Part I

WAVE

Wave is distributed energy or distributed "disturbance".

MECHANICAL WAVES

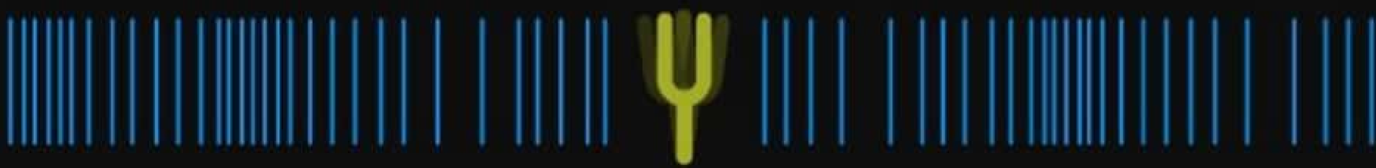
Mechanical waves orginate from a disturbance in the medium (such as a stone dropping in a pond) and the disturbance propagates through the medium.

Mechnical waves are further classified in two categories such that:

1. Transverse waves (waves on a string)

If the disturbance travels in the x direction but the particles move in a direction, perpendicular to the x axis as the wave passes, it is called transverse waves.

2. Longitudinal waves (sound waves)



Longitudinal waves are characterized by the direction of vibration (disturbance) and wave motion. They are along the same direction.

NON-MECHANICAL WAVES

These are electromagnetic waves. The motion of the electromagnetic waves in a medium depends on the electromagnetic properties of the medium.

PARTICLE VELOCITY AND ACCELERATION

$$V_P = \frac{\partial}{\partial t} y(x, t) = \frac{\partial}{\partial t} A\sin(kx - \omega t) = -\omega A \cos(kx - \omega t)$$

$$a_p = \frac{\partial}{\partial t} V_P = \frac{\partial}{\partial t} \{-\omega A \cos(kx - \omega t)\} = -\omega^2 A \sin(kx - \omega t) = -\omega^2 y$$

ENERGY CALCULATION IN WAVES

Part II

1. KINETIC ENERGY PER UNIT LENGTH

The velocity of string element in transverse direction is greatest at one mean position and zero at the extreme positions of waveform.

$$K_L = \frac{dK}{dx} = \frac{1}{2}\mu \omega^2 A^2 \cos^2(kx - \omega t)$$

- **RATE OF TRANSMISSION OF KINETIC ENERGY**

$$\frac{dK}{dx} = \frac{1}{2}\mu v \omega^2 A^2 \cos^2(kx - \omega t)$$

2. ELASTIC POTENTIAL ENERGY

The Elastic potential energy of the string element results as string element is stretched during its oscillation.

- **POTENTIAL ENERGY PER UNIT LENGTH**

$$\frac{dU}{dx} = \frac{1}{2}\mu \omega^2 A^2 \cos^2(kx - \omega t)$$

- **RATE OF TRANSMISSION OF ELASTIC POTENTIAL ENERGY**

$$\frac{dU}{dt}\Big|_{avg} = \frac{1}{2} \times \frac{1}{2}\mu v \omega^2 A^2 \frac{1}{4}\mu v \omega^2 A^2$$

3. MECHANICAL ENERGY PER UNIT LENGTH

$$E_L = \frac{dE}{dx} = 2x\frac{1}{2}\mu \omega^2 A^2 \cos^2(kx - \omega t) = \mu \omega^2 A^2 \cos^2(kx - \omega t)$$

4. AVERAGE POWER TRANSMITTED

The average power transmitted by wave is equal to time rate of transmission of mechanical energy over integral wavelengths.

$$P_{avg} = \frac{1}{2}\rho s v \omega^2 A^2$$

5. ENERGY DENSITY

$$u = \frac{1}{2}\rho v w^2 A^2$$

6. INTENSITY

Intensity of wave (I) is defined as power transmitted per unit cross section area of the medium.

$$I = \rho s v \omega^2 \frac{A^2}{2s} = \frac{1}{2}\rho v w^2 A^2$$

PHASE DIFFERENCE BETWEEN TWO PARTICLES IN THE SAME WAVE:

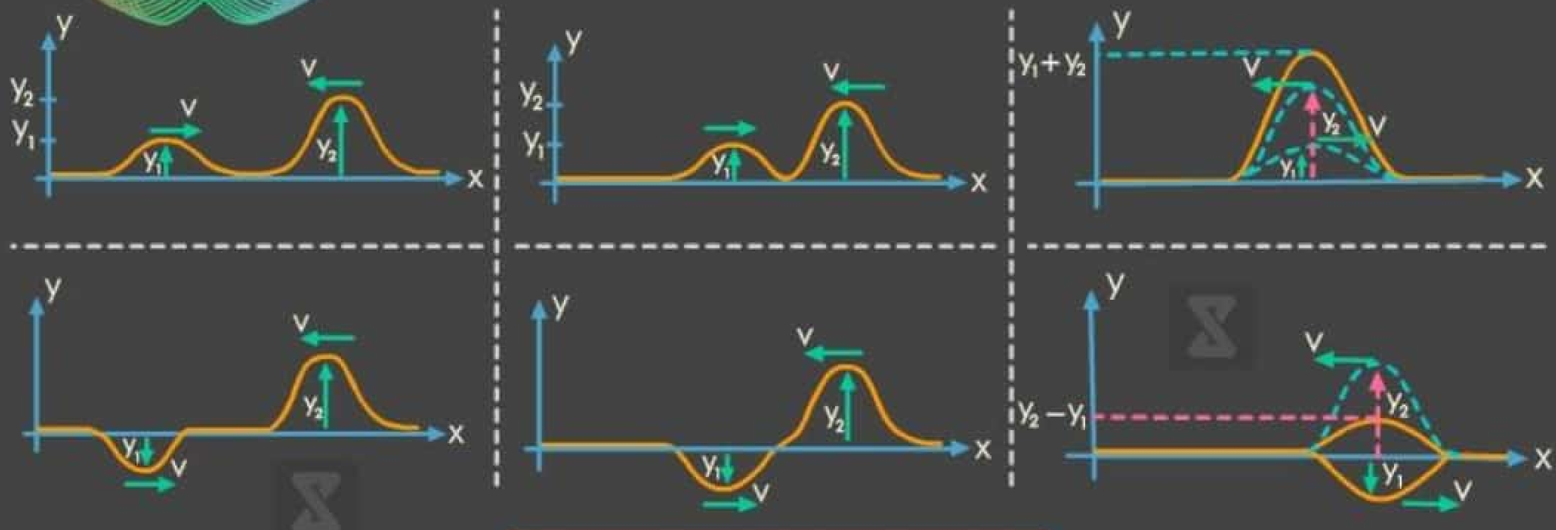
$$\Delta x \Longrightarrow \frac{\Delta\phi}{k}$$

Part I

SUPERPOSITION AND STANDING WAVES

PRINCIPLE OF SUPERPOSITION

When two or more waves superpose on a medium particle then the resultant displacement of that medium particle is given by the vector sum of the individual displacements produced by the component waves at that medium particle independently.



INTERFERENCE OF WAVES

- If the two waves are exactly in same phase, that is the shape of one wave exactly fits on to the other wave then they combine to double the displacement of every medium particle **as shown in figure (a)**. This phenomenon is called as constructive interference.
- If the superposing waves are exactly out of phase or in opposite phase then they combine to cancel all the displacements at every medium particle and medium remains in the form of a straight line **as shown in figure (b).** This phenomenon is called as destructive interference.

Figure (a) ⟶ Constructive Interference

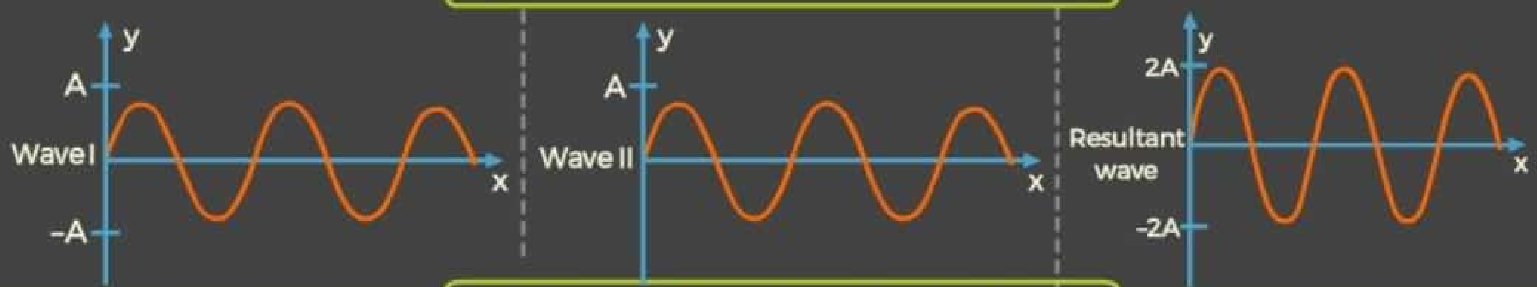
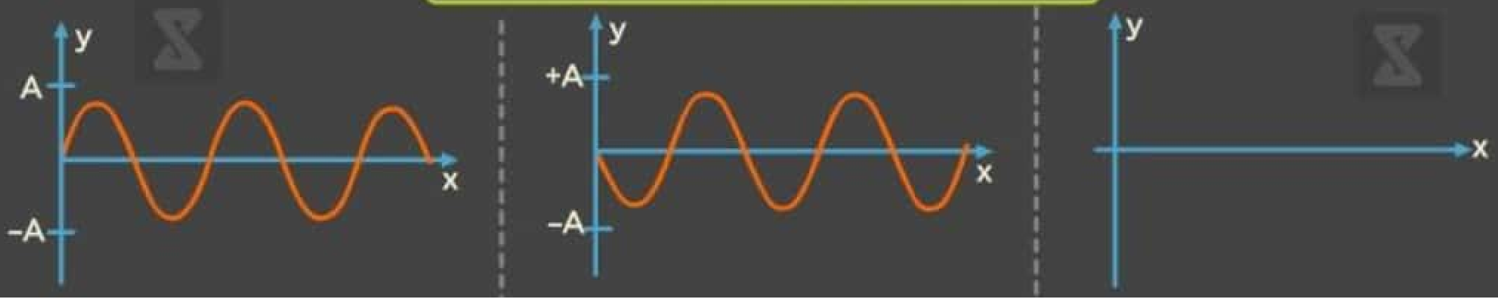


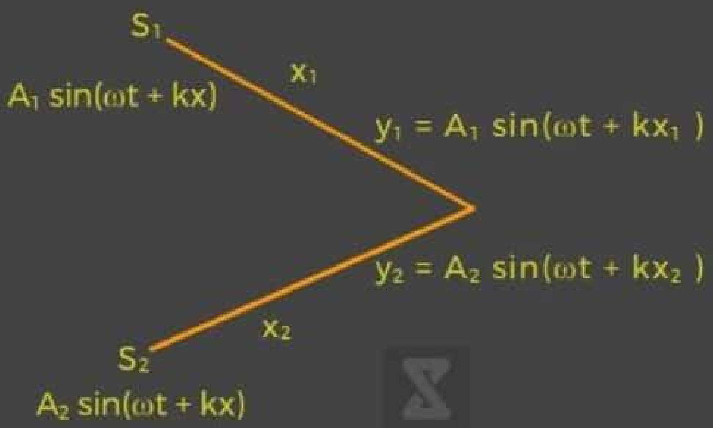
Figure (b) ⟶ Destructive Interference



SUPERPOSITION AND STANDING WAVES

Part II

ANALYTICAL TREATMENT OF INTERFERENCE OF WAVES



Whenever two or more than two waves superimpose each other, they give sum of their individual displacement.

$$y_1 = A_1 \sin(\omega t + kx_1)$$

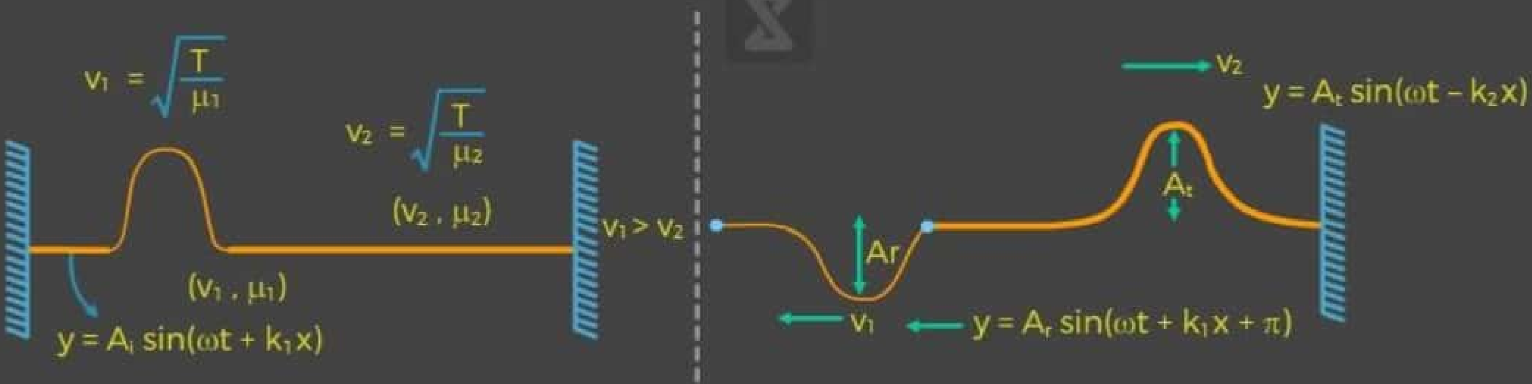
$$y_2 = A_2 \sin(\omega t + kx_2)$$

Due to superposition

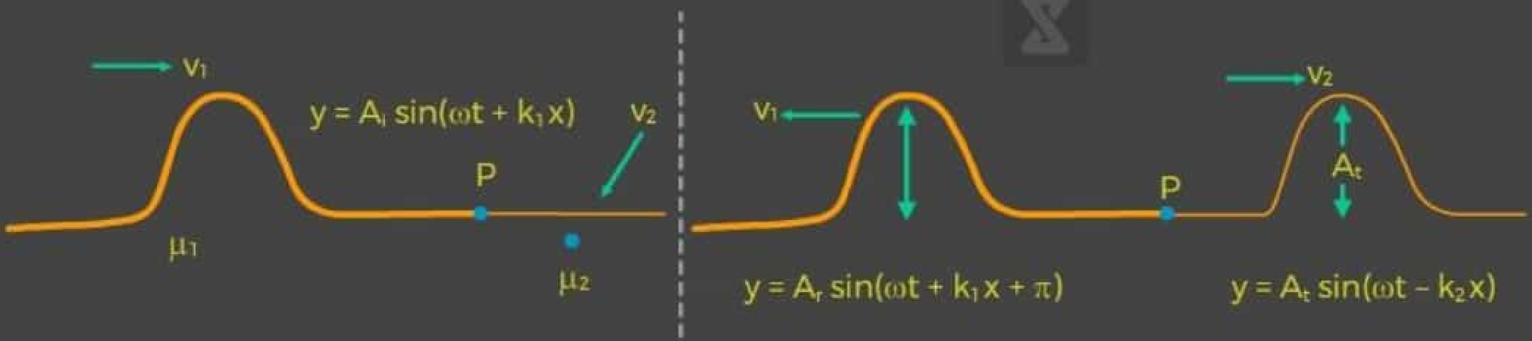
$$y_{net} = y_1 + y_2$$

REFLECTION AND TRANSMISSION BETWEEN TWO STRING

If a wave pulse is produced on a lighter string moving towards the friction, a part of the wave is reflected and a part is transmitted on the heavier string. The reflected wave is inverted with respect to the original one.



On the other hand if the wave is produced on the heavier string which moves toward the junction, a part will be reflected and a part transmitted, no inversion in waves shape will take place.



SUPERPOSITION AND STANDING WAVES

Part III

STANDING WAVES

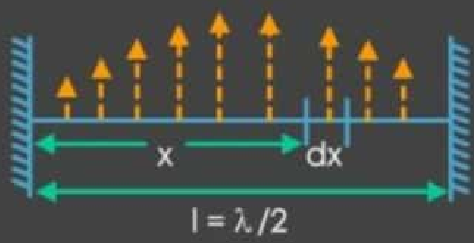
When two coherent waves travelling in opposite directions superpose then simultaneous interference of all the medium particles takes place. These waves interfere to produce a pattern of all the medium particles is what we call, a stationary wave.

ENERGY OF STANDING WAVE IN ONE LOOP

When all the particles of one loop are at extreme position then total energy in the loop is in the form of potential energy only. When the particles reaches its mean position then total potential energy converts into kinetic energy of the particles, so we can say that total energy of the loop remains constant.

Total kinetic energy at mean position is equal to total energy of the loop because potential energy at mean position is zero.

$$\text{Total K.E} = \frac{1}{2}\lambda A^2 \omega^2 \mu$$



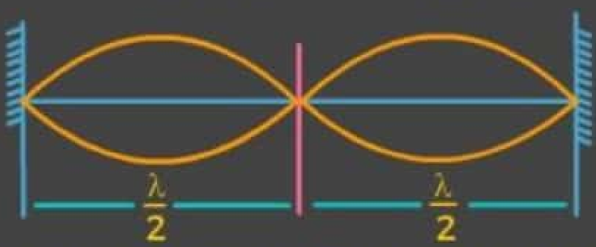
STATIONARY WAVES IN STRINGS

WHEN BOTH ENDS OF A STRING ARE FIXED

Fundamental Mode

The string vibrates in one loop in which the ends are the nodes and the centre is the antinode. This mode of vibration is known as the fundamental mode and frequency of vibration is known as the fundamental frequency or first harmonic.

$$f_1 = \frac{V}{2L}$$



First Overtone

The frequency f_2 is known as second harmonic or first overtone.

$$f_2 = \frac{V}{L}$$

Second Overtone

The frequency f_3 is known as third harmonic or second overtone.

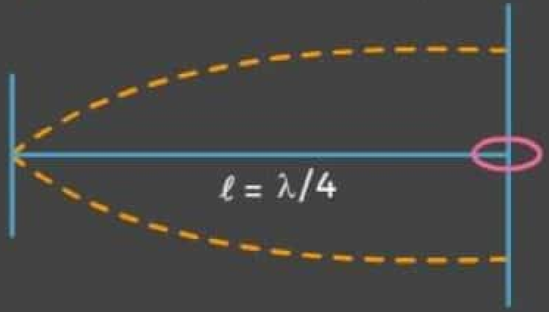
$$f_3 = \frac{3v}{2L}$$

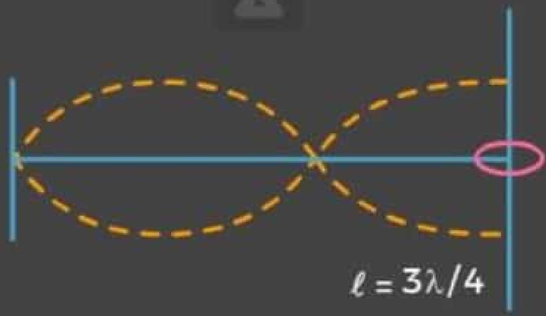
SUPERPOSITION AND STANDING WAVES

Part IV

- **When one end of the string is fixed and other is free**

Note- Free end acts as antinode





$f = \frac{1}{4\ell}\sqrt{\frac{T}{\mu}}$ fundamental or I^{st} harmonic

$f = \frac{3}{4\ell}\sqrt{\frac{T}{\mu}}$ III^{rd} harmonic or I^{st} overtone

In general : $f = \frac{(2n+1)}{4\ell}\sqrt{\frac{T}{\mu}}$ ($(2n+1)^{th}$ harmonic, n^{th} overtone)

S.No.	Travelling waves	Stationary waves
1.	These waves advance in a medium with a definite velocity	These waves remain stationary between two boundaries in the medium.
2.	In these waves, all particles of the medium oscillate with same frequency and amplitude.	In these waves, all particles except nodes oscillate with same frequency but different amplitudes. Amplitude is zero at nodes and maximum at antinodes.
3.	At any instant, phase of vibration varies continuosly from one particle to the other i.e. phase difference between two particles can have any value between 0 and $2x$	At any instant, the phase of all particles between two successive nodes is the same, but phase of particles on one side of a node is opposite to the phase of particles on the other side of the node, i.e. phase difference between any two particles can be either 0 or π
4.	In these waves, at no instant all the particles of the medium pass through their mean positions simultaneously.	In these waves, all particles of the medium pass through their mean position simultaneously twice in each time period.
5.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.

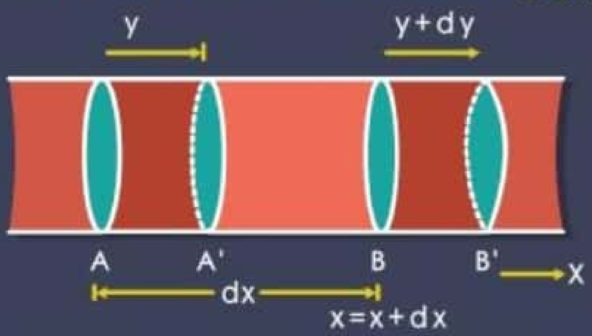
Part I

SOUND WAVE

PROPAGATION OF SOUND WAVES

Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

COMPRESSION WAVES



When a longitudinal wave is propagated in a gaseous medium, it produces compression and rarefaction in the medium periodically.

Velocity and Acceleration of particle :

General equation of wave is given by

$$y = A \sin(\omega t - kx)$$

$$V_P = \frac{\partial y}{\partial t} = A\,\omega \cos(\omega t - kx)$$

VELOCITY OF SOUND/LONGITUDINAL WAVES IN SOLIDS

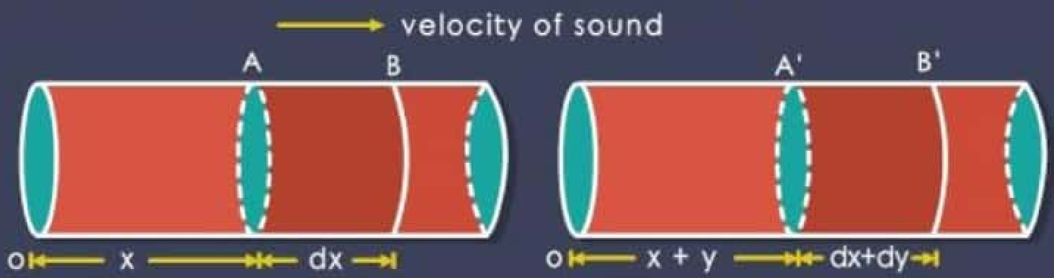
- **In Solid** $v = \sqrt{\frac{Y}{\rho}}$

  Y = Young Modulus

- **In Fluid** $v = \sqrt{\frac{B}{\rho}}$

  B = Bulk Modulus

- **In Gas** $B = -V\frac{dP}{dV}$



Newton's Formula for velocity of Sound in Gases, $v = \sqrt{\frac{P}{\rho}}$

Laplace Correction, $v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$

Effect of Temperature on Velocity of Sound, $\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$

Where,
P = Pressure
ρ = Density
V = Volume
T = Temperature

Part II

LONGITUDINAL STANDING WAVES

Two longitudinal waves of same frequency and amplitude, travelling in opposite directions interfere to produce a standing wave.

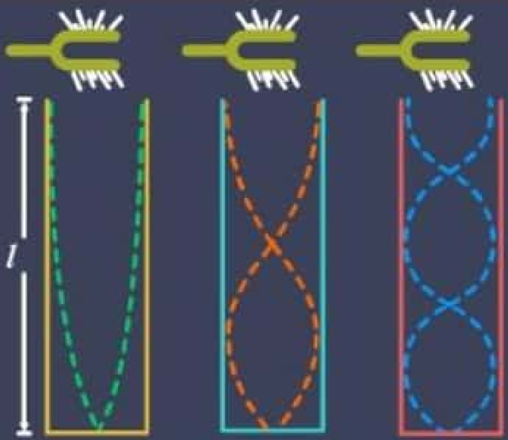
If the two interfering waves are given by:

$$p_1 = p_0 \sin(\omega t - kx) \text{ and } p_2 = p_0 \sin(\omega t + kx + \phi)$$

$$p = p_0 \sin\left(\omega t + \frac{\phi}{2}\right)$$

WAVES IN A VIBRATING AIR COLUMN

Vibration of Air in a Closed Organ Pipe



Fundamental frequency of oscillations of closed organ pipe of length l is given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{4l}$$

n_1 → Fundamental Frequency

v → Velocity

λ → Wavelength

l → Length of organ pipe

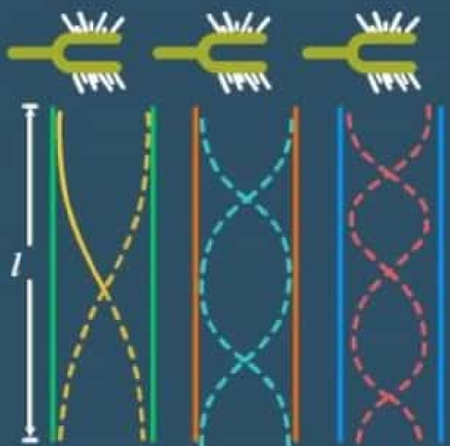
Vibration of Air in Open Organ Pipe

$$\lambda = 2l$$

The fundamental frequency of organ pipe can be given as

$$n_1 = \frac{v}{\lambda} = \frac{v}{2l}$$

$$f = \frac{nv}{2\ell}$$



End Correction

The displacment antinode at an open end of an organ pipe lies slightly outside the open end. The distance of the antinode from the open end is called end correction and its value is given by

$$e = 0.6\ r$$

where r = radius of the organ pipe, and

$$f_{closed} = \frac{v}{4(\ell + 0.6r)} \qquad f_{open} = \frac{v}{2(\ell + 1.2r)}$$

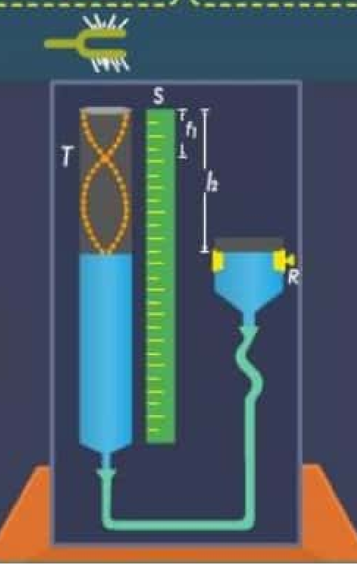
Resonance Tube

This is an apparatus used to determine the velocity of sound in air experimentally and also to compare frequencies of two tuning forks.

$$\lambda = 2(l_2 - l_1)$$

Thus, sound velocity in air can be given as

$$v = n_0 \lambda = 2n_0(l_2 - l_1)$$

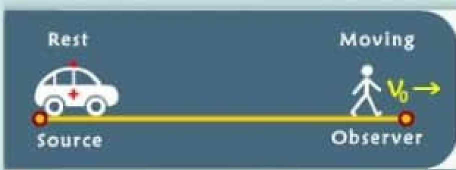


DOPPLER EFFECT

DOPPLER EFFECT: The shift in frequency of a wave emitted by a source moving relative to an observer as perceived by the observer: the shift is to higher frequencies when the source approaches and to lower frequencies when it recedes.

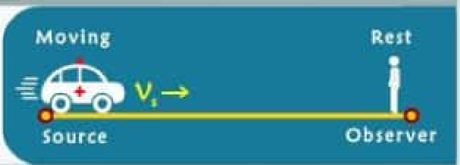
v_s = Speed of source f = Original frequency f' = Apparent frequency v_0 = Speed of observer v = Speed of sound in air

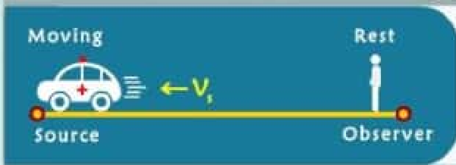
$$f' = \left(\frac{v + v_0}{v}\right) f$$



$$f' = \left(\frac{v - v_0}{v}\right) f$$

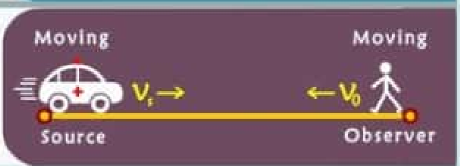
$$f' = \left(\frac{v}{v - v_s}\right) f$$

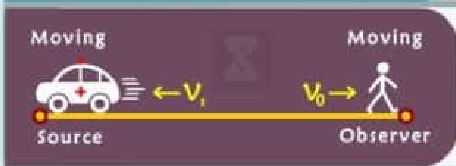




$$f' = \left(\frac{v}{v + v_s}\right) f$$

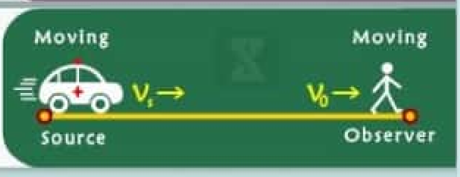
$$f' = \left(\frac{v + v_0}{v - v_s}\right) f$$

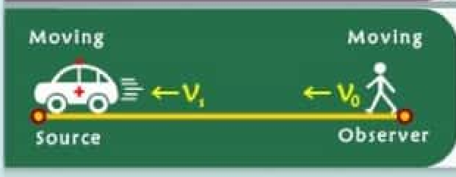




$$f' = \left(\frac{v - v_0}{v + v_s}\right) f$$

$$f' = \left(\frac{v - v_0}{v - v_s}\right) f$$





$$f' = \left(\frac{V + V_0}{V + V_s}\right) f$$

Shortcut Trick

Whenever source moves towards observer, then do substraction in denominator and vice-versa.

Whenever observer moves towards source, then do addition in numerator and vice-versa.