MATHEMATICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 68

Total Marks : 32

Max. Time : 35 min.

Topics : Definite Integration , Indefinite Integration

Type of Questions		**M.M., Min.**
Single choice Objective (no negative marking) Q.1,2,3,4	**(3 marks, 3 min.)**	**[12, 12]**
Subjective Questions (no negative marking) Q.5,6,7	**(4 marks, 5 min.)**	**[12, 15]**
Match the Following (no negative marking) Q.8	**(8 marks, 8 min.)**	**[8, 8]**

1. The value of the integral $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2\, dx$ where p, q are integers, is equal to :

(A) $-\pi$ (B) 0 (C) π (D) 2π

2. The value of the integral $\int_{\pi/3}^{\pi/2} x \sin(\pi[x] - x)\, dx$ is (where [x] denotes greater integer function)

(A) $\frac{1}{2}+\frac{\pi}{6}$ (B) $-\frac{1}{2}-\frac{\pi}{6}$ (C) $1-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$ (D) $\frac{\sqrt{3}}{2}-1-\frac{\pi}{6}$

3. If $\int_0^a \frac{dx}{\sqrt{x+a}+\sqrt{x}} = \int_0^{\pi/8} \frac{2\tan\theta}{\sin 2\theta}\, d\theta$, then value of 'a' is equal to (a > 0)

(A) $\frac{3}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{9}{16}$

4. If the value of the integral $\int_1^2 e^{x^2}\, dx$ is α, then $\int_e^{e^4} \sqrt{\ell n x}\, dx$ is equal to :

(A) $e^4 - e - \alpha$ (B) $2e^4 - e - \alpha$ (C) $2(e^4 - e) - \alpha$ (D) none of these

5. Evaluate : $\int \frac{\sqrt{1+x^2}}{1-x^2}\, dx$

6. Evaluate :

(i) $\int_0^{\pi/2} \frac{\cos x\, dx}{(1+\sin x)(2+\sin x)}$ (ii) $\int_1^2 \frac{dx}{x\left(x^4+1\right)}$ (iii) $\int_0^5 \left(|x-3| + |1-x|\right) dx$

7. Evaluate :

(i) $\int_0^{3\pi} \sin^{-1}(\sin x)dx$

(ii) $\int_{-2}^{2} \min(x-[x], -x-[-x])dx$ (where ([x] represents greatest integer less than or equal to x)

(iii) $\int_0^{\infty} \frac{x\log x}{(1+x^2)^2}dx$

8.

Column – I	**Column – II**
(A) f(x) = min {x + 1, 2 sgn (|x|) }, $\forall$ x $\in$ R, then $\int_{-5}^{4} f(x)dx$ =	(p) 3
(B) If f(x) is a continuous function for all real values of x and satisfies $\int_{n}^{n+1} f(\lvert x\rvert)dx = \frac{n^2}{2}$, $\forall$ n $\in$ I, then $\int_{-3}^{5} f(\lvert x\rvert)dx =$	(q) 0
(C) If $\int_{n}^{n+1} [x+[x+[x]]]dx = kn$, n $\in$ I (where [.] denotes the greatest integer function), then k is/are	(r) 22
(D) If f(x) = $\int_1^{x} \frac{e^t}{t}dt$, x $\in$ R^+, then the number of solutions of f′(x) = 1 is	(s) 1

Answers Key

1. (D) **2.** (C) **3.** (D) **4.** (B)

5. $-\frac{1}{\sqrt{2}}\log\left|\frac{\sqrt{1+x^2}-\sqrt{2}\,x}{\sqrt{1+x^2}+\sqrt{2}\,x}\right| - \log\left|x+\sqrt{1+x^2}\right| + c$

6. **(i)** $\ell n\frac{4}{3}$ **(ii)** $\frac{1}{4}\ln\frac{32}{17}$ **(iii)** 15

7. **(i)** $\frac{\pi^2}{4}$ **(ii)** 1 **(iii)** 0

8. (A) $\rightarrow$ (q), (B) $\rightarrow$ (r), (C) $\rightarrow$ (p), (D) $\rightarrow$ (q)