CHAPTER

8.7

WAVEGUIDES

Statement for Q.1–3:

A 2 cm by 3 cm rectangular waveguide is filled with a dielectric material with $\varepsilon_r = 6$. The waveguide is operating at 20 GHz with TM_{11} mode.

1. The cutoff frequency is

(A) 3.68 GHz (B) 22.09 GHz

(C) 9.02 GHz (D) 16.04 GHz

2. The phase constant is

(A) 816 rad/m (B) 412 rad/m

(C) 1009 rad/m (D) 168 rad/m

3. The phase velocity is

(A) 1.24×10^8 m/s (B) 1.54×10^6 m/s

(C) 3.05×10^8 m/s (D) 7.48×10^8 m/s

4. In an an-filled rectangular wave guide, the cutoff frequency of a TE_{10} mode is 5 GHz where as that of TE_{01} mode is 12 GHz. The dimensions of the guide is

(A) 3 cm by 1.25 cm (B) 1.25 cm by 3 cm

(C) 6 cm by 2.5 cm (D) 2.5 cm by 6 cm

5. Consider a 150 m long air-filled hollow rectangular waveguide with cutoff frequency 6.5 GHz. If a short pulse of 7.2 GHz is introduced into the input end of the guide, the time taken by the pulse to return the input end is

(A) 920 ns (B) 460 ns

(C) 230 ns (D) 430 ns

Statement for Q.6–7:

In an air-filled rectangular waveguide the cutoff frequencies for TM_{11} and TE_{03} modes are both equal to 12 GHz.

6. The dominant mode is

(A) TM_{10} (B) TM_{01}

(C) TE_{01} (D) TE_{10}

7. At dominant mode the cutoff frequency is

(A) 11.4 GHz (B) 4 GHz

(C) 5 GHz (D) 8 GHz

8. For an air-filled rectangular waveguide given that

$$E_z = 10 \sin\left(\frac{2\pi x}{a}\right)\sin\left(\frac{3\pi y}{b}\right)\cos(10^{12} t - \beta z) \text{ V/m}$$

If the waveguide has cross-sectional dimension $a = 6$ cm and $b = 3$ cm, then the intrinsic impedance of this mode is

(A) 373.2 Ω (B) 378.9 Ω

(C) 375.1 Ω (D) 380.0 Ω

Statement for Q.9–10:

In an air-filled waveguide, a TE mode operating at 6 GHz has

$$E_y = 15 \sin\left(\frac{2\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\sin(\omega t - 12 z) \text{ V/m}$$

9. The cutoff frequency is

(A) 4.189 GHz (B) 5.973 GHz

(C) 8.438 GHz (D) 7.946 GHz

10. The intrinsic impedance is

(A) 35.72 Ω (B) 3978 Ω

(C) 1989 Ω (D) 71.44 Ω

Statement for Q.11–12.

Consider an air-filled rectangular wave guide with $a = 2.286$ cm and $b = 1.016$ cm. The y-component of the TE mode is

$$E_y = 12\sin\left(\frac{2\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin\left(10\pi\times 10^{10}t - \beta z\right)\ \text{V/m}$$

11. The propagation constant γ is

(A) $j4094.2$ (B) $j400.7$

(C) $j2733.3$ (D) $j276.4$

12. The intrinsic impedance is

(A) 743 Ω (B) 168 Ω

(C) 986 Ω (D) 144 Ω

Statement for Q.13–14:

Consider a air-filled waveguide operating in the TE_{12} mode at a frequency 20% higher than the cutoff frequency.

13. The phase velocity is

(A) 1.66×10^8 m/s (B) 5.42×10^8 m/s

(C) 2.46×10^8 m/s (D) 9.43×10^8 m/s

14. The group velocity is

(A) 1.66×10^8 m/s (B) 4.42×10^8 m/s

(C) 2.46×10^8 m/s (D) 9.43×10^8 m/s

15. A rectangular waveguide is filled with a polyethylene ($\varepsilon_r = 2.25$) and operates at 24 GHz. The cutoff frequency of a certain mode is 16 GHz. The intrinsic impedance of this mode is

(A) 2248 Ω (B) 337.2 Ω

(C) 421.4 Ω (D) 632.2 Ω

16. The cross section of a waveguide is shown in fig. P8.7.16. It has dielectric discontinuity as shown in fig. P8.7.16. If the guide operate at 8 GHz in the dominant mode, the standing wave ratio is

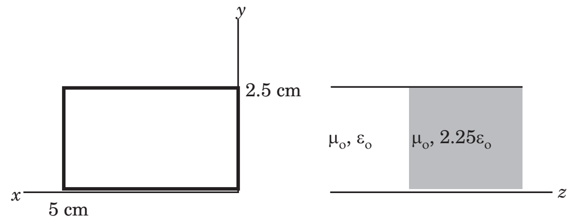

Fig. P8.7.16

(A) –3.911 (B) 2.468

(C) 1.564 (D) 4.389

Statement for Q.17–19:

Consider the rectangular cavity as shown in fig. P8.7.17–19.

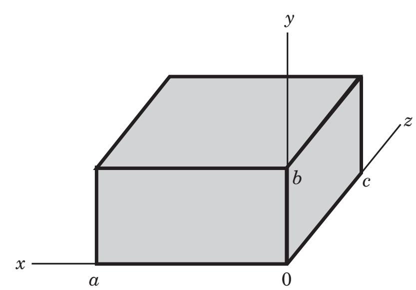

Fig. P8.7.17–19

17. If $a < b < c$, the dominant mode is

(A) TE_{011} (B) TM_{110}

(C) TE_{101} (D) TM_{101}

18. If $a > b > c$, then the dominant mode is

(A) TE_{011} (B) TM_{110}

(C) TE_{101} (D) TM_{101}

19. If $a = c > b$, then the dominant mode is

(A) TE_{011} (B) TM_{110}

(C) TE_{101} (D) TM_{101}

20. The air filled cavity resonator has dimension $a = 3$ cm, $b = 2$ cm, $c = 4$ cm. The resonant frequency for the TM_{110} mode is

(A) 5 GHz (B) 6.4 GHz

(C) 16.2 GHz (D) 9 GHz

frequency, the TM_1 mode propagates through the guide without suffering any reflective loss at the dielectric interface. This frequency is

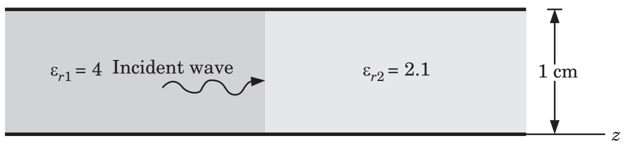

Fig. P8.7.34

(A) 8.6 GHz (B) 12.8 GHz

(C) 4.3 GHz (D) 7.5 GHz

Statement for Q.35–36:

A 6 cm × 4 cm rectangular wave guide is filled with dielectric of refractive index 1.25.

35. The range of frequencies over which single mode operation will occur is

(A) 2.24 GHz $< f <$ 3.33 GHz

(B) 2 GHz $< f <$ 3 GHz

(C) 4.48 GHz $< f <$7.70 GHz

(D) 4 GHz $< f <$ 6 GHz

36. The range of frequencies, over which guide support both TE_{10} and TE_{01} modes and no other, is

(A) 3.35 GHz $< f <$ GHz

(B) 2.5 GHz $< f <$ 3.6 GHz

(C) 3 GHz $< f <$ 3.6 GHz

(D) 2.5 GHz $< f <$ 4.02 GHz

37. Two identical rectangular waveguide are joined end to end where $a = 2b$. One guide is air filled and other is filled with a lossless dielectric of ε_r . it is found that up to a certain frequency single mode operation can be simultaneously ensured in both guide. For this frequency range, the maximum allowable value of ε_r is

(A) 4 (B) 2

(C) 1 (D) 6

38. A parallel-plate guide operates in the TEM mode only over the frequency range $0 < f < 3$ GHz. The dielectric between the plates is teflon ($\varepsilon_r = 2.1$). The maximum allowable plate separation b is

(A) 3.4 cm (B) 6.8 cm

(C) 4.3 cm (D) 8.6 cm

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SOLUTIONS

1. (A) $f_c = \dfrac{v}{2}\sqrt{\left(\dfrac{m}{a}\right)^2 + \left(\dfrac{n}{b}\right)^2}$

$$= \frac{3\times 10^8}{2\sqrt{6\times 10^{-2}}}\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2} = 3.68 \text{ GHz}$$

2. (C) $\beta_p = \beta\sqrt{1-\left(\dfrac{f_c}{f}\right)^2} = \dfrac{\omega}{v}\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}$

$$\Rightarrow \quad \beta_p = \frac{2\pi\times 20\times 10^9\sqrt{6}}{3\times 10^8}\sqrt{1-\left(\frac{3.68}{20}\right)^2} = 1009 \text{ rad/m}$$

3. (A) $v_p = \dfrac{\omega}{\beta_p} = \dfrac{2\pi\times 20\times 10^9}{1009} = 1.24\times 10^8$ m/s

4. (A) For TE_{10} mode $f_c = \dfrac{v}{2a}$,

$$a = \frac{v}{2f_c} = \frac{3\times 10^8}{2\times 5\times 10^9} = 3 \text{ cm}$$

For TE_{01} mode $f_c = \dfrac{v}{2b}$,

$$b = \frac{v}{2f_c} = \frac{3\times 10^8}{2\times 12\times 10^9} = 1.25 \text{ cm}$$

5. (D) $v = \dfrac{c}{\sqrt{1-\left(\dfrac{f_c}{f}\right)^2}} = \dfrac{3\times 10^8}{\sqrt{1-\left(\dfrac{6.5}{7.2}\right)^2}} = 6.975\times 10^8$ ms

$$t = \frac{2l}{v} = \frac{2\times 150}{6.975\times 10^8} = 430 \text{ ns}$$

6. (C) $12\times 10^9 = \dfrac{3\times 10^8}{2}\sqrt{0+\left(\dfrac{3}{b}\right)^2} \quad \Rightarrow \quad b = 3.75$ cm

$$12\times 10^9 = \frac{3\times 10^8}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{3.75\times 10^{-2}}\right)^2} \Rightarrow a = 1.32 \text{ cm}$$

Since $a < b$, the dominant mode is TE_{01} .

7. (B) $f_{c01} = \dfrac{v}{2b} = \dfrac{3\times 10^8}{2\times 3.75\times 10^{-2}} = 4$ GHz

8. (C) $E_z \neq 0$, this must be TM_{23} mode ($m = 2,\ n = 3$)

$$f_c = \frac{3\times 10^8}{2\times 10^{-2}}\sqrt{\left(\frac{2}{6}\right)^2 + \left(\frac{3}{3}\right)^2} = 15.81 \text{ GHz}$$

$$f = \frac{\omega}{2\pi} = \frac{10^{12}}{2\pi} = 159.2 \text{ GHz}$$

$$\eta_{TM} = 377\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 377\sqrt{1-\left(\frac{15.81}{159.2}\right)^2} = 375.1\,\Omega$$

9. (B) $m = 2,\ n = 1,\ \beta_p = 12,\ f = 6$ GHz

$$\beta_p = \frac{\omega}{v}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \Rightarrow 12 = \frac{2\pi\times 6\times 10^9}{3\times 10^8}\sqrt{1-\left(\frac{f_c}{6}\right)^2}$$

$\Rightarrow f_c = 5.973$ GHz

10. (B) $\eta_{TE} = \dfrac{377}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{377}{\sqrt{1-\left(\frac{5.973}{6}\right)^2}} = 3978\ \Omega$

11. (B) $m = 2,\quad n = 3,$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times 10^8}{2\times 10^{-2}}\sqrt{\left(\frac{2}{2.286}\right)^2+\left(\frac{3}{1.016}\right)^2}$$

$= 46.2$ GHz

$$f = \frac{10\pi\times 10^{10}}{2\pi} = 50\ \text{GHz}$$

$$\beta_p = \frac{\omega}{v}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \frac{2\pi\times 50\times 10^9}{3\times 10^8}\sqrt{1-\left(\frac{46.2}{50}\right)^2}$$

$= 400.7\ \text{m}^{-1},\quad \gamma = j\beta_p = j\,400.7$

12. (C) $\eta_{TE} = \dfrac{377}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{377}{\sqrt{1-\left(\frac{46.2}{50}\right)^2}} = 986\ \Omega$

13. (A) $v = c,\ f = 1.2f_c$

$$\boldsymbol{v_p} = \frac{\boldsymbol{v}}{\sqrt{\mathbf{1}-\left(\frac{\boldsymbol{f_c}}{\boldsymbol{f}}\right)^2}} = \frac{\mathbf{3\times 10^8}}{\sqrt{\mathbf{1}-\left(\frac{\boldsymbol{f_c}}{\mathbf{1.2}\boldsymbol{f_c}}\right)^2}} = \mathbf{5.42\times 10^8}\ \text{m/s}$$

14. (A) $v_g = v\sqrt{1-\left(\frac{f_c}{f}\right)^2} = c\sqrt{1-\left(\frac{f_c}{1.2f_c}\right)^2} = 1.66\times 10^8$ m/s

15. (B) $\eta = \dfrac{377}{\sqrt{\varepsilon_r}} = \dfrac{377}{1.5} = 251.33\ \Omega$

$$\eta_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{251.33}{\sqrt{1-\left(\frac{16}{24}\right)^2}} = 337.2\ \Omega$$

16. (C) Since $a > b$, the dominant mode is TE_{10} .

In free space $f_c = \dfrac{c}{2a} = \dfrac{3\times 10^8}{2\times 0.05} = 3$ GHz

$$\eta_1 = \frac{\eta_o}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-\left(\frac{3}{8}\right)^2}} = 406.7\ \Omega$$

In dielectric medium

$$f_c = \frac{c}{2a\sqrt{\varepsilon_r}} = \frac{3\times 10^8}{2\times 0.05\sqrt{2.25}} = 2\ \text{GHz}$$

$$\eta = \frac{\eta_o}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.25}} = 251.33\,\Omega,\ \eta_2 = \frac{251.33}{\sqrt{1-\left(\frac{2}{8}\right)^2}} = 259.23\,\Omega$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{259.23-406.7}{259.23+406.7} = -0.22$$

$$s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.22}{1-0.22} = 1.564$$

17. (A) $f_r = \dfrac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2+\left(\frac{p}{c}\right)^2}$

where for TM mode to z

$m = 1,\ 2,\ 3......$,

$n = 1,\ 2,\ 3......$,

$p = 0,\ 1,\ 2\ ......$

For TE mode to z

$m = 1,\ 2,\ 3.....,$

$n = 1,\ 2,\ 3.....$

$p = 1,\ 2,\ 3......$,

if $a < b < c$, then $\dfrac{1}{a} > \dfrac{1}{b} > \dfrac{1}{c}$

The lowest TM mode is TM_{110} with

$$f_{r1} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$$

The lowest TE mode is TE_{011} with

$$f_{r2} = \frac{v}{2}\sqrt{\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}$$

$f_{r2} > f_{r1}$, Hence the dominant mode is TE_{011}

18. (B) If $a > b > c$ then $\dfrac{1}{a} < \dfrac{1}{b} < \dfrac{1}{c}$

The lowest TM mode is TM_{110} with

$$f_{r1} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$$

The lowest TE mode is TE_{101} with

$$f_{r2} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{c}\right)^2}$$

$f_{r2} > f_{r1}$ Hence the dominant mode is TM_{110} .

19. (C) If $a = c > b$, then $\frac{1}{a} = \frac{1}{c} < \frac{1}{b}$

The lowest TM mode is TM_{110} with

$$f_{r1} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

The lowest TE mode is TE_{101} with

$$f_{r2} = \frac{v}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2}$$

$f_{r_2} < f_{r_1}$ Hence the dominant mode is TE_{101}.

20. (D) $f_r = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$

$$= \frac{3 \times 10^8}{2 \times 0.01}\sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{2}\right)^2} = 9 \text{ GHz}$$

21. (A) $m = n = 1$, $p = 0$, $a = b = c$, $f_r = 2$ Ghz,

$$f_r = \frac{v}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{c}\right)^2}$$

$$2 \times 10^9 = \frac{3 \times 10^8}{2}\frac{\sqrt{2}}{a} \quad \Rightarrow \quad a = 10.6 \text{ cm}$$

22. (A) $f_c = \frac{mc}{2b\sqrt{\varepsilon_r}} = \frac{2 \times 3 \times 10^8}{2 \times 0.01\sqrt{\varepsilon_r}} = 10 \times 10^9 \quad \Rightarrow \quad \varepsilon_r = 9$

23. (A) For a propagating mode $f > f_{cm}$,

$$f_{cm} = \frac{mc}{2b\sqrt{\varepsilon_r}}, \quad f > \frac{mc}{2b\sqrt{\varepsilon_r}} \quad \Rightarrow \quad m < \frac{2fb\sqrt{\varepsilon_r}}{c}$$

$$m < \frac{2 \times 30 \times 10^9 \times 0.01\sqrt{2.5}}{3 \times 10^8} \quad \Rightarrow \quad m < 3.16$$

The maximum allowed m is 3. The propagating mode will be TM_1, TE_1, TM_2, TE_2, TM_3, TE_3 and TEM Thus total 7 modes.

24. (B) $f_{cm} = \frac{mc}{2b\sqrt{\varepsilon_r}}$, $f_{c2} = 2f_{c1} = 15$ GHz

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.015} = 20 \text{ GHz}$$

$$v_{g2} = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2} = 3 \times 10^8\sqrt{1 - \left(\frac{15}{20}\right)^2} = 2 \times 10^8 \text{ m/s}$$

s

25. (A) $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$, $\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$

Mode	TE_{10}	TE_{01}	TE_{11}	TE_{20}
λ_c(cm)	14.4	6.8	6.15	7.21

$\lambda > \lambda_c$. Hence TE_{10} mode can be used.

26. (C) Let $a = kb$, $1 < k < 2$

$$f_{cmn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{1.5 \times 10^8}{a}\sqrt{m^2 + k^2n^2}$$

Dominant mode is TE_{10} , $f_{c10} = \frac{1.5 \times 10^8}{a}$

$$3 \text{ GHz} > 1.2f_c \quad \Rightarrow \quad 3 \times 10^9 > \frac{1.2 \times 1.5 \times 10^8}{a}$$

$\Rightarrow \quad a > 6$ cm

The next higher mode is TE_{01}, $f_{c01} = \frac{1.5 \times 10^8}{b}$,

$$3 \text{ GHz} < 0.8f_{c01} \quad \Rightarrow \quad 3 \times 10^9 < \frac{0.8 \times 1.5 \times 10^8}{b}$$

$\Rightarrow \quad b < 4$ cm, Thus (C) is correct option.

27. (C) $f_c = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 0.065} = 2.3$ GHz

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{3 \times 10^8}{\sqrt{1 - \left(\frac{2.3}{3}\right)^2}} = 4.7 \times 10^8 \text{ m/s}$$

28. (B) For TE_{10} mode

$$R_s = \frac{1}{\sigma_c\delta} = \sqrt{\frac{\pi f\mu}{\sigma_c}} = \sqrt{\frac{\pi \times 9 \times 10^9 \times 4\pi \times 10^{-7}}{1.1 \times 10^7}} = 0.0568$$

$$\alpha_c = \frac{R_s\left(1 + \frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right)}{b\eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{0.0568\left(1 + \frac{2 \times 1.5}{2.4}\left(\frac{3.876}{9}\right)^2\right)}{1.5 \times 10^{-2} \times 233.8\sqrt{1 - \left(\frac{3.876}{9}\right)^2}}$$

$= 0.022$

29. (B) $\frac{\sigma_d}{\omega\varepsilon} = \frac{10^{-15}}{2\pi \times 9 \times 10^9 \times 2.6 \times 8.85 \times 10^{-12}} = \frac{10^{-15}}{1.3}$

$\frac{\sigma_d}{\omega\varepsilon} << 1$, hence $v \approx \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{2.6}}$, $\eta \approx \frac{377}{\sqrt{2.6}} = 233.8$

$$f_c = \frac{3 \times 10^8}{2 \times 2.4 \times 10^{-2}\sqrt{2.6}} = 3.876 \text{ GHz}$$

$$\alpha_d = \frac{\sigma_d\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{10^{-15} \times 233.8}{2\sqrt{1 - \left(\frac{3.876}{9}\right)^2}} = 1.3 \times 10^{-13} \text{ Np/m}$$

30. (D) Dominant mode is TE_{10} mode

$$f_c = \frac{c}{2a} = \frac{3\times10^8}{2\times0.072} = 2.08 \text{ GHz}$$

$$R_s = \sqrt{\frac{\pi f \mu_o}{\sigma_c}} = \sqrt{\frac{\pi\times3\times10^9\times4\pi\times10^{-7}}{5.8\times10^7}} = 1.429\times10^{-2}\ \Omega$$

For TE_{10} mode

$$\alpha_c = \frac{R_s\left(1+\frac{2b}{a}\left(\frac{f_c}{f}\right)^2\right)}{b\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{1.429\times10^{-2}\left(1+\frac{2\times3.4}{7.2}\left(\frac{2.08}{3}\right)\right)}{377\times0.034\sqrt{1-\left(\frac{2.08}{3}\right)^2}}$$

$= 2.25\times10^{-3}$ Np/m

$$e^{-\alpha_c z} = \frac{1}{2} \Rightarrow z = \frac{1}{\alpha_c}\ln 2 = 308 \text{ m}$$

31. (B)

$$f_{cmn} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2\times0.01}\sqrt{\left(\frac{m}{8}\right)^2+\left(\frac{n}{10}\right)^2}$$

$$= 15\sqrt{\left(\frac{m}{8}\right)^2+\left(\frac{n}{10}\right)^2} \text{ GHz}$$

$f_{c10} = 1.875$ GHz

$f_{c01} = 1.5$ GHz, $f_{c11} = 2.4$ GHz

$f_{c20} = 3.75$GHz , $f_{c02} = 3$ GHz,

$f_{c21} = 4.04$ GHz, $f_{c12} = 3.54$ GHz,

$f_{c30} = 5.625$ GHz , $f_{c03} = 4.5$ GHz

If $f_c < f$, then mode will be transmit. Hence six mode will be transmitted.

32. (C) For dominant mode $(m=1, n=0)$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2\times0.04} = 3.75 \text{ GHz}$$

Since given frequency is below the cutoff frequency, 3 GHz will not be propagated and get attenuated

$$\gamma = \alpha + j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\left(\frac{\omega}{\nu}\right)^2}$$

$\beta = 0$, Since wave is attenuated,

$$\alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2-\left(\frac{\omega}{c}\right)^2} = \sqrt{\left(\frac{\pi}{0.04}\right)^2-\left(\frac{2\pi\times3\times10^9}{3\times10^8}\right)^2} = 47.1$$

33. (B) $f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

$$f_{c10} = \frac{c}{2a} = \frac{3\times10^8}{2\times0.08} = 1.875 \text{ GHz}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2-\left(\frac{\omega}{\nu}\right)^2} = \sqrt{\left(\frac{\pi}{0.08}\right)^2-\left(\frac{2\pi\times1.5\times1.875}{3\times10^8}\right)^2}$$

$= j43.9$

34. (B) The ray angle is such that the wave is interface at Brewster's angle $\theta_B = \tan^{-1}\sqrt{\frac{2.1}{4}} = 35.9°$.

The ray angle $\theta = 90° - 35.9° = 54.1°$

$$f_{c1} = \frac{c}{2b\sqrt{\varepsilon_{r1}}} = \frac{3\times10^{10}}{2\times1\times2} = 7.5 \text{ GHz}$$

$$f = \frac{f_{c1}}{\cos\theta} = \frac{7.5}{\cos 54.1°} = 12.8 \text{ GHz}$$

35. (A) $f_c = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$

$$f_{c10} = \frac{c}{a2\sqrt{\varepsilon_r}} = \frac{3\times10^8}{2\times1.25\times0.06} = 2 \text{ GHz}$$

$$f_{c01} = \frac{c}{b2\sqrt{\varepsilon_r}} = \frac{3\times10^8}{2\times1.25\times0.04} = 3 \text{ GHz}$$

2 GHz $< f <$ 3 GHz

36. (C) $f_{c11} = \frac{3\times10^8}{2\times1.25\times10^{-2}}\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{4}\right)^2} = 3.6$ GHz

3 GHz $< f <$ 3.6 GHz

37. (A) $f_c = \frac{c}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{2b}\right)^2+\left(\frac{n}{b}\right)^2}$, In guide 1 $\varepsilon_r = 1$

lowest cutoff frequency $f_{c10} = \frac{c}{2(2b)}$

Next lowest cutoff frequency $f_{c20} = \frac{c}{2b}$

In guide 2 lowest cutoff frequency $f'_{c10} = \frac{c}{2\sqrt{\varepsilon_{r2}}(2b)}$

Next lowest cutoff frequency $f'_{c20} = \frac{c}{2\sqrt{\varepsilon_{r2}}(b)}$

For single mode $f'_{c10} < f < f'_{c10}$

$$\Rightarrow \frac{c}{2(2b)} < f < \frac{c}{2\sqrt{\varepsilon_r}(b)} \Rightarrow \varepsilon_r < 4$$

38. (A) $f < f_c \Rightarrow f < \frac{v}{2b} = \frac{3\times10^8}{2\times b\times\sqrt{2.1}}$

$$\Rightarrow 3\times10^9 < \frac{3\times10^8}{2\times b\times\sqrt{2.1}} \Rightarrow b < 3.4 \text{ cm}$$

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