}} = \sqrt[6]{8}$

$\sqrt[3]{4} = 4^{\frac{1}{3}\times\frac{2}{2}} = \sqrt[6]{16}$

$\therefore$ Smallest surd is $\sqrt[3]{2}$

26.(d) $\sqrt{\frac{5}{3}} = \sqrt{\frac{5\times3}{3\times3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3} = 1.29$

27.(d) $\frac{(21)^{5.36}}{(21)^{3.47}} = (21)^x \Rightarrow (21)^{5.36-3.47} = (21)^x$

$\Rightarrow (21)^{1.89} = (21)^x$

$\Rightarrow x = 1.89$

28.(b)Given Exp. $\sqrt{\frac{28900000}{121000}} + \frac{\sqrt{24}+\sqrt{24\times9}}{\sqrt{24\times4}}$

$= \frac{170}{11} + \frac{\sqrt{24}+3\sqrt{24}}{2\sqrt{24}}$

$= \frac{170}{11} + 2 = \frac{192}{11} = 17\frac{5}{11}$

29.(d) $\sqrt{3^n} = 81 \Rightarrow (3^n)^{1/2} = 3^4$

$\Rightarrow 3^{n/2} = 3^4 \Rightarrow \frac{n}{2} = 4 \quad \therefore n = 8$

30.(b) $\sqrt[3]{\sqrt{0.000064}} = \sqrt[3]{0.008} = 0.2$

31.(a) Given Exp. $\frac{\sqrt[7]{\left((21^7)^5\right)^{\frac{1}{5}}}}{\sqrt[5]{\left((7^5)^3\right)^{\frac{1}{3}}}} = \frac{\sqrt[7]{(21^7)^{5\times\frac{1}{5}}}}{\sqrt[5]{(7^5)^{3\times\frac{1}{3}}}}$

$= \frac{\sqrt[7]{21^7}}{\sqrt[5]{7^5}} = \frac{(21^7)^{\frac{1}{7}}}{(7^5)^{1/5}} = \frac{21}{7} = 3$

32.(c) $(-3)^{(-2)^{(-2)^{(-1/4)}}} = \left(-\frac{1}{3}\right)^{(2)^{(-2)^{(-1/4)}}}$

$= \left(\frac{1}{9}\right)^{(-2)^{(-1/4)}} = (9)^{(2)^{(-1/4)}}$

$= (81)^{(-1/4)} = \left(\frac{1}{81}\right)^{(1/4)} = \left(\frac{1}{3^4}\right)^{1/4} = \frac{1}{3}$

33.(d) $\sqrt[5]{2x-7} - 3 = 0 \Rightarrow \sqrt[5]{2x-7} = 3$

$\Rightarrow \left(\sqrt[5]{2x-7}\right)^5 = 3^5 \Rightarrow 2x-7 = 243$

$\left[\because \left(\sqrt[n]{a}\right)^n = a\right]$

$\Rightarrow 2x = 250 \Rightarrow x = 125$

34.(b) Given Exp.

$= \left(x^{b-c}\right)^{(b+c-a)} \times \left(x^{c-a}\right)^{(c+a-b)} \times \left(x^{a-b}\right)^{(a+b-c)}$

$= x^{b^2-c^2-ab+ca} \times x^{c^2-a^2-bc+ab} \times x^{a^2-b^2-ac+bc}$

$= x^{b^2-c^2-ab+ca+c^2-a^2-bc+ab+a^2-b^2-ac+bc}$

$= x^0 = 1$

35.(c) Given Exp.

$$= \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}} + \frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}} + \frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$= \frac{x^a}{x^a+x^b+x^c} + \frac{x^b}{x^a+x^b+x^c} + \frac{x^c}{x^a+x^b+x^c}$$

$$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$$

36.(b) Given Exp.

$$= \frac{1}{1+\frac{a^x}{a^y}} + \frac{1}{1+\frac{a^y}{a^x}} = \frac{a^y}{a^x+a^y} + \frac{ax}{a^x+a^y}$$

$$= \frac{a^x+a^y}{a^x+a^y} = 1$$

37.(a) The given Exp. =

$3^{\frac{1}{2}\times\frac{2}{3}\times\ldots\ldots\frac{n}{n+1}} = (3)^{\frac{1}{n+1}}$

38.(c) Given Exp. $2\sqrt{x+2} + 2\sqrt{x-2} =$

$3\sqrt{x+2} - 3\sqrt{x-2}$

$$\Rightarrow 5\sqrt{x-2} = \sqrt{x+2}$$

squaring both sides,

$$25(x-2) = (x+2)$$

$$\Rightarrow 24x = 52$$

$$\Rightarrow 6x = \frac{52}{4} = 13$$

39.(b) $a^{\frac{1}{3}} = 11 \Rightarrow a = 11^3 = 1331$

$$\therefore a^2 - 331a = a(a-331)$$

$$= 1331(1331-331)$$

$$= 1331 \times 1000$$

$$= 1331000$$

40.(d) $\sqrt{\frac{x}{y}} + \sqrt{\frac{x}{y}} = \frac{10}{3} \Rightarrow \frac{x+y}{\sqrt{xy}} = \frac{10}{3}$

$$\Rightarrow \frac{10}{\sqrt{xy}} = \frac{10}{3} \qquad [\because x+y=10]$$

$$\Rightarrow \sqrt{xy} = 3 \Rightarrow x\,y = 9$$

41.(c) $2^{73} - 2^{72} - 2^{71}$

$= 2^{71}(2^2 - 2^1 - 1)$

$= 2^{71}\,(1) = 2^{71}$

42.(a) For the expression to hold true, x and y should both be positive.

43.(a) $\sqrt{156.25} + \sqrt{1.5625} - \sqrt{0.015625}$

$$= \sqrt{\frac{15625}{100}} + \sqrt{\frac{15625}{10000}} - \sqrt{\frac{15625}{1000000}}$$

$$= \frac{125}{10} + \frac{125}{100} - \frac{125}{1000}$$

$= 12.5 + 1.25 - 0.125$

$= 13.625$

44.(b) $a\sqrt{2} + b\sqrt{3} = \sqrt{49 \times 2} + \sqrt{36 \times 3}$

$- \sqrt{16 \times 3} - \sqrt{36 \times 2a}$

$$\Rightarrow a\sqrt{2} + b\sqrt{3} = 7\sqrt{2} + 6\sqrt{3} - 4\sqrt{3} - 6\sqrt{2}$$

$$\Rightarrow a\sqrt{2} + b\sqrt{3} = \sqrt{2} + 2\sqrt{3}$$

$\therefore a\sqrt{2} = \sqrt{2} \Rightarrow a = 1$ and $b\sqrt{3} = 2\sqrt{3}$

$\Rightarrow b = 2$ hence $a = 1,\ b = 2$

45.(d) Given Exp.

$$3 + \frac{1}{\sqrt{3}} + \frac{1}{(3+\sqrt{3})} \times \frac{(3-\sqrt{3})}{(3-\sqrt{3})} - \frac{1}{3-\sqrt{3}} \times \frac{(3+\sqrt{3})}{(3+\sqrt{3})}$$

$$= 3 + \frac{1}{\sqrt{3}} + \frac{1}{6}(3-\sqrt{3}) - \frac{1}{6}(3+\sqrt{3})$$

$$= 3 + \frac{1}{\sqrt{3}} + \frac{1}{2} - \frac{\sqrt{3}}{6} - \frac{1}{2} - \frac{\sqrt{3}}{6}$$

$$= 3 + \frac{1}{\sqrt{3}} - \frac{2\sqrt{3}}{6} = 3 + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{3}$$

$$= 3 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = 3$$

46.(c) $\sqrt{40 + \sqrt{9\sqrt{81}}} = \sqrt{40 + \sqrt{9 \times 9}} = \sqrt{40+9}$

$$= \sqrt{40+9} = \sqrt{49} = 7$$

47.(a) Given Exp.

$$\frac{\sqrt{3 \times 3 \times 2 \times 2 \times 2} \times \sqrt{11 \times 11 \times 3} \times \sqrt{5 \times 5 \times 7}}{\sqrt{4 \times 4 \times 2} \times \sqrt{7 \times 7 \times 3} \times \sqrt{2 \times 2 \times 3 \times 3 \times 7}}$$

$$= \frac{6\sqrt{2} \times 11\sqrt{3} \times 5\sqrt{7}}{4\sqrt{2} \times 7\sqrt{3} \times 6\sqrt{7}} = \frac{6 \times 11 \times 5}{4 \times 7 \times 6} = \frac{55}{28}$$

48.(c) $\sqrt{4^n} = (1024) \qquad \Rightarrow (4n)^{1/2} = 1024$

$$\Rightarrow \left(4^{\frac{1}{2}}\right)^n = (2)^{10} \qquad \Rightarrow 2^n = 2^{10}$$

$\therefore n = 10$

49.(b) Given Exp. $x^{(b+c)(b-c)}.\,x^{(c+a)(c-a)}\,.\,x^{(a+b)\,(a-b)}$

$$= x^{b^2-c^2}.x^{c^2-a^2}.x^{a^2-b^2}$$

$$= x^{b^2-c^2+c^2-a^2+a^2-b^2}$$

$$= x^o = 1$$

50.(a) Given Exp : $\frac{5.5^{1/2}\times 5^{3}}{5^{-3/2}} = 5^{a+2}$

$$\Rightarrow 5^{1+\frac{1}{2}+3}\times 5^{3/2} = 5^{a+2}$$

$$\Rightarrow 5^{9/2}\times 5^{3/2} = 5^{a+2}$$

$$\Rightarrow 5^{6} = 5^{a+2}$$

$$\therefore a+2=6\Rightarrow a=4$$

51.(c) $\left(\frac{3}{5}\right)^{3}\left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$ (given)

or $\left(\frac{3}{5}\right)^{3-6} = \left(\frac{3}{5}\right)^{2x-1}$

$$\Rightarrow \left(\frac{3}{5}\right)^{-3} = \left(\frac{3}{5}\right)^{2x-1}$$

$$\therefore -3 = 2x-1 \Rightarrow 2x=-2\Rightarrow x=-1$$

52.(b) $x+\frac{1}{4}\sqrt{x}+a^{2} = \left(\sqrt{x}\right)^{2}+2.\sqrt{x}.\frac{1}{8}+(a)^{2}$

Clearly $a=\frac{1}{8}$.

Then, expression $=\left(\sqrt{x}+\frac{1}{8}\right)^{2}$

53.(a) $x=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{\left(\sqrt{5}+\sqrt{3}\right)^{2}}{5-3}$

$$=\frac{8+2\sqrt{15}}{2}=4+\sqrt{15}$$

Similarly,

$$y=\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{1}{x}=4-\sqrt{15}$$

$$\therefore x+y=4+\sqrt{15}+4-\sqrt{15}=8$$

54.(c) $(0.5)^{2}=0.25,\ \sqrt{0.49}=0.7$

$\sqrt[3]{0.008}=0.2,\quad 0.23=0.23$

$$\therefore \sqrt{0.49}>(0.5)^{2}>0.23>\sqrt[3]{0.008}$$

55.(b) Expression $=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$=\frac{\left(\sqrt{3}+\sqrt{2}\right)^{2}-\left(\sqrt{3}-\sqrt{2}\right)^{2}}{\left(\sqrt{3}+\sqrt{2}\right)\ \left(\sqrt{3}-\sqrt{2}\right)}$$

$$=\frac{3+2+2\sqrt{6}-3-2+2\sqrt{6}}{\left(\sqrt{3}\right)^{2}-\left(\sqrt{2}\right)^{2}}$$

$$=\frac{4\sqrt{6}}{3-2}=4\sqrt{6}$$

56.(c) $\sqrt{12}+\sqrt{18}=\sqrt{3\times2\times2}+\sqrt{2\times3\times3}$

$$=2\sqrt{3}+3\sqrt{2}$$

$\therefore$ Required difference

$$=2\sqrt{3}+3\sqrt{2}-2\sqrt{3}-2\sqrt{2}=\sqrt{2}$$

57.(d) $2\sqrt{x}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}-\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$=\frac{\left(\sqrt{5}+\sqrt{3}\right)^{2}-\left(\sqrt{5}-\sqrt{3}\right)^{2}}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$$

$$=\frac{4.\sqrt{5}.\sqrt{3}}{5-3}=2\sqrt{15}$$

$\therefore 2\sqrt{x}=2\sqrt{15}$ Þ $x=15$

58.(d) Expression

$$=\frac{3+\sqrt{6}}{5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}}$$

$$=\frac{3+\sqrt{6}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}}$$

$$=\frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{3}\left(\sqrt{3}+\sqrt{2}\right)}{\sqrt{3}+\sqrt{2}}$$

$$=\sqrt{3}=1.732$$

59.(c) Expression $= \dfrac{(81)^{3.6} \times (9)^{2.7}}{(81)^{4.2} \times 3}$

$$= \frac{(3^4)^{3.6} \times (3^2)^{2.7}}{(3^4)^{4.2} \times 3} = \frac{3^{14.4} \times 3^{5.4}}{3^{16.8} \times 3}$$

$$\left[\because (a^m)^n = a^{mn}; a^m \times a^n = a^{m+n}; \; a^m \div a^n = a^{m-n}\right]$$

$$= \frac{3^{14.4+5.4.}}{3^{16.8+1}} = \frac{3^{19.8}}{3^{17.8}} = 3^{19.8-17.8} = 3^2 = 9$$

Alternatively:-

$$\frac{(9^2)^{3.6} \times (9)^{2.7}}{(9^2)^{4.2} \times (9)^{\frac{1}{2}}} = \frac{9^{7.2} \times 9^{2.7}}{9^{8.4} \times 9^{0.5}} = \frac{9^{9.9}}{9^{8.9}}$$

$9^{9.9-8.9} = 9$

60.(c) $(0.1)^2 = 0.01$

$\sqrt{0.0121} = \sqrt{0.11 \times 0.11} = 0.11$

$\sqrt{0.0004} = 0.02$

LEVEL - 2

1.(a) $3^x - 3^{x-1=18}$

$\Rightarrow 3^x - 3^x . 3^{-1} = 18$

$\Rightarrow 3^x\left(1 - \frac{1}{3}\right) = 18 \Rightarrow 3^x\left(\frac{2}{3}\right) = 18$

$\Rightarrow 3^x = 27 = 3^3$

$\Rightarrow x = 3 \;\therefore\; x^2 = 3^2 = 9$

2.(b) Let $a^x = b^y = c^z = k$

then, $a = k^{\frac{1}{x}}, \quad b = k^{\frac{1}{y}}$ and $c = k^{\frac{1}{z}}$

Now, $b^2 = ac$

$\Rightarrow (k^{1/y})^2 = (k^{1/x}).(k^{1/z})$

$\Rightarrow k^{2/y} = k^{\frac{1}{x}+\frac{1}{z}}$

$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z} = \frac{x+z}{xz}$

$\Rightarrow y = \frac{2xz}{x+z}$

3.(c) $a = c^z = (b^y)^z = b^{yz} = a^{xyz} \Rightarrow xyz = 1$

4.(d) $\sqrt{3+\sqrt{5}} = \sqrt{(3+\sqrt{5}) \times 2} = \sqrt{\frac{6+2\sqrt{5}}{2}}$

$= \sqrt{\frac{1^2 + (\sqrt{5})^2 + 2 \times 1 \times \sqrt{5}}{2}} = \sqrt{\frac{(1+\sqrt{5})^2}{2}}$

$= \frac{1+\sqrt{5}}{\sqrt{2}} = \sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}$

5.(a) $\sqrt{7+2\sqrt{10}} = \sqrt{7 + 2 \times \sqrt{2} \times \sqrt{5}}$

$= \sqrt{(\sqrt{2})^2 + (\sqrt{5})^2 + 2 \times \sqrt{2} \times \sqrt{5}}$

$= \sqrt{(\sqrt{5}+\sqrt{2})^2} = \sqrt{5} + \sqrt{2}$

6.(c) $\frac{1}{\sqrt{9}-\sqrt{8}} = \frac{1}{\sqrt{9}-\sqrt{8}} \times \frac{\left(\sqrt{9}+\sqrt{8}\right)}{\left(\sqrt{9}+\sqrt{8}\right)}$

$= \frac{\sqrt{9}+\sqrt{8}}{\left(\sqrt{9}\right)^2-\left(\sqrt{8}\right)^2} = \sqrt{9}+\sqrt{8}$

Similarly,

$\frac{1}{\sqrt{8}-\sqrt{7}} = \sqrt{8}+\sqrt{7}, \ \frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$

$\frac{1}{\sqrt{6}-\sqrt{5}} = \sqrt{6}+\sqrt{5}$ and $\frac{1}{\sqrt{5}-\sqrt{4}}$

$= \sqrt{5}+\sqrt{4}$

∴ Given Expression

$= \left(\sqrt{9}+\sqrt{8}\right)-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)$

$-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+\sqrt{4}\right)$

$= \left(\sqrt{9}+\sqrt{4}\right) = 3+2 = 5$

7.(d) Let $x = \sqrt{20+\sqrt{20+\sqrt{20+........\infty}}}$

$\Rightarrow x^2 = 20+\sqrt{20+\sqrt{20+.....\infty}}$

$\Rightarrow x^2 = 20+x \Rightarrow x^2-x-20 = 0$

$\Rightarrow (x-5)(x+4=0) \Rightarrow x = 5$ or -4

But the given expression is positive hence its value can not be negative.

∴ value of the expression = 5

Short-cut -

$x = \sqrt{20+\sqrt{20+\sqrt{20+........\infty}}}$

but $\Rightarrow 2\,0 = 4\times 5 = n(n+1)$

∴ $x = n + 1 = 5$ (for +ive sign)

8.(b) let $x = \sqrt{12+\sqrt{12+\sqrt{12+........\infty}}}$ = ?

but ⇒ $12 = 3\times 4 = n\,(n + 1)$

∴ $x = n + 1 = 4$ (for = +ive sign)

9.(a) $\sqrt{1+\frac{55}{729}} = 1+\frac{x}{27} \Rightarrow \sqrt{\frac{784}{729}} = 1+\frac{x}{27}$

$\Rightarrow \frac{28}{27} = 1+\frac{x}{27} \Rightarrow 1+\frac{1}{27} = 1+\frac{x}{27} \Rightarrow x = 1$

10.(a) Clearly, $65^2 = 65\times 65 = 4225$

∴ x lines between 64 and 65.

11.(b) $x = \sqrt{6+\sqrt{6+\sqrt{6+.......\infty}}}$ &$6 = n(n+1)$

$= 2\times 3$

∴ $x = n + 1 = 3$ (for +ive sign)

12.(c) $2^{x+1} + 2^{x+3} = 2560$

$\Rightarrow 2^x.2^1 + 2^x.2^3 = 2560$

$\Rightarrow 2^x(2 + 2^3) = 2560 \Rightarrow 2^x(10) = 2560$

$\Rightarrow 2^x = 256 = (2)^8 \Rightarrow x = 8$.

13.(b) $a = \left(\sqrt{8}-\sqrt{7}\right)\times\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} = \frac{1}{\sqrt{8}+\sqrt{7}}$

Similarly, $b = \frac{1}{\sqrt{7}+\sqrt{6}}$ and $c = \frac{1}{\sqrt{6}+\sqrt{5}}$

∴ $a < b < c$ [because denominator of a > denominator of b > denominator of c]

14.(b) $\sqrt[3]{(4.032)^2-(3.968)^2}$

$= \sqrt[3]{(4.032+3.968)\ (4.032-3.968)}$

$= \sqrt[3]{8\times 0.064} = \sqrt[3]{2^3\times (0.4)^3}$

$= 2\times 0.4 = 0.8$

15.(d) 19 - 17 = 2 , 13 - 11 = 2, 7 - 5 = 2 & 5 - 3 =2

∴ smallest surd = $\sqrt{19}-\sqrt{17}$

greatest surd = $\sqrt{5}-\sqrt{3}$

Alternatively : -

$\sqrt{19}-\sqrt{17} = \left(\sqrt{19}-\sqrt{17}\right)\times\frac{\left(\sqrt{19}+\sqrt{17}\right)}{\left(\sqrt{19}+\sqrt{17}\right)}$

$$=\frac{2}{\sqrt{19}+\sqrt{17}}$$

Similarly, $\sqrt{13}-\sqrt{11}=\frac{2}{\sqrt{13}+\sqrt{11}}$,

$$\sqrt{7}-\sqrt{5}=\frac{2}{\sqrt{7}+\sqrt{5}}$$

and $\sqrt{5}-\sqrt{3}=\frac{2}{\sqrt{5}+\sqrt{3}}$

∴ largest surd = $\sqrt{5}-\sqrt{3}$ because its denominator is the smallest.

16.(d) $\sqrt{4+\sqrt{15}}=\sqrt{\frac{8+2\sqrt{15}}{2}}$

$$\sqrt{\frac{8+2\times\sqrt{5}\sqrt{3}}{2}}$$

number.

17.(a) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}=a+b\sqrt{3}$

$$\Rightarrow\frac{5+2\sqrt{3}}{7+4\sqrt{3}}\times\left(\frac{7-4\sqrt{3}}{7-4\sqrt{3}}\right)=a+b\sqrt{3}$$

$$\Rightarrow\frac{35-20\sqrt{3}+14\sqrt{3}-24}{(7)^2-\left(4\sqrt{3}\right)^2}=a+b\sqrt{3}$$

$$\Rightarrow\frac{11-6\sqrt{3}}{49-48}=a+b\sqrt{3}$$

$\therefore b\sqrt{3}=-6\sqrt{3}$

$\Rightarrow b=-6$

18.(a) Let $x=\sqrt{6\sqrt{6\sqrt{6\ldots\ldots\infty}}}$

squaring both sides,

$$\Rightarrow x^{32} = 6^{31}$$

$$\Rightarrow x = 6^{\frac{31}{32}}$$

Shortcut :-

$$x = 6^{\frac{2^n-1}{2^n}} = 6^{\frac{2^5-1}{2^5}} = 6^{\frac{31}{32}}$$

Where n = no. of terms.

20.(a) Let $x = \sqrt{20-\sqrt{20-\sqrt{20........\infty}}}$

and 20 = n (n + 1) = 4 × 5

$\therefore$ x = n = 4 (for -ve sign)

21.(b) Denominator $= \sqrt{10}+\sqrt{20}+\sqrt{40} - \sqrt{5} - \sqrt{80}$

$$= \sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}$$

$$= 3\sqrt{10}-3\sqrt{5} = 3\left(\sqrt{10}-\sqrt{5}\right)$$

$$\therefore \text{Given Exp.} = \frac{15}{3\left(\sqrt{10}-\sqrt{5}\right)} \times \frac{\left(\sqrt{10}+\sqrt{5}\right)}{\left(\sqrt{10}+\sqrt{5}\right)}$$

$$= \frac{5\left(\sqrt{10}+\sqrt{5}\right)}{(10-5)} = \sqrt{10}+\sqrt{5}$$

$$= 3.162 + 2.236$$

$$= 5.398$$

22.(c) By rationalising each term,

$$\text{Given Exp. } \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + ... \frac{\sqrt{9}-\sqrt{8}}{9-8}$$

$$= \sqrt{2}-1+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+.........$$

$$..+\sqrt{8}-\sqrt{7}+\sqrt{9}-\sqrt{8}$$

$$= \sqrt{9}-1 = 3-1 = 2$$

23.(a) Given Exp. $= \dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \times \dfrac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)}$

$$+ \frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}} \times \frac{\left(\sqrt{6}+\sqrt{3}\right)}{\left(\sqrt{6}+\sqrt{3}\right)} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}$$

$$\square\times \frac{\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}-\sqrt{2}\right)}$$

$$= \sqrt{18}-\sqrt{12}+\sqrt{12}+\sqrt{6}-\sqrt{18}+\sqrt{6}$$

$$= 2\sqrt{6}$$

24.(c) Given Exp. $\left(28+2\times5\times\sqrt{3}\right)^{1/2}$ - $\left(7-2\times2\times\sqrt{3}\right)^{-1/2}$

$$= \left(5^2+\left(\sqrt{3}\right)^2+2\times5\times\sqrt{3}\right)^{1/2}$$

$$-\left(2^2+\left(\sqrt{3}\right)^2-2\times2\times\sqrt{3}\right)^{-1/2}$$

$$= \left[\left(5+\sqrt{3}\right)^2\right]^{1/2} - \left[\left(2-\sqrt{3}\right)^2\right]^{-1/2}$$

$$= \left(5+\sqrt{3}\right) - \left(2-\sqrt{3}\right)^{-1}$$

$$= 5+\sqrt{3} - \frac{1}{2-\sqrt{3}}$$

$$= 5+\sqrt{3} - \frac{1}{2-\sqrt{3}} \times \frac{\left(2+\sqrt{3}\right)}{\left(2+\sqrt{3}\right)}$$

$$= 5+\sqrt{3}-\left(2+\sqrt{3}\right) = 5-2 = 3$$

25.(b) $\sqrt{4-\sqrt{7}} = \sqrt{\dfrac{\left(4-\sqrt{7}\right)\times2}{2}}$

$$= \sqrt{\frac{8-2\times1\times\sqrt{7}}{2}}$$

$$= \sqrt{\frac{\left(\sqrt{7}-1\right)^2}{\sqrt{2}}} = \frac{\sqrt{7}-1}{\sqrt{2}}$$

$$\sqrt{8+3\sqrt{7}}=\sqrt{\frac{\left(8+3\sqrt{7}\right)\times 2}{2}}=\sqrt{\frac{16+2\times 3\times\sqrt{7}}{2}}$$

$$=\sqrt{\frac{\left(3+\sqrt{7}\right)^2}{\sqrt{2}}}=\frac{3+\sqrt{7}}{\sqrt{2}}=\frac{3}{\sqrt{2}}+\frac{\sqrt{7}}{\sqrt{2}}$$

$$\therefore \text{Given Exp.} = \frac{\left(\sqrt{7}-1\right)/\sqrt{2}}{\frac{3+\sqrt{7}}{\sqrt{2}}-2\sqrt{2}}=\frac{\sqrt{7}-1}{3+\sqrt{7}-4}$$

$$=\frac{\sqrt{7}-1}{\sqrt{7}-1}=1$$

26.(a) Since 72 $= 9\times 8$

Therefore $\sqrt{72+\sqrt{72+\sqrt{72+......\infty}}}=9$

29.(b) Let $2^x = 3^y = 6^{-z} = k$

$\Rightarrow$ $2 = k^{1/x}$ ......(i)

$\Rightarrow$ $3 = k^{1/y}$ ......(ii)

$\Rightarrow$ $6 = k^{-1/z}$......(iii)

multiplying (i) and (ii)

$6 = k^{\frac{1}{x}+\frac{1}{y}}$ ....(iv)

from (iii) & (iv)

$$k^{\frac{1}{x}+\frac{1}{y}}=k^{-\frac{1}{z}}$$

$$\Rightarrow\frac{1}{x}+\frac{1}{y}=-\frac{1}{z}\Rightarrow\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$$

30.(d) Given Exp.

$$=\frac{3+\sqrt{2\times 3}}{5\sqrt{3}-2\sqrt{4\times 3}-\sqrt{16\times 2}+\sqrt{25\times 2}}$$

$$3+\sqrt{2\times 3}$$

& $\sqrt[4]{5^y} = \sqrt{3}$

$\Rightarrow 5^{y/4} = 3^{1/2}$

$\Rightarrow 5^{1/4} = \left(3^{1/2}\right)^{1/y}$

$\Rightarrow 5^{1/4} = 3^{1/2y}$ ....(ii)

from (i) and (ii)

$3^{x/3} = 3^{1/2y}$

$\therefore \frac{x}{3} = \frac{1}{2y} \Rightarrow 2xy = 3$

33.(b) Given Exp.

$$\frac{\left(x-\sqrt{6\times4}\right)\left(\sqrt{25\times3}+\sqrt{25\times2}\right)}{\sqrt{25\times3}-\sqrt{25\times2}} = 1$$

$$\Rightarrow \frac{\left(x-2\sqrt{6}\right)\times5\left(\sqrt{3}+\sqrt{2}\right)}{5\left(\sqrt{3}-\sqrt{2}\right)} = 1$$

$$\Rightarrow \left(x-2\sqrt{6}\right)\frac{\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)} = 1$$

$$\Rightarrow \quad x-2\sqrt{6} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+2}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-2}$$

$= \left(\sqrt{3}-\sqrt{2}\right)^2$

$\Rightarrow x-2\sqrt{6} = 5-2\sqrt{6}$

$\Rightarrow x = 5$

34.(c) Given Exp

$$= \sqrt{5\times4}+\sqrt{4\times3}+\sqrt[3]{9^3}-\frac{4}{\sqrt{5}-\sqrt{3}}$$

$$\times\frac{\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}-\sqrt{9\times9}$$

$= 2\sqrt{5}+2\sqrt{3}+9-2\left(\sqrt{5}+\sqrt{3}\right)-9$

$= 0$

35.(c) Given Exp. $= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}\times\frac{\left(\sqrt{5}-\sqrt{3}\right)}{\sqrt{5}-\sqrt{3}}$ -

$$\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}\times\frac{\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)}$$

$$= \frac{1}{2}\left[\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}\right]$$

$$= \frac{1}{2}\left[2\sqrt{5}-2\sqrt{6}\right]$$

$= \sqrt{5}-\sqrt{6}$

36.(b) $a = \frac{1}{2-\sqrt{3}}+\frac{1}{3-\sqrt{8}}+\frac{1}{4-\sqrt{15}}$

$\Rightarrow a = 2+\sqrt{3}+3+\sqrt{8}+4+\sqrt{15}$

[by rationalising]

$\Rightarrow a = 9+\sqrt{3}+\sqrt{8}+\sqrt{15}$

Now, $\sqrt{3} = 1.732$, $\sqrt{8} = 2\sqrt{2} = 2\times1.414$

$= 2.828$, & $\sqrt{15} = 3.87$

$\therefore a = 9+1.732+2.828+3.87 = 17.43 < 18$

37.(d) Given Exp.

$$\frac{1}{\left(\sqrt{2}+1\right)}\times\frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}-1\right)}+\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)}\times\frac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)}$$

$$+\frac{1}{\left(\sqrt{4}+\sqrt{3}\right)}\times\frac{\left(\sqrt{4}-\sqrt{3}\right)}{\left(\sqrt{4}-\sqrt{3}\right)}+\ldots\ldots.$$

$$+\frac{1}{\left(\sqrt{121}+\sqrt{120}\right)}\times\frac{\left(\sqrt{121}-\sqrt{120}\right)}{\left(\sqrt{121}-\sqrt{120}\right)}$$

$+\left(\sqrt{121}-\sqrt{120}\right)$

$= \sqrt{121}-1 \quad = 11-1 = 10$

38. (a) $\sqrt{7\sqrt{3}+12}=\sqrt{7\sqrt{3}+4\times3}=\sqrt{\sqrt{3}\left(7+4\sqrt{3}\right)}$

$$=\sqrt{\sqrt{3}\left(7+2\times2\times\sqrt{3}\right)}$$

$$=\sqrt{\sqrt{3}\left(2^2+\left(\sqrt{3}\right)^2+2\times2\times\sqrt{3}\right)}$$

$$=\sqrt{\sqrt{3}\left(2+\sqrt{3}\right)^2}$$

$$=\sqrt{\sqrt{3}}\left(2+\sqrt{3}\right)$$

$$=\sqrt[4]{3}\left(2+\sqrt{3}\right)$$

39.(a) Given Exp:

$$\frac{3^{2n}.3^2.3^n-3^{3n}}{3^{3m}.2^3}=\frac{1}{3^3}$$

$$\Rightarrow\frac{3^{2n+n}.9-3^{3n}}{3^{3m}\times8}=3^{-3}$$

$$\Rightarrow\frac{3^{3n}(9-1)}{3^{3m}\times8}=3^{-3}$$

$$\Rightarrow 3^{3n-3m}=3^{-3}$$

$$\Rightarrow 3n-3m=-3$$

$$\Rightarrow n-m=-1$$

$$\Rightarrow m-n=1$$

40. (d)

Expression = $\sqrt{10+2\sqrt{6}+2\sqrt{10}+2\sqrt{15}}$

$$=\sqrt{10+2\times\sqrt{2}\times\sqrt{3}+2\times\sqrt{2}\times\sqrt{5}+2\times\sqrt{3}\times\sqrt{5}}$$

$$=\sqrt{2+3+5+2\times\sqrt{3}\times\sqrt{3}+2\times\sqrt{2}\times\sqrt{5}+2\times\sqrt{3}\times\sqrt{5}}$$

$$=\sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2+2\times\sqrt{2}\times\sqrt{3}+2\times\sqrt{2}\times\sqrt{5}+2\times\sqrt{3}\times\sqrt{5}}$$

$$=\sqrt{\left(\sqrt{2}+\sqrt{3}+\sqrt{5}\right)^2}=\sqrt{2}+\sqrt{3}+\sqrt{5}$$

$[(a+b+c)^2=a^2+b^2+c^2+2ab+2ac+2bc]$

41.(b) $\frac{\sqrt{7}-2}{\sqrt{7}+2}=\frac{\sqrt{7}-2}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}$

$$=\frac{\left(\sqrt{7}-2\right)^2}{7-4}=\frac{7+4-4\sqrt{7}}{3}$$

$$=\frac{11}{3}-\frac{4\sqrt{7}}{3}$$

$$\therefore\frac{\sqrt{7}-2}{\sqrt{7}+2}=a\sqrt{7}+b$$

$$\Rightarrow\frac{11}{3}-\frac{4}{3}\sqrt{7}=a\sqrt{7}+b$$

Clearly,

$a=-\frac{4}{3}$ and $b=\frac{11}{3}$

42.(a) Expression =

$$\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$$

$$=\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}\times\frac{\sqrt{6}-\sqrt{3}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

[Rationalising the respective denominators]

$$\frac{3\sqrt{2}\left(\sqrt{6}-\sqrt{3}\right)}{6-3}-\frac{4\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{6-2}+\frac{\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)}{3-2}$$

$$=\sqrt{2}\left(\sqrt{6}-\sqrt{3}\right)-\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)+\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)$$

$$=\sqrt{12}-\sqrt{6}-\sqrt{18}+\sqrt{6}+\sqrt{18}-\sqrt{12}=0$$

43.(b) $x = \frac{\sqrt{3}}{2}$

$$\therefore \sqrt{1+x} = \sqrt{1+\frac{\sqrt{3}}{2}} = \sqrt{\frac{2+\sqrt{3}}{2}\times\frac{2}{2}}$$

$$= \frac{1}{2}\sqrt{4+2.\sqrt{3}}$$

$$= \frac{1}{2}\sqrt{4+2.1.\sqrt{3}} = \frac{1}{2}\sqrt{\left((1)^2+\sqrt{3}\right)^2}$$

$$= \frac{1}{2}\sqrt{(1)^2+\left(\sqrt{3}\right)^2+2\times 1\times\sqrt{3}}$$

$$= \frac{1}{2}\sqrt{\left(1+\sqrt{3}\right)^2} = \frac{\sqrt{3}+1}{2}$$

Similary, $\sqrt{1-x} = \frac{\sqrt{3}-1}{2}$

$$\therefore \frac{\sqrt{1+x}}{1+\sqrt{1+x}} + \frac{\sqrt{1-x}}{1-\sqrt{1-x}}$$

$$= \frac{\sqrt{3}+1}{2\times\left(1+\frac{\sqrt{3}+1}{2}\right)} + \frac{\sqrt{3}-1}{2\left(1-\frac{\sqrt{3}-1}{2}\right)}$$

$$= \frac{\sqrt{3}+1}{3+\sqrt{3}} + \frac{\sqrt{3}-1}{3-\sqrt{3}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}\left(\sqrt{3}+1\right)} + \frac{\sqrt{3}-1}{\sqrt{3}\left(\sqrt{3}-1\right)}$$

$$= \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \quad = \frac{2}{\sqrt{3}}$$

44.(b) $\sqrt{2} = 1.414$ (Given)

Now, $\frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\left(\sqrt{2}-1\right)\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}$

$$\frac{\left(\sqrt{2}-1\right)^2}{2-1} = \left(\sqrt{2}-1\right)^2$$

$$= \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\left(\sqrt{2}-1\right)^2} = \sqrt{2}-1$$

$$= 1.414 - 1 = 0.414$$

45.(c) $\sqrt{\frac{\sqrt{36}-\sqrt{24}+\sqrt{24}-\sqrt{16}}{5+\sqrt{24}}}$

$$= \sqrt{\frac{6-4}{5+\sqrt{24}}} = \sqrt{\frac{2}{5+\sqrt{24}}} = \sqrt{\frac{2}{5+\sqrt{6\times 4}}}$$

$$= \sqrt{\frac{2}{5+2\sqrt{6}}} = \sqrt{\frac{2}{5+2\sqrt{6}}\times\frac{5-2\sqrt{6}}{5-2\sqrt{6}}}$$

$$= \sqrt{\frac{2\left(5-2\sqrt{6}\right)}{25-24}} = \sqrt{2\left(5-2\sqrt{6}\right)}$$

$$= \sqrt{2\left[(3)^2+\left(\sqrt{2}\right)^2-2\sqrt{3}\sqrt{2}\right]}$$

$$= \sqrt{2\left(\sqrt{3}-\sqrt{2}\right)^2} = \sqrt{2}\left(\sqrt{3}-\sqrt{2}\right) = \sqrt{6}-2$$

46.(a) $\sqrt[3]{4},\ \sqrt{2},\ \sqrt[6]{3},\ \sqrt[4]{5}$

LCM of 3, 2, 6, 4 = 12

$$\sqrt[3]{4} = (4)^{\frac{1}{3}} = (4)^{\frac{4}{12}} = \left(4^4\right)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$$

$$\sqrt{2} = (2)^{\frac{1}{2}} = (2)^{\frac{6}{12}} = \left(2^6\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$$

$$\sqrt[6]{3} = (3)^{\frac{1}{6}} = (3)^{\frac{2}{12}} = \left(3^2\right)^{\frac{1}{12}} = (9)^{\frac{1}{12}}$$

$$\sqrt[4]{5} = (5)^{\frac{1}{4}} = (5)^{\frac{3}{12}} = \left(5^3\right)^{\frac{1}{12}} = (125)^{\frac{1}{12}}$$

$$\therefore (256)^{\frac{1}{12}} > (125)^{\frac{1}{12}} > (64)^{\frac{1}{12}} > (9)^{\frac{1}{12}}$$

or, $\sqrt[3]{4} > \sqrt[4]{5} > \sqrt{2} > \sqrt[6]{3}$

47.(c) LCM of the orders of the surds = LCM of 2, 3, 5 and 7 = 210

$$5^{\frac{1}{2}} = 5^{\frac{105}{210}} = \left(5^{105}\right)^{\frac{1}{210}}$$

$$4^{\frac{1}{3}} = 4^{\frac{70}{210}} = \left(4^{70}\right)^{\frac{1}{210}}$$

$$2^{\frac{1}{5}} = 2^{\frac{42}{210}} = \left(2^{42}\right)^{\frac{1}{210}}$$

$$3^{\frac{1}{7}} = 3^{\frac{30}{210}} = \left(3^{30}\right)^{\frac{1}{210}}$$

$\therefore$ The largest number $= 5^{\frac{1}{2}} = \sqrt{5}$

48.(a) Expression

$$= \frac{\sqrt{7}}{\sqrt{16+6\sqrt{7}} - \sqrt{16-6\sqrt{7}}}$$

$$= \frac{\sqrt{7}}{\sqrt{9+7+2\times3\sqrt{7}} - \sqrt{9+7-2\times3\times\sqrt{7}}}$$

$$= \frac{\sqrt{7}}{\left(3+\sqrt{7}\right)-\left(3-\sqrt{7}\right)} = \frac{\sqrt{7}}{3+\sqrt{7}-3+\sqrt{7}} = \frac{1}{2}$$

49.(b)

$$\sqrt{7}-\sqrt{5} = \frac{\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)}{\sqrt{7}+\sqrt{5}} = \frac{2}{\sqrt{7}+\sqrt{5}}$$

Similarly,

$$\sqrt{5}-\sqrt{3} = \frac{2}{\sqrt{5}+\sqrt{3}}$$

$$\sqrt{9}-\sqrt{7} = \frac{2}{\sqrt{9}+\sqrt{7}}$$

$$\sqrt{11}-\sqrt{9} = \frac{2}{\sqrt{11}+\sqrt{9}}$$

$\therefore$ Largest number $= \sqrt{5}-\sqrt{3}$ because its denominator is the smallest.

50.(c) Let

$$x = \sqrt{7\sqrt{7\sqrt{7\sqrt{7.....}}}} \quad \therefore x = \sqrt{7x}$$

On squaring both sides,

$$x^2 = 7x$$

$$\Rightarrow x(x-7) = 0 \Rightarrow x = 7$$

$$\therefore \quad 7 = (7^3)^{y-1} = 7^{3y-3}$$

$$\Rightarrow 3y - 3 = 1 \Rightarrow 3y = 4$$

$$y = \frac{4}{3}$$

51.(d) $\dfrac{1}{\sqrt{2}+\sqrt{1}} = \dfrac{1}{\sqrt{2}+\sqrt{1}} \times \dfrac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} = \sqrt{2}-1$

$\therefore$ Expression

$$= \sqrt{2}-1+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+......$$

$$+\sqrt{99}-\sqrt{98}+\sqrt{100}-\sqrt{99}$$

$$= \sqrt{100}-1 = 10-1 = 9$$

52.(c) $x = \dfrac{\sqrt{5}-2}{\sqrt{5}+2} \times \dfrac{\left(\sqrt{5}-2\right)}{\left(\sqrt{5}-2\right)} = \dfrac{5+4-4\sqrt{5}}{5-4}$

$$= 9-4\sqrt{5}$$

$$\therefore \frac{1}{x} = 9+4\sqrt{5} \quad \left[\because (9)^2-\left(4\sqrt{5}\right)^2 = 1\right]$$

$$\therefore x+\frac{1}{x} = 18$$

$$\therefore x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

$$= (18)^2 - 2 = 322$$

$$\therefore x^4 + x^4 = x^4 + \frac{1}{x^4} = \left(x^2+\frac{1}{x^2}\right)^2 - 2$$

$= (322)^2 - 2 = 103682$ (whole number)

53.(a) $x+\dfrac{1}{x} = 3 = \text{a (let)}$

$$\therefore x^2+\frac{1}{x^2}=a^2-2=3^2-2=7$$

$$\text{and } x^3+\frac{1}{x^3}=a^3-3a=3^3-3\times3=18$$

$$\therefore x^5+\frac{1}{x^5}=\left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$$

$$=18\times7-3$$

$$=123$$

54.(b) $a=\frac{2+\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{4+3+4\sqrt{3}}{4-3}$

$$=7+4\sqrt{3}$$

$$\text{and } b=\frac{2-\sqrt{3}}{2+\sqrt{3}}=\frac{1}{a}=7-4\sqrt{3}$$

$$\therefore a^2+b^2+ab=a^2+\frac{1}{a^2}+a.\frac{1}{a}\left(\because b=\frac{1}{a}\right)$$

$$=a^2+\frac{1}{a^2}+1+1-1$$

$$=\left(a^2+\frac{1}{a^2}+2\right)-1$$

$$=\left(a+\frac{1}{a}\right)^2-1$$

$$=(a+b)^2-1$$

$$=\left(7+4\sqrt{3}+7-4\sqrt{3}\right)^2-1$$

$$=(14)^2-1$$

$$=196-1=195$$

Alternatively :-

$a + b = 14$

$ab = (7+4\sqrt{3})(7-4\sqrt{3})$

$= (7)^2 - (4\sqrt{3})^2$

$= 49 - 48$

$= 1$

$$a^2+b^2+ab = a^2+b^2+2ab-ab$$

$$= (a+b)^2 - ab$$

$$= (14)^2 - 1$$

$$= 196 - 1$$

$$= 195$$

55. (b) $a=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{5-2\sqrt{6}}{3-2}$

$$=5-2\sqrt{6}$$

$$\text{and } b=\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{1}{a}=5+2\sqrt{6}$$

$$\left[\because 5^2-\left(2\sqrt{6}\right)^2=1\right]$$

$$\therefore a+b=a+\frac{1}{a}=10$$

$$\text{Now, } \frac{a^2}{b}+\frac{b^2}{a}=a^3+\frac{1}{a^3}\quad\left(\because b=\frac{1}{a}\right)$$

$$=\left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$$

$$=(10)^3-3\times10$$

$$=970$$

56. (b) $x=2+\sqrt{3}$

$$\frac{1}{x}=\frac{1}{2+\sqrt{3}}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$=\frac{2-\sqrt{3}}{4-3}=2-\sqrt{3}$$

$$\therefore\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=x+\frac{1}{x}+2$$

$$=2+\sqrt{3}+2-\sqrt{3}+2$$

$$=6$$

$$\therefore\sqrt{x}+\frac{1}{\sqrt{x}}=\sqrt{6}$$

57.(c) $x=\frac{\sqrt{3}}{2}$

$$\because \sqrt{1+x}=\sqrt{1+\frac{\sqrt{3}}{2}}$$

$$\therefore \sqrt{\frac{2+\sqrt{3}}{2}}=\sqrt{\frac{4+2\sqrt{3}}{4}}$$

$$=\sqrt{\frac{\left(\sqrt{3}+1\right)^2}{4}}=\frac{\sqrt{3}+1}{2}$$

$$\therefore \sqrt{1-x}=\frac{\sqrt{3}-1}{2}$$

$$\therefore \sqrt{1+x}+\sqrt{1-x}$$

$$\frac{\sqrt{3}+1}{2}+\frac{\sqrt{3}-1}{2}$$

2.(c) Clearly, $b=\frac{1}{a}$

$$\text{Now, } a = \frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\left(\sqrt{5}+1\right)}{\left(\sqrt{5}+1\right)}=\frac{3+\sqrt{5}}{2}$$

$$\text{Similarly, } b=\frac{3-\sqrt{5}}{2}=\frac{1}{a}$$

$$\therefore a+b=a+\frac{1}{a}=\frac{1}{2}\left(3+\sqrt{5}+3-\sqrt{5}\right)=3$$

$$\text{now, }\frac{a^2+ab+b^2}{a^2-ab+b^2}=\frac{a^2+1+\frac{1}{a^2}}{a^2-1+\frac{1}{a^2}}\left[\because b=\frac{1}{a}\right]$$

$$\left(a^2+\frac{1}{a^2}\right)+1$$

$$\sqrt{x+2\sqrt{x+2\sqrt{x+2\sqrt{3x}}}} = \text{Integer}$$

for this, first of all, $\sqrt{3x}$ should be an integer only (b) and (d) satisfy this requirement.

If $x = 12$, then $\sqrt{3x} = 6$

Then the next point at which we need to remove the square root sign = $\sqrt{x+2\sqrt{3x}} = \sqrt{12+2\times 6}$

= $\sqrt{24}$ = Irrational no.

hence, $x \neq 12$,

now check the expression for x = 3, then we get LHS = RHS.

5.(c) $x = \dfrac{2\times 2\times\sqrt{5}\times\sqrt{3}}{\sqrt{5}+\sqrt{3}} \Rightarrow \dfrac{x}{2\sqrt{5}} = \dfrac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$\Rightarrow \frac{x}{\sqrt{20}} = \frac{2\sqrt{3}}{\sqrt{5}+\sqrt{3}} \text{ also } \frac{x}{\sqrt{12}} = \frac{2\sqrt{5}}{\sqrt{5}+\sqrt{3}}$$

Applying compodendo & dividendo

$$\therefore \frac{x+\sqrt{20}}{x-\sqrt{20}} = \frac{3\sqrt{3}+\sqrt{5}}{\sqrt{3}-\sqrt{5}} = -\frac{3\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

Similarly, $\dfrac{x+\sqrt{12}}{x-\sqrt{12}} = \dfrac{\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}}$

$$\frac{x+\sqrt{20}}{x-\sqrt{20}} + \frac{x+\sqrt{12}}{x-\sqrt{12}}$$

$$= \frac{-3\sqrt{3}-\sqrt{5}+\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

$$\frac{2\sqrt{5}-2\sqrt{3}}{\left(\sqrt{5}-\sqrt{3}\right)} = \frac{-3\sqrt{3}-\sqrt{5}+\sqrt{3}+3\sqrt{5}}{\sqrt{5}-\sqrt{3}} = 2$$

6.(c) $x = 5-\sqrt{21} = 5-\sqrt{3}.\sqrt{7}$

$$\therefore \sqrt{x} = \sqrt{5-\sqrt{3}.\sqrt{7}} = \sqrt{\frac{\left(5-\sqrt{3}.\sqrt{7}\right)\times 2}{2}}$$

$$= \sqrt{\frac{10-2.\sqrt{3}.\sqrt{7}}{2}}$$

$$= \frac{\sqrt{\left(\sqrt{7}-\sqrt{3}\right)^2}}{\sqrt{2}} = \frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}$$

$$\sqrt{32-2x} = \sqrt{32-10+2\sqrt{21}}$$

$$= \sqrt{22+2\times 1\times\sqrt{21}} = \sqrt{\left(1+\sqrt{21}\right)^2}$$

$$= 1+\sqrt{21}$$

$$\therefore \frac{\sqrt{x}}{\sqrt{32-2x}-\sqrt{21}} = \frac{\dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}}{1+\sqrt{21}-\sqrt{21}}$$

$$= \frac{1}{\sqrt{2}}\left(\sqrt{7}-\sqrt{3}\right)$$

7.(d) Given Exp :

$$a = \sqrt{3+a} \Rightarrow a^2 = 3+a \Rightarrow a^2-a-3 = 0$$

$$\Rightarrow a = \frac{1\pm\sqrt{1+12}}{2} = \frac{1\pm\sqrt{13}}{2}$$

$$a = \frac{1-\sqrt{13}}{2} < 0 \text{ (not possible)}$$

$$\therefore a = \frac{1+\sqrt{13}}{2} = \frac{1+3.6}{2} = \frac{4.6}{2} = 2.3$$

$$\therefore 2 < a < 3$$

8.(b) Expression $= \dfrac{1}{2^{\frac{2}{3}}+2^{\frac{1}{3}}+1}$

$$= \frac{2^{\frac{1}{3}}-1}{\left(2^{\frac{1}{3}}-1\right)\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}+1\right)} = \frac{2^{\frac{1}{3}}-1}{\left(2^{\frac{1}{3}}\right)^3-1}$$

$$=2^{\frac{1}{3}}-1=\sqrt[3]{2}-1$$

$$\left[\because (a-b)\left(a^2+ab+b^2\right)=a^3-b^3\right]$$

9.(b) $$x=\frac{2\sqrt{24}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow x=\frac{2\sqrt{3\times 8}}{\sqrt{3}+\sqrt{2}}=\frac{2\sqrt{3}\times\sqrt{8}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x}{\sqrt{8}}=\frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x+\sqrt{8}}{x-\sqrt{8}}=\frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}}$$

[By componendo and dividendo]

$$=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

Again,

$$\frac{x}{\sqrt{12}}=\frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x+\sqrt{12}}{x-\sqrt{12}}$$

$$=\frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}}$$

$$=\frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$\therefore \frac{x+\sqrt{8}}{x-\sqrt{8}}+\frac{x+\sqrt{12}}{x-\sqrt{12}}$$

$$=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{3}+3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{3\sqrt{3}+\sqrt{2}-\sqrt{3}-3\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$=\frac{2\sqrt{3}-2\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{2\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}-\sqrt{2}}=2$$

10.(d) $$x=\frac{2\sqrt{3}\times\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x}{\sqrt{2}}=\frac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x+\sqrt{2}}{x-\sqrt{2}}=\frac{2\sqrt{3}+\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{3}-\sqrt{2}}$$

$$=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

(By componendo and dividendo)

Similarly,

$$\frac{x}{\sqrt{3}}=\frac{2\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow \frac{x+\sqrt{3}}{x-\sqrt{3}}=\frac{2\sqrt{2}+\sqrt{3}+\sqrt{2}}{2\sqrt{2}-\sqrt{3}-\sqrt{2}}$$

$$=\frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

∵ Expression

$$=\frac{x+\sqrt{2}}{x-\sqrt{2}}+\frac{x+\sqrt{3}}{x-\sqrt{3}}$$

$$=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{\sqrt{3}+3\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$

$$=\frac{3\sqrt{3}+\sqrt{2}-\sqrt{3}-3\sqrt{2}}{\sqrt{3}-\sqrt{2}}=2$$

$$=\frac{2\left(\sqrt{3}-\sqrt{2}\right)}{\sqrt{3}-\sqrt{2}}=2$$

Answer -Key

LEVEL - 1

1. (b) 2. (a) 3. (c) 4. (d)
5. (b) 6. (a) 7. (c) 8. (b)
9. (d) 10. (b) 11. (c) 12. (d)
13. (c) 14. (b) 15. (d) 16. (a)
17. (b) 18. (b) 19. (c) 20. (d)
21. (b) 22. (a) 23. (d) 24. (d)
25. (c) 26. (d) 27. (d) 28. (b)
29. (d) 30. (b) 31. (a) 32. (c)
33. (d) 34. (b) 35. (c) 36. (b)
37. (a) 38. (c) 39. (b) 40. (d)
41. (c) 42. (a) 43. (a) 44. (b)
45. (d) 46. (c) 47. (a) 48. (c)
49. (b) 50. (a) 51. (c) 52. (b)
53. (a) 54. (c) 55. (b) 56. (c)
57. (d) 58. (d) 59. (c) 60. (c)

LEVEL - 2

1. (a) 2. (b) 3. (c) 4. (d)
5. (a) 6. (c) 7. (d) 8. (b)
9. (a) 10. (a) 11. (b) 12. (c)
13. (b) 14. (b) 15. (d) 16. (d)
17. (a) 18. (b) 19. (c) 20. (a)
21. (b) 22. (c) 23. (a) 24. (c)
25. (b) 26. (a) 27. (d) 28. (a)
29. (b) 30. (d) 31. (c) 32. (b)
33. (b) 34. (c) 35. (c) 36. (b)
37. (d) 38. (a) 39. (a) 40. (d)
41. (b) 42. (a) 43. (b) 44. (b)
45. (c) 46. (a) 47. (c) 48. (a)
49. (b) 50. (c) 51. (d) 52. (c)
53. (a) 54. (b) 55. (b) 56. (b)
57. (c) 58. (d)

LEVEL - 3

1. (b) 2. (c) 3. (d)
4. (b) 5. (c) 6. (c)
7. (d) 8. (b) 9. (b)
10. (d)