CBSE Class 10th Mathematics

Basic Sample Paper - 05

Maximum Marks:

Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. Sound of crackers is heard during festival days, but the sound of supernova explosion in space is not heard on the surface of earth because of

    a. lesser gravity

    b. the influence of the other planets

    c. large distance

    d. absence of medium

2. If $112 = q \times 6 + r$, then the possible values of r are:

a. 1, 2, 3, 4

b. 0, 1, 2, 3

c. 2, 3, 5

d. 0, 1, 2, 3, 4, 5

3. If two positive integers ‘m’ and ‘n’ can be expressed as $m = x^2y^5$ and $n = x^3y^2$,where ‘x’ and ‘y’ are prime numbers, then HCF(m, n) =

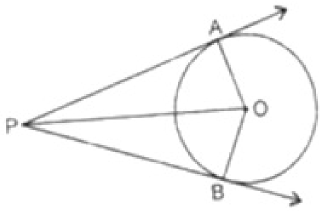
a. x^2y^2

b. x^2y^3

c. x^3y^2

d. x^3y^3

4. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, $\angle$POA is



a. 90°

b. 40°

c. 50°

d. 70°

5. The arithmetic mean of the following data is 53.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	p	13

The value of ‘p’ is

a. 17

b. 27

c. 28

d. 20

6. 1000 tickets of a lottery were sold and there are 5 prizes on these tickets. If Ramesh has purchased one lottery ticket, the probability of winning a prize is

a. $\frac{1}{100}$

b. $\frac{5}{100}$

c. $\frac{1}{200}$

d. $\frac{1}{1000}$

7. If ‘2’ is the zero of both the polynomials $3x^2 + mx - 14$ and $2x^3 + nx^2 + x - 2$, then the value of m – 2n is

a. 5

b. – 1

c. 9

d. – 9

8. The number of zeroes that the polynomial f(x) = $(x - 2)^2 + 4$ can have is

a. 0

b. 2

c. 3

d. 1

9. The points A(1, 2), B(5, 4), C(3, 8) and D(– 1, 6) are the vertices of a

   a. Rectangle

   b. Rhombus

   c. Square

   d. Parallelogram

10. The length of the median through A of ΔABC with vertices A(7, – 3), B(5, 3) and C(3, – 1) is

    a. 5 units

    b. 3 units

    c. 7 units

    d. 25 units

11. Fill in the blanks:

    The distance between the points $\left(\frac{-6}{5}, -3\right)$ and $\left(-4, \frac{-7}{5}\right)$ is ________.

12. Fill in the blanks:

    A system of two linear equations in two variables has infinitely many solutions, if their graphs ___________ each other.

    OR

    Fill in the blanks:

    The value of ‘a’ so that the point (3, a) lies on the line represented by 2x – 3y = 5 is __________.

13. Fill in the blanks:

    If x tan 45^{o}cos60^{o} = sin60^{o} cot 60^{o}, then the value of x is ________.

14. Fill in the blanks:

The maximum value of $\frac{1}{\sec\theta}$ is ________.

15. Fill in the blanks:

All ________ triangles are similar.

16. If sin A = $\frac{3}{4}$, calculate sec A.

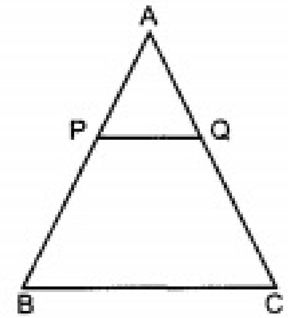
OR

Evaluate, cot 34° - tan 56°.

17. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, then find the circumference of the circle.

18. In a simultaneous throw of a pair of dice, find the probability of getting an even number on one and a multiple of 3 on the other.

19. In the fig PQ $\|$ BC and AP: PB = 1:2. Find $\frac{\text{ar}(\Delta APQ)}{\text{ar}(\Delta ABC)}$.



20. Find the common difference of the AP : $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$.

Section B

21. A bag contains 20 balls out of which x balls are red.

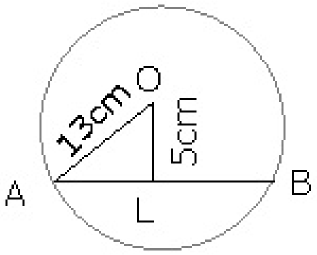
   i. If one ball is drawn at random from the bag, find the probability that it is not red.
   ii. If 4 more red balls are put into the bag, the probability of drawing a red ball will be $\frac{5}{4}$ times the probability of drawing a red ball in the first case. Find the value of x.

22. A missing helicopter is reported to have somewhere in a rectangular park having area of 700 sq km. What is the probability that it has crashed inside a circular lake of radius 10 km inside the park?

23. Two concentric circles of radii a and b (a >b) are given. Find the length of the chord of the larger circle which touches the smaller circle.

OR

In figure, if OL = 5 cm, OA = 13 cm, then length of AB is

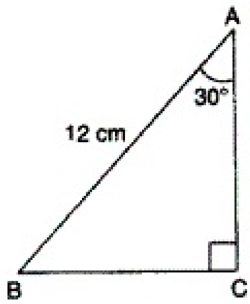


24. Prove the trigonometric identity:

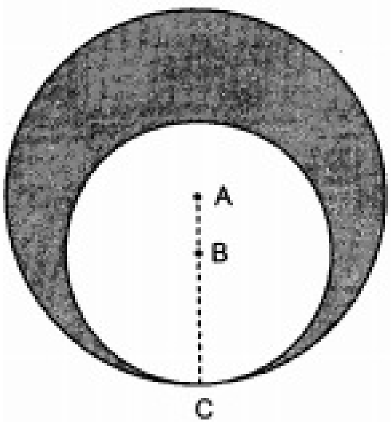
If cos A + $\cos^2$A = 1, prove that $\sin^2$A + $\sin^4$A = 1

OR

ABC is a triangle right angled at C. If $\angle A = 30^\circ$, AB = 12 cm, determine BC and AC.



25. In figure, two circles with centres A and B touch each other at the point C. If AC = 8 cm and AB = 3 cm, find the area of the shaded region.



26. Seema a home tutor used to teach maths in her locality of Paschim vihar. After teaching the chapter Polynomials, she took a class test in which she wrote the following polynomials:

    i. $5x^2 - 8x - 4$
    ii. $x^2 - 20x + 91$

    You have to find the zeros of the above quadratic polynomials by splitting the middle term.

Section C

27. Divide the polynomial $f(x) = 30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 + 2x - 4$. Also, find the quotient and remainder.

28. Draw a line segment of length 7 cm. Find a point P on it, which divides it in the ratio 3 : 5.

    OR

    Draw an isosceles $\triangle ABC$ in which BC = 5.5 cm and altitude AL = 3 cm. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.

29. Solid cylinder of brass 8 m high and 4 m diameter is melted and recast into a cone of diameter 3 m. Find the height of the cone.

30. Prove the trigonometric identity:
    (sec A - cosec A) (1 + tan A + cot A) = tan A sec A - cot A cosec A

OR

If secθ + tanθ = p, prove that $\tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

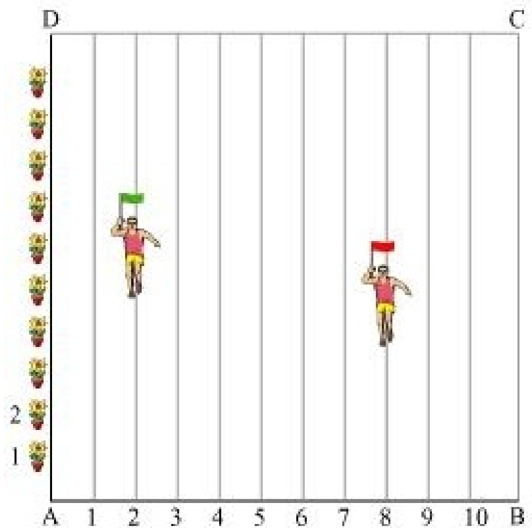
31. Find the HCF and LCM of the following positive integers by applying the prime factorization method: 15, 55, 99

OR

Find the HCF and LCM of the following positive integers by applying the prime factorization method: 15, 55, 99

32. From an external point P,two tangents,PA and PB are drawn to a circle with centre O.At one point E on the circle tangent is drawn which intersects PA and PB at C and D respectively. If PA=10 cm,find the perimeter of the triangle PCD.

33. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. Niharika runs the distance AD on the 2nd line and posts a green flag. Preet runs the distance AD on the eighth line and posts a red flag.



i. Calculate the distance Niharika and Preet posted the green flag and reg flag respectively.
ii. What is the distance between both the flags?
iii. If Rashmi has to post a blue flag exactly halfway between the line segment joining

the two flags, where should she post her flag?

34. Solve the following system of equations in x and y

ax + by = 1

$bx + ay = \frac{(a+b)^2}{a^2+b^2} - 1$ or, $bx + ay = \frac{2ab}{a^2+b^2}$

Section D

35. The perimeter of a rectangular field is 82 m and its area is 400 square metre. Find the length and breadth of the rectangle.

36. 360 bricks are stacked in the following manner, 30 bricks in the bottom row, 29 bricks in the next row, 28 bricks in the row next to it, and so on. In how many rows, 360 bricks are placed and how many bricks are there in the top row?

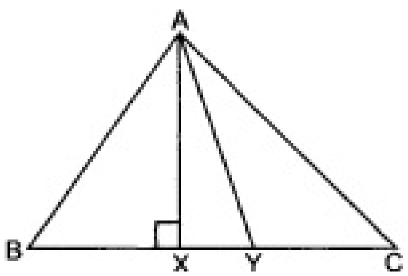
OR

Let there be an A.P. with first term 'a', common difference 'd'. If a_n denotes its n^{th} term and S_n the sum of first n terms, find. n and S_n, if a = 5, d = 3 and a_n = 50.

37. A 1.6 m tall girl stands at a distance of 3.2 m from a lamp-post and casts a shadow of 4.8 m on the ground. Find the height of the lamp-post by using

i. trigonometric ratios
ii. property of similar triangles.

38. in $\triangle ABC$, $AX \perp BC$ and Y is middle point of BC.

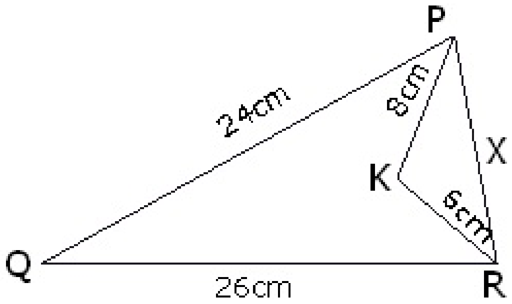


Prove that

i. $AB^2 = AY^2 + \frac{BC^2}{4} - BC.XY$
ii. $AC^2 = AY^2 + \frac{BC^2}{4} + BC.XY$

OR

In a triangle, if the square of one side is equal to the sum of the squares on the other two sides. Prove that the angle opposite to the first side is a right angle. Use the above theorem to find the measure of $\angle$ PKR in the figure given below.



39. If the diameter of cross-section of a wire is decreased by 5% how much percent will the length be increased so that the volume remains the same?

OR

A housing society used to collect rain water from the roof of its building 22 m $\times$ 20 m to a cylindrical vessel having diameter of base 2m and height 3.5 m and then pump this water into the main water tank so that all members can use it. On a particular day the rain water collected from the roof just filled the cylindrical vessel. Then, find the height of the roof.

40. Find the mean, mode and median of the following frequency distribution:

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	4	4	7	10	12	8	5

Solution

Section A

1. (d) absence of medium
   Explanation:

   Sound needs medium to travel. As there is no medium in the space, it can not travel from space to earth.

2. (d) 0, 1, 2, 3, 4, 5
   Explanation:

   For the relation x = qy + r, $0 \leqslant r < y$

   So, here r lies between $0 \leqslant r < 6$.

   Hence, r = 0, 1, 2, 3, 4, 5.

3. (a) x^2y^2
   Explanation:

   $x^2y^5 = y^3(x^2y^2)$

   $x^3y^2 = x(x^2y^2)$

   Therefore HCF (m , n) is x^2y^2

4. (c) 50°
   Explanation:

   Here ∠APB = 80°
   ∴∠AOB = 180° - 80° = 180°
   Now, since OP bisect ∠APB and ∠AOB.
   ∴∠AOP = $\frac{100^\circ}{2} = 50^\circ$

5. (c) 28

Explanation:

Class	**Mid-value** (x_i)	**Frequency** (f_i)	$f_i x_i$
0 – 20	10	12	120
20 – 40	30	15	450
40 – 60	50	32	1600
60 – 80	70	p	70p
80 – 100	90	13	1170
Total		$\sum f_i = 72 + p$	$\sum f_i x_i = 3340 + 70p$

Mean = $\frac{\sum f_i x_i}{\sum f_i}$

$\Rightarrow 53 = \frac{3340+70p}{72+p}$

$\Rightarrow$53(72+p)=3340+70p

$\Rightarrow 3340 + 70p = 3816 + 53p$

$\Rightarrow 70p - 53p = 3816 - 3340$

$\Rightarrow 17p = 476$

$\Rightarrow p = 28$

6. (c) $\frac{1}{200}$

   Explanation:

   Number of possible outcomes = 5

   Number of total outcomes = 1000

   $\therefore$ Required Probability = $\frac{5}{1000} = \frac{1}{200}$

7. (c) 9

   Explanation:

   According to the question, $p(2) = 3x^2 + mx - 14 = 0$

   $\Rightarrow 3(2)^2 + m \times 2 - 14 = 0$

   $\Rightarrow 12 + 2m - 14 = 0 \Rightarrow m = 1$

   Also $p(2) = 2x^3 + nx^2 + x - 2 = 0$

   $\Rightarrow 2 \times (2)^3 + n \times (2)^2 + 2 - 2 = 0$

   $\Rightarrow 16 + 4n = 0$

$\Rightarrow n = -4$

$\therefore m - 2n = 1 - 2 \times (-4) = 1 + 8 = 9$

8. (b) 2

   Explanation:

   $f(x) = (x-2)^2 + 4 = x^2 - 4x + 4 + 4 = x^2 - 4x + 8$

   Here the largest exponent of variable is 2,

   therefore number of zeroes of the given polynomial is 2.

9. (c) Square

   Explanation:

   Given: The points A(1, 2), B(5, 4), C(3, 8) and D(– 1, 6)

   $\therefore$ AB = $\sqrt{(5-1)^2 + (4-2)^2} = \sqrt{16+4} = 2\sqrt{5}$ units

   BC = $\sqrt{(3-5)^2 + (8-4)^2} = \sqrt{4+16} = 2\sqrt{5}$ units

   CD = $\sqrt{(-1-3)^2 + (6-8)^2} = \sqrt{16+4} = 2\sqrt{5}$ units

   AD = $\sqrt{(-1-1)^2 + (6-2)^2} = \sqrt{4+16} = 2\sqrt{5}$ units

   Therefore the 4 sides AB , BC, CD and DA are equal

   and the diagonal AC = $\sqrt{(3-1)^2 + (8-2)^2} = \sqrt{4+36} = 2\sqrt{10}$ units

   and BD = $\sqrt{(-1-5)^2 + (6-4)^2} = \sqrt{36+4} = 2\sqrt{10}$ units

   Therefore diagonals AC and BD are equal

   Since, all 4 sides are equal and both diagonals are also equal.

   Therefore, the given quadrilateral is a square.

10. (a) 5 units

    Explanation:

    ABC is a triangle with A(7, - 3), B(5, 3) and C(3, - 1)

    Let median on BC bisects BC at D. (AD is given as the median)

$\therefore$ Coordinates of D are $\left(\frac{5+3}{2}, \frac{3-1}{2}\right)$ = $(4,1)$

$\therefore$ AD = $\sqrt{(4-7)^2+(1+3)^2}$

= $\sqrt{9+16}$

= $\sqrt{25}$ = 5 units

11. $\frac{\sqrt{260}}{5} units$

12. coincide

OR

$\frac{1}{3}$

13. x = 1

14. 1

15. equilateral

16. Given: sin A = $\frac{3}{4}$

$\Rightarrow$ cos A = $\sqrt{1-\frac{9}{16}}$ [since, cos A = $\sqrt{1-sin^2A}$]

$\Rightarrow$ cos A = $\frac{\sqrt{7}}{4}$

Thus, sec A = $\frac{1}{cosA}$ = $\frac{4}{\sqrt{7}}$

OR

We have, cot 34° - tan 56°

= cot (90° - 56°) - tan 56°.

= tan56° - tan56° [Since, Cot (90° - A) = tan A]

= 0.

17. Area of square = (side)^2

= 121 cm^2

Side of square = $\sqrt{121}$

$= 11$cm

Perimeter of square = 4$\times$ side

= 4$\times$11

= 44 cm.

Circumference of the circle = Perimeter of the square = 44 cm

So, Circumference of the circle is 44 cm.

18. Favourable outcomes of an even number on one and a multiple of 3 on other = {(2,3)(2,6)(4,3)(4,6)(6,3)(6,6)(3,2)(3,4)(3,6)(6,2)(6,4)}

Therefore,number of cases favourable to the event=11

$\therefore$ Probability an even number on one and a multiple of 3 on other = $\frac{number\ of\ favourable\ outcomes}{number\ of\ total\ outcomes}=\frac{11}{36}$.

19. In ABC,

PQ $\|$ BC

$\therefore \quad \frac{\mathrm{AP}}{\mathrm{AB}}=\frac{\mathrm{AQ}}{\mathrm{AC}}$

Now in ΔAPQ and ΔABC,

$\frac{\mathrm{AP}}{\mathrm{AB}}=\frac{\mathrm{AQ}}{\mathrm{AC}}$ (As proved)

$\angle$A = $\angle$A (common angle)

$\triangle \mathrm{APQ} \sim \triangle \mathrm{ABC}$ (SAS similarity)

Since for similar triangles,the ratio of the areas is the square of the ratio of their corresponding sides.Therefore,

$$\frac{\operatorname{ar}(\Delta \mathrm{APQ})}{\operatorname{ar}(\Delta \mathrm{ABC})}=\frac{\mathrm{AP}^{2}}{\mathrm{AB}^{2}}=\frac{\mathrm{AP}^{2}}{(\mathrm{AP}+\mathrm{PB})^{2}}=\frac{1^{2}}{3^{2}}=\frac{1}{9}$$

20. Common difference(d) = $n^{th} term-(n-1)^{th} term$

$\therefore d=a_{2}-a_{1}$

$$d=\left(\frac{1-p}{p}\right)-\left(\frac{1}{p}\right)=\frac{(1-p)-(1)}{p}=\frac{-p}{p}=-1$$

d=-1

Section B

21. i. Let the number of red balls = x

P(a red ball)=$\frac{x}{20}$

P(not a red ball) = $1-\frac{x}{20}$

ii.Total number of balls = 24, red balls = x + 4

P(red ball) = $\frac{x+4}{24}$

According to the question,

$\frac{x+4}{24}=\frac{5}{4}\times\frac{x}{20}$

$x+4=\frac{5x}{80}\times 24$

$x+4=\frac{x}{16}\times 24$

$x+4=\frac{3x}{2}$

x=8

22. The area of park = 700 km^2

Total outcomes = 700

Area of the lake = $\pi\times r^2$

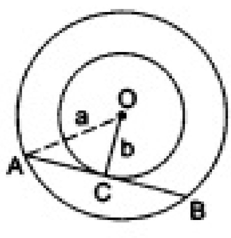
= 3.14 $\times$ 10^2

= 314 km^2

So expected outcome = 314

Probability = $\frac{\text{Excepted outcome}}{\text{total outcome}}=\frac{314}{700}$

= 0.448

23. 

Let O be the common centre of the two circles

and AB be the chord of the larger circle which touches the smaller circle at C.

Join OA and OC.

Then, OA = a and OC = b.

Now, $\mathrm{OC}\perp AB$ and OC bisects AB [$\because$ the chord of the larger circle touching the smaller circle, is bisected at the point of contact].

In right $\triangle ACO$, we have

$OA^2 = OC^2 + AC^2$ [by Pythagoras' theorem]

$\Rightarrow AC = \sqrt{OA^2 - OC^2} = \sqrt{a^2 - b^2}$.

$\therefore AB = 2AC = 2\sqrt{a^2 - b^2}$ [$\because$ C is the midpoint of AB]

i.e, required length of the chord AB = $2\sqrt{a^2 - b^2}$

OR

$AB = 2\ AL = 2\sqrt{OA^2 - OL^2}$

$= 2\sqrt{13^2 - 5^2}$

$= 2\sqrt{169 - 25} = 2\sqrt{144}$

$= 2 \times 12 = 24$ cm

24. $\cos A + \cos^2 A = 1$ (Given)

$\Rightarrow \cos A = 1 - \cos^2 A$

$\Rightarrow \cos A = \sin^2 A$ [$\because 1 - \cos^2 A = \sin^2 A$]...... (i)

L.H.S = $\sin^2 A + \sin^4 A$

$= \sin^2 A + (\sin^2 A)^2$

$= \sin^2 A + (\cos A)^2$

$= \sin^2 A + \cos^2 A$ [$\because \sin^2 A + \cos^2 A = 1$]

= 1 = R.H.S

therefore, $\sin^2 A + \sin^4 A = 1$

Hence proved.

OR

In right triangle ABC,

$\sin 30^\circ = \frac{\mathrm{BC}}{AB} \Rightarrow \frac{1}{2} = \frac{\mathrm{BC}}{12}$

$\Rightarrow BC = \frac{12}{2} = 6\mathrm{cm}$

Again, $\cos 30^\circ = \frac{AC}{AB}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AC}{12} \Rightarrow AC = 6\sqrt{3}\mathrm{cm}$

25. AC = 8 cm, AB = 3 cm

Then BC = 8 - 3 = 5 cm

Therefore, radius of big circle=AC=8 cm

and radius of small circle=BC=5 cm

$\therefore$ Area of shaded region = Area of big circle - Area of small circle

$= \pi(AC)^2 - \pi(BC)^2$

$= \frac{22}{7}(8)^2 - \frac{22}{7}(5)^2$

$= \frac{22}{7}[64 - 25]$

$= \frac{22}{7} \times 39$

= 122.57 cm^2

26. i. $5x^2 - 8x - 4 = 0$

$5x^2 - 10x + 2x - 4 = 0$

$5x(x - 2) + 2(x - 2) = 0$

$(x - 2)(5x + 2) = 0$

x = $-\frac{2}{5}, 2$

Thus, zeros of the polynomial are $-\frac{2}{5}, 2$

ii. $x^2 - 20x + 91 = 0$

$x^2 - 13x - 7x + 91 = 0$

$x(x - 13) - 7(x - 13) = 0$

$(x - 13)(x - 7) = 0$

x = 13, 7

Thus, zeros of the polynomial are 13, 7

Section C

27. Using long division method, we obtain

$$
\begin{array}{r|l|l}
3x^2 + 2x - 4 & 30x^4 + 11x^3 - 82x^2 - 12x + 48 & 10x^2 - 3x - 12 \\
 & 30x^4 + 20x^3 - 40x^2 & \\
 & - \quad - \quad + & \\
\hline
 & -9x^3 - 42x^2 - 12x + 48 & \\
 & -9x^3 - 6x^2 + 12x & \\
 & + \quad + \quad - & \\
\hline
 & -36x^2 - 24x + 48 & \\
 & -36x^2 - 24x + 48 & \\
 & + \quad + \quad - & \\
\hline
 & 0 &
\end{array}
$$

Clearly, quotient q(x) = $10x^2 - 3x - 12$ and remainder r(x) = 0.

Also,

g(x) q(x) + r(x) = $(3x^2 + 2x - 4)(10x^2 - 3x - 12) + 0$

$= 3x^2 (10x^2 - 3x - 12) + 2x (10x^2 - 3x - 12) - 4 (10x^2 - 3x - 12) + 0$

$= 30x^4 - 9x^3 - 36x^2 + 20x^3 - 6x^2 - 24x - 40x^2 + 12x + 48 + 0$

$= 30x^4 - 9x^3 + 20x^3 - 36x^2 - 6x^2 - 40x^2 - 24x + 12x + 48 + 0$

$= 30x^4 + 11x^3 - 82x^2 - 12x + 48$

i.e. f(x) = g(x) q(x) + r(x) = $30x^4 + 11x^3 - 82x^2 - 12x + 48$ = f(x)

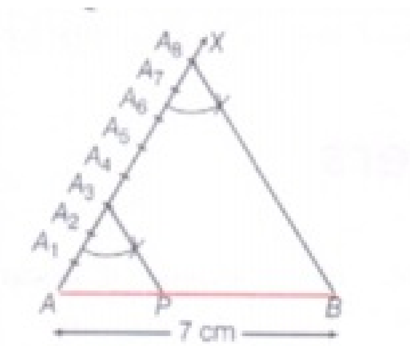
or, Dividend = Quotient $\times$ Divisor + Remainder.

28. We have to draw a line segment of length 7 cm.Then,we have to find a point P on it, which divides it in the ratio 3 : 5.

Steps of construction:

i. Draw a line segment $AB = 7$ cm.
ii. Draw a ray AX, making an acute $\angle BAX$ with AB.
iii. Mark $3 + 5 = 8$ points, i.e, $A_1, A_2, A_3, A_4 \dots A_8$ on AX, such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 \dots = A_7A_8$
iv. Join A_8 B
v. From A_3, draw $A_3P || A_8B$ which intersects AB at point P [by making an angle at A_3 equal to $\angle AA_8B$

Then, P is the point on AB which divides it in the ratio 3:5. So, $AP : PB = 3 : 5$



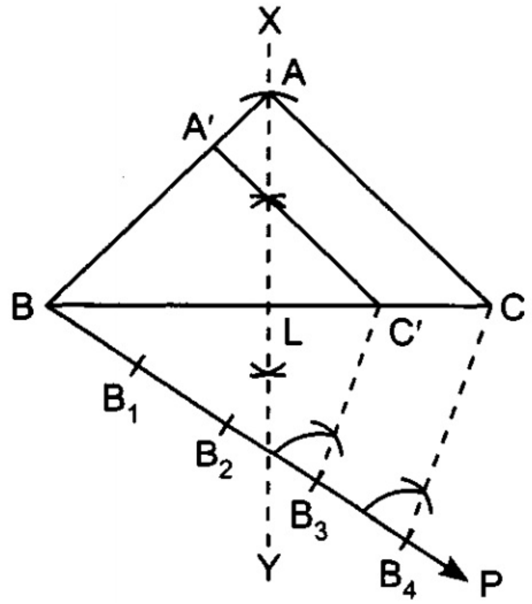
Justification: In $\triangle ABA_8$, we have $A_3P || A_8B$

$\therefore \frac{AP}{PB} = \frac{AA_3}{A_3A_8}$ [by basic proportionality theorem]

By construction, $\frac{AA_3}{A_3A_8} = \frac{3}{5}$

Hence, $\frac{AP}{PB} = \frac{3}{5}$

OR



Steps of Construction:

i. Draw a line segment $BC = 5.5cm.$
ii. Perpendicular bisector XY of BC is drawn intersecting BC at L.
iii. Point A is marked on XL such that $AL = 3cm.$
iv. AB and AC are joined.
v. On BP, points B_1, B_2, B_3 and B_4 are marked such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
vi. B_4C is joined.
vii. B_3C' is drawn parallel to B_4C intersecting BC at C'.
viii. $C'A'$ is drawn parallel to CA intersecting BA at A'.
ix. $A'BC'$ is the required triangle.

29. Solid cylinder of brass 8 m high and 4 m diameter is melted and recast into a cone of diameter 3 m. We have to find the height of the cone.

We have,

	Cylinder	**Cone**
Radii	r_1 = 2m	r_2 = 1.5 m

Heights	$h_1 = 8$ m	$h_2 = ?$
Volumes	V_1	V_2

Clearly, Volume of the cone = Volume of the cylinder

i.e., $V_1 = V_2$

$\Rightarrow \quad \frac{1}{3}\pi r_2^2 h_2 = \pi r_1^2 h_1$

$\Rightarrow \quad r_2^2 h_2 = 3r_1^2 h_1$

$\Rightarrow \quad h_2 = \frac{3r_1^2 h_1}{r_2^2} \Rightarrow h_2 = \frac{3\times 2^2\times 8}{(1.5)^2}\text{m} \Rightarrow h_2 = \frac{96}{2.25}\text{m} = 42.66\text{m}$

Hence, the height of the cone is 42.66 m.

30. LHS = (secA - cosecA) (1 + tanA + cotA)

$= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right)\left(1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= \left(\frac{\sin A - \cos A}{\cos A\ \ \sin A}\right)\left(\frac{\cos A \sin A + \sin^2 A + \cos^2 A}{\cos A \sin A}\right)$

$= \frac{(\sin A - \cos A)(\sin^2 A + \cos A \sin A + \cos^2 A)}{\cos^2 A \sin^2 A}$

$= \frac{\sin^3 A - \cos^3 A}{\cos^2 A\ \sin^2 A}\ \left[\because a^3 - b^3 = (a-b)(a^2+ab+b^2)\right]$

RHS = tanA secA - cotA cosecA

$= \frac{\sin A}{\cos A}\frac{1}{\cos A} - \frac{\cos A}{\sin A}\frac{1}{\sin A}$

$= \frac{\sin A}{\cos^2 A} - \frac{\cos A}{\sin^2 A} = \frac{\sin^3 A - \cos^3 A}{\cos^2 A \sin^2 A}$

LHS = RHS

Hence proved.

OR

Given, secθ + tanθ = p

$\Rightarrow$ secθ = p - tanθ ....(1)

Now,

sec$^2\theta$ - tan$^2\theta$ = 1

$\Rightarrow$ (secθ + tanθ)(secθ - tanθ) = 1

$\Rightarrow$ p(secθ - tanθ) = 1

$\Rightarrow$ secθ - tanθ = $\frac{1}{p}$ ....(2)

From (1) and (2),

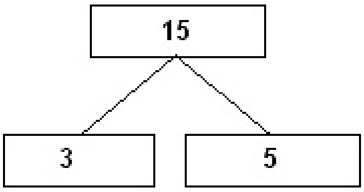
p - tanθ - tanθ = $\frac{1}{p}$

$\Rightarrow$ p - 2tanθ = $\frac{1}{p}$

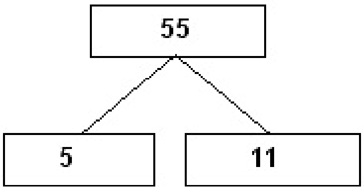
$\Rightarrow$ 2tanθ = $p - \frac{1}{p}$

$\Rightarrow \tan\theta = \frac{1}{2}\left(p - \frac{1}{p}\right)$

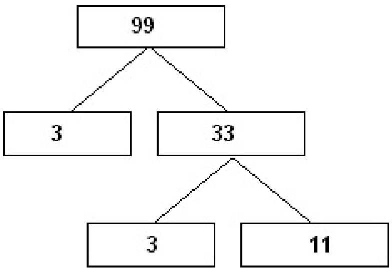
31.



so, 15 = 3 $\times$ 5



So, 55 = 5 $\times$ 11



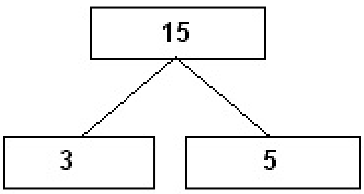
So, 99 = $3 \times 3 \times 11 = 3^2 \times 11$

Therefore,

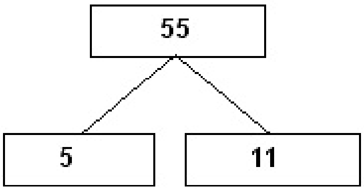
HCF (15 ,55, 99) = 1

LCM (15, 55, 99) = $3^2 \times 5 \times 11 = 495$.

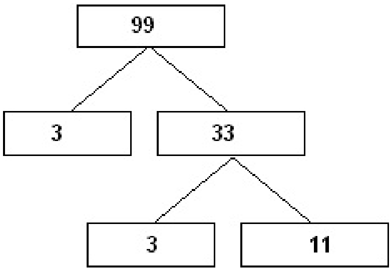
OR



so, 15 = 3 $\times$ 5



So, 55 = 5 $\times$ 11



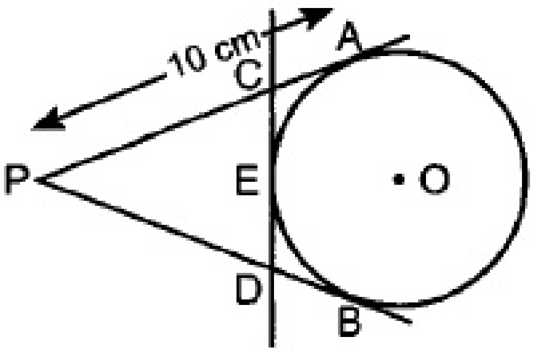
So, 99 = $3 \times 3 \times 11 = 3^2 \times 11$

Therefore,

HCF (15 ,55, 99) = 1

LCM (15, 55, 99) = $3^2 \times 5 \times 11 = 495$.

32. Given,



PA = 10 cm.

PA = PB [If P is external point] ...(i)[From an external point tangents drawn to a circle are equal in length]

If C is external point, then CA = CE

If D is external point, then

DB = DE ...(ii)

Perimeter of triangle $\triangle$PCD

$= PC + CD + PD$
$= PC + CE + ED + PD$
$= PC + CA + DB + PD$
$= PA + PB$
$= PA + PA$
$= 2PA$
$= 2 \times 10 = 20\text{cm}$ [From (i)]

33. i. It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\frac{1}{4} \times 100$ = 25m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).
Similarly, Preet posted a red flag at the distance AD i.e., $\frac{1}{5} \times 100$ = 20m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

ii. According to distance formula,
Distance between these flags by using the distance formula, D
= $[(8 - 2)^2 + (25 - 20)^2]^{1/2}$ = $(36 + 25)^{1/2}$ = $\sqrt{61}m$

iii. The point at which Rashmi should post her blue flat is the mid-point of the line joining these points. Let this point be A(x, y)
Now by midpoint formula,
$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$
$x = \frac{2+8}{2} = 5$
$y = \frac{25+20}{2} = 22.5$
Hence, A(x, y) = (5, 22.5)
Therefore, Rashmi should post her blue flag at 22.5 m on 5th line.

34. The given system of equations are
ax + by - 1 = 0 .... (i)
$bx + ay = \frac{2ab}{a^2+b^2}$ .... (ii)
By cross-multiplication, of equations (i) and (ii) ,we have
$\frac{x}{b\times-\frac{2ab}{a^2+b^2}-a\times-1} = \frac{-y}{a\times-\frac{2ab}{a^2+b^2}-b\times-1} = \frac{1}{a\times a-b\times b}$
$\Rightarrow \quad \frac{x}{\frac{-2ab^2}{a^2+b^2}+a} = \frac{-y}{\frac{-2a^2b}{a^2+b^2}+b} = \frac{1}{a^2-b^2}$

$$\Rightarrow \frac{x}{\frac{-2ab^2+a^3+ab^2}{a^2+b^2}} = \frac{-y}{\frac{-2a^2b+a^2b+b^3}{a^2+b^2}} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \frac{x}{\frac{a^3-ab^2}{a^2+b^2}} = \frac{-y}{\frac{-a^2b+b^3}{a^2+b^2}} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \frac{x}{\frac{a\left(a^2-b^2\right)}{a^2+b^2}} = \frac{-y}{\frac{b\left(a^2-b^2\right)}{a^2+b^2}} = \frac{1}{a^2-b^2}$$

$$\Rightarrow \quad x = \frac{a\left(a^2-b^2\right)}{a^2+b^2} \times \frac{1}{a^2-b^2} \text{ and } y = \frac{b\left(a^2-b^2\right)}{a^2+b^2} \times \frac{1}{a^2-b^2}$$

$$\Rightarrow \quad x = \frac{a}{a^2+b^2} \text{ and } y = \frac{b}{a^2+b^2}$$

Hence, the solution of the given system of equations is $x = \frac{a}{a^2+b^2}, y = \frac{b}{a^2+b^2}$.

Section D

35. Perimeter = 82 m

$\Rightarrow$ 2 (l +b) = 82 m

or, l + b = 41 m

Area = 400 m^2

$\Rightarrow$ l $\times$ b = 400 m^2

Let length be x m. Then,

breadth = (41- x) m

Now, $x(41-x) = 400$

$41x - x^2 = 400$

$x^2 - 41x + 400 = 0$

(x - 16)(x - 25) = 0

x = 16 or x = 25

Hence, if length = 16 m, then breadth = 25 m

or, if length = 25 m, then breadth = 16 m

36. Number of bricks in the bottom row=30. in the next row=29, and so on.

therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,... , which is an AP with first term , a=30 and common difference, d= 29 - 30 = -1.

Suppose number of rows is n, then sum of number of bricks in n rows should be 360

i.e. $S_n = 360$

$\Rightarrow \frac{n}{2}[2 \times 30 + (n-1)(-1)] = 360 \quad \{S_n = \frac{n}{2}(2a + (n-1)d)\}$

$\Rightarrow 720 = n(60 - n + 1)$

$\Rightarrow 720 = 60\text{n - n}^2 + n$

$\Rightarrow n^2 - 61n + 720 = 0$

$\Rightarrow n^2 - 16n - 45n + 720 = 0$ [by factorisation]

$\Rightarrow n(n-16) - 45(n-16) = 0$

$\Rightarrow (n-16)(n-45) = 0$

$\Rightarrow (n-16) = 0$ or $(n-45) = 0$

$\Rightarrow n = 16$ or $n = 45$

Hence, number of rows is either 45 or 16.

When, $n = 16$,

$a_{16} = 30 + (16-1)(-1) \quad \{a_n = a + (n-1)d\}$

$= 30 - 15 = 15$

When, $n = 45$

$a_{45} = 30 + (45-1)(-1) \quad \{a_n = a + (n-1)d\}$

$= 30 - 44 = -14$ [$\because$ The number of logs cannot be neagtive]

Hence, the number of rows is 16 and number of logs in the top row is 15.

OR

Given,

First term(a) = 5

Common difference(d) = 3

and, nth term (a_n) = 50

$\Rightarrow$a + (n - 1)d = 50

$\Rightarrow$5 + (n - 1)(3) = 50

$\Rightarrow$5 + 3n - 3 = 50

$\Rightarrow$3n = 50 - 5 + 3

$\Rightarrow$3n = 48

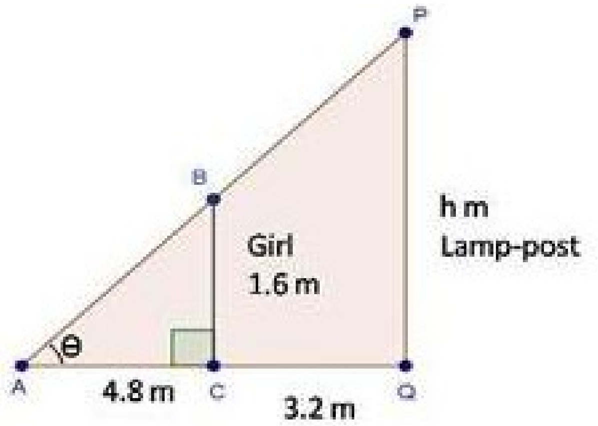
$\Rightarrow \quad n = \frac{48}{3} = 16$

Therefore, S_n = $\frac{n}{2}[a + a_n]$

$= \frac{16}{2}[5 + 50]$

$= 8 \times 55$

= 440

37. 

Suppose BC be the height of girl of length 1.6 m

Let PQ be the height of lamp - post of length = h m

Suppose AC be the length of shadow = 4.8 m

Distance between girl and lamp - post = 3.2 m

(i) Using trigonometric ratios

In right ΔACB

$\tan\theta = \frac{BC}{AC}$

$\Rightarrow \tan\theta = \frac{1.6}{4.8} = \frac{1}{3}$ ...(1)

In right ΔPQA

$\tan\theta = \frac{PQ}{AQ}$

$\Rightarrow \tan\theta = \frac{h}{8}$ ...(2)

Compare equation (1) and equation (2)

$\frac{h}{8} = \frac{1}{3}$

$\Rightarrow h = \frac{8}{3}m$ = 2.67 meter

(ii) Using similar triangles

In ΔACB and ΔAQP

$\angle A = \angle A$

and, $\angle ACB$ and $\angle AQP$ [each 90^0]

Then, $\Delta ACB - \Delta AQP$ [by AA similarity]

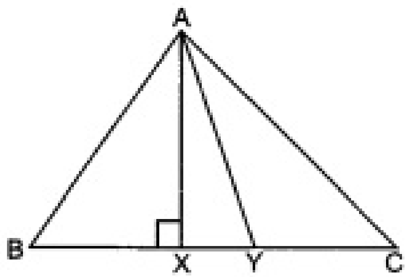
$\therefore \frac{AC}{AQ} = \frac{BC}{PQ}$ [c.p.c.t]

$\Rightarrow \frac{4.8}{8} = \frac{1.6}{h}$

$\Rightarrow h = \frac{1.6\times8}{4.8} = \frac{8}{3}m$ = 2.67 M

Hence, length of the pole is 2.67 m(nearly)

38. (i) In right $\triangle$AXB



$AB^2 = AX^2 + BX^2$ ..(1)

In right $\triangle$AXY

$AY^2 = AX^2 + XY^2$ ..(2)

subtract (2) from (1), we get

$AB^2 - AY^2 = AX^2 + BX^2 - AX^2 - XY^2$

$\Rightarrow AB^2 - AY^2 = BX^2 - XY^2$

$\Rightarrow AB^2 - AY^2 = (BX - XY)(BX + XY)$

$\Rightarrow AB^2 - AY^2 = [(BY - XY) - XY](BY)$

$\Rightarrow AB^2 - AY^2 = (\frac{1}{2}BC - 2XY)\frac{1}{2}BC$ [Y is the mid-point of BC]

$\Rightarrow$ AB^2 - AY^2 = $\frac{1}{4}$BC^2 - BC.XY

$\Rightarrow$ AB^2 = AY^2 + $\frac{1}{4}$BC^2 - BC.XY Hence proved

(ii) In right $\triangle$AXC

AC^2 = AX^2 + XC^2 ..(3)

In right $\triangle$AXY,

AC^2 - AY^2 = XC^2 - XY^2

(3) - (4), we get

AC^2 - AY^2 = XC^2 - XY^2

$\Rightarrow$ AC^2 - AY^2 = (XC - XY)(XC + XY)

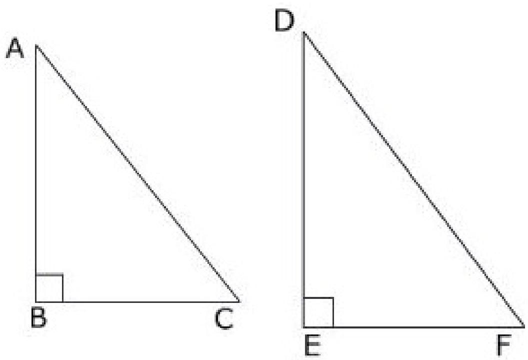
= YC(YC + XY + XY)

$\Rightarrow$ AC^2 - AY^2 = $\frac{1}{2}$BC($\frac{1}{2}$BC + 2XY) (Y is mid-point of BC)

$\Rightarrow$ AC^2 - AY^2 = $\frac{1}{4}$BC^2 + BC.XY

$\Rightarrow$ AC^2 = + AY^2 + $\frac{1}{4}$BC^2 + BC.XY Hence proved.

OR



i. Given: In $\triangle$ABC such that

   AC^2 = AB^2 + BC^2

   To prove: Triangle ABC is right angled at B

   Construction: Construct a triangle DEF such that

   DE = AB, EF = BC and $\angle E = 90^\circ$

   Proof: $\because \triangle$ DEF is a right angled triangle right angled at E [construction]

   $\therefore$ By Pythagoras theorem, we have

   DF^2 = DE^2 + EF^2

   $\Rightarrow$ DF^2 = AB^2 + BC^2 [$\because$ DE = AB and EF = BC]

$\Rightarrow DF^2 = AC^2 [\because AB^2 + BC^2 = AC^2]$

$\Rightarrow$ DF = AC

Thus,in $\triangle$ ABC and $\triangle$DEF, we have

AB = DE

BC = EF

and AC = DF [By Construction and (i)]

$\therefore \triangle ABC \cong \triangle DEF$ (SSS)

$\Rightarrow \angle B = \angle E = 90^{\circ}$

Hence, $\triangle$ ABC is a right triangle.

ii. In $\triangle$QPR , $\angle$ QPR = 90°

$\Rightarrow 24^2 + x^2 = 26^2$

$\Rightarrow$ x = 10

$\Rightarrow$ PR = 10 cm

Now in $\triangle$PKR, $PR^2 = PK^2 + KR^2$[as $10^2 = 8^2 + 6^2$]

$\therefore$ PKR is right angled at K

$\Rightarrow \angle PKR = 90^{\circ}$

39. Let r be the radius of cross-section of wire and h be its length. Then
Volume of cylinder = $\pi r^2 h$........(i)

Where r is the original radius of the wire and h is the original height (length) of the wire. When the diameter of the wire is decreased by 5%

$\therefore$ New diameter = $2r - \frac{5}{100} \times 2r$

= $2r \times \frac{95}{100}$

$\Rightarrow$ New radius = $\frac{95r}{100}$

Let the new length be h_1. Then,

Volume = $\pi \left(\frac{95r}{100}\right)^2 h_1$........(ii)

From (i) and (ii), we obtain

$$\pi r^2 h = \pi \left(\frac{95r}{100}\right)^2 h_1 \Rightarrow h = \frac{10000}{9025} h_1$$

$\therefore$ h = 1.108 × h_1

or, h = (1+ 0.108) h_1

Hence, if we increase the height by 10% the volume remains the same.

OR

Dimension of the roof is 22m $\times$ 20 m

Diameter of the cylinderical vessel = 2m

Radius of the cylinder (R) = 1 cm

Height of the cylinderical vessel (H) = 3.5 m

Let the height of the roof be h

Volume of water thus collected on the roof = $22 \times 20 \times h$

Volume of the cylinderical vessel = $\pi R^2 h$

= $\pi \times (1)^2 \times 3.5$

= 3.5π

Volume of water collected on on the roof = Volume of the cylinderical vessel

$22 \times 20 \times h$ = 3.5π

$\Rightarrow$ h $= \frac{3.5\pi}{22 \times 20} = 0.025\text{m}$ = 2.5 cm

Hence, the height of the roof is 2.5 cm.

40. i. To find mean:

Taking the assuming mean a = 35, we can compute x_i and $f_i x_i$ as following:

Daily wages (in Rs)	**Frequency (f_i)**	x_i	$d_i = x_i - 35$	$u_i = \frac{x_i - 35}{h}$	$f_i u_i$
0 - 10	4	5	-30	-3	-12
10 - 20	4	15	-20	-2	-8
20 - 30	7	25	-10	-1	-7
30 - 40	10	35	0	0	0
40 - 50	12	45	10	1	12
50 - 60	8	55	20	2	16
60 - 70	5	65	30	3	15

	Σf_i = 50				$\sum f_i u_i$ = 16

Here a = 35

Σf_i = 50

$\sum f_i u_i$ = 16

So $\overline{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$ = 35 + $\left(\frac{16}{50}\right) \times 10$ = 35 + 3.2 = 38.2

ii. To find mode:

As we may observe that maximum class frequency is 12 belonging to class interval 40 - 50.

So, modal class = 40 - 50

Lower limit (l) of modal class = 40

Frequency (f_1) of modal class = 12

Class size (h) = 10

Frequency (f_0) of class preceding the modal class = 10

Frequency (f_2) of class succeeding the modal class = 8

Mode = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$ = 40 + $\left(\frac{12-10}{2\times12-12-8}\right)$

$= 40 + \frac{2}{24-20} = 40 + \frac{2}{4}$ = 40 + 0.5 = 40.5

iii. To find median:

We know that

3 median = mode + 2 mean

= 40.5 + 2 $\times$ 38.2

= 40.5 + 76.4

= 116.9

So Median $= \frac{116.9}{3}$ = 38.97