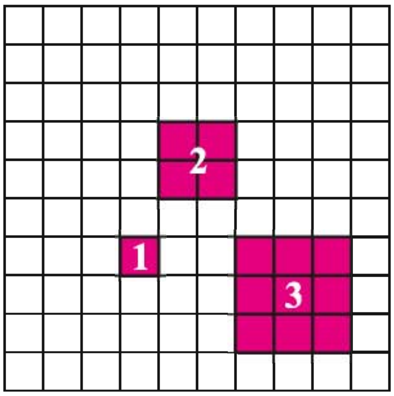
CHAPTER 3

SQUARE AND SQUARE ROOT

3.1 Sonu and Dinu are making squares on a square sheet and writing their area by counting the squares.



Area of a square (Square 1) of size unity = 1 Square Unit

Area of a square of size 2 units = 4 Square Unit

Area of a square of size 3 units = 9 Square Units

You also make squares of 4, 5, 6, 7, 8, 9, 10 units and write their area by counting the squares.

Complete the table given below:

Side of Square	1	2	3	4	5	6					
Area of Square	1	4	9								

Table 3.1

What is special in the numbers 1, 4, 9, 16, 25, ………… and similar numbers?

Since we can express these numbers as $1 = 1\times 1 = 1^2$; $4 = 2\times 2 = 2^2$; $9 = 3\times 3 = 3^2$, we find that these can be obtained by multiplying a number by itself. Such numbers are called as **Square Numbers.**

In general, for $s = r^2$, s is a square number. Is 24 a square number?

Think of following numbers and their squares and fill in the blanks:

Numbers	Square
1	$1\times 1 = 1$
2	$2\times 2 = 4$
3	$3\times 3 = 9$
4	$4\times 4 = 16$
5	$5\times 5 = 25$
6	……………
7	……………
8	……………
9	……………
10	……………

Table 3.2

From the above table it is evident that there are only 10 numbers which are square between 1 to 100 which are square of a number and rest of the numbers are not squares.

Numbers 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100 are square numbers and are called perfect square numbers.

Do and learn:

Write the perfect square number lying between

(i) 20 and 30 (ii) 40 and 50

3.2 Properties of Square Numbers

Square numbers of the numbers from 1 to 20 are given in the table given below:

Numbers	Squares	Numbers	Squares
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

Table 3.3

Write the digits on ones place in square from above table in the form of set A:

A= {0, 1, 4, ……}

Write the numbers between 0 and 9 in B which are not in A:

B = {2, 3, ………}

On the basis of numbers in sets A and set B we can say that the numbers with 2, 3, 7, 8 lying at ones place cannot be square numbers.

Do and learn

(1) Tell on the basis of digits at ones place, which numbers are not perfect square?

(i) 2304 (ii) 402 (iii) 3003 (iv) 100 (v) 1008

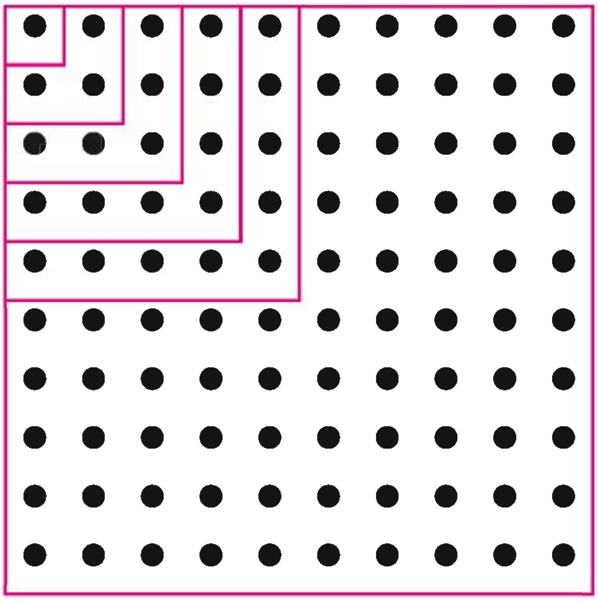
(2) Tell three numbers for which you can say with confidence that they cannot be perfect square.

(i) …………… (ii) …………… (iii) ……………

Perfect squares of even and odd numbers given in table 3.3 are of which type?
Square of odd numbers – Even / Odd
Square of even numbers – Even / Odd
From the above activity, we can say that the **squares of even numbers are even and squares of odd numbers are odd**.

1. Interesting forms of square numbers:

(i)



In figure, squares are drawn starting from the North-West corner. Look at the squares carefully and write the number of black circles:

First square	1	=	1	=	1^2
Second square	1+3	=	4	=	2^2
Third square	1+3+5	=	9	=	3^2
Fourth square	1+3+5+7	=	…	=	…
Fifth square	1+3+5+7+9	=	…	=	…
Sixth square	……………..	=	…	=	…
Seventh square	……………..	=	…	=	…

We have seen that First square = First odd number $= 1^2$
Second square = Sum of first two odd numbers $= 2^2$
Third square = Sum of first three odd numbers $= 3^2$
Proceeding in the similar fashion, we find that the sum of first eight odd numbers will be $= 8^2 = 64$.

2. Look at the squares of 1, 11, 111, ...

$1^2 = 1$

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

$11111^2 =$ ....

$111111^2 =$ ....

3. Write two adjacent numbers, like 4 and 5

Square them $4^2 = 16$, $5^2 = 25$

Difference of squares $= 25 - 16 = 9$

Sum of the numbers $= 4 + 5 = 9$

Write few more similar adjacent number.

You will find that the difference of square of two adjacent numbers = sum of numbers

4. Pythagorean Triplets: $3^2 + 4^2$

$9 + 16 = 25 = (5)^2$

$6^2 + 8^2$

$36 + 64 = 100 = (10)^2$

You will see that in each example there is a triplet and the square of greatest number in the triplet is equal to the sum of squares of other two numbers. Such numbers are known as **Pythagorean Triplets**.

In the above example 3,4,5 and 6, 8,10 are Pythagorean triplets.

Example 1 Check whether 9, 40, 41 is a Pythagorean triplet or not?

Solution $(9)^2 + (40)^2$

$= 81 + 1600$

$= 1681 = (41)^2$

Hence, $(9)^2 + (40)^2 = (41)^2$ which proves that 9, 40, 41 is a Pythagorean triplet.

Exercise 3.1

1. What will be unit place digit in the squares of following numbers:

(i) 24 (ii) 17 (iii) 100 (iv) 55 (v) 111

(vi) 1023 (vii) 5678 (viii) 12796 (ix) 2412

2. Find the squares of numbers given below:

(i) 18 (ii) 11 (iii) 107 (iv) 15 (v) 200 (vi) 27

3. Which of the following numbers has their square an even number:
(i) 235 (ii) 395 (iv) 5508
(iv) 2001 (v) 82003 (vi) 10224
4. Find the following sums without any operation:
(i) 1+3+5+7
(ii) 1+3+5+7+9+11+13
(iii) 1+3+5+7+9+11+13+15+17+19
5. Write 64 as sum of eight odd numbers.
6. How many numbers are there between squares of following numbers?
(i) 10 and 11 (ii) 17 and 18 (iii) 30 and 31
7. Check whether given three numbers are Pythagorean triplets or not?
(i) 9, 12, 15 (ii) 7, 11, 13 (iii) 10, 24, 26

3.3 Square Root

Look at the squares of following numbers:

$(4)^2 = 4 \times 4 = 16$

$(5)^2 = 5 \times 5 = 25$

$(6)^2 = 6 \times 6 = 36$

We see in the above examples that square of 4 is 16. On the contrary we can say that square root of 16 is 4. Similarly, square of 5 is 25 and square root of 25 is 5. i.e., square root is inverse operation of square.

Square root is denoted by the sign "$\sqrt{\ }$"

For example: Square root of 81 = $\sqrt{81} = 9$.

Do and learn ◆ From the table 3.3, what will be the square roots of following?
(i) 49 (ii) 64 (iii) 100

We have seen in the earlier example that sum of first 'n' odd numbers is 'n^2'.

For example: $5^2 = 1+3+5+7+9$

The method in which we add first five odd numbers and obtain the square of 5, we subtract odd numbers from 25 and obtain the square root of 25. Let us see

$25-1=24 \quad 24-3=21 \quad 21-5=16 \quad 16-7=9 \quad 9-9=0$

By successive subtraction of first five odd numbers from 25 we get remainder 0. This means square root of 25 is 5. i.e., $\sqrt{25}=5$

You also try to find the square roots of some perfect square numbers using this process.

3.4 Finding Square Root by the Method of Prime Factorization

Given below are factors of some numbers and their squares

Numbers	Prime Factors of Numbers	Square Numbers	Prime Factors of Square Numbers
6	2×3	36	$2 \times 2 \times 3 \times 3$
8	$2 \times 2 \times 2$	64	$2 \times 2 \times 2 \times 2 \times 2 \times 2$
12	$2 \times 2 \times 3$	144	$2 \times 2 \times 2 \times 2 \times 3 \times 3$

You will find that prime factors of a number repeats twice in the square of the number. For example: Prime factors of 6 are 2×3 then 2×2 and 3×3 comes in the prime factors of 36.

Contrary to this number of prime factors in the square root is half of the number of prime factors of square.

Let us find the square root of perfect square number 144.

We know that the prime factors of 144 are

2	144
2	72
2	36
2	18
2	9
2	3
	1

$144 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

Making the pairs of prime factors we find that

$144 = (2 \times 2 \times 3)^2$

$144 = 2 \times 2 \times 3$

$144 = 12$

Similarly, concentrate upon prime factors of 192.

$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Here all the factors are not in pairs. Hence 192 is not a perfect square. If we want to make it perfect square, we need to multiply it by 3 or divide it by 3.

$192 \times 3 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

$\sqrt{192 \times 3} = 2 \times 2 \times 2 \times 3$

$\sqrt{576} = 24$

Also, $\frac{192}{3} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}{3}$

$\sqrt{\frac{192}{3}} = 2 \times 2 \times 2 = 8$

Example 3 Find the square root of 6400.

Solution

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

$$6400 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{5 \times 5}$$

$$\sqrt{6400} = 2 \times 2 \times 2 \times 2 \times 5$$

$$= 80$$

Example 4 Is 60 a perfect square?

Solution

2	60
2	30
3	15
5	5
	1

$$60 = 2 \times 2 \times 3 \times 5$$

3 and 5 prime factors are not in pairs. Hence 60 is not a perfect square. We can see in the exact form that there is only one zero.

Example 5 Is 1800 a perfect square? If not then find the least multiple of 1800 which is a perfect square and find the square root of new number.

Solution We know that $1800 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times \underline{5 \times 5}$.

According to prime factors 2 does not exists in pairs. Hence 1800 is not a perfect square. If we make one more pair of 2 then it will become a perfect square. Therefore, multiplying1800 by 2 we get

$$1800 \times 2 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5}.$$

Now every prime factor is in pair. Hence,

$1800 \times 2 = 3600$ a perfect square.

$$\sqrt{3600} = 2 \times 2 \times 3 \times 5$$

$$= 60$$

Example 6 Find the smallest square number which is divisible by each 6, 9, and 15.

Solution We will solve it in two steps. First we will find LCM for the number divisible by 6, 9, and 15 and then find the multiple of LCM which is perfect square. LCM of 6, 9, 16 $= 2 \times 3 \times 3 \times 5$
$= 90$

2	6, 9, 15
3	3, 9, 15
3	1, 3, 15
5	1, 1, 5
	1, 1, 1

Since the factors of 90 are not in pairs we multiply it by 2 and 5 to complete pairs so that $90 \times 2 \times 5 = 2 \times \underline{2} \times 3 \times 3 \times 5 \times \underline{5}$.

Thus, 900 is the smallest perfect square divisible by 6, 9, and 15.

Exercise 3.2

1. What can be the unit place digit in the square root of following numbers
   (i) 9604 (ii) 65536 (iii) 998001 (iv) 60481729
2. Estimate and tell which numbers cannot be perfect squares?
   (i) 48 (ii) 81 (iii) 102 (iv) 24636
3. Find the square root by prime factorization method.
   (i) 1296 (ii) 729 (iii) 1764 (iv) 3969 (v) 4356 (vi) 1600
4. Numbers given below are not perfect squares. Find the smallest whole number to which when multiplied makes it perfect square.
   (i) 252 (ii) 396 (iii) 1620
5. Numbers given below are not perfect squares. Use prime factorization method to find the smallest number by which these can be divided to make them perfect squares.
   (i) 1000 (ii) 867 (iii) 4375
6. Rose plants are to be planted in a square shape garden. Number of plants in each row is equal to the number of rows. If 2401 plants are there in the garden then find the number of rows.
7. Find the smallest square number which is completely divisible by 4, 9 and 10

3.5 Finding Square Root by Division Method

When numbers are big then prime factorization method becomes lengthy and cumbersome. For this we use division method to find the square root.

Example 7 Focus on the following steps in finding the square root of 576.

Solution **Step 1** Make pairs of digits by starting from unit place in the number.

e.g. in 576 $\overline{5}\ \overline{76}$

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{5}\ \overline{76} \\ & -4 \\ \cline{2-2} & 1 \end{array}$$

Step 2 Choose the greatest number whose square is either Equal or less than the extreme left digit or pair of digits. i.e., we need to choose a number whose square is less than 5, which is 2.

$$(2)^2 < 5 < (3)^2$$

Put this number as quotient at the top and its square below 5 and then subtract.

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{5}\ \overline{76} \\ +2 & -4 \\ \cline{2-2} 4 & 176 \end{array}$$

Step 3 Again, write the next pair after the remainder as we do in case of simple division. (Note that we write only one digit in case of simple division but a pair in the evaluation of square root.)

Step 4 Add the divisor with the same number and write it below.

Step 5 In our example we need to write a digit (any digit between 0 and 9) after the divisor 4 so that the new divisor can be (40, 41, 42, ……,, 49) and at the same time we write this digit after the quotient 2 at the top. The product of new divisor and this digit should be a number which is less than or equal to our dividend 176.

$$\begin{array}{r|l} & 2\,4 \\ \hline 2 & \overline{5}\ \overline{76} \\ +2 & 4 \\ \cline{2-2} 44 & 176 \\ 4 & 176 \\ \cline{2-2} & 0 \end{array}$$

Step 6 At this point 44 × 4 = 176. Now, since remainder is zero and no digit is left, we write the square root of 576 as 576 = 24

Example 8 Find the square root of 7056 by using division.

Solution **Step 1** Make pairs of digits by starting from unit place in the number.

$$\begin{array}{|l} \hline \overline{70}\ \overline{56} \end{array}$$

Step 2 We choose the greatest number whose square is less than or equal to 70.

$$(8)^2 < 70 < (9)^2$$

We write this number as quotient and its square 64 below 70.

Step 3 – Divisor 8 is added again to 8 and we get a new divisor 16

$$\begin{array}{r|l} & 8 \\ \hline 8 & \overline{70}\ \overline{56} \\ +8 & 64 \\ \cline{2-2} 16 & 6 \end{array}$$

Step 4 Now, we write next pair of digits 56. We get new dividend 656.

Step 5 Again we choose a digit from among (0 - 9) so that the divisor becomes (160, 161, ……, 169). We multiply this number by the same digit so that the product is less than or equal to 656.

```
       8
   8 | 70 56
  +8 | 64
  16 |  6
```

```
       84
   8 | 70 56
  +8 | 64
 164 |  6 56
   4 | -6 56
     |  0 00
```

This number is 4 in this example because $164 \times 4 = 656$.
Hence we find that 7056 = 84

Example 9 Area of a square ground is 1089 m^2. Find the side of the ground.

Solution Area of square ground $= \sqrt{1089}\,m^2$

Therefore, side of the ground $= \sqrt{1089}$

Hence, $= 1089 = 33$ m.

Thus, the side of the ground = 33 m.

```
       3 3
   3 | 10 89
   3 |  9
  63 |  1 89
   3 |  1 89
     |     0
```

Example 10 Find the smallest number which when subtracted from 1989 makes it a perfect square. Also find the square root of that number.

Solution Let us try to find the square of the number 1989.
Here we see that 1989 is 53 more than a perfect square.
Therefore, by subtracting 53 from 1989 will make it a perfect square. $1989 - 53 = 1936$.

$\therefore \quad \sqrt{1936} = 44$

```
       4 4
   4 | 19 89
  +4 | 16
  84 |  3 89
  +4 |  3 36
     |    53
```

Similarly if we want to find that number which when added to 1989 makes it a perfect square then we will consider the square of 45 instead of 44. We have $45^2 = 2025$. Thus, we need to add 2025-1986 = 36 in 1989 so as to make it a perfect square.

Example 11 Find the greatest four digit number which is a perfect square.

Solution We know that the greatest four digit number is 9999. We try to find the square root of it using division method. The remainder is 198, which shows that the square 99^2 is 198 less than 9999. Therefore, the required number is 9999-198 = 9801.

	9 9
9	$\overline{99}\ \overline{99}$
+9	81
189	18 99
9	17 01
	1 98

3.6 Square Root of Decimal Numbers

Example 12 Consider the number $\sqrt{51.84}$

Solution **Step 1** - We follow making of pairs of digits in evaluating the square root of decimal numbers too. Since there are two parts in decimal numbers, namely integer part and decimal part. Pairing of integer part will be same as was in case of earlier examples i.e., starting from unit place. But, in case of decimal part pairing starts from one-tenth place. One-tenth and one-hundredth is one pair one-thousandth and ten-thousandth is another pair etc.

In this example the pairs will be $\overline{51}$ and $\overline{84}$

	7
7	$\overline{51}.\overline{84}$
7	49
	284

Step 2 – As earlier, choose a number whose square is less than or equal to 51, i.e., $(7)^2 < 51 < (8)^2$ and we will write it at divisor column as well as at quotient place.

Step 3 – We will write the product of 7 by 7 below the divi dend and sum of 7 and 7 at the divisor column.

Step 4 – Remainder is 2. Next time we write 84 on the right of it so as to get 284 as dividend. Since 84 was the decimal part of the number we put decimal in quotient.

Step 5 – Now, as before, we choose a number from (0 – 9) and put after 4 in 14 so that the new divisor becomes either of (140, 141, 142, ….., 149) which when multiplied by the same number gives the number not greater than 284. This show our quotient. In this example, the number will be 2 so that 142 × 2 = 284.

	7.2
7	$\overline{51}.\overline{84}$
7	49
142	284
2	284
144	0 00

Hence, $\sqrt{51.84} = 7.2$

Where to proceed?

Consider the number 176.341. Put the bars over integer part and decimal part. Now consider176; we start from the unit place near decimal and move towards left. First bar is over 76 and second bar is over 1. For 0.341, we start from the decimal and move towards right.

First bar is over 34 and in order to put second bar over 1 we put 0 so that it becomes $0.\overline{34}\,\overline{10}$

3.7 Estimating the Square Root

(i) Number of digits in square root
Consider following table

$1^2 = 1$	$99^2 = 9801$
$9^2 = 81$	$100^2 = 10000$
$10^2 = 100$	$999^2 = 9898001$

How many digits are there in the square of one-digit number? 1 or 2
How many digits are there in the square of two-digit number? 3 or 4
How many digits are there in the square of three-digit number? ....

Contrary to this, there will be one digit in the square root of one-digit numbers and 1 or 2 digits in the square root of two-digit numbers and so on.

Do and learn:

How many digits will be there in the square of following numbers?

(i)1369 (ii)15376 (iii) 6031936

We frequently need to evaluate square roots in our daily life.

There are 350 students in a school. They are to be arranged in square configuration and remaining students will look after general arrangements of Independence Day celebration. Here we need to estimate perfect square we know that $100 < 350 < 400$ and

$\sqrt{100} = 10$ and $\sqrt{400} = 20$

Therefore, $10 < \sqrt{350} < 20$ but still we are not near to square number.

We know that

$18^2 = 324$ and $19^2 = 361$.

Thus $18 < \sqrt{350} < 19$

Hence, we can make 18 rows and 18 columns of students out of 350 students

Remaining 16 students will look after general arrangements.

Exercise 3.3

1. Find the square root of the following number using division method:
   (i) 441 (ii) 576 (iii) 1225 (iv) 2916 (v) 4624 (vi) 7921
2. Find the square root of the following numbers without really calculating.
   (i) 121 (ii) 256 (iii) 4489 (iv) 60025
3. Find the square root of the following decimal numbers.
   (i) 6.25 (ii) 2.89 (iii) 32.49 (iv) 31.36 (v) 57.76
4. What is to be added in the following numbers so that they become perfect squares?
   (i) 420 (ii) 2000 (iii) 837 (iv) 3500
5. What is to be subtracted from the following numbers so that they become prefect square numbers?
   (i) 555 (ii) 252 (iii) 1650 (iv) 6410
6. Chairs are to be arranged for a wedding function in square configuration. 1000 chairs are available. How many additional chairs will be required for square configuration? At the same time, also find the number of chairs in each row.
7. Area of a square farm is 361 m^2. How much wire will be required for fencing the four sides?
8. Find the smallest number that can divide 2352 to make it a perfect square.

We Learnt

1. Normally if a number *m* is expressed as n^2(where *m* and *n* both are natural numbers) then *m* is a square number. For example: $n = 5$ and $m = 5^2 = 25$.
2. The number in which the unit place digit is 2, 3, 7, 8 can never be square numbers, i.e., unit place digit in all the square numbers is either of the numbers 0, 1, 4, 5, 6, or 9.
3. Number of zeros in the end in a square number is always even
4. Square root is the inverse operation of square.
5. A perfect square has two square roots one positive and other negative. Positive square root is denoted by $\sqrt{}$