3. Pairs of Equations

Questions Pg-54

1. Question

Raju bought seven notebooks of two hundred pages and five of hundred pages, for 107 rupees. Joseph bought five notebooks of two hundred pages and seven of hundred pages, for 97 rupees. What is the price of each kind of notebook?

Answer

Let the price of two hundred pages notebook be ‘x’ and price of hundred pages book be ‘y’.

According to the question,

Raju bought 7 notebooks of two hundred pages and five of hundred pages for 107 rupees

⇒ 7x + 5y = 107 ... (1)

Joseph bought five notebooks of two hundred pages and seven of hundred pages, for 97 rupees

⇒ 5x + 7y = 97... (2)

Equating (1) and (2)

7x + 5y = 107

5x + 7y = 97

Multiply Equation (1) by 7 and equation (2) by 5

Then, Subtract equation (2) from (1)

$$\begin{array}{rl} 49x + 35y = 749 & \ldots (3) \\ \underline{-25x - 35y = -485} & \ldots (4) \\ 24x \quad = 264 & \end{array}$$

$$x = \frac{264}{24}$$

x = 11

Put x = 11 in Equation (3)

49 × 11 + 35 y = 749

⇒ 539 + 35y = 749

⇒ 35y = 749 – 539

⇒ 35y = 210

$$\Rightarrow y = \frac{210}{35}$$

⇒ y = 6

Hence, price of two hundred pages notebook is 11rupees and price of hundred pages notebook is 6 rupees.

2. Question

Four times a number and three times a number added together make 43. Two times the second number, subtracted from three times the first give 11. What are the numbers?

Answer

Let the first number be ‘x’ and second number be ‘y’.

According to the question,

Four times a number and three times a number added together make 43

⇒ 4x + 3y = 43 … (1)

Two times the second number, subtracted from three times the first give 11

⇒ 3x – 2y = 11 … (2)

Equating equation (1) and (2)

4x + 3y = 43 … (1)

3x – 2y = 11 … (2)

Multiply equation (1) by 2 and (2) by 3

$$\begin{array}{rl} 8x + 6y = 86 & \ldots (3) \\ \underline{9x - 6y = 33} & \ldots (4) \\ 17x = 119 & \end{array}$$

$$x = \frac{119}{17}$$

x = 7

Put x = 7 in equation (4)

9 × 7 – 6y = 33

⇒ 63 – 6y = 33

⇒ 63 = 33 + 6y

⇒ 6y = 63 – 33

⇒ 6y = 30

$$\Rightarrow y = \frac{30}{6}$$

⇒ y = 5

Hence the two numbers are 7 and 5.

3. Question

The sum of the digits of a two–digit number is 11. The number got by interchanging the digits is 27 more than the original number. What is this number?

Answer

Let the digit in the unit's place be 'x' and the digit in the tens place be 'y'.

According to the question,

x + y = 11 … (1)

Then, the number = 10y + x = 11

The number got by interchanging the digits is 27 more than the original number

⇒ 10x + y = 10y + x + 27

⇒ 10x + y – 10y – x = 27

⇒ 9x – 9y = 27

Divide by 9 both sides

⇒ x – y = 3 … (2)

Equating equation (1) and (2)

$$\begin{array}{l} x + y = 11 \\ \underline{x - y = 3} \\ 2x = 14 \end{array}$$

$$x = \frac{14}{2}$$

x = 7

Put x = 7 in Equation (1)

7 + y = 11

y = 11 – 7

y = 4

∴ the number = 10y + x

= 10 × 4 + 7

= 40 + 7

= 47

4. Question

Four years ago, Rahim’s age was three times Ramu’s age. After two years, it would just be double. What are their ages now?

Answer

Let present age of Rahim be ‘x’ and present age of Ramu be ‘y’

According to the question,

Four years ago, Rahim’s age was three times Ramu’s age

⇒ (x – 4) = 3(y – 3)

⇒ x – 4 = 3y – 9

⇒ x – 3y = –9 + 4

⇒ x – 3y = –5 … (1)

After two years, it would just be double

⇒ (x + 2) = 2(y + 2)

⇒ x + 2 = 2y + 4

⇒ x – 2y = 4 – 2

⇒ x – 2y = 2 … (2)

Subtract equation (2) from (1)

$$\begin{array}{r} x - 3y = -5 \\ \underline{-x + 2y = 2} \\ -y = -3 \end{array}$$

y = 3

Put y = 3 in Equation (1)

x – 3 × 3 = –5

⇒ x – 9 = –5

⇒ x = –5 + 9

⇒ x = 4

Hence, Rahim is of 4years and Ramu is 3 years old.

5. Question

If the length of a rectangle is increased by 5 metres and breadth decreased by 3 metres, the area would decrease by 5 square metres. If the length is increased by 3 metres and breadth increased by 2 metres, the area would increase by 50 square metres. What are the length and breadth?

Answer

Let the length be ‘x’ and breadth be ‘y’.

Area of rectangle = length × breadth

= x × y

= xy

According to the question,

The length of a rectangle is increased by 5 metres and breadth decreased by 3 metres, the area would decrease by 5 square metres

⇒ (x + 5) × (y – 3) = xy – 5

⇒ x (y – 3) + 5(y – 3) = xy – 5

⇒ xy – 3x + 5y – 15 = xy – 5

⇒ xy – 3x + 5y – xy = –5 + 15

⇒ –3x + 5y = 10 … (1)

the length is increased by 3 metres and breadth increased by 2 metres, the area would increase by 50 square metres

⇒ (x + 3) × (y + 2) = xy + 50

⇒ x (y + 2) + 3(y + 2) = xy + 50

⇒ xy + 2x + 3y + 6 = xy + 50

⇒ xy + 2x + 3y – xy = 50 – 6

⇒ 2x + 3y = 44 … (2)

Equating equation (1) and (2)

–3x + 5y = 10 … (1)

2x + 3y = 44 … (2)

Multiplying equation (1) by 2 and equation (2) by 3 and equate

$$\begin{array}{ll} -6x + 10y = 20 & \ldots (3) \\ \underline{6x + 9y = 132} & \ldots (4) \\ 19y = 152 & \end{array}$$

$$y = \frac{152}{19}$$

y = 8

Put y = 8 in Equation (3)

–6x + 10 × 8 = 20

⇒ –6x + 80 = 20

⇒ –6x = 20 – 80

⇒ –6x = –60

⇒ 6x = 60

$$\Rightarrow x = \frac{60}{6}$$

⇒ x = 10

∴ The length of rectangle is 10 metres and breadth are 8 metres.

Questions Pg-56

1. Question

A 10-metre-long rope is to be cut into two pieces and a square is to be made using each. The difference in the areas enclosed must be $1\frac{1}{4}$ square metres. How should it be cut?

Answer

Let the larger part of rope be ‘x’ and smaller part be ‘y’.

According to question,

x + y = 10 ... (1)

Area of larger square – area of smaller square $= 1\frac{1}{4} = \frac{5}{4}$

$\Rightarrow x^2 - y^2 = \frac{5}{4}$

$\Rightarrow (x+y)(x-y) = \frac{5}{4}$

$\Rightarrow 10\,(x - y) = \frac{5}{4}$

$\Rightarrow (x-y) = \frac{5}{4} \times \frac{1}{10}$

$\Rightarrow (x-y) = \frac{1}{8}$

⇒ 8(x – y) = 1

⇒ 8x – 8y = 1 ... (2)

Equating equation (1) and (2)

x + y = 10

8x – 8y = 1

Multiply equation (1) by 8

8x + 8y = 80 ... (3)

8x – 8y = 1 ... (2)

16x = 81

$x = \frac{81}{16} = 5\frac{1}{16}$

Put $x = \frac{81}{16}$ in equation (2)

$8 \times \frac{81}{16} - 8y = 1$

$\Rightarrow \frac{81}{2} - 8y = 1$

$\Rightarrow \frac{81}{2} = 1 + 8y$

$\Rightarrow 8y = \frac{81}{2} - 1$

$\Rightarrow 8y = \frac{81-2}{2}$

$\Rightarrow 8y = \frac{79}{2}$

$\Rightarrow y = \frac{79}{2} \times \frac{1}{8}$

$\Rightarrow y = \frac{79}{16} = 4\frac{15}{16}$

∴ The length of the rope to after cutting should be $5\frac{1}{16}$ and $4\frac{15}{16}$ respectively.

2. Question

The length of a rectangle is 1 metre more than its breadth. Its area is $3\frac{3}{4}$ square metres. What are its length and breadth?

Answer

Let the length be ‘x’ and breadth be ‘y’.

Length = Breadth + 1

⇒ x = y + 1 ... (1)

Area of rectangle = Length × breadth

$3\frac{3}{4} = x \times y$

$\Rightarrow \frac{15}{4} = x \times y$

$\Rightarrow \frac{15}{4} = (y+1)y$

$\Rightarrow \frac{15}{4} = y^2 + y$

⇒ 15 = 4(y^2 + y)

⇒ 15 = 4y^2 + 4y

⇒ 4y^2 + 4y – 15 = 0

⇒ 4y^2 – 6y + 10y – 15 = 0

⇒ 2y (2y – 3) + 5(2y – 3) = 0

⇒ (2y – 3) (2y + 5) = 0

2y – 3 = 0 or 2y + 5 = 0

$2y = 3 \text{ or } 2y = -5$

$y = \frac{3}{2} \text{ or } y = -\frac{5}{2}$

Hence, side of a rectangle cannot be negative. Therefore, $y = \frac{3}{2}$

Put $y = \frac{3}{2}$ in equation (1)

$\Rightarrow x = \frac{3}{2} + 1$

$\Rightarrow x = \frac{3+2}{2}$

$\Rightarrow x = \frac{5}{2}$

$\therefore$ The length is $\frac{5}{2}$ metre and breadth are $\frac{3}{2}$ metre.

3. Question

The hypotenuse of a right triangle is $6\frac{1}{2}$ centimetres and its area are $7\frac{1}{2}$ square centimetres. Calculate the lengths of its perpendicular sides.

Answer

Let the base be 'x' and perpendicular be 'y'.

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$\Rightarrow 7\frac{1}{2} = \frac{1}{2} \times x \times y$

$\Rightarrow \frac{15}{2} = \frac{1}{2}xy$

$\Rightarrow \frac{15}{2} \times 2 = xy$

$\Rightarrow 15 = xy$ ... (1)

By Pythagoras Theorem,

$(\text{Base})^2 + (\text{Height})^2 = (\text{hypotenuse})^2$

$\Rightarrow x^2 + y^2 = \left(6\frac{1}{2}\right)^2$

$\Rightarrow x^2 + y^2 = \left(\frac{13}{2}\right)^2$

$\Rightarrow x^2 + y^2 = \frac{169}{4}$

$\Rightarrow (x+y)^2 - 2xy = \frac{169}{4}$

$\Rightarrow (x+y)^2 - 2 \times 15 = \frac{169}{4}$

$\Rightarrow (x+y)^2 - 30 = \frac{169}{4}$

$\Rightarrow (x+y)^2 = \frac{169}{4} + 30$

$\Rightarrow (x+y)^2 = \frac{169+120}{4}$

$\Rightarrow (x+y)^2 = \frac{289}{4}$

$\Rightarrow x+y = \sqrt{\frac{289}{4}}$

$\Rightarrow x+y = \frac{17}{2}$ ... (2)

From equation (1),

$\Rightarrow x + \frac{15}{x} = \frac{17}{2}$

$\Rightarrow \frac{x^2+15}{x} = \frac{17}{2}$

$\Rightarrow 2(x^2 + 15) = 17 \times x$

$\Rightarrow 2x^2 + 30 = 17x$

$\Rightarrow 2x^2 - 17x + 30 = 0$

$\Rightarrow 2x^2 - 12x - 5x + 30 = 0$

$\Rightarrow 2x(x - 6) - 5(x - 6) = 0$

$\Rightarrow (2x - 5)(x - 6) = 0$

$2x - 5 = 0$ or $x - 6 = 0$

$2x = 5$ or $x = 6$

$x = \frac{5}{2}$ or $x = 6$

Put $x = \frac{5}{2}$ in equation (2)

$\Rightarrow \frac{5}{2} + y = \frac{17}{2}$

$\Rightarrow y = \frac{17}{2} - \frac{5}{2}$

$\Rightarrow y = \frac{12}{2} = 6$

Put $x = 6$ in equation 2

$\Rightarrow 6 + y = \frac{17}{2}$

$\Rightarrow y = \frac{17}{2} - 6$

$\Rightarrow y = \frac{17-12}{2} = \frac{5}{2}$

$\therefore$ Base of triangle can be $\frac{5}{2}$ or 6

Perpendicular of triangle can be $\frac{5}{2}$ or 6.