# CHAPTER

# 5.3

# THE LAPLACE TRANSFORM

**Statement for Q.1-12:**

Determine the Laplace transform of given signal.

**1.** $x(t) = u(t-2)$

(A) $\frac{-e^{-2s}}{s}$ (B) $\frac{e^{-2s}}{s}$

(C) $\frac{e^{-2s}}{1+s}$ (D) 0

**2.** $x(t) = u(t+2)$

(A) $\frac{1}{s}$ (B) $-\frac{1}{s}$

(C) $\frac{e^{-2s}}{s}$ (D) $\frac{-e^{-2s}}{s}$

**3.** $x(t) = e^{-2t}u(t+1)$

(A) $\frac{1}{s+2}$ (B) $\frac{e^{-s}}{s+2}$

(C) $\frac{e^{-(s+2)}}{s+2}$ (D) $\frac{-e^{-s}}{s+2}$

**4.** $x(t) = e^{2t}u(-t+2)$

(A) $\frac{e^{2(s-2)}-1}{s-2}$ (B) $\frac{e^{-2s}}{s+2}$

(C) $\frac{1-e^{-2(s-2)}}{s-2}$ (D) $\frac{e^{-2s}}{s-2}$

**5.** $x(t) = \sin 5t$

(A) $\frac{5}{s^2+5}$ (B) $\frac{s}{s^2+5}$

(C) $\frac{5}{s^2+25}$ (D) $\frac{s}{s^2+25}$

**6.** $x(t) = u(t) - u(t-2)$

(A) $\frac{e^{-2s}-1}{s}$ (B) $\frac{1-e^{-2s}}{s}$

(C) $\frac{2}{s}$ (D) $\frac{-2}{s}$

**7.** $x(t) = \frac{d}{dt}\{te^{-t}u(t)\}$

(A) $\frac{1}{s(s+1)^2}$ (B) $\frac{s}{(s+1)^2}$

(C) $\frac{e^{-s}}{s+1}$ (D) $\frac{e^{-s}}{(s+1)^2}$

**8.** $x(t) = tu(t) * \cos 2\pi t\, u(t)$

(A) $\frac{1}{s(s^2+4\pi^2)}$ (B) $\frac{2\pi}{s^2(s^2+4\pi^2)}$

(C) $\frac{1}{s^2(s^2+4\pi^2)}$ (D) $\frac{s^3}{s^2+4\pi^2}$

**9.** $x(t) = t^3u(t)$

(A) $\frac{3}{s^4}$ (B) $\frac{-3}{s^4}$

(C) $\frac{6}{s^4}$ (D) $-\frac{6}{s^4}$

**10.** $x(t) = u(t-1) * e^{-2t}u(t-1)$

(A) $\frac{e^{-2(s+1)}}{2s+1}$ (B) $\frac{e^{-2(s+1)}}{s+1}$

(C) $\frac{e^{-(s+2)}}{s+2}$ (D) $\frac{e^{-2(s+1)}}{s+2}$

**11.** $x(t) = \int_0^t e^{-3\tau} \cos 2\tau \, d\tau$

(A) $\frac{-(s+3)}{s((s+3)^2+4)}$ (B) $\frac{(s+3)}{s((s+3)^2+4)}$

(C) $\frac{s(s+3)}{(s+3)^2+4}$ (D) $\frac{-s(s+3)}{(s+3)^2+4}$

**12.** $x(t) = t\frac{d}{dt}\{e^{-t} \cos t \, u(t)\}$

(A) $\frac{-(s^2+4s+2)}{(s^2+2s+2)^2}$ (B) $\frac{(s^2+4s+2)}{(s^2+2s+2)^2}$

(C) $\frac{(s^2+2s+2)}{(s^2+4s+2)^2}$ (D) $\frac{-(s^2+2s+2)}{(s^2+4s+2)^2}$

**Statement for Q.13–24:**

Determine the time signal $x(t)$ corresponding to given $X(s)$ and choose correct option.

**13.** $X(s) = \frac{s+3}{s^2+3s+2}$

(A) $(2e^{-2t} + e^{-t})u(t)$ (B) $(2e^{-t} - e^{-2t})u(t)$

(C) $(2e^{-2t} - e^{-t})u(t)$ (D) $(2e^{-t} + e^{-2t})u(t)$

**14.** $X(s) = \frac{2s^2+10s+11}{s^2+5s+6}$

(A) $2\delta(t) + (e^{-3t} - e^{-2t})u(t)$

(B) $2\delta(t) + (e^{-2t} - e^{-3t})u(t)$

(C) $2\delta(t) + (e^{-2t} + e^{-3t})u(t)$

(D) $2\delta(t) - (e^{-2t} + e^{-3t})u(t)$

**15.** $X(s) = \frac{2s-1}{s^2+2s+1}$

(A) $(3e^{-t} - 2te^{-t})u(t)$

(B) $(3e^{-t} + 2te^{-t})u(t)$

(C) $(2e^{-t} - 3te^{-t})u(t)$

(D) $(2e^{-t} + 3te^{-t})u(t)$

**16.** $X(s) = \frac{5s+4}{s^3+3s^2+2s}$

(A) $(2 + e^{-t} + 3e^{-2t})u(t)$

(B) $(2 + e^{-t} - 3e^{-2t})u(t)$

(C) $(3 + e^{-t} - 3e^{-2t})u(t)$

(D) $(3 + e^{-t} + 3e^{-2t})u(t)$

**17.** $X(s) = \frac{s^2-3}{(s+2)(s^2+2s+1)}$

(A) $(e^{-2t} - 2te^{-t})u(t)$ (B) $(e^{-2t} + 2te^{-t})u(t)$

(C) $(e^{-t} - 2te^{-2t})u(t)$ (D) $(e^{-t} + 2te^{-2t})u(t)$

**18.** $X(s) = \frac{3s+2}{s^2+2s+10}$

(A) $\left(3e^{-t} \cos 3t - \frac{1}{3}e^{-t} \sin 3t\right)u(t)$

(B) $\left(3e^{-t} \sin 3t - \frac{1}{3}e^{-t} \cos 3t\right)u(t)$

(C) $(3e^{-t} \cos 3t - e^{-t} \sin 3t)u(t)$

(D) $(3e^{-t} \sin 3t + 3e^{-t} \cos 3t)u(t)$

**19.** $X(s) = \frac{4s^2+8s+10}{(s+2)(s^2+2s+5)}$

(A) $(2e^{-2t} + 2e^{-t} \sin 2t - 2e^{-t} \cos 2t)u(t)$

(B) $(2e^{-2t} + 2e^{-t} \cos 2t - 2e^{-t} \sin 2t)u(t)$

(C) $(2e^{-2t} + 2e^{-t} \cos 2t - e^{-t} \sin 2t)u(t)$

(D) $(2e^{-2t} + 2e^{-t} \sin 2t - e^{-t} \cos 2t)u(t)$

**20.** $X(s) = \frac{3s^2+10s+10}{(s+2)(s^2+6s+10)}$

(A) $(e^{-2t} + 2e^{-3t} \cos t + 2e^{-3t} \sin t)u(t)$

(B) $(e^{-2t} + 2e^{-3t} \cos t - 6e^{-3t} \sin t)u(t)$

(C) $(e^{-2t} + 2e^{-3t} \cos t - 2e^{-3t} \sin t)u(t)$

(D) $(9e^{-2t} - 6e^{-3t} \cos t + 3e^{-3t} \sin t)u(t)$

**21.** $X(s) = \frac{2s^2+11s+16+e^{-2s}}{(s^2+5s+6)}$

(A) $2\delta(t) + (3e^{-2t} - 2e^{-3t})u(t-2)$

(B) $2\delta(t) + (2e^{-2t} - e^{-3t} + e^{-2(t-2)} + e^{-3(t-2)})u(t)$

(C) $2\delta(t) + (2e^{-2t} - e^{-3t})u(t) + (e^{-2t} - e^{-3t})u(t-2)$

(D) $2\delta(t) + (2e^{-2t} - e^{-3t})u(t) + (e^{-2(t-2)} - e^{-3(t-2)})u(t-2)$

**22.** $X(s) = s\frac{d^2}{ds^2}\left(\frac{1}{s^2+9}\right) + \frac{1}{s+3}$

(A) $\left(e^{-3t} + \frac{2t}{3}\sin 3t + \frac{t^2}{9}\cos 3t\right)u(t)$

(B) $(e^{-3t} + 2t \sin 3t + t^2 \cos 3t)u(t)$

(C) $\left(e^{-3t} + \frac{2t}{3}\sin 3t + t^2 \cos 3t\right)u(t)$

(D) $(e^{-3t} + t^2 \sin 3t + 2t \cos 3t)u(t)$

**23.** $X(s) = \dfrac{1}{(2s+1)^2 + 4}$

(A) $e^{-0.5t} \sin t\, u(t)$ (B) $\dfrac{1}{2} e^{-t} \sin t\, u(t)$

(C) $\dfrac{1}{4} e^{-0.5t} \sin t\, u(t)$ (D) $e^{-t} \sin t\, u(t)$

**24.** $X(s) = e^{-2s} \dfrac{d}{ds}\left(\dfrac{1}{(s+1)^2}\right)$

(A) $-te^{-t}u(1-t)$ (B) $-te^{-t}u(t-1)$

(C) $-(t-2)^2 e^{-(t-2)}u(t-2)$ (D) $te^{-t}u(t-1)$

**Statement for Q.25–29:**

Given the transform pair below. Determine the time signal $y(t)$ and choose correct option.

$$\cos 2t\, u(t) \xleftrightarrow{L} X(s).$$

**25.** $Y(s) = (s+1)X(s)$

(A) $[\cos 2t - 2\sin 2t]u(t)$ (B) $\left(\cos 2t + \dfrac{\sin 2t}{2}\right)u(t)$

(C) $[\cos 2t + 2\sin 2t]u(t)$ (D) $\left(\cos 2t - \dfrac{\sin 2t}{2}\right)u(t)$

**26.** $Y(s) = X(3s)$

(A) $\cos\left(\dfrac{2}{3}t\right)u(t)$ (B) $\dfrac{1}{3}\cos\left(\dfrac{2}{3}t\right)u(t)$

(C) $\cos 6t\, u(t)$ (D) $\dfrac{1}{3}\cos 6t\, u(t)$

**27.** $Y(s) = X(s+2)$

(A) $\cos 2(t-2)\, u(t)$ (B) $e^{2t} \cos 2t\, u(t)$

(C) $\cos 2(t+2)\, u(t)$ (D) $e^{-2t} \cos 2t\, u(t)$

**28.** $Y(s) = \dfrac{X(s)}{s^2}$

(A) $4 \cos 2t\, u(t)$ (B) $\dfrac{1 - \cos 2t}{4} u(t)$

(C) $t^2 \cos 2t\, u(t)$ (D) $\dfrac{\cos 2t}{t^2} u(t)$

**29.** $Y(s) = \dfrac{d}{ds}[e^{-3s}X(s)]$

(A) $t\cos 2(t-3)\, u(t-3)$ (B) $t\cos 2(t-3)\, u(t)$

(C) $-t\cos 2(t-3)\, u(t-3)$ (D) $-t\cos 2(t-3)\, u(t)$

**Statement for Q.30–33:**

Given the transform pair

$$x(t)u(t) \xleftrightarrow{L} \frac{2s}{s^2+2}.$$

Determine the Laplace transform $Y(s)$ of the given time signal in question and choose correct option.

**30.** $y(t) = x(t-2)$

(A) $\dfrac{2se^{-2s}}{s^2+2}$ (B) $\dfrac{2se^{2s}}{s^2+2}$

(C) $\dfrac{2(s-2)}{(s-2)^2+1}$ (D) $\dfrac{2(s+2)}{(s+2)^2+1}$

**31.** $y(t) = x(t) * \dfrac{dx(t)}{dt}$

(A) $\dfrac{4s^3}{(s^2+2)^2}$ (B) $\dfrac{4}{(s^2+2)^2}$

(C) $\dfrac{-4s^3}{(s^2+2)^2}$ (D) $\dfrac{4}{(s^2+2)^2}$

**32.** $y(t) = e^{-t}x(t)$

(A) $\dfrac{2(s+1)}{(s+1)^2+2}$ (B) $\dfrac{2(s+1)}{s^2+2s+2}$

(C) $\dfrac{2(s+1)}{s^2+2s+4}$ (D) $\dfrac{2(s+1)}{s^2+2s}$

**33.** $y(t) = 2tx(t)$

(A) $\dfrac{8-4s^2}{(s^2+2)^2}$ (B) $\dfrac{4s^2-8}{(s^2+2)^2}$

(C) $\dfrac{4s^2}{s^2+1}$ (D) $\dfrac{s^2}{s^2+1}$

**Statement for Q.34–43:**

Determine the bilateral laplace transform and choose correct option.

**34.** $x(t) = e^{-t}u(t+2)$

(A) $\dfrac{e^{2(s+1)}}{s+1}$, Re $(s) > -1$

(B) $\dfrac{1}{1+s}$, Re $(s) < -1$

(C) $\dfrac{e^{2(s+1)}}{s+1}$, Re $(s) < -1$

(D) $\dfrac{1}{1+s}$, Re $(s) > -1$

**35.** $x(t) = u(-t+3)$

(A) $\frac{1-e^{-3s}}{s}$, $\quad$ Re $(s) > 0$

(B) $\frac{-e^{-3s}}{s}$, $\quad$ Re $(s) < 0$

(C) $\frac{1-e^{-3s}}{s}$, $\quad$ Re $(s) < 0$

(D) $\frac{-e^{-3s}}{s}$, $\quad$ Re $(s) > 0$

**36.** $y(t) = \delta(t+1)$

(A) $e^s$, Re$(s) > 0$ $\quad$ (B) $e^s$, Re $(s) < 0$

(C) $e^s$, all $s$ $\quad$ (D) None of above

**37.** $x(t) = \sin t\, u(t)$

(A) $\frac{1}{(1+s^2)}$, $\quad$ Re $(s) < 0$

(B) $\frac{1}{(1+s^2)}$, $\quad$ Re $(s) > 0$

(C) $\frac{-1}{(1+s^2)}$, $\quad$ Re $(s) < 0$

(D) $\frac{-1}{(1+s^2)}$, $\quad$ Re $(s) > 0$

**38.** $x(t) = e^{-\frac{t}{2}}u(t) + e^{-t}u(t) + e^{t}u(-t)$

(A) $\frac{6s^2+2s-2}{(2s+1)(s^2-1)}$, $\quad$ Re $(s) < -0.5$

(B) $\frac{6s^2+2s-2}{(2s+1)(s^2-1)}$, $\quad -1 >$ Re $(s) > 1$

(C) $\frac{1}{s+0.5} + \frac{1}{s+1} + \frac{1}{s-1}$, $\quad -1 <$ Re $(s) < 1$

(D) $\frac{1}{s+0.5} + \frac{1}{s+1} - \frac{1}{s-1}$, $\quad -0.5 <$ Re $(s) < 1$

**39.** $x(t) = e^{t}\cos 2t\, u(-t) + e^{-t}u(t) + e^{\frac{t}{2}}u(t)$

(A) $\frac{(1-s)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad 0.5 <$ Re $(s) < 1$

(B) $\frac{(1-s)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad -1 <$ Re $(s) < 1$

(C) $\frac{(s-1)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad 0.5 <$ Re $(s) < 1$

(D) $\frac{(s-1)}{(s-1)^2+4} + \frac{1}{s+1} + \frac{1}{s-0.5}$, $\quad -1 <$ Re $(s) < 1$

**40.** $x(t) = e^{(3t+6)}u(t+3)$

(A) $\frac{e^{3s}}{s-3}$, $\quad$ Re $(s) > 3$

(B) $\frac{e^{3s}}{s-3}$, $\quad$ Re $(s) < 3$

(C) $\frac{e^{3(s-1)}}{s-3}$, $\quad$ Re $(s) > 3$

(D) $\frac{e^{3(s-1)}}{s-3}$, $\quad$ Re $(s) < 3$

**41.** $x(t) = \cos 3t\, u(-t) * e^{-t}u(t)$

(A) $\frac{-s}{(s+1)(s^2+9)}$, $\quad$ Re $(s) > 0$

(B) $\frac{-s}{(s+1)(s^2+9)}$, $\quad -1 <$ Re $(s) < 0$

(C) $\frac{s}{(s+1)(s^2+9)}$, $\quad -1 <$ Re $(s) < 0$

(D) $\frac{s}{(s+1)(s^2+9)}$, $\quad$ Re $(s) > 0$

**42.** $x(t) = e^{t}\sin(2t+4)\, u(t+2)$

(A) $\frac{e^{2(s-1)}}{(s-1)^2+4}$, $\quad$ Re $(s) > 1$

(B) $\frac{e^{2(s-1)}}{(s-1)^2+4}$, $\quad$ Re $(s) < 1$

(C) $\frac{e^{(s-2)}}{(s-1)^2+4}$, $\quad$ Re $(s) > 1$

(D) $\frac{e^{(s-2)}}{(s-1)^2+4}$, $\quad$ Re $(s) < 1$

**43.** $x(t) = e^{t}\frac{d}{dt}[e^{-2t}u(-t)]$

(A) $\frac{1-s}{s+1}$, $\quad$ Re $(s) < -1$

(B) $\frac{1-s}{s+1}$, $\quad$ Re $(s) > -1$

(C) $\frac{s-1}{s+1}$, $\quad$ Re $(s) < -1$

(D) $\frac{s-1}{s+1}$, $\quad$ Re $(s) > -1$

**Statement for Q.44–49:**

Determine the corresponding time signal for given bilateral Laplace transform.

**44.** $X(s) = \frac{e^{5s}}{s+2}$ with ROC: Re $(s) < -2$

(A) $e^{-2(t+5)}u(t+5)$

(A) $\frac{1}{2}e^{-t}\sin t\,u(t)$ (B) $2e^{-t}\cos t\,u(t)$

(C) $2e^{-t}\cos t\,u(t) + \frac{1}{2}e^{-t}\sin t\,u(t)$

(D) $\frac{1}{2}e^{-t}\cos t\,u(t-1) + 2e^{-t}\sin t\,u(t-1)$

**58.** $\frac{d^3y(t)}{dt^3} + 4\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} = x(t)$

All initial condition are zero, $x(t) = 10e^{-2t}$

(A) $\left[\frac{5}{3} + 5e^{-t} - 5e^{-2t} + \frac{5}{3}e^{-3t}\right]u(t)$

(B) $\left[\frac{5}{3} - 5e^{-t} + 5e^{-2t} + \frac{5}{3}e^{-3t}\right]u(t)$

(C) $\frac{5}{3}u(t) - 5u(t-1) + 5u(t-2) + \frac{5}{3}u(t-3)$

(D) $\frac{5}{3}u(t) + 5u(t-1) - 5u(t-2) + \frac{5}{3}u(t-3)$

**59.** The transform function $H(s)$ of a causal system is

$$H(s) = \frac{2s^2 + 2s - 2}{s^2 - 1}$$

The impulse response is

(A) $2\delta(t) - (e^{-t} + e^{t})u(-t)$

(B) $2\delta(t) - (e^{-t} + e^{t})u(t)$

(C) $2\delta(t) + e^{-t}u(t) - e^{t}u(-t)$

(D) $2\delta(t) + (e^{-t} + e^{t})u(t)$

**60.** The transfer function $H(s)$ of a stable system is

$$H(s) = \frac{2s - 1}{s^2 + 2s + 1}$$

The impulse response is

(A) $2u(-t+1) - 3tu(-t+1)$

(B) $(3te^{-t} - 2e^{-t})u(t)$

(C) $2u(t+1) - 3tu(t+1)$

(D) $(2e^{-t} - 3te^{-t})u(t)$

**61.** The transfer function $H(s)$ of a stable system is

$$H(s) = \frac{s^2 + 5s - 9}{(s+1)(s^2 - 2s + 10)}$$

The impulse response is

(A) $-e^{-t}u(t) + (e^{t}\sin 3t + 2e^{t}\cos 3t)u(t)$

(B) $-e^{-t}u(t) - (e^{t}\sin 3t + 2e^{t}\cos 3t)u(-t)$

(C) $-e^{-t}u(t) - (e^{t}\sin 3t + 2e^{t}\cos 3t)u(t)$

(D) $-e^{-t}u(t) + (e^{t}\sin 3t + 2e^{t}\cos 3t)u(-t)$

**62.** A stable system has input $x(t)$ and output $y(t) = e^{-2t}\cos t\,u(t)$. The impulse response of the system is

(A) $\delta(t) - (e^{-2t}\cos t + e^{-2t}\sin t)u(t)$

(B) $\delta(t) - (e^{-2t}\cos t + e^{-2t}\sin t)u(t-2)$

(C) $\delta(t) - (e^{2t}\cos t + e^{2t}\sin t)u(t)$

(D) $\delta(t) - (e^{2t}\cos t + e^{2t}\sin t)u(t+2)$

**63.** The relation ship between the input $x(t)$ and output $y(t)$ of a causal system is described by the differential equation

$$\frac{dy(t)}{dt} + 10y(t) = 10x(t)$$

The impulse response of the system is

(A) $-10e^{-10t}u(-t+10)$ (B) $10e^{-10t}u(t)$

(C) $10e^{-10t}u(-t+10)$ (D) $-10e^{-10t}u(t)$

**64.** The relationship between the input $x(t)$ and output $y(t)$ of a causal system is defined as

$$\frac{d^2y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = -4x(t) + 5\frac{dx(t)}{dt}.$$

The impulse response of system is

(A) $3e^{-t}u(t) + 2e^{2t}u(-t)$

(B) $(3e^{-t} + 2e^{2t})u(t)$

(C) $3e^{-t}u(t) - 2e^{2t}u(-t)$

(D) $(3e^{-t} - 2e^{2t})u(-t)$

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# SOLUTIONS

**1.** (B) $X(s)=\int_0^\infty x(t)e^{-st}dt=\int_2^\infty e^{-st}dt=\frac{e^{-2s}}{s}$

**2.** (A) $X(s)=\int_0^\infty x(t)e^{-3t}dt=\int_0^\infty u(t+2)^{-3t}dt=\int_0^\infty e^{-3t}dt=\frac{1}{s}$

**3.** (A) $X(s)=\int_0^\infty e^{-2t}e^{-st}dt=\frac{1}{s+2}$

**4.** (C) $X(s)=\int_0^\infty x(t)e^{-st}dt=\int_0^\infty e^{2t}u(-t+2)e^{-st}dt$

$=\int_0^2 e^{t(2-s)}dt=\frac{e^{2(2-s)}-1}{2-s}=\frac{1-e^{-2(2-s)}}{s-2}$

**5.** (C) $X(s)=\int_0^\infty \frac{(e^{j5t}-e^{-j5t})}{2j}e^{-st}dt=\frac{5}{s^2+25}$

**6.** (B) $X(s)=\int_0^2 e^{-st}dt=\frac{1-e^{-2s}}{s}$

**7.** (B) $p(t)=te^{-t}u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{(s+1)^2}$

$x(t)=\frac{d}{dt}p(t)\overset{L}{\longleftrightarrow}X(s)=\frac{s}{(s+1)^2}$

**8.** (A) $p(t)=tu(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{s^2}$

$q(t)=\cos 2\pi t\,u(t)\overset{L}{\longleftrightarrow}Q(s)=\frac{s}{s^2+4\pi^2}$

$x(t)=p(t)*q(t)\overset{L}{\longleftrightarrow}X(s)=P(s)Q(s)$

$\Rightarrow\quad X(s)=\frac{1}{s(s^2+4\pi^2)}$

**9.** (C) $p(t)=tu(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{s^2}$

$q(t)=-tp(t)\overset{L}{\longleftrightarrow}Q(s)=\frac{d}{ds}P(s)=\frac{-2}{s^3}$

$x(t)=-tq(t)\overset{L}{\longleftrightarrow}X(s)=\frac{d}{ds}Q(s)=\frac{6}{s^4}$

$t^n u(t)\overset{L}{\longleftrightarrow}\frac{n!}{s^{n+1}}$

**10.** (D) $p(t)=u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{1}{s}$

$q(t)=u(t-1)\overset{L}{\longleftrightarrow}Q(s)=\frac{e^{-s}}{s^2}$

$r(t)=e^{-2t}u(t)\overset{L}{\longleftrightarrow}R(s)=\frac{1}{s+2}$

$v(t)=e^{-2t}u(t-1)\overset{L}{\longleftrightarrow}V(s)=\frac{e^{-(s+2)}}{s^2}$

$x(t)=q(t)*v(t)\overset{L}{\longleftrightarrow}X(s)=Q(s)V(s)$

$\Rightarrow\quad X(s)=\frac{e^{-2(s+1)}}{s+2}$

**11.** (B) $p(t)=e^{-3t}\cos 2t\,u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{s+3}{(s+3)^2+4}$

$\int_{-\infty}^{t}p(\tau)d\tau\overset{L}{\longleftrightarrow}\frac{1}{s}\int_{-\infty}^{0}p(\tau)d\tau+\frac{P(s)}{s}$

$\Rightarrow\quad X(s)=\frac{(s+3)}{s[(s+3)^2+4]}$

**12.** (A) $p(t)=e^{-t}\cos t\,u(t)\overset{L}{\longleftrightarrow}P(s)=\frac{s+1}{(s+1)^2+1}$

$q(t)=\frac{d}{dt}p(t)\overset{L}{\longleftrightarrow}Q(s)=\frac{s(s+1)}{(s+1)^2+1}$

$x(t)=tq(t)\overset{L}{\longleftrightarrow}X(s)=-\frac{d}{ds}Q(s)$

$\Rightarrow\quad X(s)=\frac{-(s^2+4s+2)}{(s^2+2s+2)^2}$

**13.** (B) $X(s)=\frac{s+3}{(s^2+3s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$

$A=\left.\frac{s+3}{s+2}\right|_{s=-1}=2,\ B=\left.\frac{s+3}{s+1}\right|_{s=-2}=-1$

$x(t)=[2e^{-t}-e^{-2t}]u(t)$

**14.** (A) $X(s)=2-\frac{1}{(s+2)(s+3)}=2-\frac{1}{(s+2)}+\frac{1}{(s+3)}$

$x(t)=2\delta(t)+(e^{-3t}-e^{-3t})u(t)$

**15.** (C) $X(s)=\frac{2s-1}{s^2+2s+1}=\frac{A}{(s+1)}+\frac{B}{(s+1)^2}$

$B=(2s-2)|_{s=-1}=-3,\ A=2$

$x(t)=x(t)=[2e^{-t}-3te^{-t}]u(t)$

**16.** (B) $X(s)=\frac{5s+4}{s^3+3s^2+2s}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+2}$

$A=sX(s)|_{s=0}=2,\ B=(s+1)X(s)|_{s=-1}=1,$

$C=(s+2)X(s)|_{s=-2}=-3$

$x(t)=[2+e^{-t}-3e^{-2t}]u(t)$

**17.** (C) $X(s)=\frac{s^2-3}{(s+2)(s^2+2s+1)}$

$=\frac{A}{(s+2)}+\frac{B}{(s+1)}+\frac{C}{(s+1)^2}$

$$A = (s+2)X(s)\big|_{s=-2} = 1,\ C = (s+1)^2 X(s)\big|_{s=-1} = -2$$
$$A + B = 1 \Rightarrow B = 0$$
$$x(t) = [e^{-2t} - te^{-t}]u(t)$$

**18.** (A) $X(s) = \dfrac{3s+2}{s^2+2s+10} = \dfrac{3(s+1)}{(s+1)^2+3^2} - \dfrac{1}{(s+1)^2+3^2}$

$$x(t) = \left[3e^{-t}\cos 3t - \frac{1}{3}e^{-t}\sin 3t\right]u(t)$$

**19.** (C) $X(s) = \dfrac{4s^2+8s+10}{(s+2)(s^2+2s+5)}$

$$= \frac{A}{(s+2)} + \frac{B(s+1)}{(s+1)^2+2^2} + \frac{C}{(s+1)^2+2^2}$$
$$A = (s+2)X(s)\big|_{s=-2} = 2$$
$$A + B = 4 \Rightarrow B = 2$$
$$5A + 2B + 2C = 10 \Rightarrow C = -2$$
$$x(t) = [2e^{-2t} + 2e^{-t}\cos 2t - e^{-t}\sin 2t]u(t)$$

**20.** (B) $X(s) = \dfrac{3s^2+10s+10}{(s+2)(s^2+6s+10)}$

$$= \frac{A}{(s+2)} + \frac{B(s+3)}{(s+3)^2+1} + \frac{C}{(s+3)^2+1}$$
$$A = (s+2)X(s)\big|_{s=-2} = 1,\ A + B = 3 \Rightarrow B = 2$$
$$10A + 6B + 2C = 10 \Rightarrow C = -6$$
$$x(t) = [e^{-2t} + 2e^{-3t}\cos t - 6e^{-3t}\sin t]u(t)$$

**21.** (D) $X(s) = \dfrac{2s^2+11s+16+e^{-2s}}{(s^2+5s+6)}$

$$= 2 + \frac{A}{(s+2)} + \frac{B}{(s+3)} + \frac{e^{-2s}}{(s+2)} - \frac{e^{-2s}}{(s+3)}$$
$$A = \frac{(s+2)(2s^2+11s+16)}{(s^2+5s+6)}\bigg|_{s=-2} = 2$$
$$B = \frac{(s+3)(2s^2+11s+16)}{(s^2+5s+6)}\bigg|_{s=-3} = -1$$
$$x(t) = 2\delta(t) + [2e^{-2t} - e^{-3t}]u(t) + [e^{-2(t-2)} - e^{-3(t-2)}]u(t-2)$$

**22.** (C) $P(s) = \dfrac{1}{s^2+9} \overset{L}{\longleftrightarrow} p(t) = \dfrac{1}{3}\sin 3t\, u(t)$

$$Q(s) = \frac{d^2}{ds^2}P(s) \overset{L}{\longleftrightarrow} q(t) = (-1)^2 t^2 p(t) = \frac{t^2}{3}\sin 3t\, u(t)$$
$$R(s) = sQ(s) \overset{L}{\longleftrightarrow} r(t) = \frac{d}{dt}q(t) - q(0^-)$$
$$= \frac{2t}{3}\sin 3t\, u(t) + t^2\cos 3t\, u(t)$$
$$V(s) = \frac{1}{s+3} \overset{L}{\longleftrightarrow} v(t) = e^{-3t}u(t)$$
$$x(t) = v(t) + r(t) = \left[\frac{2t}{3}\sin 3t\, u(t) + t^2\cos 3t\, u(t) + e^{-3t}\right]u(t)$$

**23.** (C) $P\left(\dfrac{s}{a}\right) \overset{L}{\longleftrightarrow} ap(at)$

$$\frac{1}{(s+1)^2+4} \overset{L}{\longleftrightarrow} \frac{1}{2}e^{-t}\sin 2t\, u(t)$$
$$x(t) \overset{L}{\longleftrightarrow} \frac{1}{4}e^{-0.5t}\sin t\, u(t)$$

**24.** (C) $P(s) = \dfrac{1}{(s+1)^2} \overset{L}{\longleftrightarrow} p(t) = te^{-t}u(t)$

$$Q(s) = \frac{d}{ds}P(s) \overset{L}{\longleftrightarrow} q(t) = -tp(t) = -t^2e^{-t}u(t)$$
$$X(s) = e^{-2s}Q(s) \overset{L}{\longleftrightarrow} x(t) = q(t-2)$$
$$\Rightarrow x(t) = -(t-2)\,e^{(t-2)}u(t-2)$$

**25.** (A) $sX(s) + X(s) \overset{L}{\longleftrightarrow} \dfrac{dx(t)}{dt} + x(t)$

$$\Rightarrow y(t) = (-2\sin 2t + \cos 2t)u(t)$$

**26.** (B) $X\left(\dfrac{s}{a}\right) \overset{L}{\longleftrightarrow} ax(at)$

$$X(3s) \overset{L}{\longleftrightarrow} \frac{1}{3}\cos\left(\frac{2}{3}t\right)u(t)$$

**27.** (D) $X(s+2) \overset{L}{\longleftrightarrow} e^{-2t}x(t)$

**28.** (B) $P(s) = \dfrac{X(s)}{s} \overset{L}{\longleftrightarrow} \displaystyle\int_{-\infty}^{t} x(\tau)d\tau$

$$\overset{L}{\longleftrightarrow} \int_{-\infty}^{t} \cos 2\tau\, u(\tau)d\tau = \frac{\sin 2t}{2}$$
$$\frac{P(s)}{s} \overset{L}{\longleftrightarrow} \int_{0}^{t} \frac{\sin 2\tau}{2}d\tau = \frac{1-\cos 2t}{4}u(t),$$

**29.** (C) $P(s) = e^{-3s}X(s) \overset{L}{\longleftrightarrow} p(t) = x(t-3)$

$$= \cos 2(t-3)\, u(t-3)$$
$$Q(s) = \frac{d}{ds}P(s) \overset{L}{\longleftrightarrow} q(t) = -p(t)$$
$$= -t\cos 2(t-3)\, u(t-3).$$

**30.** (A) $x(t-2) \overset{L}{\longleftrightarrow} e^{-2s}X(s),\ Y(s) = \dfrac{2se^{-2s}}{s^2+2}$

**31.** (A) $p(t) = \dfrac{d}{dt}x(t) \overset{L}{\longleftrightarrow} P(s) = sX(s)$

$$y(t) = x(t) * p(t) \overset{L}{\longleftrightarrow} Y(s) = P(s)X(s) = s(X(s))^2$$

**32.** (A) $e^{-t}x(t) \overset{L}{\longleftrightarrow} X(s+1) = \dfrac{2(s+1)}{(s+1)^2+2}$

**33.** (B) $2tx(t) \xleftrightarrow{L} -2\dfrac{d}{ds}X(s) = \dfrac{4s^2-8}{(s^2+2)^2}$

**34.** (A) $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

$= \int_{-2}^{\infty} e^{-t}e^{-st}dt = \int_{-2}^{\infty} e^{-t(s+1)}dt = \dfrac{e^{2(s+1)}}{s+1}$ , $\text{Re}(s) > -1$

**35.** (B) $X(s) = \int_{-\infty}^{\infty} u(-t+3)e^{-st}dt = \int_{-\infty}^{3} e^{-st}dt = \dfrac{-e^{-3s}}{s}$ $\text{Re}(s) < 0$

**36.** (C) $Y(s) = \int_{-\infty}^{\infty} \delta(t+1)e^{-st}dt = e^{s}$, All $s$

**37.** (B) $X(s) = \int_{0}^{\infty} \dfrac{(e^{jt}-e^{-jt})}{2j} e^{-st}dt$

$= \dfrac{1}{2j}\int_{0}^{\infty} e^{t(j-s)}dt - \dfrac{1}{2j}\int_{0}^{\infty} e^{-t(j+s)}dt = \dfrac{1}{1+s^2}$, $\text{Re}(s) > 0$

**38.** (D) $X(s) = \int_{0}^{\infty} e^{-\frac{t}{2}}e^{-st} + \int_{0}^{\infty} e^{-t}e^{-st}dt + \int_{-\infty}^{0} e^{t}e^{-st}dt$

$= \dfrac{1}{s+0.5} + \dfrac{1}{s+1} - \dfrac{1}{s-1}$

$\text{Re}(s) > -0.5$, $\text{Re}(s) > -1$, $\text{Re}(s) < 1$

$\Rightarrow$ $-0.5 < \text{Re}(s) < 1$

**39.** (A) $X(s) = \int_{-\infty}^{0} e^{t}\dfrac{(e^{jt}-e^{-jt})}{2j} e^{-st}dt + \int_{0}^{\infty} e^{-t}e^{-st}dt + \int_{0}^{\infty} e^{-\frac{t}{2}}e^{-st}dt$

$\text{Re}(s) < 1$, $\text{Re}(s) > -1$, $\text{Re}(s) > 0.5$

Therefore $0.5 < \text{Re}(s) < 1$

$X(s) = -\dfrac{s-1}{(s-1)^2+4} + \dfrac{1}{(s+1)} + \dfrac{1}{s-0.5}$

**40.** (C) $x(t) = e^{-3}e^{-3(t+3)}u(t+3)$

$p(t) = e^{3t}u(t) \xleftrightarrow{L} P(s) = \dfrac{1}{s-3}$

$q(t) = p(t+3) \xleftrightarrow{L} Q(s) = e^{3s}P(s) = \dfrac{e^{3s}}{s-3}$

$X(s) = \dfrac{e^{3(s-1)}}{s-3}$, $\text{Re}(s) > 3$

**41.** (B) $p(t) * q(t) \xleftrightarrow{L} P(s)Q(s)$

$X(s) = \dfrac{-s}{s^2+9}\left(\dfrac{1}{s+1}\right)$

$\text{Re}(s) > -1$, $\text{Re}(s) < 0$

$\Rightarrow$ $-1 < \text{Re}(s) < 0$

**42.** (A) $x(t) = e^{-2}e^{t+2}\sin(2t+4)\,u(t+2)$

$p(t+2) \xleftrightarrow{L} e^{2s}P(s)$,

$X(s) = \dfrac{e^{2s}e^{-2}}{(s-1)^2+4}$, $\text{Re}(s) > 1$

**43.** (A) $p(t) = e^{-2t}u(-t) \xleftrightarrow{L} P(s) = \dfrac{-1}{s+2}$, $\text{Re}(s) < -2$

$q(t) = \dfrac{d}{dt}p(t) \xleftrightarrow{L} Q(s) = sP(s)$

$x(t) = e^{t}q(t) \xleftrightarrow{L} X(s) = Q(s-1) = \dfrac{1-s}{1+s}$

$\text{Re}(s) < -1$ thus left-sided .

**44.** (C) Left-sided

$P(s) = \dfrac{1}{s+2} \xleftrightarrow{L} p(t) = -e^{-2t}u(-t)$

$X(s) = e^{5s}P(s) \xleftrightarrow{L} x(t) = p(t+5)$

$\Rightarrow$ $x(t) = -e^{-2(t+5)}u(-(t+5))$

**45.** (A) Right-sided

$P(s) = \dfrac{1}{(s-3)} \xleftrightarrow{L} p(t) = e^{3t}u(t)$

$X(s) = \dfrac{d^2}{ds^2}P(s) \xleftrightarrow{L} x(t) = t^2e^{3t}u(t)$

**46.** (D) Left-sided

$x(t) = -u(-t) + u(-t+1) + \delta(t+2)$

**47.** (C) Right-sided, $P(s) = \dfrac{1}{s} \xleftrightarrow{L} p(t) = u(t)$

$Q(s) = e^{-3s}P(s) \xleftrightarrow{L} q(t) = p(t-3) = u(t-3)$

$R(s) = \dfrac{d}{ds}Q(s) \xleftrightarrow{L} r(t) = -tq(t) = -tu(t-3)$

$V(s) = \dfrac{1}{s}R(s) \xleftrightarrow{L} v(t) = \int_{-\infty}^{t} r(\tau)d\tau$

$\Rightarrow$ $v(t) = -\int_{3}^{t} tdt = -\dfrac{1}{2}(t^2-9)$

$X(s) = \dfrac{1}{s}v(s) \xleftrightarrow{L} x(t) = -\dfrac{1}{2}\int_{-\infty}^{t}(t^2-9)$

$\Rightarrow$ $x(t) = \left[\dfrac{-1}{6}(t^3-27) + \dfrac{9}{2}(t-3)\right]u(t-3)$

**48.** (B) $X(s) = \dfrac{-s-4}{s^2+3s+2} = \dfrac{-3}{(s+1)} + \dfrac{2}{s+2}$

Left-sided, $x(t) = 3e^{-t}u(-t) - 2e^{-2t}u(-t)$

**49.** (A) $X(s) = \dfrac{5}{(s+1)} - \dfrac{1}{(s+1)^2}$

Left-sided, $x(t) = -5u(-t) + te^{-t}u(-t)$

**50.** (D) $x(0^+) = \lim_{x\to\infty} sX(s) = \dfrac{s}{s^2+5s-2} = 0$

**51.** (A) $x(0^+) = \lim_{s\to\infty} sX(s) = \dfrac{s^2+2s}{s^2+2s-3} = 1$

**52.** (D) $x(0^+) = \lim_{s\to\infty} sX(s) = \dfrac{e^{-2s}(6s^3+s^2)}{s^2+2s-2} = 0$

**53.** (A) $x(\infty) = \lim_{s\to 0} sX(s) = \dfrac{2s^3+3s}{s^2+5s+1} = 0$

**54.** (C) $x(\infty) = \lim_{s\to 0} sX(s) = \dfrac{s+2}{s^2+3s+1} = 2$

**55.** (B) $x(\infty) = \lim_{s\to 0} sX(s) = \dfrac{e^{-3s}(2s^2+1)}{s^2+5s+4} = \dfrac{1}{4}$

**56.** (C) $sY(s) - y(0^-) + 10Y(s) = 10(s)$

$y(0^-) = 1, \quad X(s) = \dfrac{1}{s}$

$Y(s) = \dfrac{10}{s(s+1)} + \dfrac{1}{(s+1)} = \dfrac{1}{s}$

$\Rightarrow \quad y(t) = u(t)$

**57.** (C) $s^2Y(s) - 2s + 2sY(s) - 2 + 5Y(s) = 1$

$(s^2+2s+5)Y(s) = 3 + 2s$

$Y(s) = \dfrac{2s+3}{s^2+2s+5} = \dfrac{2(s+1)}{(s+1)^2+2^2} + \dfrac{1}{(s+1)^2+2^2}$

$\Rightarrow \quad y(t) = 2e^{-t}\cos t\, u(t) + \dfrac{1}{2}e^{-t}\sin t\, u(t)$

**58.** (B) $s^3Y(s) + 4s^2Y(s) + 3sY(s) = \dfrac{10}{(s+2)}$

$Y(s) = \dfrac{10}{s(s+1)(s+2)(s+3)} = \dfrac{A}{s} + \dfrac{B}{(s+1)} + \dfrac{C}{(s+2)} + \dfrac{D}{s+3}$

$A = sY(s)\big|_{s=0} = \dfrac{5}{3},\ B = (s+1)Y(s)\big|_{s=-1} = -5,$

$C = (s+2)Y(s)\big|_{s=-2} = 5,\ D = (s+3)Y(s)\big|_{s=0} = \dfrac{5}{3}$

$\Rightarrow \quad y(t) = \left[\dfrac{5}{3} - 5e^{-t} + 5e^{-2t} + \dfrac{5}{3}e^{-3t}\right]u(t)$

**59.** (D) For a causal system $h(t) = 0$ for $t < 0$

$H(s) = 2 + \dfrac{1}{s+1} + \dfrac{1}{s-1}$

$\Rightarrow \quad h(t) = 2\delta(t) + (e^{-t} + e^{t})u(t)$

**60.** (D) $H(s) = \dfrac{2}{s+1} - \dfrac{3}{(s+1)^2}$, System is stable

$h(t) = (2e^{-t} - 3te^{-t})u(t).$

**61.** (A) $H(s) = \dfrac{-1}{(s+1)} + \dfrac{2(s-1)}{(s-1)^2+3^2} + \dfrac{3}{(s-1)^2+3^2}$

System is stable

$h(t) = -e^{-t}u(t) + (2e^{t}\cos 3t + e^{t}\sin 3t)u(-t)$

**62.** (A) $X(s) = \dfrac{1}{s+1}, \quad Y(s) = \dfrac{(s+2)}{(s+2)^2+1}$

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{(s+1)(s+2)}{(s+2)^2+1}$

$= 1 - \dfrac{(s+2)}{(s+2)^2+1} - \dfrac{1}{(s+2)^2+1}$

$h(t) = \delta(t) - (e^{-2t}\cos t + e^{-2t}\sin t)u(t)$

**63.** (B) $sY(s) + 10Y(s) = 10X(s)$

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{10}{s+10}$

$\Rightarrow \quad h(t) = 10e^{-10t}u(t)$

**64.** (B) $Y(s)(s^2 - s - 2) = X(s)(5s-4)$

$H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{5s-4}{s^2-s-2} = \dfrac{3}{s+1} + \dfrac{2}{s-2}$

$h(t) = 3e^{-t}u(t) + 2e^{2t}u(t).$

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