Vector Algebra

True/False

1) Scalar product of two perpendicular vectors is zero.
2) Vector product of two collinear vectors is zero.
3) Scalar triple product of three coplanar vectors is 10.
4) $|\vec{a}.\vec{b}| \leq |\vec{a}||\vec{b}|$ is triangle inequality.
5) $|\vec{a}+\vec{b}| \leq |\vec{a}|+|\vec{b}|$ is Cauchy-Schwartz inequality.
6) $\hat{\imath}\times\hat{\imath}=\hat{\jmath}\times\hat{\jmath}=\hat{k}\times\hat{k}=\vec{0}$.
7) $\hat{\imath}.\hat{\imath}=\hat{\jmath}.\hat{\jmath}=\hat{k}.\hat{k}=0$
8) $\vec{a}\times\vec{b}$ is parallel to both the vectors $\vec{a}$ and $\vec{b}$.
9) Vectors $3\hat{\imath}+2\hat{\jmath}-\hat{k}$ is parallel to the vector $\hat{\imath}+6\hat{\jmath}-3\hat{k}$.
10) Area of a parallelogram can be calculated by using vector product two vectors.

Multiple Choice Questions

1 If $\vec{a}.\vec{b}=|\vec{a}\times\vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

2 Magnitude of the vector $\frac{1}{\sqrt{3}}\hat{\imath}+\frac{1}{\sqrt{3}}\hat{\jmath}+\frac{1}{\sqrt{3}}\hat{k}$ is :

(a)-1 (b)1 (c)0 (d)$\frac{1}{3}$

3 If $\sqrt{3}\,\vec{a}.\vec{b}=|\vec{a}\times\vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

4 If $\vec{a}.\vec{b}=\sqrt{3}|\vec{a}\times\vec{b}|$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

5 If $\vec{a}.\vec{b}=0$ then angle between vector $\vec{a}$ and vector $\vec{b}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

6 Name of the inequality $|\vec{a}.\vec{b}| \leq |\vec{a}||\vec{b}|$ is :

(a)Cauchy-Schwartz Inequality (b)Triangle Inequality
(c)Rolle's Theorem (d)Lagrange's Mean Value Theorem

7 Magnitude of vector $\vec{a}=3\hat{\imath}+\hat{\jmath}+\hat{k}$ is :

(a) 3 (b) $\sqrt{10}$ (c) $\sqrt{11}$ (d) $\sqrt{12}$

8 Projection of $\vec{a}=3\hat{\imath}+\hat{\jmath}+\hat{k}$ on $\vec{b}=\hat{\imath}-2\hat{\jmath}-\hat{k}$ is :

(a)$\frac{2}{\sqrt{6}}$ (b)0 (c)$\frac{1}{\sqrt{6}}$ (d)$\sqrt{6}$

9 If $\vec{a}$ is a non-zero vector then $|\vec{a}\times\vec{a}|$ is equal to

(a)$|\vec{a}|$ (b)$|\vec{a}|^2$ (c)1 (d)0

10 If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} - \hat{k}$ then $\vec{a}.\vec{b}$ is equal to

(a)1 (b)0 (c)-1 (d)3

2 & 4 Marks Questions

1. Find the volume of a rectangular parallelepiped whose edges are given by the vectors $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + \hat{k}, \vec{b} = 3\hat{\imath} - \hat{\jmath} - 5\hat{k}, \vec{c} = -9\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$.
2. Find the scalar triple product of the vectors $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} - 5\hat{k}, \vec{b} = \hat{\imath} - 2\hat{\jmath} + 5\hat{k}, \vec{c} = 7\hat{\imath} - \hat{\jmath} + 4\hat{k}$
3. For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\vec{a}.\vec{b}| \leq |\vec{a}||\vec{b}|$. Also write the name of inequality.
4. For any two vectors $\vec{a}$ and $\vec{b}$ prove that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$. Also write the name of inequality.
5. Adjacent sides of a parallelogram are given by $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 5\hat{k}$. Find a unit vector along a diagonal of the parallelogram.
6. Adjacent sides of a parallelogram are given by $6\hat{\imath} - \hat{\jmath} + 5\hat{k}$ and $\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$. Find the area of parallelogram.
7. Find the area of triangle whose diagonals are given by the vectors $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + \hat{\jmath} - 7\hat{k}$.
8. Find the value of p if the vectors $p\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, $\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ are coplanar.
9. Find a vector of magnitude 8units along $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + \hat{k}$
10. Find a unit vector along $\vec{a} = 5\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$
11. If $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + \hat{k}, \vec{b} = 3\hat{\imath} - \hat{\jmath} - 5\hat{k}$ then find $|\vec{a} \times \vec{b}|$.
12. Find the projection of $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + \hat{k}$ on $\vec{b} = 3\hat{\imath} - \hat{\jmath} - 5\hat{k}$.
13. Find the area of parallelogram whose diagonals are given by vectors:

(i) $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = \hat{i} - \hat{k}$

(ii) $\vec{a} = \hat{i} + \hat{j} - 4\hat{k}$ & $\vec{b} = \hat{i} + 8\hat{j} + 2\hat{k}$

14. Find the area of triangle whose vertices are :

(i) $A(2,3,5), B(3,5,8), C(2,7,8)$

(ii) $A(1,2,4), B(3,1,-2), C(4,3,1)$

(iii) $P(1,1,1), Q(1,2,3), R(2,3,1)$

15. Show that the following vectors are co-planar :

(i) $-2\hat{i}-2\hat{j}+4\hat{k}\ ,-2\hat{i}+4\hat{j}-2\hat{k}\ ,\ 4\hat{i}-2\hat{j}-2\hat{k}$

(ii) $\hat{i}+2\hat{j}+\hat{k}\ ,\ 3\hat{i}+2\hat{j}-7\hat{k}\ ,\ 5\hat{i}+6\hat{j}-5\hat{k}$

16. For what value of λ are the following vectors coplanar :

(i) $\hat{i}+2\hat{j}+3\hat{k}\ ,\ 3\hat{i}-\hat{j}+2\hat{k}\ ,-2\hat{i}+\lambda\hat{j}+\hat{k}\ \&\ 6\hat{i}-4\hat{j}+2\hat{k}$

(ii) $\hat{i}+2\hat{j}+3\hat{k}\ ,\ 3\hat{i}-\lambda\hat{j}+2\hat{k}\ ,-2\hat{i}+3\hat{j}+\hat{k}\ \&\ 6\hat{i}-4\hat{j}+2\hat{k}$

17. Find x such that the four points $A(3,2,1), B(4,x,5), C(4,2,-2),\ D(6,5,-1)$ are coplanar.

18. Find the volume of the parallelepiped whose sides are given by the vectors :

(i) $3\hat{i}+4\hat{j}\ ,\ 2\hat{i}+3\hat{j}+4\hat{k}\ ,\ 5\hat{k}$

(ii) $\hat{i}+2\hat{j}+3\hat{k}\ ,\ 2\hat{i}+3\hat{j}+4\hat{k}\ ,-\hat{i}+2\hat{j}-3\hat{k}$

19. Find the unit vector perpendicular to the diagonal of parallelogram whose adjacent sides are given by vectors :

(i) $\vec{a}=2\hat{i}-3\hat{j}+\hat{k}\ \&\ \vec{b}=\hat{i}+7\hat{j}-\hat{k}$

(ii) $\vec{a}=5\hat{i}+\hat{j}-\hat{k}\ \&\ \vec{b}=\hat{i}+\hat{j}+\hat{k}$

20. Find the angle between vectors $\vec{a}\ \&\ \vec{b}$ such that :

(i) $|\vec{a}|=\sqrt{3}\ ,\ |\vec{b}|=2\ \&\ \vec{a}.\vec{b}=3$

(ii) $|\vec{a}|=\sqrt{3}\ ,\ |\vec{b}|=2\ \&\ \vec{a}.\vec{b}=\sqrt{6}$

(iii) $|\vec{a}|=5\ ,\ |\vec{b}|=2\ \&\ \vec{a}.\vec{b}=5$

Prepared By : Vaibhav (Lecturer Maths), Govt. Multipurpose Sec. School, Patiala