9. SURDS AND INDICES

IMPORTANT FACTS AND FORMULAE

1. LAWS OF INDICES :

(i) $a^m \times a^n = a^{m+n}$ (ii) $\dfrac{a^m}{a^n} = a^{m-n}$ (iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^n = a^n b^n$ (v) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ (vi) $a^0 = 1$

2. SURDS : *Let a be a rational number and n be a positive integer such that* $a^{\frac{1}{n}} = \sqrt[n]{a}$ *is irrational. Then,* $\sqrt[n]{a}$ *is called a surd of order n.*

3. LAWS OF SURDS :

(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$ (ii) $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$ (iii) $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iv) $(\sqrt[n]{a})^n = a$ (v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ (vi) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$.

SOLVED EXAMPLES

Ex. 1. *Simplify :* (i) $(27)^{\frac{2}{3}}$ (ii) $(1024)^{-\frac{4}{5}}$ (iii) $\left(\dfrac{8}{125}\right)^{-\frac{4}{3}}$.

Sol. (i) $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{\left(3 \times \frac{2}{3}\right)} = 3^2 = 9.$

(ii) $(1024)^{-\frac{4}{5}} = (4^5)^{-\frac{4}{5}} = 4^{\left\{5 \times \frac{(-4)}{5}\right\}} = 4^{-4} = \dfrac{1}{4^4} = \dfrac{1}{256}.$

(iii) $\left(\dfrac{8}{125}\right)^{-\frac{4}{3}} = \left\{\left(\dfrac{2}{5}\right)^3\right\}^{-\frac{4}{3}} = \left(\dfrac{2}{5}\right)^{\left\{3 \times \frac{(-4)}{3}\right\}} = \left(\dfrac{2}{5}\right)^{-4} = \left(\dfrac{5}{2}\right)^4 = \dfrac{5^4}{2^4} = \dfrac{625}{16}.$

Ex. 2. *Evaluate :* (i) $(.00032)^{\frac{3}{5}}$ (ii) $(256)^{0.16} \times (16)^{0.18}$.

Sol. (i) $(0.00032)^{\frac{3}{5}} = \left(\dfrac{32}{100000}\right)^{\frac{3}{5}} = \left(\dfrac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\dfrac{2}{10}\right)^5\right\}^{\frac{3}{5}} = \left(\dfrac{1}{5}\right)^{\left(5 \times \frac{3}{5}\right)} = \left(\dfrac{1}{5}\right)^3 = \dfrac{1}{125}.$

(ii) $(256)^{0.16} \times (16)^{0.18} = \{(16)^2\}^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18}$

$= (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32 + 0.18)} = (16)^{0.5} = (16)^{\frac{1}{2}} = 4.$

Ex. 3. ***What is the quotient when $(x^{-1} - 1)$ is divided by $(x - 1)$?***

Sol. $\dfrac{x^{-1}-1}{x-1} = \dfrac{\frac{1}{x}-1}{x-1} = \dfrac{(1-x)}{x} \times \dfrac{1}{(x-1)} = -\dfrac{1}{x}$.

Hence, the required quotient is $-\dfrac{1}{x}$.

Ex. 4. ***If $2^{x-1} + 2^{x+1} = 1280$, then find the value of x.***

Sol. $2^{x-1} + 2^{x+1} = 1280 \Leftrightarrow 2^{x-1}(1+2^2) = 1280$

$$\Leftrightarrow 2^{x-1} = \frac{1280}{5} = 256 = 2^8 \Leftrightarrow x - 1 = 8 \Leftrightarrow x = 9.$$

Hence, $x = 9$.

Ex. 5. ***Find the value of $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$.***

Sol. $\left[5\left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5\left\{(2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{4}} = \left[5\left\{2^{\left(3\times\frac{1}{3}\right)} + 3^{\left(3\times\frac{1}{3}\right)}\right\}^3\right]^{\frac{1}{4}}$

$$= (5(2+3)^3)^{\frac{1}{4}} = (5 \times 5^3)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{\left(4\times\frac{1}{4}\right)} = 5^1 = 5.$$

Ex. 6. ***Find the value of $\left\{(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right\}$.***

Sol. $\left[(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right] = \left[(4^2)^{\frac{3}{2}} + (4^2)^{-\frac{3}{2}}\right] = 4^{\left(2\times\frac{3}{2}\right)} + 4^{\left\{2\times\frac{(-3)}{2}\right\}}$

$$= 4^3 + 4^{-3} = 4^3 + \frac{1}{4^3} = \left(64 + \frac{1}{64}\right) = \frac{4097}{64}.$$

Ex. 7. ***If $\left(\frac{1}{5}\right)^{3y} = 0.008$, then find the value of $(0.25)^y$.***

Sol. $\left(\frac{1}{5}\right)^{3y} = 0.008 = \frac{8}{1000} = \frac{1}{125} = \left(\frac{1}{5}\right)^3 \Leftrightarrow 3y = 3 \Leftrightarrow y = 1.$

$\therefore (0.25)^y = (0.25)^1 = 0.25.$

Ex. 8. ***Find the value of $\dfrac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}}$.***

Sol. $\dfrac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}} = \dfrac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \dfrac{3^{\left(5\times\frac{n}{5}\right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \dfrac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$

$$= \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9.$$

Ex. 9. ***Find the value of $\left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right)$*** (N.I.F.T. 2003)

Sol. Putting $2^{\frac{1}{4}} = x$, we get :

$$\left(2^{\frac{1}{4}} - 1\right)\left(2^{\frac{3}{4}} + 2^{\frac{1}{2}} + 2^{\frac{1}{4}} + 1\right) = (x-1)(x^3 + x^2 + x + 1), \text{ where } x = 2^{\frac{1}{4}}$$

$$= (x-1)[x^2(x+1) + (x+1)]$$

$$= (x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1)$$

$$= (x^4 - 1) = \left[\left(2^{\frac{1}{4}}\right)^4 - 1\right] = \left[2^{\left(\frac{1}{4}\times 4\right)} - 1\right] = (2-1) = 1.$$

Ex. 10. ***Find the value of*** $\dfrac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

Sol.

$$\frac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = \frac{6^{\frac{2}{3}} \times (6^7)^{\frac{1}{3}}}{(6^6)^{\frac{1}{3}}} = \frac{6^{\frac{2}{3}} \times 6^{\left(7\times\frac{1}{3}\right)}}{6^{\left(6\times\frac{1}{3}\right)}} = \frac{6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}\right)}}{6^2}$$

$$= 6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3}-2\right)} = 6^{\frac{2}{3}} \times 6^{\frac{1}{3}} = 6^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 6^1 = 6.$$

Ex. 11. ***If*** $x = y^a$, $y = z^b$ ***and*** $z = x^c$, ***then find the value of abc.***

Sol. $z^1 = x^c = (y^a)^c$ $\quad [\because x = y^a]$

$= y^{(ac)} = (z^b)^{ac}$ $\quad [\because y = z^b]$

$= z^{b(ac)} = z^{abc}$

$\therefore abc = 1.$

Ex. 12. ***Simplify :*** $\left(\dfrac{x^a}{x^b}\right)^{(a^2+b^2+ab)} \times \left(\dfrac{x^b}{x^c}\right)^{(b^2+c^2+bc)} \times \left(\dfrac{x^c}{x^a}\right)^{(c^2+a^2+ca)}$

Sol. Given Expression $= [x^{(a-b)}]^{(a^2+b^2+ab)} \cdot [x^{(b-c)}]^{(b^2+c^2+bc)} \cdot [x^{(c-a)}]^{(c^2+a^2+ca)}$

$= x^{(a-b)(a^2+b^2+ab)} \cdot x^{(b-c)(b^2+c^2+bc)} \cdot x^{(c-a)(c^2+a^2+ca)}$

$= x^{(a^3-b^3)} \cdot x^{(b^3-c^3)} \cdot x^{(c^3-a^3)} = x^{(a^3-b^3+b^3-c^3+c^3-a^3)} = x^0 = 1.$

Ex. 13. ***Which is larger*** $\sqrt{2}$ ***or*** $\sqrt[3]{3}$ ***?***

Sol. Given surds are of order 2 and 3. Their L.C.M. is 6.

Changing each to a surd of order 6, we get :

$$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2}\times\frac{3}{3}\right)} = 2^{\frac{3}{6}} = (2^3)^{\frac{1}{6}} = (8)^{\frac{1}{6}} = \sqrt[6]{8}$$

$$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3}\times\frac{2}{2}\right)} = 3^{\frac{2}{6}} = (3^2)^{\frac{1}{6}} = (9)^{\frac{1}{6}} = \sqrt[6]{9}.$$

Clearly, $\sqrt[6]{9} > \sqrt[6]{8}$ and hence $\sqrt[3]{3} > \sqrt{2}$.

Ex. 14. ***Find the largest from among*** $\sqrt[4]{6}$, $\sqrt{2}$ ***and*** $\sqrt[3]{4}$.

Sol. Given surds are of order 4, 2 and 3 respectively. Their L.C.M. is 12.

Changing each to a surd of order 12, we get :

$\sqrt[4]{6} = 6^{\frac{1}{4}} = 6^{\left(\frac{1}{4} \times \frac{3}{3}\right)} = \left(6^{\frac{3}{12}}\right) = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$,

$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2} \times \frac{6}{6}\right)} = \left(2^{\frac{6}{12}}\right) = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$,

$\sqrt[3]{4} = 4^{\frac{1}{3}} = 4^{\left(\frac{1}{3} \times \frac{4}{4}\right)} = \left(4^{\frac{4}{12}}\right) = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$.

Clearly, $(256)^{\frac{1}{12}} > (216)^{\frac{1}{12}} > (64)^{\frac{1}{12}}$.

$\therefore$ Largest one is $(256)^{\frac{1}{12}}$ *i.e.*, $\sqrt[3]{4}$.

EXERCISE 9

Directions : *Mark (✓) against the correct answer* :

1. The value of $(256)^{\frac{5}{4}}$ is :
   (*a*) 512 (*b*) 984 (*c*) 1024 (*d*) 1032

2. The value of $(\sqrt{8})^{\frac{1}{3}}$ is :
   (*a*) 2 (*b*) 4 (*c*) $\sqrt{2}$ (*d*) 8

3. The value of $\left(\frac{32}{243}\right)^{-\frac{4}{5}}$ is :
   (*a*) $\frac{4}{9}$ (*b*) $\frac{9}{4}$ (*c*) $\frac{16}{81}$ (*d*) $\frac{81}{16}$

4. The value of $\left(-\frac{1}{216}\right)^{-\frac{2}{3}}$ is :
   (*a*) 36 (*b*) -36 (*c*) $\frac{1}{36}$ (*d*) $-\frac{1}{36}$

5. The value of $5^{\frac{1}{4}} \times (125)^{0.25}$ is :
   (*a*) $\sqrt{5}$ (*b*) 5 (*c*) $5\sqrt{5}$ (*d*) 25

6. The value of $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(32)^{-\frac{1}{5}}}$ is : **(M.B.A. 2003)**
   (*a*) 102 (*b*) 105 (*c*) 107 (*d*) 109

7. The value of $[(10)^{150} \div (10)^{146}]$ is : **(Bank P.O. 2002)**
   (*a*) 1000 (*b*) 10000 (*c*) 100000 (*d*) 10^6

8. $(2.4 \times 10^3) \div (8 \times 10^{-2}) = ?$
   (*a*) 3×10^{-5} (*b*) 3×10^4 (*c*) 3×10^5 (*d*) 30

9. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} \div \left(\frac{1}{27}\right)^{-\frac{4}{3}} = ?$
   (*a*) $\frac{3}{4}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{4}{9}$ (*d*) $\frac{1}{8}$

10. $(1000)^7 \div 10^{18} = ?$ **(Bank P.O. 2003)**

(*a*) 10 (*b*) 100 (*c*) 1000 (*d*) 10000

11. $(256)^{0.16} \times (256)^{0.09} = ?$ **(S.S.C. 2004)**

(*a*) 4 (*b*) 16 (*c*) 64 (*d*) 256.25

12. $(0.04)^{-1.5} = ?$ **(Bank P.O. 2003)**

(*a*) 25 (*b*) 125 (*c*) 250 (*d*) 625

13. $(17)^{3.5} \times (17)^? = 17^8$ **(Bank P.O. 2003)**

(*a*) 2.29 (*b*) 2.75 (*c*) 4.25 (*d*) 4.5

14. $49 \times 49 \times 49 \times 49 = 7^?$

(*a*) 4 (*b*) 7 (*c*) 8 (*d*) 16

15. The value of $(8^{-25} - 8^{-26})$ is

(*a*) 7×8^{-25} (*b*) 7×8^{-26} (*c*) 8×8^{-26} (*d*) None of these

16. $(64)^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}} = ?$ **(Bank P.O. 2002)**

(*a*) $\frac{1}{8}$ (*b*) $\frac{3}{8}$ (*c*) $\frac{1}{16}$ (*d*) $\frac{3}{16}$ (*e*) None of these

17. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^?$: **(Bank P.O. 2003)**

(*a*) 3.5 (*b*) 4.5 (*c*) 6 (*d*) 7 (*e*) None of these

18. $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^?$ **(Bank P.O. 2003)**

(*a*) 8.5 (*b*) 13 (*c*) 16 (*d*) 17.5 (*e*) None of these

19. The value of $\frac{(243)^{0.13} \times (243)^{0.07}}{(7)^{0.25} \times (49)^{0.075} \times (343)^{0.2}}$ is : **(C.B.I. 2003)**

(*a*) $\frac{3}{7}$ (*b*) $\frac{7}{3}$ (*c*) $1\frac{3}{7}$ (*d*) $2\frac{2}{7}$

20. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$, then the value of x is : **(M.B.A. 2003)**

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) $\frac{7}{2}$

21. If $2^{2n-1} = \frac{1}{8^{n-3}}$, then the value of n is :

(*a*) 3 (*b*) 2 (*c*) 0 (*d*) -2

22. If $5^a = 3125$, then the value of $5^{(a-3)}$ is :

(*a*) 25 (*b*) 125 (*c*) 625 (*d*) 1625

23. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is :

(*a*) 4 (*b*) 5 (*c*) 6 (*d*) 8

24. If $\sqrt{2^n} = 64$, then the value of n is :

(*a*) 2 (*b*) 4 (*c*) 6 (*d*) 12

25. If $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3}$, then the value of n is :

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

26. If $\frac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27$, then the value of n is :

(*a*) 0 (*b*) 2 (*c*) 3 (*d*) 4

27. If $2^{n+4} - 2^{n+2} = 3$, then n is equal to :

(a) 0 (b) 2 (c) -1 (d) -2

28. If $2^{n-1} + 2^{n+1} = 320$, then n is equal to :

(a) 6 (b) 8 (c) 5 (d) 7

29. If $3^x - 3^{x-1} = 18$, then the value of x^x is :

(a) 3 (b) 8 (c) 27 (d) 216

30. $\dfrac{2^{n+4} - 2 \times 2^n}{2 \times 2^{(n+3)}} + 2^{-3}$ is equal to :

(a) 2^{n+1} (b) $\left(\dfrac{9}{8} - 2^n\right)$ (c) $\left(-2^{n+1} + \dfrac{1}{8}\right)$ (d) 1

31. If $x = 3 + 2\sqrt{2}$, then the value of $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)$ is : **(C.B.I. 2003)**

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $3\sqrt{3}$

32. Given that $10^{0.48} = x$, $10^{0.70} = y$ and $x^z = y^2$, then the value of z is close to :

(a) 1.45 (b) 1.88 (c) 2.9 (d) 3.7

(C.B.I. 2003)

33. If m and n are whole numbers such that $m^n = 121$, then the value of $(m-1)^{n+1}$ is : **(S.S.C. 2001)**

(a) 1 (b) 10 (c) 121 (d) 1000

34. $\dfrac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}} = ?$ **(S.S.C. 2004)**

(a) 1 (b) 3 (c) 9 (d) 3^n

35. Number of prime factors in $(216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}}$ is :

(a) 6 (b) 7 (c) 8 (d) None of these

36. Number of prime factors in $\dfrac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}}$ is :

(a) 56 (b) 66 (c) 112 (d) None of these

37. $\dfrac{1}{1 + a^{(n-m)}} + \dfrac{1}{1 + a^{(m-n)}} = ?$ **(M.B.A. 2003)**

(a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) a^{m+n}

38. $\dfrac{1}{1 + x^{(b-a)} + x^{(c-a)}} + \dfrac{1}{1 + x^{(a-b)} + x^{(c-b)}} + \dfrac{1}{1 + x^{(b-c)} + x^{(a-c)}} = ?$ **(M.B.A. 2003)**

(a) 0 (b) 1 (c) x^{a-b-c} (d) None of these

39. $\left(\dfrac{x^b}{x^c}\right)^{(b+c-a)} \cdot \left(\dfrac{x^c}{x^a}\right)^{(c+a-b)} \cdot \left(\dfrac{x^a}{x^b}\right)^{(a+b-c)} = ?$ **(L.I.C. 2003)**

(a) x^{abc} (b) 1 (c) $x^{ab+bc+ca}$ (d) x^{a+b+c}

40. $\left(\dfrac{x^a}{x^b}\right)^{(a+b)} \cdot \left(\dfrac{x^b}{x^c}\right)^{(b+c)} \cdot \left(\dfrac{x^c}{x^a}\right)^{(c+a)} = ?$

(a) 0 (b) x^{abc} (c) x^{a+b+c} (d) 1

41. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}} \cdot \left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} = ?$

(a) 1 (b) $x^{\frac{1}{abc}}$ (c) $x^{\frac{1}{(ab+bc+ca)}}$ (d) None of these

42. If $abc = 1$, then $\left(\frac{1}{1+a+b^{-1}} + \frac{1}{1+b+c^{-1}} + \frac{1}{1+c+a^{-1}}\right) = ?$

(a) 0 (b) 1 (c) $\frac{1}{ab}$ (d) ab

43. If a, b, c are real numbers, then the value of $\sqrt{a^{-1}b} \cdot \sqrt{b^{-1}c} \cdot \sqrt{c^{-1}a}$ is :

(a) abc (b) $\sqrt{abc}$ (c) $\frac{1}{abc}$ (d) 1

44. If $3^{(x-y)} = 27$ and $3^{(x+y)} = 243$, then x is equal to : **(R.R.B. 2003)**

(a) 0 (b) 2 (c) 4 (d) 6

45. If $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{x-1} = \frac{2}{3}$, then the value of x is :

(a) 1 (b) 2 (c) 3 (d) 4

46. If $2^x = \sqrt[3]{32}$, then x is equal to :

(a) 5 (b) 3 (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

47. If $2^x \times 8^{\frac{1}{5}} = 2^{\frac{1}{5}}$, then x is equal to :

(a) $\frac{1}{5}$ (b) $-\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $-\frac{2}{5}$

48. If $5^{(x+3)} = (25)^{(3x-4)}$, then the value of x is :

(a) $\frac{5}{11}$ (b) $\frac{11}{5}$ (c) $\frac{11}{3}$ (d) $\frac{13}{5}$

49. If $a^x = b^y = c^z$ and $b^2 = ac$, then y equals :

(a) $\frac{xz}{x+z}$ (b) $\frac{xz}{2(x-z)}$ (c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(x+z)}$

50. If $2^x = 3^y = 6^{-z}$, then $\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right)$ is equal to :

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{1}{2}$

51. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of xyz is :

(a) 0 (b) 1 (c) $\frac{1}{abc}$ (d) abc

52. If $2^x = 4^y = 8^z$ and $\left(\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z}\right) = \frac{24}{7}$, then the value of z is :

(a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$

53. The largest number from among $\sqrt{2}$, $\sqrt[3]{3}$ and $\sqrt[4]{4}$ is :

(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$ (c) $\sqrt[4]{4}$ (d) All are equal

54. If $x = 5 + 2\sqrt{6}$, then $\dfrac{(x-1)}{\sqrt{x}}$ is equal to :

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

ANSWERS

1. (c)	2. (c)	3. (d)	4. (a)	5. (b)	6. (a)	7. (b)	8. (b)	9. (c)
10. (c)	11. (a)	12. (b)	13. (d)	14. (c)	15. (b)	16. (c)	17. (d)	18. (b)
19. (a)	20. (c)	21. (b)	22. (a)	23. (a)	24. (d)	25. (d)	26. (c)	27. (d)
28. (d)	29. (c)	30. (d)	31. (b)	32. (c)	33. (d)	34. (c)	35. (b)	36. (b)
37. (c)	38. (b)	39. (b)	40. (d)	41. (a)	42. (b)	43. (d)	44. (c)	45. (d)
46. (d)	47. (d)	48. (b)	49. (d)	50. (a)	51. (b)	52. (c)	53. (b)	54. (b)

SOLUTIONS

1. $(256)^{\frac{5}{4}} = (4^4)^{\frac{5}{4}} = 4^{\left(4 \times \frac{5}{4}\right)} = 4^5 = 1024.$

2. $(\sqrt{8})^{\frac{1}{3}} = \left(8^{\frac{1}{2}}\right)^{\frac{1}{3}} = 8^{\left(\frac{1}{2} \times \frac{1}{3}\right)} = 8^{\frac{1}{6}} = (2^3)^{\frac{1}{6}} = 2^{\left(3 \times \frac{1}{6}\right)} = 2^{\frac{1}{2}} = \sqrt{2}.$

3. $\left(\dfrac{32}{243}\right)^{-\frac{4}{5}} = \left\{\left(\dfrac{2}{3}\right)^5\right\}^{-\frac{4}{5}} = \left(\dfrac{2}{3}\right)^{5 \times \frac{(-4)}{5}} = \left(\dfrac{2}{3}\right)^{(-4)} = \left(\dfrac{3}{2}\right)^4 = \dfrac{3^4}{2^4} = \dfrac{81}{16}.$

4. $\left(-\dfrac{1}{216}\right)^{-\frac{2}{3}} = \left[\left(-\dfrac{1}{6}\right)^3\right]^{-\frac{2}{3}} = \left(-\dfrac{1}{6}\right)^{3 \times \frac{(-2)}{3}} = \left(-\dfrac{1}{6}\right)^{-2} = \dfrac{1}{\left(-\dfrac{1}{6}\right)^2} = \dfrac{1}{\left(\dfrac{1}{36}\right)} = 36.$

5. $5^{\frac{1}{4}} \times (125)^{0.25} = 5^{0.25} \times (5^3)^{0.25} = 5^{0.25} \times 5^{(3 \times 0.25)} = 5^{0.25} \times 5^{0.75} = 5^{(0.25 + 0.75)} = 5^1 = 5.$

6. $\dfrac{1}{(216)^{-\frac{2}{3}}} + \dfrac{1}{(256)^{-\frac{3}{4}}} + \dfrac{1}{(32)^{-\frac{1}{5}}} = \dfrac{1}{(6^3)^{-\frac{2}{3}}} + \dfrac{1}{(4^4)^{\left(-\frac{3}{4}\right)}} + \dfrac{1}{(2^5)^{-\frac{1}{5}}}$

$= \dfrac{1}{6^{3 \times \frac{(-2)}{3}}} + \dfrac{1}{4^{4 \times \frac{(-3)}{4}}} + \dfrac{1}{2^{5 \times \frac{(-1)}{5}}} = \dfrac{1}{6^{-2}} + \dfrac{1}{4^{-3}} + \dfrac{1}{2^{-1}}$

$= (6^2 + 4^3 + 2^1) = (36 + 64 + 2) = 102.$

7. $(10)^{150} \div (10)^{146} = \dfrac{(10)^{150}}{(10)^{146}} = (10)^{(150 - 146)} = 10^4 = 10000.$

8. $(2.4 \times 10^3) \div (8 \times 10^{-2}) = \dfrac{2.4 \times 10^3}{8 \times 10^{-2}} = \dfrac{24 \times 10^2}{8 \times 10^{-2}} = (3 \times 10^4).$

9. $\left(\dfrac{1}{216}\right)^{-\frac{2}{3}} \div \left(\dfrac{1}{27}\right)^{-\frac{4}{3}} = (216)^{\frac{2}{3}} \div (27)^{\frac{4}{3}} = \dfrac{(216)^{\frac{2}{3}}}{(27)^{\frac{4}{3}}} = \dfrac{(6^3)^{\frac{2}{3}}}{(3^3)^{\frac{4}{3}}} = \dfrac{6^{\left(3 \times \frac{2}{3}\right)}}{3^{\left(3 \times \frac{4}{3}\right)}} = \dfrac{6^2}{3^4} = \dfrac{36}{81} = \dfrac{4}{9}.$

10. $(1000)^7 \div 10^{18} = \dfrac{(1000)^7}{10^{18}} = \dfrac{(10^3)^7}{10^{18}} = \dfrac{10^{(3\times 7)}}{10^{18}} = \dfrac{10^{21}}{10^{18}} = (10)^{(21-18)} = 10^3 = 1000.$

11. $(256)^{0.16} \times (256)^{0.09} = (256)^{(0.16+0.09)} = (256)^{0.25} = (256)^{\left(\frac{25}{100}\right)}$

$$= (256)^{\frac{1}{4}} = (4^4)^{\frac{1}{4}} = 4^{\left(4\times\frac{1}{4}\right)} = 4^1 = 4.$$

12. $(0.04)^{-1.5} = \left(\dfrac{4}{100}\right)^{-1.5} = \left(\dfrac{1}{25}\right)^{-\frac{3}{2}} = (25)^{\frac{3}{2}} = (5^2)^{\frac{3}{2}} = 5^{\left(2\times\frac{3}{2}\right)} = 5^3 = 125.$

13. Let $(17)^{3.5} \times (17)^x = 17^8$. Then, $(17)^{3.5+x} = (17)^8$.

$\therefore\ 3.5 + x = 8 \iff x = (8 - 3.5) \iff x = 4.5.$

14. $49 \times 49 \times 49 \times 49 = (7^2 \times 7^2 \times 7^2 \times 7^2) = 7^{(2+2+2+2)} = 7^8.$

So, the correct answer is 8.

15. $8^{-25} - 8^{-26} = \left(\dfrac{1}{8^{25}} - \dfrac{1}{8^{26}}\right) = \dfrac{(8-1)}{8^{26}} = 7 \times 8^{-26}.$

16. $(64)^{-\frac{1}{2}} - (-32)^{-\frac{4}{5}} = (8^2)^{-\frac{1}{2}} - \{(-2)^5\}^{-\frac{4}{5}} = 8^{2\times\frac{(-1)}{2}} - (-2)^{5\times\frac{(-4)}{5}} = 8^{-1} - (-2)^{-4}$

$$= \frac{1}{8} - \frac{1}{(-2)^4} = \left(\frac{1}{8} - \frac{1}{16}\right) = \frac{1}{16}.$$

17. $(18)^{3.5} \div (27)^{3.5} \times 6^{3.5} = 2^x$

$\iff (18)^{3.5} \times \dfrac{1}{(27)^{3.5}} \times 6^{3.5} = 2^x \iff (3^2 \times 2)^{3.5} \times \dfrac{1}{(3^3)^{3.5}} \times (2\times 3)^{3.5} = 2^x$

$\iff 3^{(2\times 3.5)} \times 2^{3.5} \times \dfrac{1}{3^{(3\times 3.5)}} \times 2^{3.5} \times 3^{3.5} = 2^x$

$\iff 3^7 \times 2^{3.5} \times \dfrac{1}{3^{10.5}} \times 2^{3.5} \times 3^{3.5} = 2^x \iff 2^7 = 2^x \iff x = 7.$

18. Let $(25)^{7.5} \times (5)^{2.5} \div (125)^{1.5} = 5^x$. Then, $\dfrac{(5^2)^{7.5} \times (5)^{2.5}}{(5^3)^{1.5}} = 5^x \iff \dfrac{5^{(2\times 7.5)} \times 5^{2.5}}{5^{(3\times 1.5)}} = 5^x$

$\iff \dfrac{5^{15} \times 5^{2.5}}{5^{4.5}} = 5^x \iff 5^x = 5^{(15+2.5-4.5)} = 5^{13} \iff x = 13.$

19. $\dfrac{(243)^{0.13} \times (243)^{0.07}}{7^{0.25} \times (49)^{0.075} \times (343)^{0.2}} = \dfrac{(243)^{(0.13+0.07)}}{7^{0.25} \times (7^2)^{0.075} \times (7^3)^{0.2}}$

$$= \frac{(243)^{0.2}}{7^{0.25} \times 7^{(2\times 0.075)} \times 7^{(3\times 0.2)}} = \frac{(3^5)^{0.2}}{7^{0.25} \times 7^{0.15} \times 7^{0.6}}$$

$$= \frac{3^{(5\times 0.2)}}{7^{(0.25+0.15+0.6)}} = \frac{3^1}{7^1} = \frac{3}{7}.$$

20. $\left(\dfrac{a}{b}\right)^{x-1} = \left(\dfrac{b}{a}\right)^{x-3} \iff \left(\dfrac{a}{b}\right)^{x-1} = \left(\dfrac{a}{b}\right)^{-(x-3)} = \left(\dfrac{a}{b}\right)^{(3-x)}$

$$\iff x - 1 = 3 - x \iff 2x = 4 \iff x = 2.$$

21. $2^{2n-1} = \dfrac{1}{8^{n-3}} \iff 2^{2n-1} = \dfrac{1}{(2^3)^{n-3}} = \dfrac{1}{2^{3(n-3)}} = \dfrac{1}{2^{(3n-9)}} = 2^{(9-3n)}$

$$\iff 2n - 1 = 9 - 3n \iff 5n = 10 \iff n = 2.$$

22. $5^a = 3125 \Leftrightarrow 5^a = 5^5 \Leftrightarrow a = 5.$
$\therefore\ 5^{(a-3)} = 5^{(5-3)} = 5^2 = 25.$

23. $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2} \Leftrightarrow \dfrac{5 \times 5^{\frac{1}{2}} \times 5^3}{5^{-\frac{3}{2}}} = 5^{a+2} \Leftrightarrow 5^{\left(1+\frac{1}{2}+3+\frac{3}{2}\right)} = 5^{a+2}$

$\Leftrightarrow 5^6 = 5^{a+2} \Leftrightarrow a + 2 = 6 \Leftrightarrow a = 4.$

24. $\sqrt{2^n} = 64 \Leftrightarrow (2^n)^{\frac{1}{2}} = 2^6 \Leftrightarrow 2^{\frac{n}{2}} = 2^6 \Leftrightarrow \dfrac{n}{2} = 6 \Leftrightarrow n = 12.$

25. $(\sqrt{3})^5 \times 9^2 = 3^n \times 3\sqrt{3} \Leftrightarrow \left(3^{\frac{1}{2}}\right)^5 \times (3^2)^2 = 3^n \times 3 \times 3^{\frac{1}{2}} \Leftrightarrow 3^{\left(\frac{1}{2}\times 5\right)} \times 3^{(2\times 2)} = 3^{\left(n+1+\frac{1}{2}\right)}$

$\Leftrightarrow 3^{\left(\frac{5}{2}+4\right)} = 3^{\left(n+\frac{3}{2}\right)} \Leftrightarrow n + \dfrac{3}{2} = \dfrac{13}{2} \Leftrightarrow n = \left(\dfrac{13}{2} - \dfrac{3}{2}\right) = \dfrac{10}{2} = 5.$

26. $\dfrac{9^n \times 3^5 \times (27)^3}{3 \times (81)^4} = 27 \Leftrightarrow \dfrac{(3^2)^n \times 3^5 \times (3^3)^3}{3 \times (3^4)^4} = 3^3 \Leftrightarrow \dfrac{3^{2n} \times 3^5 \times 3^{(3\times 3)}}{3 \times 3^{(4\times 4)}} = 3^3$

$\Leftrightarrow \dfrac{3^{2n+5+9}}{3 \times 3^{16}} = 3^3 \Leftrightarrow \dfrac{3^{2n+14}}{3^{17}} = 3^3 \Leftrightarrow 3^{(2n+14-17)} = 3^3$

$\Leftrightarrow 3^{2n-3} = 3^3 \Leftrightarrow 2n - 3 = 3 \Leftrightarrow 2n = 6 \Leftrightarrow n = 3.$

27. $2^{n+4} - 2^{n+2} = 3 \Leftrightarrow 2^{n+2}(2^2 - 1) = 3 \Leftrightarrow 2^{n+2} = 1 = 2^0 \Leftrightarrow n + 2 = 0 \Leftrightarrow n = -2.$

28. $2^{n-1} + 2^{n+1} = 320 \Leftrightarrow 2^{n-1}(1 + 2^2) = 320 \Leftrightarrow 5 \times 2^{n-1} = 320$

$\Leftrightarrow 2^{n-1} = \dfrac{320}{5} = 64 = 2^6 \Leftrightarrow n - 1 = 6 \Leftrightarrow n = 7.$

29. $3^x - 3^{x-1} = 18 \Leftrightarrow 3^{x-1}(3-1) = 18 \Leftrightarrow 3^{x-1} = 9 = 3^2 \Leftrightarrow x - 1 = 2 \Leftrightarrow x = 3.$
$\therefore\ x^x = 3^3 = 27.$

30. $\dfrac{2^{n+4} - 2 \times 2^n}{2 \times 2^{n+3}} + 2^{-3} = \dfrac{2^{n+4} - 2^{n+1}}{2^{n+4}} + \dfrac{1}{2^3} = \dfrac{2^{n+1}(2^3 - 1)}{2^{n+4}} + \dfrac{1}{2^3}$

$= \dfrac{2^{n+1} \times 7}{2^{n+1} \times 2^3} + \dfrac{1}{2^3} = \left(\dfrac{7}{8} + \dfrac{1}{8}\right) = \dfrac{8}{8} = 1.$

31. $\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^2 = x + \dfrac{1}{x} - 2 = (3 + 2\sqrt{2}) + \dfrac{1}{(3 + 2\sqrt{2})} - 2$

$= (3 + 2\sqrt{2}) + \dfrac{1}{(3 + 2\sqrt{2})} \times \dfrac{(3 - 2\sqrt{2})}{(3 - 2\sqrt{2})} - 2 = (3 + 2\sqrt{2}) + (3 - 2\sqrt{2}) - 2 = 4.$

$\therefore\ \left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right) = 2.$

32. $x^z = y^2 \Leftrightarrow (10^{0.48})^z = (10^{0.70})^2 \Leftrightarrow 10^{(0.48z)} = 10^{(2\times 0.70)} = 10^{1.40}$

$\Leftrightarrow 0.48z = 1.40 \Leftrightarrow z = \dfrac{140}{48} = \dfrac{35}{12} = 2.9$ (approx.).

33. We know that $11^2 = 121$. Putting $m = 11$ and $n = 2$, we get :
$(m-1)^{n+1} = (11-1)^{(2+1)} = 10^3 = 1000.$

34. Given Expression $= \dfrac{(243)^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}} = \dfrac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \dfrac{3^{\left(5 \times \frac{n}{5}\right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$

$= \dfrac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \dfrac{3^{(n+2n+1)}}{3^{(2n+n-1)}} = \dfrac{3^{3n+1}}{3^{3n-1}} = 3^{(3n+1-3n+1)} = 3^2 = 9.$

35. $(216)^{\frac{3}{5}} \times (2500)^{\frac{2}{5}} \times (300)^{\frac{1}{5}} = (3^3 \times 2^3)^{\frac{3}{5}} \times (5^4 \times 2^2)^{\frac{2}{5}} \times (5^2 \times 2^2 \times 3)^{\frac{1}{5}}$

$= 3^{\left(3 \times \frac{3}{5}\right)} \times 2^{\left(3 \times \frac{3}{5}\right)} \times 5^{\left(4 \times \frac{2}{5}\right)} \times 2^{\left(2 \times \frac{2}{5}\right)} \times 5^{\left(2 \times \frac{1}{5}\right)} \times 2^{\left(2 \times \frac{1}{5}\right)} \times 3^{\frac{1}{5}}$

$= 3^{\frac{9}{5}} \times 2^{\frac{9}{5}} \times 5^{\frac{8}{5}} \times 2^{\frac{4}{5}} \times 5^{\frac{2}{5}} \times 2^{\frac{2}{5}} \times 3^{\frac{1}{5}}$

$= 3^{\left(\frac{9}{5}+\frac{1}{5}\right)} \times 2^{\left(\frac{9}{5}+\frac{4}{5}+\frac{2}{5}\right)} \times 5^{\left(\frac{8}{5}+\frac{2}{5}\right)} = 3^2 \times 2^3 \times 5^2.$

Hence, the number of prime factors = (2 + 3 + 2) = 7.

36. $\dfrac{6^{12} \times (35)^{28} \times (15)^{16}}{(14)^{12} \times (21)^{11}} = \dfrac{(2 \times 3)^{12} \times (5 \times 7)^{28} \times (3 \times 5)^{16}}{(2 \times 7)^{12} \times (3 \times 7)^{11}} = \dfrac{2^{12} \times 3^{12} \times 5^{28} \times 7^{28} \times 3^{16} \times 5^{16}}{2^{12} \times 7^{12} \times 3^{11} \times 7^{11}}$

$= 2^{(12-12)} \times 3^{(12+16-11)} \times 5^{(28+16)} \times 7^{(28-12-11)}$

$= 2^0 \times 3^{17} \times 5^{44} \times 7^{-5} = \dfrac{3^{17} \times 5^{44}}{7^5}$

Number of prime factors = 17 + 44 + 5 = 66.

37. $\dfrac{1}{1+a^{(n-m)}} + \dfrac{1}{1+a^{(m-n)}} = \dfrac{1}{\left(1+\dfrac{a^n}{a^m}\right)} + \dfrac{1}{\left(1+\dfrac{a^m}{a^n}\right)}$

$= \dfrac{a^m}{(a^m+a^n)} + \dfrac{a^n}{(a^m+a^n)} = \dfrac{(a^m+a^n)}{(a^m+a^n)} = 1.$

38. Given Exp. $= \dfrac{1}{\left(1+\dfrac{x^b}{x^a}+\dfrac{x^c}{x^a}\right)} + \dfrac{1}{\left(1+\dfrac{x^a}{x^b}+\dfrac{x^c}{x^b}\right)} + \dfrac{1}{\left(1+\dfrac{x^b}{x^c}+\dfrac{x^a}{x^c}\right)}$

$= \dfrac{x^a}{(x^a+x^b+x^c)} + \dfrac{x^b}{(x^a+x^b+x^c)} + \dfrac{x^c}{(x^a+x^b+x^c)} = \dfrac{(x^a+x^b+x^c)}{(x^a+x^b+x^c)} = 1.$

39. Given Exp. $= x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)} \cdot x^{(a-b)(a+b-c)}$

$= x^{(b-c)(b+c)-a(b-c)} \cdot x^{(c-a)(c+a)-b(c-a)} \cdot x^{(a-b)(a+b)-c(a-b)}$

$= x^{(b^2-c^2+c^2-a^2+a^2-b^2)} \cdot x^{-a(b-c)-b(c-a)-c(a-b)} = (x^0 \times x^0) = (1 \times 1) = 1.$

40. Given Exp. $= x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c-a)(c+a)}$

$= x^{(a^2-b^2)} \cdot x^{(b^2-c^2)} \cdot x^{(c^2-a^2)} = x^{(a^2-b^2+b^2-c^2+c^2-a^2)} = x^0 = 1.$

41. Given Exp. $= \{x^{(a-b)}\}^{\frac{1}{ab}} \cdot \{x^{(b-c)}\}^{\frac{1}{bc}} \cdot \{x^{(c-a)}\}^{\frac{1}{ca}} = x^{\frac{(a-b)}{ab}} \cdot x^{\frac{(b-c)}{bc}} \cdot x^{\frac{(c-a)}{ca}}$

$= x^{\left\{\frac{(a-b)}{ab}+\frac{(b-c)}{bc}+\frac{(c-a)}{ca}\right\}} = x^{\left(\frac{1}{b}-\frac{1}{a}\right)+\left(\frac{1}{c}-\frac{1}{b}\right)+\left(\frac{1}{a}-\frac{1}{c}\right)} = x^0 = 1.$

42. Given Exp. $= \dfrac{1}{1+a+b^{-1}} + \dfrac{1}{1+b+c^{-1}} + \dfrac{1}{1+c+a^{-1}}$

$$= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{b^{-1}+1+b^{-1}c^{-1}} + \frac{a}{a+ac+1}$$

$$= \frac{1}{1+a+b^{-1}} + \frac{b^{-1}}{1+b^{-1}+a} + \frac{a}{a+b^{-1}+1} = \frac{1+a+b^{-1}}{1+a+b^{-1}} = 1.$$

$[\because \; abc = 1 \Rightarrow (bc)^{-1} = a \Rightarrow b^{-1}c^{-1} = a \text{ and } ac = b^{-1}]$

43. $\sqrt{a^{-1}b}\cdot\sqrt{b^{-1}c}\cdot\sqrt{c^{-1}a} = (a^{-1})^{\frac{1}{2}}\cdot b^{\frac{1}{2}}\cdot(b^{-1})^{\frac{1}{2}}\cdot c^{\frac{1}{2}}\cdot(c^{-1})^{\frac{1}{2}}\cdot a^{\frac{1}{2}}$

$$= (a^{-1}a)^{\frac{1}{2}}\cdot(b\cdot b^{-1})^{\frac{1}{2}}\cdot(c\cdot c^{-1})^{\frac{1}{2}} = (1)^{\frac{1}{2}}\cdot(1)^{\frac{1}{2}}\cdot(1)^{\frac{1}{2}} = (1\times1\times1) = 1.$$

44. $3^{x-y} = 27 = 3^3 \;\Leftrightarrow\; x - y = 3$ ...(*i*)

$3^{x+y} = 243 = 3^5 \;\Leftrightarrow\; x + y = 5$ ...(*ii*)

On solving (*i*) and (*ii*), we get $x = 4$.

45. $\left(\dfrac{9}{4}\right)^x\cdot\left(\dfrac{8}{27}\right)^{x-1} = \dfrac{2}{3} \Leftrightarrow \dfrac{9^x}{4^x}\times\dfrac{8^{x-1}}{(27)^{x-1}} = \dfrac{2}{3}$

$$\Leftrightarrow \frac{(3^2)^x}{(2^2)^x}\times\frac{(2^3)^{(x-1)}}{(3^3)^{(x-1)}} = \frac{2}{3} \Leftrightarrow \frac{3^{2x}\times2^{3(x-1)}}{2^{2x}\times3^{3(x-1)}} = \frac{2}{3}$$

$$\Leftrightarrow \frac{2^{(3x-3-2x)}}{3^{(3x-3-2x)}} = \frac{2}{3} \Leftrightarrow \frac{2^{(x-3)}}{3^{(x-3)}} = \frac{2}{3} \Leftrightarrow \left(\frac{2}{3}\right)^{(x-3)} = \left(\frac{2}{3}\right)^1 \Leftrightarrow x-3=1 \Leftrightarrow x=4.$$

46. $2^x = \sqrt[3]{32} \;\Leftrightarrow\; 2^x = (32)^{\frac{1}{3}} = (2^5)^{\frac{1}{3}} = 2^{\frac{5}{3}} \;\Leftrightarrow\; x = \dfrac{5}{3}.$

47. $2^x\times8^{\frac{1}{5}} = 2^{\frac{1}{5}} \Leftrightarrow 2^x\times(2^3)^{\frac{1}{5}} = 2^{\frac{1}{5}} \Leftrightarrow 2^x\times2^{\frac{3}{5}} = 2^{\frac{1}{5}} \Leftrightarrow 2^{\left(x+\frac{3}{5}\right)} = 2^{\frac{1}{5}}$

$$\Leftrightarrow x+\frac{3}{5} = \frac{1}{5} \Leftrightarrow x = \left(\frac{1}{5}-\frac{3}{5}\right) = \frac{-2}{5}.$$

48. $5^{(x+3)} = 25^{(3x-4)} \;\Leftrightarrow\; 5^{(x+3)} = (5^2)^{(3x-4)}$

$$\Leftrightarrow 5^{(x+3)} = 5^{2(3x-4)} \Leftrightarrow 5^{(x+3)} = 5^{(6x-8)}$$

$$\Leftrightarrow x+3 = 6x-8 \Leftrightarrow 5x = 11 \Leftrightarrow x = \frac{11}{5}.$$

49. Let $a^x = b^y = c^z = k$. Then, $a = k^{\frac{1}{x}}$, $b = k^{\frac{1}{y}}$ and $c = k^{\frac{1}{z}}$.

$$\therefore\; b^2 = ac \Leftrightarrow \left(k^{\frac{1}{y}}\right)^2 = k^{\frac{1}{x}}\times k^{\frac{1}{z}} \Leftrightarrow k^{\left(\frac{2}{y}\right)} = k^{\left(\frac{1}{x}+\frac{1}{z}\right)}$$

$$\therefore\; \frac{2}{y} = \frac{(x+z)}{xz} \Leftrightarrow \frac{y}{2} = \frac{xz}{(x+z)} \Leftrightarrow y = \frac{2xz}{(x+z)}.$$

50. Let $2^x = 3^y = 6^{-z} = k \;\Leftrightarrow\; 2 = k^{\frac{1}{x}}$, $3 = k^{\frac{1}{y}}$ and $6 = k^{-\frac{1}{z}}$.

Now, $2\times3 = 6 \;\Leftrightarrow\; k^{\frac{1}{x}}\times k^{\frac{1}{y}} = k^{-\frac{1}{z}} \;\Leftrightarrow\; k^{\left(\frac{1}{x}+\frac{1}{y}\right)} = k^{-\frac{1}{z}}$

$$\therefore\; \frac{1}{x}+\frac{1}{y} = -\frac{1}{z} \Leftrightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0.$$

51. $a^1 = c^z = (b^y)^z = b^{yz} = (a^x)^{yz} = a^{xyz}$. $\therefore \quad xyz = 1$.

52. $2^x = 4^y = 8^z \quad \Leftrightarrow \quad 2^x = 2^{2y} = 2^{3z} \quad \Leftrightarrow \quad x = 2y = 3z$.

$$\therefore \quad \frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7} \Leftrightarrow \frac{1}{6z} + \frac{1}{6z} + \frac{1}{6z} = \frac{24}{7} \Leftrightarrow \frac{3}{6z} = \frac{24}{7} \Leftrightarrow z = \left(\frac{3}{6} \times \frac{7}{24}\right) = \frac{7}{48}.$$

53. L.C.M. of 2, 3, 4 is 12.

$$\sqrt{2} = 2^{\frac{1}{2}} = 2^{\left(\frac{1}{2} \times \frac{6}{6}\right)} = 2^{\frac{6}{12}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}$$

$$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3} \times \frac{4}{4}\right)} = 3^{\frac{4}{12}} = (3^4)^{\frac{1}{12}} = (81)^{\frac{1}{12}} = \sqrt[12]{81}$$

$$\sqrt[4]{4} = 4^{\frac{1}{4}} = 4^{\left(\frac{1}{4} \times \frac{3}{3}\right)} = 4^{\frac{3}{12}} = (4^3)^{\frac{1}{12}} = (64)^{\frac{1}{12}} = \sqrt[12]{64}$$

Clearly, $\sqrt[12]{81}$, *i.e.*, $\sqrt[3]{3}$ is the largest.

54. $x = 5 + 2\sqrt{6} = 3 + 2 + 2\sqrt{6} = (\sqrt{3})^2 + (\sqrt{2})^2 + 2 \times \sqrt{3} \times \sqrt{2} = (\sqrt{3} + \sqrt{2})^2$.

Also, $(x - 1) = 4 + 2\sqrt{6} = 2(2 + \sqrt{6}) = 2\sqrt{2}\,(\sqrt{2} + \sqrt{3})$.

$$\therefore \quad \frac{(x-1)}{\sqrt{x}} = \frac{2\sqrt{2}\,(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})} = 2\sqrt{2}.$$