4
CHAPTER

Method of Indeterminate Analysis

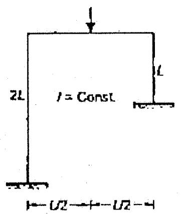
Q.1 Deformation in structures are caused by
(a) external forces only
(b) temperature
(c) wind and earthquake
(d) forces and temperature

Q.2 In the case of a roller support the net constraint along which of the following direction must be zero?
(a) Parallel to the axis of the member
(b) Parallel to the plane on which the roller rests
(c) Perpendicular to the roller
(d) None of these

Q.3 The extensional rigidity of an element is
(a) AE/L (b) $4\ EI/L$
(c) $6\ EI/L^2$ (d) AE
where
A = area of cross section,
I = sectional moment of interia,
L = length and
E = Young's modulus

Q.4 Unequal settlements in the supports of a statically determinate structure develop
(a) reactions from supports
(b) member forces
(c) no reactions
(d) forces in limited members

Q.5 A rigid-jointed plane frame as shown below



(a) will sway to left (b) will sway to right
(c) will not sway (d) none of these

Q.6 Consider the following statements related to Castigliano's theorem:
1. It provides a powerful tool for the analysis of statically indeterminate structures
2. It is based on the energy concept
3. It can be readily derived from Betti's generalized reciprocal theorem

Which of these statement is/are correct?
(a) both 1 and 2 (b) both 1 and 3
(c) only 1 (d) 1, 2 and 3

Q.7 Consider the following statements:
1. Column analogy method is a force method of analysis.
2. Flexibility of a structure is the displacement caused by a unit force.
3. Stiffness of a structure is the force required for unit displacement.

Which of these statements is/are correct?
(a) both 1 and 2 (b) both 1 and 3
(c) both 2 and 3 (d) 1, 2 and 3

Q.8 The total strain energy of a member on account of axial force (S) is given by

(a) $\int \frac{Sdx}{2AE}$ (b) $\int \frac{S^2dx}{2EI}$

(c) $\int \frac{S^2dx}{EA}$ (d) $\int \frac{S^2dx}{2EA}$

Q.9 The carryover factor in a prismatic member whose far end is hinged is
(a) 0 (b) 1/2
(c) 3/4 (d) 1

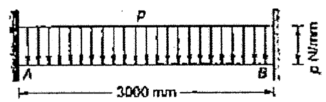
Q.10 If one end of a prismatic beam AB with fixed ends is given a transverse displacement Δ without any rotation, then the transverse reactions at A or B due to displacement is

(a) $\frac{6EI\Delta}{L^2}$ (b) $\frac{6EI\Delta}{L^3}$

(c) $\frac{12EI\Delta}{L^2}$ (d) $\frac{12EI\Delta}{L^3}$

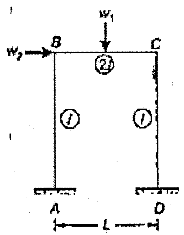
Q.11 The three moments equation is applicable only when
(a) the beam is prismatic
(b) there is no settlement of supports
(c) there is no discontinuity such as hinges within the span
(d) the spans are equal

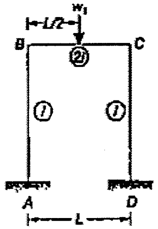
Q.12 The fixed-ended beam AB has a span of 3000 mm and is of uniform cross-section. The plastic moment capacity (M_p) of the beam's cross-section is 1.5×10^8 N-mm The beam will behave like a simply supported beam when the magnitude of the distributed load p attains a value equal to

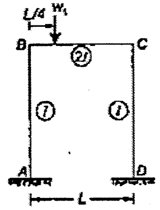


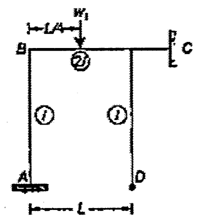
(a) 100 N/mm (b) 20 N/mm
(c) 200 N/mm (d) 500 N/mm

Q.13 Which of the following structure will experience sway?

1.


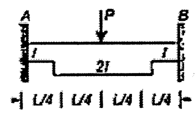
2.


3.


4.


Select the correct answer using the codes given below:
Codes:
(a) 1 and 2 (b) 1 and 3
(c) 3 and 4 (d) 2 and 4

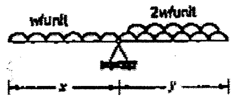
Q.14 In the structure shown in the given figure, the fixed end moment at joint A is



(a) $\frac{PL}{16}$ (b) $\frac{5PL}{48}$

(c) $\frac{7PL}{64}$ (d) $\frac{PL}{8}$

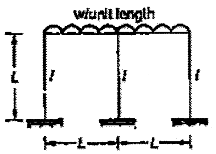
Q.15 The structure shown in the given figure is stable if



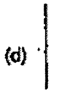
(a) $x = 2y$ (b) $2x = y$

(c) $\sqrt{2x} = y$ (d) $x = \sqrt{2}y$

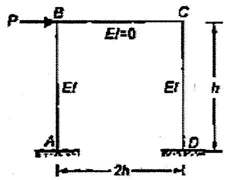
Q.16 The correct bending moment diagram for the middle column of the frame as shown is



(a)  (b) 

(c)  (d) 

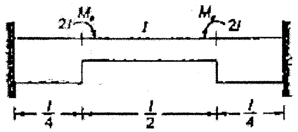
Q.17 If the flexural rigidity of the beam BC of the portal frame shown in the given figure is assumed to be zero, then the horizontal displacement of the beam would be



(a) $\frac{Ph^3}{3EI}$ (b) $\frac{Ph^3}{24EI}$

(c) $\frac{Ph^3}{12EI}$ (d) $\frac{Ph^3}{6EI}$

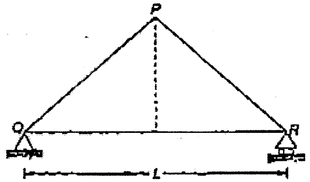
Q.18 What is the fixed end moment for the beam shown in the figure given below?



(a) M_0 (b) $\frac{2M_0}{3}$

(c) $\frac{M_0}{3}$ (d) $\frac{M_0}{6}$

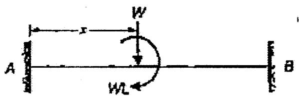
Q.19 A statically determinate truss PQR is subjected to a temperature rise ΔT. A, E and α are constant for all members. The force in member QR due to this temperature increase is



(a) $EA \propto DT$ (b) $\sqrt{2}EA\alpha\Delta T$

(c) $EA\Delta T/L$ (d) None of these

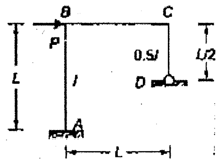
Q.20 A fixed end beam carries a load W and clockwise moment WL at a distance 'x' from one of the fixed ends. Then the value of 'x' such that the moment at one of the end is zero will be



(a) $\frac{L}{(\sqrt{2}+1)}$ (b) $\frac{L}{2}$

(c) $\frac{\sqrt{2}L}{4}$ (d) $\frac{L}{(\sqrt{2}-1)}$

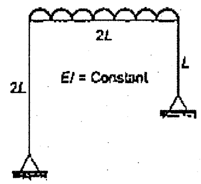
Q.21 The given figure shows a portal frame with one end fixed and other hinged. The ratio of the fixed end moments $\frac{M_{DA}}{M_{CD}}$ due to side sway will be



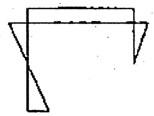
(a) 1.0 (b) 2.0

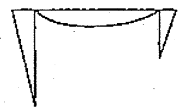
(c) 2.5 (d) 1.0

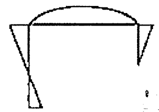
Q.22 The given figure shows a portal frame with loads.

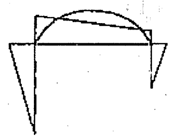


The bending moment diagram for this frame will be

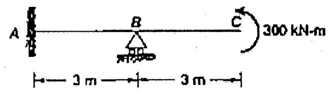
(a) 

(b) 

(c) 

(d) 

Q.23 A propped cantilever of uniform flexural rigidity is loaded as shown in the given figure. The bending moment at fixed end A is



(a) 150 kN-m hogging
(b) 300 kN-m sagging
(c) 150 kN-m sagging
(d) 300 kN-m hogging

Q.24 A fixed beam AB of span L carries a uniformly distributed load w per unit length. During loading, the support B sinks downwards by an amount δ. If $\delta = \frac{wL^4}{72EI}$, what is the fixed end moment at B?

(a) $\frac{wL^2}{12}$ (b) $\frac{wL^2}{6}$

(c) $\frac{6EI\delta}{L^2}$ (d) Zero

Q.25 Match List-I (Names of persons with whom the methods of analysis are associated) with List-II (Method) and select the correct answer using the codes given below the lists:

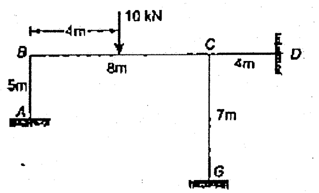
List-I
1. Clapeyron
2. Hardy cross
3. Lame
4. Euler

List-II
A. Moment distribution method
B. Method for determining crippling load on a column
C. Theorem of Three moment
D. Thick cylinders

Codes:

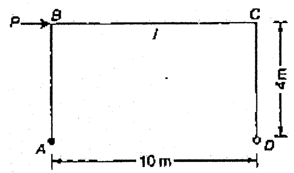
	1	2	3	4
(a)	C	A	B	D
(b)	A	C	D	B
(c)	C	A	D	B
(d)	A	B	D	C

Q.26 The distribution factors for members *CB*, *CD* and *CG* for the frame shown in the figure (*EI* constant) will be respectively



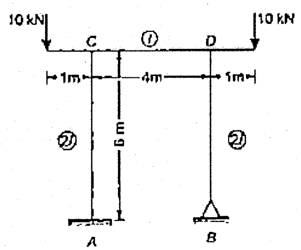
(a) 0.24, 0.28 and 0.48
(b) 0.24, 0.48 and 0.28
(c) 0.48, 0.24 and 0.28
(d) 0.28, 0.48 and 0.24

Q.27 A portal frame is shown in the given figure. If $\theta_B = \theta_C = \dfrac{400}{EI}$ radians, then the value of moment at *B* will be



(a) 120 kNm
(b) 240 kNm
(c) 360 kNm
(d) 480 kNm

Q.28 The possible direction of sway of the rigid frame as shown in figure is



(a) is towards left
(b) is towards right
(c) does not exist as there is no sway
(d) cannot be ascertained

Q.29 Match List-I (Method) with List-II (Factor) and select the correct answer using the code given below the lists:

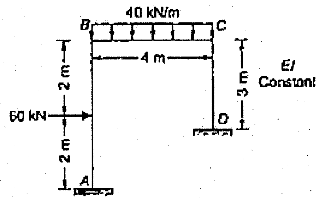
List-I
A. Moment distribution method
B. Slope deflection method
C. Kani's method
D. Force method

List-II
1. Rotation factor
2. Flexibility
3. Hardy Cross
4. Displacements
5. Stiffness matrices

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	2	1	5	3
(c)	2	4	1	3
(d)	3	1	5	2

Q.30 The figure below shows a rigid frame. If *D* is lateral translation of the joints, slope deflection equation for the member *BA* can be written as:



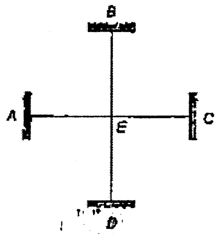
(a) $M_{BA} = -30 + \dfrac{2EI}{4}\left(20 - \dfrac{3\Delta}{4}\right)$

(b) $M_{BA} = -30 - \dfrac{2EI}{4}\left(20 + \dfrac{3\Delta}{4}\right)$

(c) $M_{BA} = -30 + \dfrac{2EI}{4}\left(20 - \dfrac{3\Delta}{4}\right)$

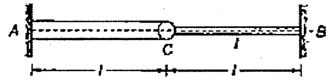
(d) $M_{BA} = 30 + \dfrac{2EI}{4}\left(20 + \dfrac{3\Delta}{4}\right)$

Q.31 Four identical beams *AE*, *BE*, *CE* and *DE* has been rigidly jointed at *E*. The point *C* slips and rotates along with member firmly fixed at *E*.



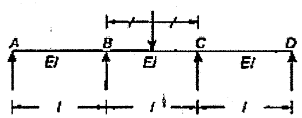
Which one among the following is correct?
(a) There is no moment on the members.
(b) Except at *C*, there is no moment on the members of frame.
(c) Except at *C* and *E* for member *EC*, no moment will be there on other members.
(d) All the members are subjected to moment.

Q.32 A fixed beam *AB* with a central hinge *C* is built of two components. *AC* is rigid and *CB* has a moment of inertia *I*. When support *A* yields and rotates through θ, what is the moment at *B*?



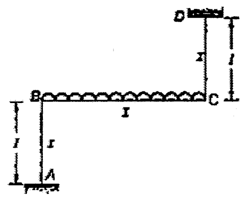
(a) $\dfrac{EI\theta}{l}$
(b) $\dfrac{2EI\theta}{l}$
(c) $\dfrac{3EI\theta}{l}$
(d) $\dfrac{4EI\theta}{l}$

Q.33 For a symmetrical continuous beam as shown in the figure given below, which one of the following is correct in respect of distribution factors at *B*?



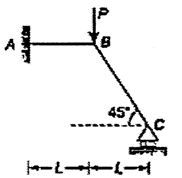
(a) $\delta_{BA} : \delta_{BC} = 3:4$
(b) $\delta_{BA} : \delta_{BC} = 1:2$
(c) $\delta_{BA} : \delta_{BC} = 3:2$
(d) $\delta_{BA} : \delta_{BC} = 3:8$

Q.34 A plane frame is loaded as shown in figure. The rotations are indicated as θ_B and θ_C and sway is indicated by the symbol Δ. For the given frame which of the following statement is correct?



(a) $\theta_B = \theta_C$; Δ is present
(b) $\theta_B = -\theta_C$; Δ is present
(c) $\theta_B = \theta_C$; Δ is absent
(d) $\theta_B = \theta_C$; Δ is absent

Q.35 The shear equation for the portal frame as shown below is



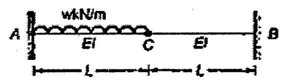
(a) $\dfrac{M_{AB}+M_{BC}}{L\sqrt{2}} + \dfrac{M_{BC}}{L} + P = 0$

(b) $\dfrac{M_{AB}+M_{BA}}{L} - \dfrac{M_{BC}}{L} + P = 0$

(c) $\dfrac{M_{AB}+M_{BA}}{L} - \dfrac{M_{BC}}{L\sqrt{2}} + P = 0$

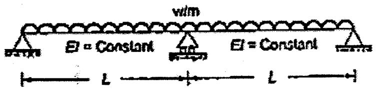
(d) $\dfrac{M_{AB}+M_{BA}}{L} + \dfrac{M_{BC}}{L} + P = 0$

Q.36 What is the reaction on the hinge *C* for the beam as shown in the figure?



(a) $\dfrac{3}{8}wL$
(b) $\dfrac{1}{2}wL$
(c) $\dfrac{1}{4}wL$
(d) $\dfrac{3}{16}wL$

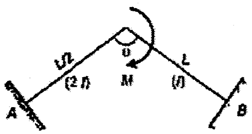
Q.37 If a continuous beam as shown in figure below is subjected to uniformly distributed load of w/m run, then the value of central support reaction becomes zero if central support sinks by



(a) $\frac{7wL^4}{24EI}$ (b) $\frac{5wL^4}{384EI}$

(c) $\frac{10wL^4}{384EI}$ (d) $\frac{5}{24}\frac{wL^4}{EI}$

Q.38 A plane rigid jointed steel frame with fixed supports is acted upon by a couple M as shown below. In order to find moment induced at the fixed supports, moment distribution was carried out. The ratio of moment at support A to that at B is



(a) 1 : 2 (b) 2 : 1

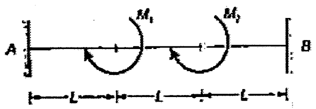
(c) 1 : 4 (d) 4 : 1

Q.39 A two span continuous beam ABC ($AB = BC$) is fixed at A and simply supported at B and C. The beam is only loaded in the span BC. The flexural rigidities of two spans are equal and supports are at the same level. If M_A and M_B are the moments at support A and B respectively, then which of the following relationships is correct?

(a) $M_A = M_B$ (b) $M_A = -2M_B$

(c) $M_A = 0.5\ M_B$ (d) $M_A = -0.5\ M_B$

Q.40 The magnitude of fixed end moment at support 'A' of the fixed beam loaded as shown below is



(a) $\frac{M_1+M_2}{3}$ (b) $\frac{M_1+M_2}{2}$

(c) $\frac{M_1}{3}$ (d) $\frac{M_2}{3}$

Q.41 The following methods are used for structural analysis:

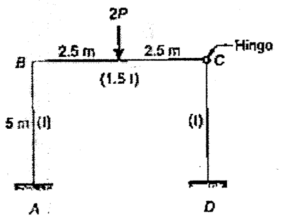
1. Macaulay's method
2. Column analogy method
3. Kani's method
4. Method of sections

Which of the above methods are used for indeterminate structural analysis?

(a) 1 and 2 (b) 1 and 3

(c) 2 and 3 (d) 2, 3 and 4

Q.42 The rotation at joint C of portal frame as shown in figure for members CB and CD will be



(a) Same (b) Different

(c) Zero (d) None of these

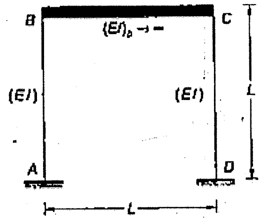
Q.43 A propped cantilever AB carries a uniformly distributed load of q/unit length. In this condition the moment reaction $M_A = \frac{qL^2}{8}$.

What is the clockwise moment required at B to make the slope of the deflection curve equal to zero?

(a) $\frac{qL^2}{8}$ (b) $\frac{qL^2}{16}$

(c) $\frac{qL^2}{12}$ (d) $\frac{qL^2}{4}$

Q.44 A portal frame is shown in figure. Beam is having infinite flexural rigidity and all members are inextensible.



If a point load P is applied at B in the direction from B to C then magnitude of moment M_{AB} will be

(a) PL (b) $\frac{PL}{2}$

(c) $\frac{PL}{4}$ (d) $\frac{PL}{8}$

Q.45 Which of the following statement about moment distribution method is correct?

(a) It is based on simultaneous equations of equilibrium.

(b) It is not applicable for beam with variable cross-section.

(c) The carryover factor cannot be greater than one.

(d) This method yield exact results.

Q.46 Consider the following statements: Williot - Mohr diagram is used to determine the deflection in

1. an arch
2. a truss
3. a rigid frame

Which of these statement(s) is(are) correct?

(a) only 1 (b) only 2

(c) only 3 (d) 1, 2 and 3

Q.47 A fixed beam AB of flexural rigidity EI and span L carries a uniformly distributed load, w per unit length. The amount of vertical deflection that the support B must settle for no fixed end moment at B is

(a) $\frac{wl^4}{24EI}$ (b) $\frac{wl^4}{36EI}$

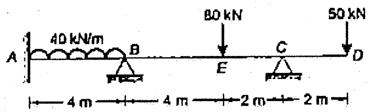
(c) $\frac{wl^4}{72EI}$ (d) $\frac{wl^4}{84EI}$

Q.48 A two span continuous beam ABC was subjected to concentrated load of 50 kN at middle of span BC. Under the action of load, the middle of the span BC underwent a downward deflection of 0.05 m and the middle of span AB underwent an upward deflection of 0.02 m. When 20 kN and 30 kN loads are applied simultaneously at middle of spans AB and BC respectively, then deflection at middle of span BC is

(a) 0.035 m (b) 0.024 m

(c) 0.022 m (d) 0.044 m

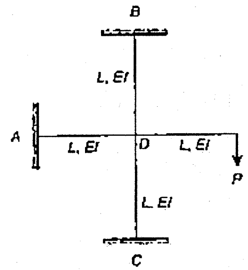
Q.49 A continuous beam $ABCD$ is shown in figure. The restraining moment at C is ________.



(a) 28.89 kN-m (b) 40.89 kN-m

(c) 171.11 kN-m (d) Zero

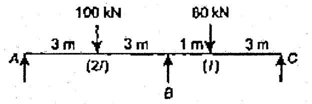
Q.50 In the following diagram, all the members have same flexural rigidity. The fixed end moment developed at support A will be



(a) $\frac{PL}{8}$ (b) $\frac{PL}{6}$

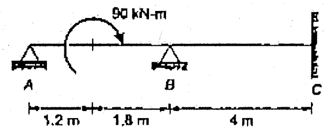
(c) $\frac{PL}{4}$ (d) $\frac{PL}{3}$

Q.51 Based on figure as shown below, the reaction at A is



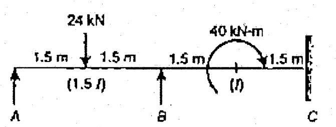
(a) 30 kN (b) 37 kN
(c) 40 kN (d) 45 kN

Q.52 In the figure as given below, fixed end moment at B, is



(a) 28.80 kN-m (b) 10.80 kN-m
(c) 0 (d) 12 kN-m

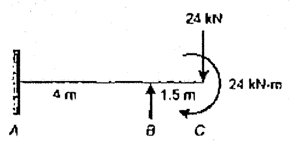
Q.53 On the basis of figure as shown below, the relative stiffness of members BA and BC will be respectively



(a) $\frac{8I}{24}, \frac{9I}{24}$ (b) $\frac{8I}{24}, \frac{17I}{24}$

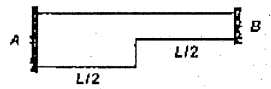
(c) $\frac{9I}{24}, \frac{17I}{24}$ (d) $\frac{9I}{24}, \frac{8I}{24}$

Q.54 In the figure given below, the vertical reaction at A will be



(a) 22.5 kN (b) 46.5 kN
(c) 48 kN (d) 50 kN

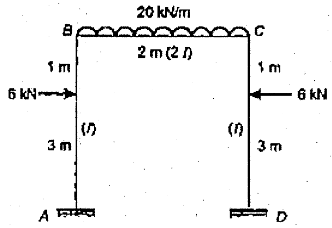
Q.55 In the figure shown below, what will be the carry over factor from A to B?



(a) $\frac{1}{2}$ (b) 1.0

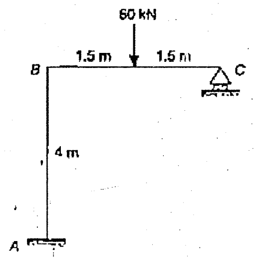
(c) $<\frac{1}{2}$ (d) $>\frac{1}{2}$

Q.56 In the figure given below, if slope at B is i_b and slope at C is i_c, then



(a) $i_b = i_c$ (b) $i_b = -i_c$
(c) $i_b = 2i_c$ (d) $i_c = 2i_b$

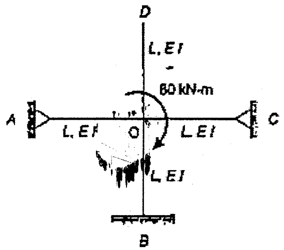
Q.57 In the figure given below, if δ is the 'deflection' and the slope at B is i_b, then M_{ab} will be



(a) $\frac{1}{2}EIi_b - \frac{3}{8}EI\delta$ (b) $EIi_b - \frac{3}{8}EI\delta$

(c) $EIi_b - \frac{3}{4}EI\delta$ (d) $EIi_b - \frac{1}{2}EI\delta$

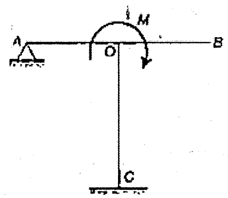
Q.58 Consider the frame as shown in the figure. A moment of 80 kN-m acts at the joint O. Given that the support at B fails at any moment greater than 18 kN-m, then the given structure is



(a) unsafe
(b) safe
(c) just safe
(d) cannot be determined from the given data

Q.59 Assertion (A): In the analysis of rigid frames, the usual practice is to consider the strain energy due to flexure only.
Reason (R): The strain energies due to axial and shear forces are usually quite small as compared to that of flexure.
(a) Both Assertion (A) and Reason (R) are individually true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are individually true but Reason (R) is NOT the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

Q.60 For the simple frame, whose all the members are identical and are of length L and flexural rigidity EI, is shown below.

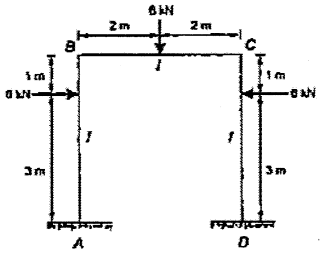


The correct set of bending moments (M_{OA}, M_{OB}, M_{OC}) that will develop at O in the three beam elements OA, OB and OC respectively, are

(a) $\left(\frac{3M}{11}, \frac{4M}{11}, \frac{4M}{11}\right)$ (b) $\left(\frac{3M}{8}, \frac{M}{8}, \frac{4M}{8}\right)$

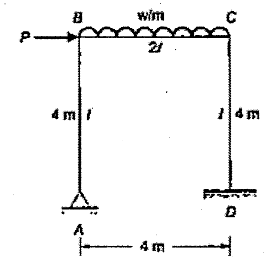
(c) $\left(\frac{M}{3}, \frac{M}{3}, \frac{M}{3}\right)$ (d) $\left(\frac{3M}{7}, 0, \frac{4M}{7}\right)$

Q.61 The slope deflection equation at end B of the member AB for the frame shown in figure is given by

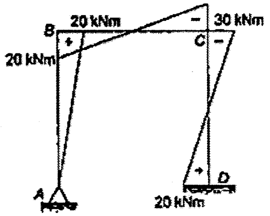


(a) $M_{BA} = 3.375 + EI\theta_B$
(b) $M_{BA} = -3.375 + EI\theta_B$
(c) $M_{BA} = -1.125 + 0.5EI\theta_B$
(d) $M_{BA} = 3.375 + 0.5EI\theta_B$

Q.62 A portal frame is as shown in figure (i). The end moment diagram for the same portal frame is shown in figure (ii). The value of sway force P is



(i)



(ii)

+ → Sagging on outer face
– → Hogging

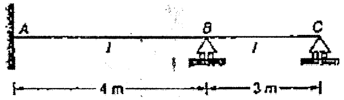
(a) 7.5 kN (b) 10 kN
(c) 17.5 kN (d) 25 kN

Q.63 Statement (I): In a symmetrical portal frame with one end fixed and the other end hinged, the fixed end bending moment induced at each end of the horizontal member of the frame due to the sinking of hinge support by an amount Δ is directly proportional to Δ.

Statement (II): The fixed end bending moment induced at each end of the horizontal member of the frame is given by $\frac{6EI\Delta}{L^2}$.

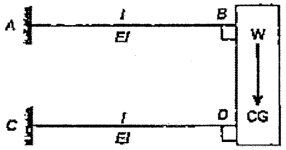
(a) Both Statement (I) and Statement (II) are individually true and Statement (II) is the correct explanation of Statement (I).
(b) Both Statement (I) and Statement (II) are individually true but Statement (II) is NOT the correct explanation of Statement (I).
(c) Statement (I) is true but Statement (II) is false.
(d) Statement (I) is false but Statement (II) is true.

Q.64 For the beam shown in figure, moment required to produce unit slope at B is _____ EI kNm.



(a) 3 (b) 2
(c) 4 (d) 7

Q.65 Find the vertical deflection δ of the rigid block as shown in figure.



(a) $\frac{wl^3}{12EI}$ (b) $\frac{wl^3}{24EI}$
(c) $\frac{wl^3}{36EI}$ (d) $\frac{wl^3}{6EI}$

■■■■

Answers **Method of Indeterminate Analysis**

1. (d)	2. (b)	3. (d)	4. (c)	5. (a)	6. (d)	7. (d)	8. (d)	9. (a)	10. (d)
11. (c)	12. (c)	13. (b)	14. (b)	15. (d)	16. (d)	17. (d)	18. (b)	19. (d)	20. (a)
21. (a)	22. (d)	23. (a)	24. (d)	25. (c)	26. (b)	27. (b)	28. (c)	29. (a)	30. (b)
31. (d)	32. (c)	33. (a)	34. (b)	35. (b)	36. (d)	37. (d)	38. (d)	39. (c)	40. (d)
41. (c)	42. (b)	43. (c)	44. (c)	45. (c)	46. (b)	47. (c)	48. (c)	49. (a)	50. (b)
51. (b)	52. (a)	53. (d)	54. (a)	55. (c)	56. (b)	57. (a)	58. (b)	59. (a)	60. (d)
61. (a)	62. (c)	63. (a)	64. (b)	65. (b)					

Explanations **Method of Indeterminate Analysis**

1. (d)
Deformation in structures are caused by forces and temperature.

3. (d)
Product of E and A is known as extensional rigidity.

4. (c)
In statically determinate structure, structure develop no reactions on settlement while in statically indeterminate structure, structure develops reactions.

5. (a)
A frame will sway towards the side which has lesser I/L (relative stiffness)

8. (d)
Strain energy due to axial loading $= \int \frac{S^2 dx}{2AE}$

Strain energy due to bending $= \int \frac{M_x^2 ds}{2EI}$

Strain energy due to shear $= \int \frac{q}{2G} dV$

Strain energy due to shear force $= \int \frac{S^2 ds}{2GA_s}$

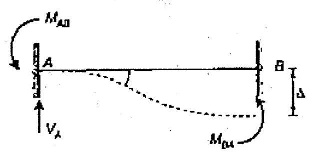
where A_s = Shear area

Strain energy due to torsion $= \frac{T^2 L}{2GI_p}$

Strain energy in hollow circular shaft
$= \frac{\tau^2_{max} \cdot V}{4G} \times \frac{D^2 + d^2}{D^2}$

D = outer diameter of circular shaft
d = inner diameter of circular shaft

10. (d)



$$M_{AB} = M_{BA} = \frac{-6EI\Delta}{L^2}$$

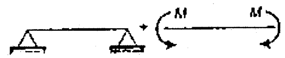
$$\Sigma M_B = 0$$
$$V_A \times L = M_{AB} + M_{BA}$$
$$V_A \times L = \frac{6EI\Delta}{L^2} \times 2$$
$$V_A = \frac{12EI\Delta}{L^3}$$

11. (c)
Three moment equation was developed by Clepeyron and Bertot. It can be used for beams having different moment of inertia, settled supports, unequal spans but no discontinuity within the span.

12. (c)
$$\frac{pL^2}{12} = M_p = 1.5 \times 10^8 \text{ Nmm}$$
$$\Rightarrow p = \frac{12 \times 1.5 \times 10^8}{(3000)^2} = 200 \text{ N/mm}$$

14. (b)
$$\frac{\text{Area of bending moment diagram for half of beam}}{EI} = \text{zero}$$

Dividing fixed beam into two parts



$$\frac{1}{2} \cdot \frac{1}{4}\left(\frac{PL}{4EI}\right) + \frac{1}{2}\left(\frac{PL}{4} + \frac{PL}{2}\right) \cdot \frac{1}{2EI} \cdot \frac{L}{4} = \frac{ML}{4EI} + \frac{ML}{8EI}$$

$$\Rightarrow M = \frac{5PL}{48}$$

15. (d)
$$\frac{wx^2}{2} = \frac{2wy^2}{2} \Rightarrow x = \sqrt{2}y$$

16. (d)
There will be no moment in central column.

17. (d)
As the beam BC has no rigidity, so end B of column AB behaves as free end.

For a cantilever beam subjected to concentrated load at free end, the stiffness is $\frac{3EI}{l^3}$.

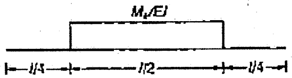
Stiffness of given system

$$= \frac{3EI}{h^3}+\frac{3EI}{h^3}=\frac{6EI}{h^3}$$

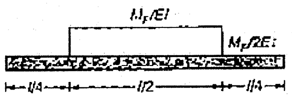
So displacement of point $B=\frac{Ph^3}{6EI}$

18. (b)
As the beam and loading are symmetrical so fixed end moments will also be same. From Mohr's theorem (the end rotations being zero) the area of free and fixed bending moment diagrams will be equal.

Free moment diagram



Fixed moment diagram



$$\frac{M_0}{EI}\times\frac{l}{2} = \frac{M_F}{2EI}\times\frac{l}{4}\times 2+\frac{M_F}{EI}\times\frac{l}{2}$$

$M_0=\frac{3}{2}M_F$ or $M_F=\frac{2}{3}M_0$.

19. (d)
In a determinate structure no stresses or forces are generated due to temperature change.

20. (a)

$$\frac{Wx(L-x)^2}{L^2} = \frac{WL(L-x)[2x-(L-x)]}{L^2}$$

$\Rightarrow x^2+2xL-L^2=0$

$\Rightarrow x=\frac{L}{(\sqrt{2}+1)}$

21. (a)
Due to sway, the deflection of point B will be equal to that of point C.

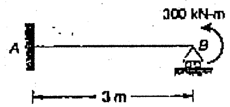
$$M_{BA} = \frac{6EI\Delta}{L^2}$$

$$M_{CD} = \frac{3E(0.5I)\Delta}{(L/2)^2}=\frac{6EI\Delta}{L^2}$$

$$\therefore \frac{M_{BA}}{M_{CD}} = 1.0$$

22. (d)
Due to unsymmetrical nature of frame, the lateral translation will occur towards left. Therefore horizontal force of equal and opposite nature will develop on the supports. So correct bending moment diagram will be (*d*).

23. (a)



The moment is transferred to point B directly.

If $M_{BA}=300$ kN-m

the $M_{AB}=\frac{1}{2}\times300=150$ kN-m as the carry over factor for beam is $\frac{1}{2}$. The direction of moment will be anti-clockwise i.e. hogging.

24. (d)
Fixed end moment at B due to

UDL $=\frac{wL^2}{12}$ (clockwise)

Fixed end moment at B due to sinking of support B

$=\frac{6EI\delta}{L^2}$ (Anti-clockwise)

$=\frac{6EI\times wL^4}{L^2\times72EI}$

$=\frac{wL^2}{12}$ (Anti-clockwise)

Hence final fixing moment at B

$$=\frac{wL^2}{12}-\frac{wL^2}{12}=0$$

26. (b)

Member	Relative stiffness	Distribution factor
CB	EI/8	0.24
CD	EI/4	0.48
CG	EI/7	0.28
Total	29EI/56	1.00

27. (b)
The deformed shape of structure will have translation at B and C.

$$M_{BC}=0+\frac{2EI}{10}(2\theta_B+\theta_C)$$

$$=\frac{2EI}{10}\times3\times\frac{400}{EI}=240\text{ kN-m}$$

30. (b)
Taking:

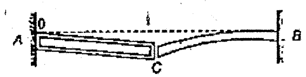
- anticlockwise end moment positive
- anticlockwise rotation at member end positive
- anticlockwise chord rotation positive

$$FEM_{BA} = \frac{60\times2\times2^2}{4^2}=-30\text{ kN-m}$$

$$M_{BA} = \left(-30+\frac{2EI}{4}\left(2\theta_B-\frac{3\Delta}{4}\right)\right)$$

31. (d)
Since the members are firmly fixed at E. It is a rigid joint at which all the members will have same rotation. So from slope deflection equations, all the members will have moment.

32. (c)
When support A yields and rotates through θ, the whole beam AC rotates due to its rigidity. The deflection diagram of beam AB is shown below.



The rotation at end C of beam BC is $\frac{3EI}{l}$.

Therefore moment at B is $\frac{3EI\theta}{l}$.

33. (a)
At joint B.

Stiffness of BA, $k_{BA}=\frac{3EI}{l}$

Stiffness of BC, $k_{BC}=\frac{4EI}{l}$

The distribution factors.

$$\delta_{BA}=\frac{k_{BA}}{k_{BA}+k_{BC}}=\frac{3EI/l}{\frac{3EI}{l}+\frac{4EI}{l}}=\frac{3}{7}$$

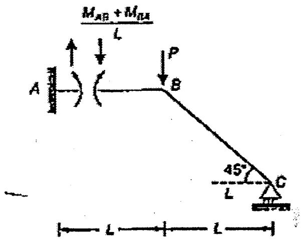
$$\delta_{BC}=\frac{k_{BC}}{k_{BA}+k_{BC}}=\frac{4EI/l}{\frac{3EI}{l}+\frac{4EI}{l}}=\frac{4}{7}$$

$\delta_{BA}:\delta_{BC}=3:4$

34. (b)
The structure is symmetrical but loading is anti-symmetrical about mid point of BC. If a symmetric structure is subjected to antisymmetric loading, the structural response is antisymmetric.

Hence, $\theta_B=-\theta_C$ and Δ is present.

35. (b)

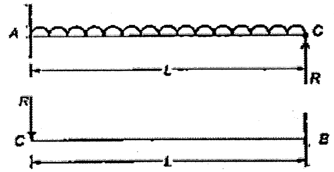


Taking moment of all forces about C

$$\frac{M_{AB}+M_{BA}}{L}\times L-M_{BC}+P\times L=0$$

$$\frac{M_{AB}+M_{BA}}{L}-\frac{M_{BC}}{L}+P=0$$

36. (d)
Let R be the reaction at hinge C.



By compatibility,

$$(\Delta_C)_{AC} = (\Delta_C)_{CB}$$

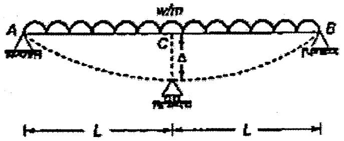
$$\frac{wL^4}{8EI} - \frac{RL^3}{3EI} = \frac{RL^3}{3EI}$$

$$\frac{2RL^3}{3EI} = \frac{wL^4}{8EI}$$

$$R = \frac{3}{16}wL$$

Hence option (d) is correct.

37. (d)



$$\Delta = \frac{5}{384}\frac{wL_{AB}^4}{EI}$$

$$\Delta = \frac{5}{384} \times \frac{w \times (2L)^4}{EI} = \frac{5 \times 16wL^4}{384EI}$$

$$\Delta = \frac{wL^4}{4.8EI} = \frac{5}{24}\frac{wL^4}{EI}$$

Hence the value of central support reaction become zero if central support sinks by $\frac{wL^4}{4.8EI}$.

38. (d)

$$\frac{M_A}{M_B} = \frac{I_A}{I_B} \times \frac{L_B}{L_A} = \frac{2I}{I} \times \frac{L}{L/2} = 4:1$$

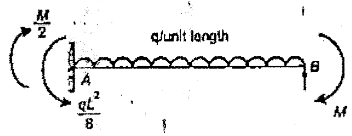
40. (d)

$$M_A = \frac{M_1(2l)}{(3l)^2}(2l - 2l) + \frac{M_2(l)}{(3l)^2}(2 \times l - l)$$

$$= 0 + \frac{M_2}{3} = \frac{M_2}{3}$$

42. (b)
Rotation at joint C for members CB and CD will be different as flexural rigidity of both members are different.

43. (c)
Let the clockwise moment required to make the slope of the deflection curve equal to zero at B be M. Thus a carry over moment of magnitude $\frac{M}{2}$ will be induced at A in clockwise direction.
Taking moment about $B = 0$

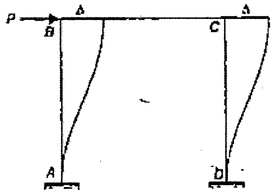


$$\Rightarrow \frac{qL^2}{8} - \frac{M}{2} - M = 0$$

$$\Rightarrow \frac{3}{2}M = \frac{qL^2}{8}$$

$$\Rightarrow M = \frac{qL^2}{12}$$

44. (c)
$(EI)_b \to \infty$, means beam will not bend any more because its flexural rigidity is infinite so, column AB will behave as the column shown in figure below



As we know that

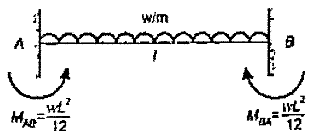
$$\frac{24EI\,\Delta}{L^3} = P$$

$$\Rightarrow \Delta = \frac{PL^3}{24EI}$$

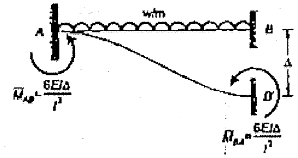
$$\text{Now } M_{AB} = \frac{6EI\Delta}{L^2}$$

$$= \frac{6EI}{L^2} \times \frac{PL^3}{24EI} = \frac{PL}{4}$$

47. (c)



If support B settles by Δ then



$$\therefore \bar{M}_{BA} = M_{BA}$$

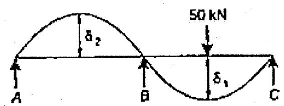
$$\Rightarrow \frac{6EI\Delta}{l^2} = \frac{wl^2}{12}$$

$$\Rightarrow \Delta = \frac{wl^4}{72EI}$$

∴ Downward vertical deflection (Δ)

$$= \frac{wl^4}{72EI}$$

48. (c)



Using Maxwell's reciprocal theorem,
Deflection at middle of span BC

$$= \frac{\delta_1}{50} \times 30 - \frac{\delta_2}{50} \times 20$$

$$= \left(\frac{0.05}{50} \times 30 - \frac{0.02}{50} \times 20\right) \text{ m} = 0.022 \text{ m}$$

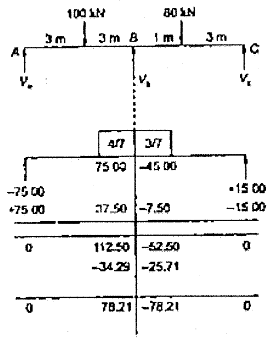
50. (b)
The eccentric load P can be replaced by load P and moment PL at the joint D. The distribution factor for the memeber AD, BD and CD will be $\frac{1}{3}$ for each.

$$\therefore M_{DA} = \frac{PL}{3}$$

Carry over factor for the beam AD is $\frac{1}{2}$

$$\therefore M_{AD} = \frac{PL}{6}$$

51. (b)



Thus, $V_a \times 6 - 100 \times 3 = -78.21$

∴ $V_a = 36.965$

$= 37$ kN

52. (a)

$$\bar{M}_{ba} = \frac{Ma(3b - l)}{l^2}$$

$$= \frac{90 \times 1.2(3 \times 1.8 - 3)}{3^2}$$

$$= 28.80 \text{ kN-m}$$

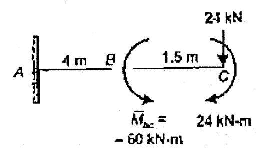
53. (d)
Relative stiffness of member

$$BA = \frac{3}{4} \times \frac{1.5I}{3} = \frac{9I}{24}$$

Relative stiffness of member

$$BC = \frac{8I}{24}$$

54. (a)



$\bar{M}_{bc} = -(24 \times 1.5 + 24) = -60$ kNm

B.M at $A = +30$ (carry over moment)

$V_b \times 4 - 24 \times 5.5 - 24 = 30$

$$V_b = 46.5\text{ kN}(\uparrow)$$
$$V_a = 22.5\text{ kN}(\downarrow)$$

55. (c)

The carry-over factor for prismatic beam is 1/2. When the beam is non-prismatic, the more stiff section wil be able to take more moment and it will transfer lesser amount of moment to less stiff section. Thus, carry over factor is expected to be less than 1/2.

56. (b)

As the frame & loading are symmetrical,

so, $i_c = -i_b$

57. (a)

$$\bar{M}_{cb} = \bar{M}_{bc} = 0$$

$$\bar{M}_{bc} = \frac{-60\times 3}{8} = -22.5\text{ kNm}$$

$$M_{ab} = \bar{M}_{ab} + \frac{2EI}{l}\left(2i_a + i_b - \frac{3\delta}{l}\right)$$

where $l = 4$ m

$$= \frac{1}{2}EIi_b - \frac{3}{8}EI\delta \qquad (\because\ i_a = 0)$$

58. (b)

Stiffness of $AO = \frac{3EI}{L}$

Stiffness of $CO = \frac{3EI}{L}$

Stiffness $BO = \frac{4EI}{L}$

Distribution factors for

$$OA : OC : OB = \frac{3}{10} : \frac{3}{10} : \frac{4}{10}$$

$\therefore$ Moment OB = Moment at $O\times$ distribution factor of OB

$$= 80\times\frac{4}{10} = 32\text{ kN-m}$$

$$\therefore \quad M_{BO} = \frac{1}{2}M_{OB}$$

$$= 16\text{ kN-m} < 18\text{ kN-m}$$

Hence safe.

60. (d)

Joint	Member	Relative stiffness	Total stiffness	DF
O	OA	$\frac{3}{4}\frac{I}{L}$	$\frac{7}{4}\frac{I}{L}$	$\frac{3}{7}$
	OB	0		0
	OC	$\frac{I}{L}$		$\frac{4}{7}$

Distribute the moments, on respective members, according to the distribution factor.

61. (a)

$$\bar{M}_{BA} = +\frac{6\times 3^2\times 1}{4^2} = +3.375\text{ kNm}$$

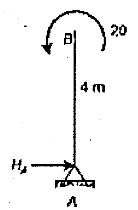
$$M_{BA} = \bar{M}_{BA} + \frac{2EI}{4}(2\theta_B + \theta_A)$$

$$= +3.375 + \frac{1}{2}EI(2\theta_B + 0)$$

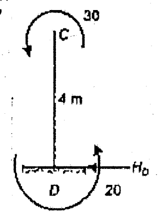
$$= 3.375 + EI\theta_B$$

(Note that the frame and loading are symmetrical and so there will not be any sway).

62. (c)



$$-H_A\times 4 = 20$$
$$H_A = -5\text{ kN}$$
$$H_A = 5\text{ kN}\leftarrow$$



$$H_D\times 4 = 20 + 30$$

$$H_D = \frac{50}{4} = 12.5\text{kN}\leftarrow$$

$$\therefore \quad P = H_A + H_D = 5 + 12.5 = 17.5\text{ kN}$$

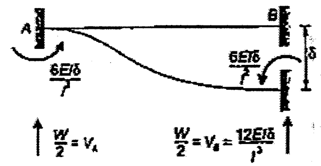
$$\Rightarrow \quad P = 17.5\text{ kN}$$

64. (b)

$$m = \left(\frac{3EI}{L}\right)_{BC} + \left(\frac{4EI}{L}\right)_{BA}$$

$$= \frac{3EI}{3} + \frac{4EI}{4} = 2\,EI\text{ kNm}$$

65. (b)



$$\therefore \quad \delta = \frac{wl^3}{24EI}$$