CHAPTER 12

BASIC OPERATIONS & FACTORIZATION

CONSTANT

A symbol having a fixed numerical value is called a constant.
Examples:

$8, -7, \frac{5}{9}, \pi$ etc. are all constants.

VARIABLES

A symbol which may be assigned different numerical values is known as a variable.

Example:

We know that area of circle is given by the formula $A = \pi r^2$ where r is the radius of the circle. Here π is constant while A and r are variables.

ALGEBRAIC EXPRESSIONS

A combination of constants and variables, connected by some or all of the operations +, –, × and ÷, is known as an algebraic expression.

Terms of an Algebraic Expression

The several parts of an algebraic expression separated by + or – operations are called the terms of the expression.

Examples:

(i) $5 + 9x - 7x^2 y + \frac{3}{7}xy$

(ii) $x^3 + 3x^2y + 3xy^2 + y^3 + 7$

POLYNOMIALS

An algebraic expression in which the variables involved have only non-negative integral powers is called a polynomial.

General Form

$p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is a polynomial is variable x, where $a_0, a_1, a_2, a_3....a_n$ are real numbers and n is non-negative integer.

Examples:

(i) $6x^3 - 4x^2 + 7x - 3$ is a polynomial in one variable x.

(ii) $9y^5 + 6y^4 + 7y^3 + 10y^2 - 8y + \frac{2}{7}$ is a polynomial in one variable y.

(iii) $3 + 2x^2 - 6x^2y + 5xy^2$ is a polynomial in two variables x and y.

(v) $5 + 8x^{5/2} + 7x^3$ is an expression but not a polynomial since it contains a term containing $x^{5/2}$ where $\frac{5}{2}$ is not a non-negative integer.

COEFFICIENTS

In the polynomial $6x^3 - 5x^2 + 5x - 7$ we say that coefficients of x^3, x^2 and x are 6, –5 and 5 respectively and we also say that –7 is the constant term in it.

DEGREE OF A POLYNOMIAL IN ONE VARIABLE

In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

Examples:

(i) $3x + 5$ is a polynomial in x of degree 1.

(ii) $4y^2 - \frac{7}{2}y + 5$ is a polynomial in y of degree 2.

Degree of a Polynomial in Two or More Variables

In case of polynomials in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of polynomial.

Examples:

(i) $7x^2 - 5x^2y^2 + 3xy + 7y + 9$ is a polynomial in x and y of degree 4.
(ii) $4x^3y^3 - 5xy^2 + 2x^4 - 7$ is a polynomial in x and y of degree 6.

POLYNOMIALS OF VARIOUS DEGREES

(1) Linear Polynomial: A polynomial of degree 1 is called a linear a linear polynomial.

Examples:

(i) $2x + 7$ is a linear polynomial in x.
(ii) $2x + y + 7$ is a linear polynomial in x and y.

(2) Quadratic Polynomial: A polynomial of degree 2 is called a quadratic polynomial.

Examples:

(i) $x^2 + 2x + 3$ is a quadratic polynomial in x.
(ii) $ny + yz + zx$ is a quadratic polynomial in x, y and z.

(3) Cubic Polynomial: A polynomial of degree 3 is called a cubic polynomial.

Examples:

(i) $5x^3 - 3x^2 + 7x + 7$ is a cubic polynomial in x and y.
(ii) $4x^2y + 7xy^2 + 7$ is a cubic polynomial in x and y.

(4) Biquadratic Polynomial: A polynomial of degree 4 is called a biquadratic polynomial.

Examples:

(i) $x^4 - 7x^3 + 15x^2 + 7x - 9$ is a biquadratic polynomial in x.

(ii) $x^2y^2 + xy^3 + y^4 - 8xy + 2y^2 + 9$ is a biquadratic polynomial in x and y.

NUMBER OF TERMS IN A POLYNOMIAL

(i) **Monomial:** A polynomial containing one non-zero term is called a monomial.

Example:

$5, 3x, \dfrac{7xy}{4}$ are all monomials.

(ii) **Binomial:** A polynomial containing two non-zero terms is called a binomial.

Example:

$(7 + 5x), (x - 7y), (5x^2y + 3yz)$ are all binomials.

(iii) **Trinomial:** A polynomial containing three nonzero terms is called a trinomial.

Example:

$(8 + 5x + x^2), 3x - 5xy + 7y^2$ are all trinomials.

CONSTANT POLYNOMIAL

A polynomial containing one term only, consisting of a constant is called a constant polynomial.

Examples:

$7, -5, \dfrac{7}{9}$ etc. are all constant polynomials.

Clearly, the degree of a non-zero constant polynomial is zero.

ZERO POLYNOMIAL

A polynomial consisting of one term, namely zero only, is called a zero polynomial. The degree of a zero polynomial is not defined.

Zeros of A Polynomial

Let $p(x)$ be a polynomial. If $p(a) = 0$, then we say that a is a zero of the polynomial $p(x)$. Finding the zeros of a polynomial $p(x)$ means solving the equation $p(x) = 0$.

Example ***1:*** **If $f(t) = 3t^2 - 10t + 6$, find $f(0)$.**

Solution: $f(t) = 3t^2 - 10t + 6$

$\Rightarrow f(0) = 3 \times 0^2 - 10 \times 0 + 6 = 6$

Example ***2:*** **If $p(x) = 2x^2 - 5x + 4$, find $p(2)$**

Solution: $p(x) = 2x^2 - 5x + 4$

$\Rightarrow p(2) = (2 \times 2^2 - 5 \times 2 + 4) = 8 - 10 + 4 = 2$

Example ***3:*** **Find a zero of polynomial $p(x) = x - 7$**

Solution: $p(x) = x - 7$

Now, $p(x) = 0 \Rightarrow x - 7 = 0 \Rightarrow x = 7$

$\therefore$ 7 is a zero of polynomial $p(x)$

ADDITION AND DIFFERENCE OF TWO POLYNOMIALS

Addition of two polynomials is determined by arranging terms of same degrees with signs and adding the co-efficients. The operation of subtraction is similar to the operation of addition. Only difference is that the signs of the polynomial to be subtracted are changed and then operation of addition is performed.

Example: If $p(x)$
$= x^4 - 5x^3 + 3x + 9$ and $q(x) = 2x^4 - 3x^3 + 5x - 4$
Then, $p(x) + q(x)$
$= (x^4 - 5x^3 + 3x + 9) + (2x^4 - 3x^3 + 5x - 4)$
$= (x^4 + 2x^4) + (-5x^3 - 3x^3) + (3x + 5x) + (9 - 4)$
$= 3x^4 - 8x^3 + 8x + 5$

For the sake of convenience, the above operation can be written in the following form:

$$\begin{array}{l} p(x) = x^4 - 5x^3 + 3x + 9 \\ q(x) = 2x^4 - 3x^3 + 5x - 4 \\ \hline \therefore p(x) + q(x) = 3x^4 - 8x^3 + 8x + 5 \end{array}$$

and, $p(x) - q(x)$
$= (x^4 - 5x^3 + 3x + 9) - (2x^4 - 3x^3 + 5x - 4)$
$= (x^4 - 5x^3 + 3x + 9) + (-2x^4 + 3x^3 - 5x + 4)$
$= (x^4 - 2x^4) + (-5x^3 + 3x^3) + (3x - 5x) + (9 + 4)$
$= -x^4 - 2x^3 - 2x + 13$

For the sake of convenience,

$$\begin{array}{lccccccccc} \Rightarrow & p(x) & = & x^4 & - & 5x^3 & + & 3x & + & 9 \\ & q(x) & = & 2x^4 & - & 3x^3 & + & 5x & - & 4 \\ & - & & - & & + & & - & & + \\ \hline \end{array}$$

$\therefore p(x) - q(x) = -x^4 - 2x^3 - 2x + 13$

MULTIPLICATION OF TWO POLYNOMIALS

To determine the product of two polynomials, distributive law of multiplication is used first and then grouping is made of terms of same degrees for addition and subtraction.

$$\begin{array}{r} x^3 - 6x^2 + x + 1 \\ x^2 - 3x + 2 \\ \hline x^5 - 6x^4 + x^3 + x^2 \qquad\qquad \\ - 3x^4 + 18x^3 - 3x^2 - 3x \qquad \\ + 2x^3 - 12x^2 + 2x + 2 \\ \hline x^5 - 9x^4 + 21x^3 - 14x^2 - x + 2 \end{array}$$

Remember

✧ $(-x) \times (-x) = +x^2$
$(+x) \times (-x) = -x^2$
$(x) \times (x) = x^2$ etc.

Or, $(-) \times (-) = +$
$(-) \times (+) = -$
$(+) \times (-) = -$
$(+) \times (+) = +$

DIVISION OF POLYNOMIAL BY ANOTHER POLYNOMIAL

Let $p(x)$ and $q(x)$ be two polynomials and $q(x) \neq 0$. If we find two polynomials $g(x)$ and $r(x)$ such that

$p(x) = g(x)\ q(x) + r(x)$

i.e. Dividend = Divisor × Quotient + Remainder

Where degree of $r(x)$ < degree of $g(x)$, then we say that on dividing $p(x)$ by $q(x)$, the quotient is $g(x)$ and remainder is $r(x)$. If remainder $r(x)$ = zero, we say that $q(x)$ is a factor of $p(x)$.

Let's take a few examples to illustrate the method of division of a polynomial by a polynomial of lesser degree.

Example 4: Divide $p(x) = x^3 + 3x^2 - 12x + 4$ by $g(x) = x - 2$.

Solution:

$$\begin{array}{r|l} & x^2 + 5x - 2 \\ \hline x - 2 & x^3 + 3x^2 - 12x + 4 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & 5x^2 - 12x + 4 \\ & 5x^2 - 10x \\ & - \quad + \\ \hline & -2x + 4 \\ & -2x + 4 \\ & + \quad - \\ \hline & 0 \end{array}$$

NOTE :

It is to be noted that the degree of $q(x)$ is less than that of $p(x)$ and polynomial of higher degree is always divided by a polynomial of lower degree. The operation of division ends when the remainder is either zero or the degree of remainder is less than that of divisor.

In the above example, the quotient is $x^2 + 5x - 2$ and remainder is zero. As the remainder is zero, $(x - 2)$ is a factor of $x^3 + 3x^2 - 12x + 4$.

Example 5: Divide $p(x) = x^3 - 14x^2 + 37x - 60$ by $g(x) = x - 2$.

Solution:

$$\begin{array}{r|l} & x^2 - 12x + 13 \\ \hline x - 2 & x^3 - 14x^2 + 37x - 60 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -12x^2 + 37x - 60 \\ & -12x^2 + 24x \\ & + \quad - \\ \hline & 13x - 60 \\ & 13x - 26 \\ & - \quad + \\ \hline & -34 \end{array}$$

Here, quotient = $x^2 - 12x + 13$ and remainder = – 34 Since remainder ≠ 0, then $(x - 2)$ is not a factor of $x^3 - 14x^2 + 37x - 60$.

Remainder Theorem

Let $f(x)$ be a polynomial of degree $n \geq 1$, and let a be any real number. When $f(x)$ is divided by $(x - a)$, then the remainder is $f(a)$.

NOTE :

(i) If a polynomial $p(x)$ is divided by $(x + a)$, the remainder is the value of $p(x)$ at $x = -a$ i.e. $p(-a)$
$[\because x + a = 0 \Rightarrow x = -a]$

(ii) If a polynomial $p(x)$ is divided by $(ax - b)$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$ i.e. $p\left(\frac{b}{a}\right)$.
$[\because ax - b = 0 \Rightarrow x = \frac{b}{a}]$

(iii) If a polynomial $p(x)$ is dividied by $(ax + b)$, then remainder is the value of $p(x)$ at $x = -\frac{b}{a}$ i.e. $p\left(-\frac{b}{a}\right)$
$[\because ax + b = 0 \Rightarrow x = -\frac{b}{a}]$

(iv) If a polynomial $p(x)$ is divided by $b - ax$, the remainder is the value of $p(x)$ at $x = \frac{b}{a}$ i.e. $p\left(\frac{b}{a}\right)$
$[\because b - ax = 0 \Rightarrow x = \frac{b}{a}]$

Example 6: Let $p(x) = x^4 - 3x^2 + 2x + 5$. Find remainder when $p(x)$ is divided by $(x - 1)$.

Solution:

$$\begin{array}{r|l} & x^3 + x^2 - 2x \\ \hline x - 1 & x^4 + 0x^3 - 3x^2 + 2x + 5 \\ & x^4 - x^3 \\ & - \quad + \\ \hline & x^3 - 3x^2 + 2x + 5 \\ & x^3 - x^2 \\ & - \quad + \\ \hline & -2x^2 + 2x + 5 \\ & -2x^2 + 2x \\ & + \quad - \\ \hline & 5 \end{array}$$

Here, remainder = 5

Find the value of p(1) from the above example.

$p(1) = 1 - 3 \times 1 + 2 \times 1 + 5 = 5$

Thus, remainder obtained on dividing $p(x)$ by $(x - 1)$ is same as $p(1)$.

FACTOR THEOREM

Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be a real number such that $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

Conversely, if $(x-a)$ is a factor of $p(x)$, then $p(a) = 0$

$p(x)$ when divided by $(x-a)$ gives the remainder equal to $p(a)$.

NOTE :

(i) $(x + a)$ is a factor of a polynomial (if and only if) $p(-a) = 0$

(ii) $(ax - b)$ is a factor of a polynomial if $p\left(\frac{b}{a}\right) = 0$

(iii) $(ax + b)$ is a factor of a polynomial $p(x)$ if $P\left(-\frac{b}{a}\right) = 0$

(iv) $(x-a)(x-b)$ are factors of a polynomial $p(x)$ if $p(a) = 0$ and $p(b) = 0$

FACTORISATION

To express a given polynomial as the product of polynomials, each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorisation.

Examples:

(i) $x^2 - 16 = (x-4)(x+4)$

(ii) $x^2 - 3x + 2 = (x-2)(x-1)$

Formulae for Factorisation

(i) $(x + y)^2 = x^2 + y^2 + 2xy$

(ii) $(x - y)^2 = x^2 + y^2 - 2xy$

(iii) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(iv) $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

(v) $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

(vi) $x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$

Thus, for factorisation, we have

(i) $x^2 - y^2 = (x - y)(x + y)$

(ii) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(iii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Methods of Factorisation

Method-1: When each term of an expression has a common factor, we divide each term by this factor and take it out as a multiple as shown below:

Example: $x(x-y)^3 + 3x^2y(x-y)$

$= x(x-y)[(x-y)^2 + 3xy]$

$= x(x-y)[(x^2 - y^2 - 2xy + 3xy]$

$= x(x-y)(x^2 + y^2 + xy)$

Method 2: Sometimes in a given expression it is not possible to take out a common factor directly. However the terms of the expression are grouped in such a manner that we may have a common factor. This can now be factorised as discussed above.

Example: Factorise: $6ab - b^2 + 12ac - 2bc$

Solution: $6ab - b^2 + 12ac - 2bc$

$= (6ab + 12ac) - (b^2 + 2bc)$

$= 6a(b + 2c) - b(b + 2c)$

$= (b + 2c)(6a - b)$

Method 3: Factorisation of Quadratic Trinomials

Case I: Polynomial of the form $x^2 + bx + c$. We find integers p and q such that $p + q = b$ and $pq = c$. Then,

$x^2 + bx + c = x^2 + (p + q)x + pq$

$= x^2 + px + qx + pq = x(x + p) + q(x + p)$

$= (x + q)(x + p)$

Case II: Polynomial of the form $ax^2 + bx + c$.

In this case, we find integers p and q such that $p + q = b$ and $pq = ac$.

Then, $ax^2 + bx + c = ax^2 + (p + q)x + \frac{pq}{a}$

$= a^2x^2 + apx + aqx + pq = ax(ax + p) + q(ax + p)$

$= (ax + p)(ax + q)$

Example 7: Factorise: $x^2 + 9x + 18$

Solution: We try to split 9 into two parts whose sum is 9 and product is 18.

Clearly, $6 + 3 = 9$ and $6 \times 3 = 18$

$\therefore x^2 + 9x + 18 = x^2 + 6x + 3x + 18$

$= x(x + 6) + 3(x + 6) = (x + 3)(x + 6)$

Example 8: Factorise: $x^2 + 5x - 24$

Solution: We try to split 5 into two parts whose sum is 5 and product is -24.

Clearly, $8 + (-3) = 5$ and $8 \times (-3) = -24$

$\therefore x^2 + 5x - 24 = x^2 - 3x + 8x - 24$

$(x^2 - 3x) + (8x - 24)$

$= x(x - 3) + 8(x - 3) = (x - 3)(x + 8)$

Example 9: Factorise : $\sqrt{3}x^2 + 11x + 6\sqrt{3}$

Solution: Here, $\sqrt{3} \times 6\sqrt{3} = 18$

So, we try to split 11 into two parts whose sum is 11 and product is 18.

Clearly, $9 + 2 = 11$ and $9 \times 2 = 18$

$\therefore \sqrt{3}x^2 + 11x + 6\sqrt{3} = \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3}$

$= \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3})$

$= (x + 3\sqrt{3})(\sqrt{3}x + 2)$

Method 4: Factorisation of $x^3 + y^3 + z^3 - 3xyz$

Theorem:

$x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Theorem: If $x + y + z = 0$ then prove that

$x^3 + y^3 + z^3 = 3xyz$

Example 10: Factorise: $a^3 - 8b^3 + 64c^3 + 24abc$

Solution: $a^3 - 8b^3 + 64c^3 + 24abc$

$= a^3 + (-2b)^3 + (4c)^3 - 3a \times (-2b)(4c)$

$= x^3 + y^3 + z^3 - 3xyz$

Where, $a = x$, $-2b = y$ and $4c = z$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= [a + (-2b) + 4c][a^2 + (-2b)^2 + (4c)^2 - a(-2b) - (-2b)(4c) - a(4c)]$

$= (a - 2b + 4c)(a^2 + 4b^2 + 16c^2 + 2ab + 8bc - 4ac)$

Example 11: Factorise : $(p-q)^3 + (q-r)^3 + (r-p)^3$

Solution: If $(p - q) = x$, $(q - r) = y$ and $r - p = z$,

then, $x + y + z = p - q + q - r + r - p = 0$

$\therefore x^3 + y^3 + z^3 = 3xyz = 3(p-q)(q-r)(r-p)$

Example 12: If p = 2 – a, find the value of, $a^3 + 6ap + p^3 - 8$

Solution: $p = 2 - a \Rightarrow a + p + (-2) = 0$

$\Rightarrow x + y + z = 0$

Where, a = x, p = y and –2 = z

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow a^3 + p^3 + (-2)^3 = 3 \times a \times p \times (-2)$

$\Rightarrow a^3 + 6ap + p^3 - 8 = 0$

OTHER SOLVED EXAMPLES

1. **Factorise:** $x^2 - \left(a + \frac{1}{a}\right)x + 1$

Solution: $x^2 - \left(a + \frac{1}{a}\right)x + 1$

$= x^2 - ax - \frac{x}{a} + 1 = x(x-a) - \frac{1}{a}(x-a)$

$= (x-a)\left(x - \frac{1}{a}\right)$

2. **Factorise:** $x^5 - x$

Solution: $x^5 - x$

$= x(x^4 - 1) = x[(x^2)^2 - 1] = x(x^2 + 1)(x^2 - 1)$

$= x(x^2 - 1)(x + 1)(x - 1)$

3. **Factorise:** $x^2 - y^2 - 2y - 1$

Solution: $x^2 - y^2 - 2y - 1$

$= x^2 - (y^2 + 2y + 1)$

$= x^2 - (y + 1)^2$

$= (x + y + 1)(x - y + 1)$

4. **Factorise:** $4x^2 - \frac{2x(a^2 - 1)}{a} - 1$

Solution: $4x^2 - 2x\left(\frac{a^2 - 1}{a}\right) - 1$

$= 4x^2 - 2xa + \frac{2x}{a} - 1 = 2x(2x - a) + \frac{1}{a}(2x - a)$

$= (2x - a)\left(2x + \frac{1}{a}\right)$

5. **Factorise:** $(2a + 3b)^2 - 14(2a + 3b)(3a - b)(3a - b) - 32(3a - b)^2$

Solution: Let (2a + 3b) = x and (3a – b) = y

$\therefore$ Expression $= x^2 - 14xy - 32y^2$

$= x^2 - 16xy + 2xy - 32y^2$

$= x(x - 16y) + 2y(x - 16y)$

$= (x - 16y)(x + 2y)$

$= [(2a + 3b) - 16(3a - b)][2a + 3b) + 2(3a - b)]$

$=(19b - 46a)(8a + b)$

6. **Factorise:** $(2x - 3y)^2 - 10x + 15y - 6$

Solution: $(2x - 3y)^2 - 5(2x - 3y) - 6$

Let (2x – 3y) = a

$= a^2 - 5a - 6$

$= a^2 - 6a + a - 6$

$= a(a - 6) + 1(a - 6)$

$= (2x - 3y - 6)(2x - 3y + 1)$ (Putting the value of *a*)

7. **Factorise:** $7(a + b)^2 + 48(a + b)ab - 7a^2b^2$

Solution: $7(a + b)^2 + 48(a + b)ab - 7a^2b^2$

Let (a + b) = x

$\therefore$ Expression $= 7x^2 + 48xab - 7a^2b^2$

$= 7x^2 + 49xab - xab - 7a^2b^2$

$= 7x(x + 7ab) - ab(x + 7ab)$

$= (x + 7ab)(x - 7ab)$

(Putting the value of x,

$= (a + b + 7ab)[7(a + b) - ab]$

$= (a + b + 7ab)[7a + 7b - ab]$

8. **Factorise:** $\left(\frac{a}{b} + \frac{b}{a}\right)^4 - 2\left(\frac{a^2}{b^2} - \frac{b^2}{a^2}\right)^2 + \left(\frac{a}{b} - \frac{b}{a}\right)^4 - c^2$

Solution: $\left(\frac{a}{b} + \frac{b}{a}\right)^4 - 2\left(\frac{a^2}{b^2} - \frac{b^2}{a^2}\right)^2 + \left(\frac{a}{b} - \frac{b}{a}\right)^4 - c^2$

$= \left(\frac{a}{b} + \frac{b}{a}\right)^4 - 2\left(\frac{a}{b} + \frac{b}{a}\right)^2\left(\frac{a}{b} - \frac{b}{a}\right)^2 + \left(\frac{a}{b} - \frac{b}{a}\right)^4 - c^2$

Let $\frac{a}{b} + \frac{b}{a} = x$ and $\frac{a}{b} - \frac{b}{a} = y = x^4 - 2x^2y^2 + y4 - c^2$

$= (x^2 - y^2)^2 - c^2 = \left[\left(\frac{a}{b} + \frac{b}{a}\right)^2 - \left(\frac{a}{b} - \frac{b}{a}\right)^2\right]^2 - c^2$

(Putting the value of x, y)

$\left[\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\frac{a}{b} \times \frac{b}{a} - \frac{a^2}{b^2} - \frac{b^2}{a^2} + 2\frac{a}{b} \times \frac{b}{a}\right]^2 - c^2$

$= (4)^2 - c^2 = (4 + c)(4 - c)$

9. **Factorise:** $512xy^{10} + xy^{10}$

Solution: $512xy^{10} + xy^{10}$

$= xy[512y^9 + x^9] = xy[8^3y^9 + x^9] = xy[(8y^3)^3 + (x^3)^3]$

$= xy(8y^3 + x^3)(64y^6 - 8y^3x^3 + x^6)$

$= xy[(2y)^3 + x^3](64y^6 - 8y^3x^3 + x^6)$

$= xy(2y + x)(4y^2 - 2xy + x^2)(64y^6 - 8x^3y^3 + x^6)$

10. **Factorise:** $8x^3 + 27y^3 - 6x - 9y$

Solution: $8x^3 + 27y^3 - 6x - 9y$

$= (2x)^3 + (3y)^3 - 3(2x + 3y)$

$= (2x + 3y)[(2x)^2 - 2x \times 3y + (3y)^2] - 3(2x + 3y)$

$= (2x + 3y)(4x^2 - 6xy + 9y^2) - 3(2x + 3y)$

$= (2x + 3y)(4x^2 - 6xy + 9y^2 - 3)$

EXERCISE

1. When the polynomial $f(x) = x^4 + 2x^3 - 3x^2 + x - 1$ is divided by $(x - 2)$ what will be the remainder?
(a) 21 (b) 22
(c) 23 (d) 29
2. When $f(x) = x^3 - 3x^2 + 4x + 50$ is divided by $(x + 3)$, the remainder is
(a) 16 (b) – 16
(c) 15 (d) – 15
3. When polynomial $f(x) = 4x^3 - 12x^2 + 14x - 3$ is divided by $2x - 1$, the remainder is :
(a) $\frac{2}{3}$ (b) $\frac{1}{3}$
(c) $\frac{3}{2}$ (d) $\frac{2}{7}$
4. If polynomials $2x^3 + ax^2 + 3x - 5$ and $x^3 + x^2 - 2x + a$ are divided by $(x - 2)$, the same remainders are obtained. Find the value of a.
(a) – 3 (b) 3
(c) – 4 (d) – 9
5. If the polynomials $f(x)\ x^4 - 2x^3 + 3x^2 - ax + b$ is divided by $(x - 1)$ and $(x + 1)$, the remainders are respectively 5 and 19. The values of a and b are:
(a) a = 8, b = 7 (b) a = 5, b = 8
(c) a = 8, b = 5 (d) a = 6, b = 8
6. The factor of polynomial $f(x) = x^3 + x^2 - 17x + 15$ is
(a) $x - 3$ (b) $x - 5$
(c) $x + 3$ (d) $x + 1$
7. For what value of a, $(x - a)$ is a factor of $f(x) = x^5 - a^2x^3 + 2x + a - 3$?
(a) – 1 (b) 1
(c) – 2 (d) 2
8. For what value of a, $(x + a)$ is a factor of polynomial $f(x) = x^3 + ax^2 - 2x + a + 6$?
(a) 2 (b) 3
(c) – 2 (d) – 3
9. For what value of k, $(2x^4 + 3x^3 + 2kx^2 + 3x + 6)$ is exactly divisible by $(x + 2)$?
(a) 1 (b) 2
(c) – 2 (d) – 1
10. For what value of a and b, $(x^3 - 10x^2 + ax + b)$ is exactly divisible by $(x - 1)$ and $(x - 2)$?
(a) a = 23, b = –14 (b) a = –23, b = 14
(c) a = 21, b = –14 (d) a = –21, b = 15
11. If $a^2 + b^2 + c^2 = 2a - 2b - 2$, then the value of $3a - 2b + c$ is
(a) 0 (b) 3
(c) 5 (d) 2
12. If $a + b + c = 3$, $a^2 + b^2 + c^2 = 6$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$, where a, b, c are all non-zero, then 'abc' is equal to
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$
13. If $a^2 - 4a - 1 = 0$, $a \neq 0$, then the value of $a^2 + 3a + \frac{1}{a^2} - \frac{3}{a}$ is
(a) 24 (b) 26
(c) 28 (d) 30
14. If $x = 2 + \sqrt{3}$, then $x^2 + \frac{1}{x^2}$ is equal to
(a) 10 (b) 12
(c) –12 (d) 14
15. If a = 4.965, b = 2.343 and c = 2.622, then the value of $a^3 - b^3 - c^3 - 3abc$ is
(a) –2 (b) –1
(c) 0 (d) 9.93^2
16. If $x + y + z = 0$, then the value of $\frac{x^2 + y^2 + z^2}{x^2 - yz}$ is
(a) –1 (b) 0
(c) 1 (d) 2
17. If $a = \sqrt{6} + \sqrt{5}$ $b = \sqrt{6} - \sqrt{5}$ then $2a^2 - 5ab + 2b^2 =$
(a) 38 (b) 39
(c) 40 (d) 41
18. If $p = \frac{5}{18}$
then $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$ is equal to
(a) $\frac{4}{27}$ (b) $\frac{5}{27}$
(c) $\frac{8}{27}$ (d) $\frac{10}{27}$
19. If $x = \frac{4ab}{a+b}$, then the value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$ is equal to:
(a) 0 (b) 1
(c) 2 (d) None of these

20. If $\frac{x}{(2x+y+z)} = \frac{y}{(x+2y+z)} = \frac{z}{(x+y+2z)} = a$, then find 'a' if $x+y+z \neq 0$

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{2}$

21. If $a+b+c=0$, then find the value of

$$\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}.$$

(a) 0 (b) 1
(c) 6 (d) None of these

22. If $a^2 = b+c, b^2 = c+a, c^2 = a+b$, then the value of

$$\frac{1}{1+a}+\frac{1}{b+1}+\frac{1}{1+c}$$

(a) abc (b) $a^2 b^2 c^2$
(c) 1 (d) 0

23. $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$, then find the value of

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$$

(a) 1 (b) 2
(c) 3 (d) 4

24. If $x = b+c-2a$, $y = c+a-2b$, $z = a+b-2c$, then the value of $x^2+y^2-z^2+2xy$ is

(a) 0 (b) $a+b+c$
(c) $a-b+c$ (d) $a+b-c$

25. $1.5x = 0.04y$, then value of $\frac{y^2-x^2}{y^2+2xy+x^2}$ is

(a) $\frac{730}{77}$ (b) $\frac{73}{77}$
(c) $\frac{73}{770}$ (d) $\frac{74}{77}$

26. If $x^2+y^2-4x+4y+8=0$, then the value of $x-y$ is

(a) 4 (b) -4
(c) 0 (d) 8

27. When x^5+1 is divided by $(x-2)$, the remainder is:

(a) 15 (b) 17
(c) 31 (d) 33

28. If $x+y=2z$ then the value of $\frac{x}{x-z}+\frac{z}{y-z}$ is

(a) 1 (b) 3
(c) $\frac{1}{2}$ (d) 2

29. If $\frac{p}{b-c}=\frac{q}{c-a}=\frac{r}{a-b}$, then $p+q+r=$?

(a) 0 (b) 1
(c) -1 (d) -2

30. If $a+b+c=2s$ then find the value of $(s-a)^3+(s-b)^3+3(s-a)(s-b)c$ is

(a) c (b) c^2
(c) c^3 (d) $2c^2$

31. If $a+b+c=0$, find the value of $\frac{a+b}{c}-\frac{2b}{c+a}+\frac{b+c}{a}$.

(a) 0 (b) 1
(c) -1 (d) 2

32. If $a+b+c=8$, then the value of $(a-4)^3+(b-3)^3+(c-1)^3-3(a-4)(b-3)(c-1)$ is

(a) 2 (b) 4
(c) 1 (d) 0

33. If $x^4+\frac{1}{x^4}=119$ amd $x>1$, then the value of $x^3-\frac{1}{x^3}$ is

(a) 54 (b) 18
(c) 72 (d) 36

34. If $5a+\frac{1}{3a}=5$, then the value of $9a^2+\frac{1}{25a^2}$ is

(a) $\frac{51}{5}$ (b) $\frac{29}{5}$
(c) $\frac{52}{5}$ (d) $\frac{39}{5}$

35. If $a+b+c=0$, the value of

$$\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right) \text{ is}$$

(a) 2 (b) 3
(c) 4 (d) 5

36. $(y-z)^3+(z-x)^3+(x-y)^3$ is equal to

(a) $3(y-z)(z+x)(y-x)$ (b) $(x-y)(y+z)(x-z)$
(c) $3(y-z)(z-x)(x-y)$ (d) $(y-z)(z-x)(x-y)$

37. If $x+\frac{1}{x}=5$, then the value of $x^3+\frac{1}{x^3}$ is:

(a) 125 (b) 110
(c) 45 (d) 75

38. If $x+\frac{4}{x}=4$, find the value of $x^3+\frac{4}{x^3}$.

(a) 8 (b) $8\frac{1}{2}$
(c) 16 (d) $16\frac{1}{2}$

39. If $x = 3 + 2\sqrt{2}$, then the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ is
(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) $3\sqrt{3}$

40. If $a^2 + b^2 + c^2 = 2a - 2b - 2$, then the value of $3a - 2b + c$ is
(a) 0 (b) 3
(c) 5 (d) 2

41. If $a + b + c = 3$, $a^2 + b^2 + c^2 = 6$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$, where a, b, c are all non-zero, then 'abc' is equal to
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

42. If $x = \frac{4ab}{a+b}$, then the value of $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b}$ is equal to:
(a) 0 (b) 1
(c) 2 (d) None of these

43. If $\frac{x}{(2x+y+z)} = \frac{y}{(x+2y+z)} = \frac{z}{(x+y+2z)} = a$, then find 'a' if $x + y + z \neq 0$
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{8}$ (d) $\frac{1}{2}$

44. If $a + b + c = 0$, then find the value of
$$\frac{a^2}{a^2 - bc} + \frac{b^2}{b^2 - ca} + \frac{c^2}{c^2 - ab}.$$
(a) 0 (b) 1
(c) 6 (d) None of these

45. If $a^2 = b + c$, $b^2 = c + a$, $c^2 = a + b$, then the value of
$$\frac{1}{1+a} + \frac{1}{b+1} + \frac{1}{1+c}$$
(a) abc (b) $a^2 b^2 c^2$
(c) 1 (d) 0

46. $\frac{a}{1-a} + \frac{b}{1-b} + \frac{c}{1-c} = 1$, then find the value of
$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$$
(a) 1 (b) 2
(c) 3 (d) 4

47. If $x = b + c - 2a$, $y = c + a - 2b$, $z = a + b - 2c$, then the value of $x^2 + y^2 - z^2 + 2xy$ is
(a) 0 (b) $a + b + c$
(c) $a - b + c$ (d) $a + b - c$

48. $1.5x = 0.04y$, then value of $\frac{y^2 - x^2}{y^2 + 2xy + x^2}$ is
(a) $\frac{730}{77}$ (b) $\frac{73}{77}$
(c) $\frac{73}{770}$ (d) $\frac{74}{77}$

49. If $x^2 + y^2 - 4x + 4y + 8 = 0$, then the value of $x - y$ is
(a) 4 (b) -4
(c) 0 (d) 8

50. When $x^5 + 1$ is divided by $(x - 2)$, the remainder is:
(a) 15 (b) 17
(c) 31 (d) 33

51. If $x + y = 2z$ then the value of $\frac{x}{x-z} + \frac{z}{y-z}$ is
(a) 1 (b) 3
(c) $\frac{1}{2}$ (d) 2

52. If $\frac{p}{b-c} = \frac{q}{c-a} = \frac{r}{a-b}$, then $p + q + r = ?$
(a) 0 (b) 1
(c) -1 (d) -2

53. If $a + b + c = 2s$ then find the value of $(s-a)^3 + (s-b)^3 + 3(s-a)(s-b)c$ is
(a) c (b) c^2
(c) c^3 (d) $2c^2$

54. If $a + b + c = 0$, find the value of $\frac{a+b}{c} - \frac{2b}{c+a} + \frac{b+c}{a}$.
(a) 0 (b) 1
(c) -1 (d) 2

55. If $a + b + c = 8$, then the value of $(a-4)^3 + (b-3)^3 + (c-1)^3 - 3(a-4)(b-3)(c-1)$ is
(a) 2 (b) 4
(c) 1 (d) 0

56. If $a + b + c = 0$, the value of
$$\left(\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right) \text{ is}$$
(a) 2 (b) 3
(c) 4 (d) 5

57. If $a = 331$, $b = 336$ and $c = -667$, then the value of $a^3 + b^3 + c^3 - 3abc$ is
(a) 1 (b) 6
(c) 3 (d) 0

58. $(y-z)^3 + (z-x)^3 + (x-y)^3$ is equal to
(a) $3(y-z)(z+x)(y-x)$ (b) $(x-y)(y+z)(x-z)$
(c) $3(y-z)(z-x)(x-y)$ (d) $(y-z)(z-x)(x-y)$

59. If $x + \frac{4}{x} = 4$, find the value of $x^3 + \frac{4}{x^3}$.

(a) 8 (b) $8\frac{1}{2}$

(c) 16 (d) $16\frac{1}{2}$

60. If $x = 3 + 2\sqrt{2}$, then the value of $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ is

(a) 1 (b) 2

(c) $2\sqrt{2}$ (d) $3\sqrt{3}$

61. If $x - \frac{1}{x} = \frac{1}{3}$, then what is $9x^2 + \frac{9}{x^2}$ is equal to?

(a) 18 (b) 19
(c) 20 (d) 21

62. If $a + b + c = 6$ and $a^2 + b^2 + c^2 = 26$, then what is $ab + bc + ca$ equal to?
(a) 0 (b) 2
(c) 4 (d) 5

63. If $x + y + z = 0$, then what is $\frac{xyz}{(x+y)(y+z)(z+x)}$ equal to (where, $x \neq -y$, $y \neq -z$, $z \neq -x$)?
(a) –1 (b) 1
(c) $xy + yz + zx$ (d) None of these

64. If $x^3 + 5x^2 + 10k$ leaves remainder $-2x$ when divided by $x^2 + 2$, then what is the value of k?
(a) –2 (b) –1
(c) 1 (d) 2

65. If $x + \frac{1}{x} = a$, then what is the value of $x^3 + x^2 + \frac{1}{x^3} + \frac{1}{x^2}$?
(a) $a^3 + a^2$ (b) $a^3 + a^2 - 5a$
(c) $a^3 + a^2 - 3a - 2$ (d) $a^3 + a^2 - 4a - 2$

66. What is the value of

$$\frac{\left[(2.247)^3 + (1.730)^3 + (1.023)^3 - 3 \times 2.247 \times 1.730 \times 1.023\right]}{\left[(2.247)^2 + (1.730)^2 + (1.023)^2 - 2.247 \times 1.730 - 1.730 \times 1.023 - 2.247 \times 1.023\right]}?$$

(a) 1.730 (b) 4
(c) 5 (d) 5.247

67. If $0.764\, y = 1.236\, x$, then what is the value of $\left(\frac{y-x}{y+1}\right)$?

(a) 0.764 (b) 0.236
(c) 2 (d) 0.472

68. The factor(s) of $5px - 10qy + 2rpx = 4qry$ is/are
(a) Only $(5 + 2r)$
(b) Only $(px - 2qy)$
(c) Both $(5 + 2r)$ and $(px - 2qy)$
(d) Neither $(5 + 2r)$ nor $(px - 2qy)$

69. What is the value of

$$\frac{725 \times 725 \times 725 + 371 \times 371 \times 371}{725 \times 725 - 725 \times 371 + 371 \times 371}?$$

(a) 9610 (b) 1960
(c) 1096 (d) 1016

70. Consider the following statements :
I. $x + 3$ is the factor of $x^3 + 2x^2 + 3x + 8$.
II. $x - 2$ is the factor of $x^3 + 2x^2 + 3x + 8$.
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

71. What is $\frac{(x^2 + y^2)(x - y) - (x - y)^3}{x^2y - xy^2}$ equal to?

(a) 1 (b) 2
(c) 4 (d) –2

72. The quantity which must be added to $(1 - x)(1 + x^2)$ to obtain x^3 is
(a) $2x^3 + 3x^2 + x + 1$ (b) $2x^3 + x^2 + x - 1$
(c) $2x^3 - x^2 + 1 - 1$ (d) $-x^2 + x - 1$

73. $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$ is divisible by
(a) Only $(y - z)$
(b) Only $(z - x)$
(c) Both $(y - z)$ and $(z - x)$
(d) Neither $(y - z)$ nor $(z - x)$

74. What should be added to be $x(x + a)(x + 2a)(x + 3a)$, so that the sum be a perfect square?
(a) $9a^2$ (b) $4a^2$
(c) a^4 (d) None of these

75. If $3x^4 - 2x^3 + 3x^2 - 2x + 3$ is divided by $(3x + 2)$, then the remainder is

(a) 0 (b) $\frac{185}{27}$

(c) $\frac{181}{25}$ (d) $\frac{3}{4}$

76. If $\left(x^2 + \frac{1}{x^2}\right) = \frac{17}{4}$, then what is $\left(x^3 - \frac{1}{x^3}\right)$ equal to?

(a) $\frac{75}{16}$ (b) $\frac{63}{8}$

(c) $\frac{95}{8}$ (d) None of these

77. What is the remainder when $x^5 - 5x^2 + 125$ is divided by $x + 5$?
(a) 0 (b) 125
(c) –3125 (d) 3125

78. Consider the following statements :
1. $(a - b - c)$ is one of the factors of $3abc + b^3 + c^3 - a^3$.
2. $(b + c - 1)$ is one of the factors of $3bc + b^3 + c^3 - 1$

Which of the above statements is / are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

79. The expression $x^3q^2 - x^3pt + 4x^2pt - 4x^2q^2 + 3xq^2 - 3xpt$ is divisible by
(a) $(x - 1)$ only
(b) $(x - 3)$ only
(c) both $(x - 1)$ and $(x-3)$
(d) neither $(x - 1)$ nor $(x - 3)$

80. If $\frac{p}{x}+\frac{q}{y}=m$ and $\frac{q}{x}+\frac{p}{y}=n$, then what is $\frac{x}{y}$ equal to? **(CDS)**

(a) $\frac{np+mq}{mp+nq}$ (b) $\frac{np+mq}{mp-nq}$
(c) $\frac{np-mq}{mp-nq}$ (d) $\frac{np-mq}{mp+nq}$

81. If $a^2 - by - cz = 0$, $ax - b^2 + cz = 0$ and $ax + by - c^2 = 0$, then the value of $\frac{x}{a+x}+\frac{y}{b+y}+\frac{z}{c+z}$ will be **(CDS)**
(a) $a + b + c$ (b) 3
(c) 1 (d) 0

82. If $(s - a) + (s - b) + (s - c) = s$, then the value of $\frac{(s-a)^2+(s-b)^2+(s-c)+s^2,}{a^2+b^2+c^2}$ will be **(CDS)**
(a) 3 (b) 1
(c) 0 (d) –1

83. If the polynomial $x^6 + px^5 + qx^4 - x^2 - x - 3$ is divisible by $(x^4 - 1)$, then the value of $p^2 + q^2$ is **(CDS)**
(a) 1 (b) 9
(c) 10 (d) 13

84. Let p and q be non-zero integers. Consider the polynomial
$$A(x) = x^2 + px + q$$
It is given that $(x - m)$ and $(x - km)$ are aimple factora of $A(x)$, where m is a non-zero integer and k is a positive integer, $k \geq 2$. Which one of the following is correct? **(CDS)**
(a) $(k + 1)^2p^2 = kq$ (b) $(k + 1)^2q = kp^2$
(c) $k^2q = (k + 1)p^2$ (d) $k^2p^2 = (k + 1)^2q$

85. Let m be a non-zero integer and n be a positive integer. Let R be the remainder obtained on dividing the polynomial $x^2 + m^2$ by $(x - m)$. Then **(CDS)**
(a) R is a non zero even integer
(b) R is odd, if m is odd
(c) $R + t^3$ for some integer s, if n is even
(d) $R = t^3$ for some integer t, if 3 divides n

86. If $a^3 = 117 + b^3$ and $a = 3 + b$, then the value of $a + b$ is (given that $a > 0$ and $b > 0$) **(CDS)**
(a) 7 (b) 9
(c) 11 (d) 13

87. For what value of k is $(x + 1)$ a factor of $x^3 + kx^2 - x + 2$?
(a) 4 (b) 3 **(CDS)**
(c) 1 (d) –2

88. Which of the points 8 $P(5, - 1)$, $Q(3, -2)$ and $R(1, 1)$ lie in the solution of the system of inequations $x + y \leq 4$ and $x - y \geq 2$?
(a) Q and R only (b) P and R only **(CDS)**
(c) P and Q only (d) P Q and R

89. Which one of the following is correct? **(CDS)**
(a) $(x + 2)$ is a factor of $x^4 - 6x^3 + 12x^2 - 24x + 32$
(b) $(x + 2)$ is a factor of $x^4 + 6x^3 - 12x^2 + 24x - 32$
(c) $(x - 2)$ is a factor of $x^4 - 6x^3 + 12x^2 - 24x + 32$
(d) $(x - 2)$ is a factor of $x^4 + 6x^3 - 12x^2 + 24x - 32$

90. Let x and y be positive integers such that x > y. The expressions 3x + 2y and 2x + 3y when divided by 5 leave remainders 2 and 3 respectively. What is the remainder when (x – y) is divided by 5? **(CDS)**
(a) 4 (b) 2
(c) 1 (d) 0

HINTS & SOLUTIONS

1. (a) Here, $x-2=0 \Rightarrow x=2$
By Remainder Theorem, when polynomial $f(x)$ is divided by $(x-2)$, the remainder is $f(2)$.
$\therefore f(2)=2^4+2\times 2^3-3\times 2^2+2-1$
[**Remember:** x has been replaced by 2]
$=16+16-12+2-1=21$
$\therefore$ Remainder = 21

2. (b) Divisor = $x+3$
$\therefore x+3=0 \Rightarrow x=-3$
By Remainder Theorem,
Remainder $=f(-3)=[(-3)^3-3(-3)^2+4(-3)+50]$
$=(-27-27-12+50)=-16$

3. (c) Here, $2x-1=0 \Rightarrow x=\frac{1}{2}$
By Remainder Theorem,
Remainder
$$=f\left(\frac{1}{2}\right)=\left[4\times\left(\frac{1}{2}\right)^3-12\left(\frac{1}{2}\right)^2+14\left(\frac{1}{2}\right)-3\right]$$
$$=\left(\frac{1}{2}-3+7-3\right)=\frac{3}{2}$$

4. (a) $f(x)=2x^3+ax^2+3x-5$
$g(x)=x^3+x^2-2x+a$
By Remainder Theorem,
$f(2)=2\times 2^3+a\times 2^2+3\times 2-5)=17+4a$
Again, $g(2)=(2^3+2^2-2\times 2+a)=8+a$
$\therefore 17+4a=8+a$
$\Rightarrow 3a=-9$
$\Rightarrow a=-3$

5. (b) $f(x)=x^4-2x^3+3x^2-ax+b$
$f(1)=1-2+3-a+b=2-a+b$ $[x-1=0 \Rightarrow x=1]$
$f(-1)=1+2+3+a+b=6+a+b$
$[x+1=0 \Rightarrow x=-1]$
$\therefore 2-a+b=5 \Rightarrow b-a=3$ ... (i)
and, $6+a+b=19 \Rightarrow a+b=13$ ... (ii)
By adding equations (i) and (ii),
$2b=16 \Rightarrow b=8$
From equation (ii),
$a+b=13 \Rightarrow a=13-8=5$

6. (a) By Factor Theorem,
If $f(a)=0$, then $(x-a)$ is a factor of $f(x)$.
$\therefore f(3)=3^3+3^2-17\times 3+15$
$=27+9-51+15=0$
$\therefore (x-3)$, is a factor of $f(x)$.
Remember: $f(5)\neq 0; f(-3)\neq 0; f(-1)\neq 0$

7. (b) $(x-a)$, is a factor of polynomial
$x^5-a^2x^3+2x+a-3$
$\therefore f(a)=0$
$\Rightarrow a^5-a^5+2a+a-3=0$
$\Rightarrow 3a=3 \Rightarrow a=1$

8. (c) Here, $x+a=0 \Rightarrow x=-a$
$\therefore f(-a)=0$
$\Rightarrow (-a)^3+a(-a)^2-2(-a)+a+6=0$
$\Rightarrow 3a=-6 \Rightarrow a=-2$

9. (d) Here, $x+2=0 \Rightarrow x=-2$
By Factor Theorem,
$f(-2)=0$
$\Rightarrow 2(-2)^4+3(-2)^3+2k(-2)^2+3(-2)+6=0$
$\Rightarrow 32-24+8k-6+6=8k+8=0$
$\Rightarrow 8k=-8 \Rightarrow k=-1$

10. (a) $f(x)=x^3-10x^2+ax+b$
By Factor Theorem,
$f(1)=1-10+a+b=a+b-9$
$[\because x-1=0 \Rightarrow x=1]$
$\therefore f(1)=0 \Rightarrow a+b=9$ ...(i)
$f(2)=8-40+2a+b=2a+b-32$
$\therefore f(2)=0 \Rightarrow 2a+b=32$ ...(ii)
From equation (ii) – equation (i),
$a=23$
From equation (i),
$b=9-23=-14$

11. (c) $a^2+b^2+c^2=2a-2b-2$
$(a^2-2a+1)+(b^2+2b+1)+c^2=0$
$(a-1)^2+(b+1)^2+c^2=0$
This equation is possible if
$a-1=0, b+1=0$ and $c=0$
$a=1, b=-1, c=0$
$3a-2b+c=3\times 1-2\times(-1)+0$
$=3+2=5$

12. (b) $a+b+c=3$
Squaring both sides
$a^2+b^2+c^2+2(ab+bc+ac)=9$
$6+2(ab+bc+ca)=9$
$ab+bc+ca=\frac{3}{2}$ ...(1)
given $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$
$\Rightarrow ab+bc+ac=abc=\frac{3}{2}$ [from (1)]

13. (d) $a^2-4a-1=0$
$a^2-4a=1$
$a(a-4)=1$
$a-4=\frac{1}{a}$
$a-\frac{1}{a}=4$ ...(1)
We have $a^2+3a+\frac{1}{a^2}-\frac{3}{a}$

$$\left(a^2+\frac{1}{a^2}\right)+3\left(a-\frac{1}{a}\right)$$

$$\left(a-\frac{1}{a}\right)^2+3\left(a-\frac{1}{a}\right)+2$$

$4^2+3\times4+2=30$

14. (d) $x=2+\sqrt{3}$

$$\frac{1}{x}=\frac{1}{2+\sqrt{3}}\times\frac{2-\sqrt{3}}{2-\sqrt{3}}=2-\sqrt{3}$$

$$x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2$$

$$=\left(2+\sqrt{3}+2-\sqrt{3}\right)^2-2$$

$=16-2=14$

15. (c) $a=4.965\approx5, b=2.343\approx2$

$c=2.622$

$a-b=c$

taking cube both sides

$a^3-b^3-3a^2b+3ab^2=c^3$

$a^3-b^3-c^3-3ab(a-b)=0$

$a^3-b^3-c^3-3abc=0$

16. (d) $x+y+z=0$

$y+z=-x$

$y^2+z^2+2yz=x^2$

$\Rightarrow y^2+z^2=x^2-2yz$ ...(1)

$$\frac{x^2+y^2+z^2}{x^2-yz}=\frac{x^2-2yz+x^2}{x^2-yz}=\frac{2(x^2-yz)}{x^2-yz}$$

$=2$

17. (b) $2a^2-5ab+2b^2\Rightarrow 2(a^2-2ab+b^2)-ab$

$\Rightarrow 2(a-b)^2-ab$

$$\Rightarrow 2\left(\sqrt{6}+\sqrt{5}-\sqrt{6}+\sqrt{5}\right)^2-\left(\sqrt{6}-\sqrt{5}\right)\left(\sqrt{6}-\sqrt{5}\right)$$

$\Rightarrow 2\times4\times5-1=39$

18. (c) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3(3p)^2.\frac{1}{6}+3(3p)\left(\frac{1}{6}\right)^2$$

$$=\left(3P-\frac{1}{6}\right)^3$$

$$=\left(3\times\frac{5}{18}-\frac{1}{6}\right)^3=\frac{8}{27}$$

19. (c) Given, $x=\frac{4ab}{a+b}$

$$\Rightarrow \frac{x}{2a}=\frac{2b}{a+b}$$

Applying componendo and dividendo, we get

$$\frac{x+2a}{x-2a}=\frac{2b+a+b}{2b-a-b}=\frac{a+3b}{b-a} \quad \text{...(i)}$$

Also, $\frac{x}{2b}=\frac{2a}{a+b}$

Applying componendo and dividendo, we get

$$\frac{x+2b}{x-2b}=\frac{2a+a+b}{2a-a-b}=\frac{3a+b}{a-b} \quad \text{...(ii)}$$

Add (i) & (ii),

$$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}=\frac{a+3b}{b-a}+\frac{3a+b}{a-b}$$

$$=\frac{1}{b-a}[a+3b-3a-b]=\frac{2(b-a)}{(b-a)}=2$$

20. (b) Given,

$$\frac{x}{2x+y+z}=\frac{y}{x+2y+z}=\frac{z}{x+y+2z}=a$$

$\Rightarrow$ $x=a(2x+y+z)$, $y=a(x+2y+z)$ and $z=a(x+y+2z)$

$\Rightarrow$ $x+y+z=a(4x+4y+4z)=4a(x+y+z)$

$\Rightarrow$ $4a=1$ or $a=\frac{1}{4}$.

21. (d) Given, $a+b+c=0$

$\Rightarrow$ $a^2=(b+c)^2$

Now, consider $\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}$

$$=\frac{(b+c)^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}$$

$$=\frac{(b+c)^2}{(b+c)^2-bc}+\frac{b^2}{b^2+c(b+c)}+\frac{c^2}{c^2+b(b+c)}$$

$$=\frac{(b+c)^2}{b^2+c^2+bc}+\frac{b^2}{b^2+c^2+bc}+\frac{c^2}{c^2+b^2+bc}$$

$$=\frac{b^2+c^2+2bc+b^2+c^2}{b^2+c^2+bc}=\frac{2(b^2+c^2+bc)}{b^2+c^2+bc}=2$$

22. (c) $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}$ .....(i)

Given that,

$a^2=b+c$

$a+a^2=a+b+c$

$a(a+1)=a+b+c$

$$a+1=\frac{a+b+c}{a}$$

$$\frac{1}{a+1}=\frac{a}{a+b+c}$$

Similarly,

$$\frac{1}{b+1}=\frac{b}{a+b+c}$$

$$\frac{1}{c+1}=\frac{c}{a+b+c}$$

Put in eq. (i)

$$\therefore \quad \frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}=\frac{a+b+c}{a+b+c}=1$$

23. (d) Given that,

$$\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$$

$$\Rightarrow \frac{a}{1-a}+1+\frac{b}{1-b}+1+\frac{c}{1-c}+1=4$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=4$$

24. (a) $x^2+y^2-z^2+2xy$

$= x^2+y^2+2xy-z^2$

$= (x+y)^2-z^2 = (x+y+z)(x+y-z)$

$= (b+c-2a+c+a-2b+a+b-2c)(x+y-z)$

$= 0$

25. (b) $\frac{x}{y}=\frac{0.04}{1.5}=\frac{4}{150}=\frac{2}{75}$

$$\frac{y}{x}=\frac{75}{2}$$

Now, $\frac{y^2-x^2}{y^2+2xy+x^2}$

$$=\frac{(y-x)(y+x)}{(y+x)^2}$$

$$=\frac{y-x}{y+x}=\frac{\frac{y}{x}-1}{\frac{y}{x}+1}$$

$$=\frac{\frac{75}{2}-1}{\frac{75}{2}+1}=\frac{73}{77}$$

26. (c) $x^2+y^2-4x-4y+8=0$

$\Rightarrow x^2-4x+4+y^2-4y+4=0$

$\Rightarrow (x-2)^2+(y-2)^2=0$

$\Rightarrow x=2$ and $y=2$

$\therefore x-y=2-2=0$

27. (d) Let $f(x)=x^5+1$

Since $(x-2)$ is the factor of (x^5+1), hence from Remainder Theorem, we have, $f(2)=(2)^5+1=33$

Hence, the remainder = 33

28. (a) $x+y=2z \Rightarrow x=2z-y$

Subtract 'z' from both sides $\Rightarrow x-z=2z-y-z=z-y$

$$\therefore \frac{x}{x-z}+\frac{z}{y-z}$$

$$=\frac{x}{x-z}-\frac{z}{z-y}=\frac{x}{x-z}-\frac{z}{x-z}=\frac{x-z}{x-z}=1$$

29. (a) $\because \frac{p}{b-c}=\frac{q}{c-a}=\frac{r}{a-b}=k$ (Let)

$\therefore p=k(b-c),\ q=k(c-a),\ r=k(a-b)$

Expression $= p+q+r$

$= k(b-c)+k(c-a)+k(a-b)$

$= k(b-c+c-a+a-b)$

$= k\times 0=0$

30. (c) $a+b+c=2s$

$c=2s-a-b=(s-a)+(s-b)$

$\therefore (s-a)^3+(s-b)^3+3(s-a)(s-b)c$

$=(s-a)^3+(s-b)^3+3(s-a)(s-b)[(s-a)+(s-b)]$

(Put the value of c)

$=[(s-a)+(s-b)]^3=(2s-a-b)^3$

$(a+b+c-a-b)^3=c^3$ (Put the value of 2s)

31. (a) $a+b+c=0$

i.e. $a=-(b+c);\ b=-(c+a);\ c=-(a+b)$

Now, $\frac{a+b}{c}-\frac{2b}{c+a}+\frac{b+c}{a}$

$$\Rightarrow \frac{a+b}{-(a+b)}-\frac{2[-(c+a)]}{c+a}+\frac{b+c}{-(b+c)}$$

$\Rightarrow -1+2-1=0$

32. (d) We have $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

Here $x=a-4,\ y=b-3,\ z=c-1$

So, given expression is $(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$=(a-4+b-3+c-1)(x^2+y^2+z^2-xy-yz-zx)$

$=(a+b+c-8)(x^2+y^2+z^2-xy-yz-zx)$

$=(8-8)(x^2+y^2+z^2-xy-yz-zx)$

$=0$

33. (d) $x^4+\frac{1}{x^4}=119$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2-2=119 \Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=121$$

$$\Rightarrow x^2+\frac{1}{x^2}=11 \Rightarrow \left(x-\frac{1}{x}\right)^2+2=11$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2=9 \Rightarrow x-\frac{1}{x}=3$$

Cubing both sides,

$$\left(x-\frac{1}{x}\right)^3 = 27$$

$$\Rightarrow x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)=27 \Rightarrow x^3-\frac{1}{x^3}-3\times 3=27$$

$$\Rightarrow x^3-\frac{1}{x^3}=27+9=36$$

34. (d) $5a+\frac{1}{3a}=5$

Multiply by $\frac{3}{5}$ on both sides

$$\frac{3}{5}\left(5a+\frac{1}{3a}\right)=5\times\frac{3}{5}$$

$$3a+\frac{1}{5a}=3$$

Squaring on both sides

$$9a^2+\frac{1}{25a^2}+2\times 3a\times\frac{1}{5a}=9$$

$$\Rightarrow 9a^2+\frac{1}{25a^2}=9-\frac{6}{5}=\frac{39}{5}$$

35. (b) If $a+b+c=0$

then $a^3+b^3+c^3=3abc$

Dividing both sides by abc

$$\frac{a^3}{abc}+\frac{b^3}{abc}+\frac{c^3}{abc}=\frac{3abc}{abc}$$

$$\frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}=3$$

36. (c) If $a+b+c=0$,

then, $a^3+b^3+c^3=3\,abc$

Here, $y-z+z-x+x-y=0$

$\therefore$ $(y-z)^3+(z-x)^3+(x-y)^3$

$=3(y-z)(z-x)(x-y)$

37. (b) Using $(a+b)^3=a^3+b^3+3ab(a+b)$

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow (5)^3=\left(x^3+\frac{1}{x^3}\right)+15$$

or $x^3+\frac{1}{x^3}=125-15=110$

38. (b) $x+\frac{4}{x}=4$

$x^2+4=4x \Rightarrow x^2-4x+4=0 \Rightarrow (x-2)^2=0$

$x=2$

$$x^3+\frac{4}{x^3}=(2)^3+\frac{4}{(2)^3} \Rightarrow 8+\frac{4}{8} \Rightarrow 8+\frac{1}{2} \Rightarrow 8\frac{1}{2}$$

39. (b) $x=3+2\sqrt{2}$

$x=2+1+2\sqrt{2}$

$x=(\sqrt{2})^2+(1)^2+2.1.\sqrt{2}$

$x=(\sqrt{2}+1)^2$

$\sqrt{x}=(\sqrt{2}+1)$ ...(1)

$$\frac{1}{\sqrt{x}}=\frac{1}{\sqrt{2}+1}\times\frac{\sqrt{2}-1}{\sqrt{2}-1}=\frac{\sqrt{2}-1}{2-1}=\sqrt{2}-1$$

Now, $\sqrt{x}-\frac{1}{\sqrt{x}}=\sqrt{2}+1-(\sqrt{2}-1)=\sqrt{2}+1-\sqrt{2}+1$

$$\sqrt{x}-\frac{1}{\sqrt{x}}=2$$

40. (c) $a^2+b^2+c^2=2a-2b-2$

$(a^2-2a+1)+(b^2+2b+1)+c^2=0$

$(a-1)^2+(b+1)^2+c^2=0$

This equation is possible if

$a-1=0$, $b+1=0$ and $c=0$

$a=1, b=-1, c=0$

$3a-2b+c=3\times 1-2\times(-1)+0$

$=3+2=5$

41. (b) $a+b+c=3$

Squaring both sides

$a^2+b^2+c^2+2(ab+bc+ac)=9$

$6+2(ab+bc+ca)=9$

$ab+bc+ca=\frac{3}{2}$ ...(1)

given $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$

$\Rightarrow ab+bc+ac=abc=\frac{3}{2}$ [from (1)]

42. (c) Given, $x=\frac{4ab}{a+b}$

$$\Rightarrow \frac{x}{2a}=\frac{2b}{a+b}$$

Applying componendo and dividendo, we get

$$\frac{x+2a}{x-2a}=\frac{2b+a+b}{2b-a-b}=\frac{a+3b}{b-a} \quad \text{...(i)}$$

Also, $\frac{x}{2b}=\frac{2a}{a+b}$

Applying componendo and dividendo, we get

$$\frac{x+2b}{x-2b}=\frac{2a+a+b}{2a-a-b}=\frac{3a+b}{a-b} \quad \text{...(ii)}$$

Add (i) & (ii),

$$\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}=\frac{a+3b}{b-a}+\frac{3a+b}{a-b}$$

$$=\frac{1}{b-a}[a+3b-3a-b]=\frac{2(b-a)}{(b-a)}=2$$

43. (b) Given,

$$\frac{x}{2x+y+z}=\frac{y}{x+2y+z}=\frac{z}{x+y+2z}=a$$

$\Rightarrow$ $x=a(2x+y+z)$, $y=a(x+2y+z)$ and $z=a(x+y+2z)$

$\Rightarrow$ $x+y+z=a(4x+4y+4z)=4a(x+y+z)$

$\Rightarrow$ $4a=1$ or $a=\frac{1}{4}$.

44. (d) Given, $a+b+c=0$

$\Rightarrow$ $a^2=(b+c)^2$

Now, consider $\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}$

$$=\frac{(b+c)^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}$$

$$=\frac{(b+c)^2}{(b+c)^2-bc}+\frac{b^2}{b^2+c(b+c)}+\frac{c^2}{c^2+b(b+c)}$$

$$=\frac{(b+c)^2}{b^2+c^2+bc}+\frac{b^2}{b^2+c^2+bc}+\frac{c^2}{c^2+b^2+bc}$$

$$=\frac{b^2+c^2+2bc+b^2+c^2}{b^2+c^2+bc}=\frac{2(b^2+c^2+bc)}{b^2+c^2+bc}=2$$

45. (c) $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}$ .....(i)

Given that,

$a^2=b+c$

$a+a^2=a+b+c$

$a(a+1)=a+b+c$

$a+1=\frac{a+b+c}{a}$

$\frac{1}{a+1}=\frac{a}{a+b+c}$

Similarly,

$\frac{1}{b+1}=\frac{b}{a+b+c}$

$\frac{1}{c+1}=\frac{c}{a+b+c}$

Put in eq. (i)

$$\therefore \frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}=\frac{a+b+c}{a+b+c}=1$$

46. (d) Given that,

$$\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$$

$$\Rightarrow \frac{a}{1-a}+1+\frac{b}{1-b}+1+\frac{c}{1-c}+1=4$$

$$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=4$$

47. (a) $x^2+y^2-z^2+2xy$

$=x^2+y^2+2xy-z^2$

$=(x+y)^2-z^2=(x+y+z)(x+y-z)$

$=(b+c-2a+c+a-2b+a+b-2c)(x+y-z)$

$=0$

48. (b) $\frac{x}{y}=\frac{0.04}{1.5}=\frac{4}{150}=\frac{2}{75}$

$\frac{y}{x}=\frac{75}{2}$

Now, $\frac{y^2-x^2}{y^2+2xy+x^2}$

$=\frac{(y-x)(y+x)}{(y+x)^2}$

$=\frac{y-x}{y+x}=\frac{\frac{y}{x}-1}{\frac{y}{x}+1}$

$=\frac{\frac{75}{2}-1}{\frac{75}{2}+1}=\frac{73}{77}$

49. (c) $x^2+y^2-4x-4y+8=0$

$\Rightarrow$ $x^2-4x+4+y^2-4y+4=0$

$\Rightarrow$ $(x-2)^2+(y-2)^2=0$

$\Rightarrow$ $x=2$ and $y=2$

$\therefore$ $x-y=2-2=0$

50. (d) Let $f(x)=x^5+1$

Since $(x-2)$ is the factor of (x^5+1), hence from Remainder Theorem, we have, $f(2)=(2)^5+1=33$

Hence, the remainder = 33

51. (a) $x+y=2z \Rightarrow x=2z-y$

Subtract 'z' from both sides $\Rightarrow x-z=2z-y-z=z-y$

$\therefore \frac{x}{x-z}+\frac{z}{y-z}$

$= \frac{x}{x-z} - \frac{z}{z-y} = \frac{x}{x-z} - \frac{z}{x-z} = \frac{x-z}{x-z} = 1$

52. (a) $\because \frac{p}{b-c} = \frac{q}{c-a} = \frac{r}{a-b} = k$ (Let)

$\therefore$ $p = k(b-c)$, $q = k(c-a)$, $r = k(a-b)$

Expression $= p + q + r$

$= k(b-c) + k(c-a) + k(a-b)$

$= k(b - c + c - a + a - b)$

$= k \times 0 = 0$

53. (c) $a + b + c = 2s$

$c = 2s - a - b = (s-a) + (s-b)$

$\therefore (s-a)^3 + (s-b)^3 + 3(s-a)(s-b)c$

$= (s-a)^3 + (s-b)^3 + 3(s-a)(s-b)[(s-a) + (s-b)]$

(Put the value of c)

$= [(s-a) + (s-b)]^3 = (2s - a - b)^3$

$(a + b + c - a - b)^3 = c^3$ (Put the value of 2s)

54. (a) $a + b + c = 0$

i.e. $a = -(b+c)$; $b = -(c+a)$; $c = -(a+b)$

Now, $\frac{a+b}{c} - \frac{2b}{c+a} + \frac{b+c}{a}$

$\Rightarrow \frac{a+b}{-(a+b)} - \frac{2[-(c+a)]}{c+a} + \frac{b+c}{-(b+c)}$

$\Rightarrow -1 + 2 - 1 = 0$

55. (d) We have $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Here $x = a - 4$, $y = b - 3$, $z = c - 1$

So, given expression is $(x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (a - 4 + b - 3 + c - 1)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (a + b + c - 8)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= (8 - 8)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= 0$

56. (b) If $a + b + c = 0$

then $a^3 + b^3 + c^3 = 3abc$

Dividing both sides by abc

$\frac{a^3}{abc} + \frac{b^3}{abc} + \frac{c^3}{abc} = \frac{3abc}{abc}$

$\frac{a^2}{bc} + \frac{b^2}{ac} + \frac{c^2}{ab} = 3$

57. (d) Here, $a + b + c = 0$

$\Rightarrow 331 + 336 - 667 = 0$

$\therefore a^3 + b^3 + c^3 - 3abc = 0$

58. (c) If $a + b + c = 0$,

then, $a^3 + b^3 + c^3 = 3abc$

Here, $y - z + z - x + x - y = 0$

$\therefore$ $(y-z)^3 + (z-x)^3 + (x-y)^3$

$= 3(y-z)(z-x)(x-y)$

59. (b) $x + \frac{4}{x} = 4$

$x^2 + 4 = 4x \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0$

$x = 2$

$x^3 + \frac{4}{x^3} = (2)^3 + \frac{4}{(2)^3} \Rightarrow 8 + \frac{4}{8} \Rightarrow 8 + \frac{1}{2} \Rightarrow 8\frac{1}{2}$

60. (b) $x = 3 + 2\sqrt{2}$

$x = 2 + 1 + 2\sqrt{2}$

$x = (\sqrt{2})^2 + (1)^2 + 2.1.\sqrt{2}$

$x = (\sqrt{2} + 1)^2$

$\sqrt{x} = (\sqrt{2} + 1)$ ...(1)

$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$

Now, $\sqrt{x} - \frac{1}{\sqrt{x}} = \sqrt{2} + 1 - (\sqrt{2} - 1) = \sqrt{2} + 1 - \sqrt{2} + 1$

$\sqrt{x} - \frac{1}{\sqrt{x}} = 2$

61. (b) Given, $x - \frac{1}{x} = \frac{1}{3}$

$\Rightarrow 3x - \frac{3}{x} = 1$

Squaring both sides,

$9x^2 + \frac{9}{x^2} - 2 \times 9 = 1$

$\Rightarrow 9x^2 + \frac{9}{x^2} = 19$

62. (d) $\because (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(6)^2 = 26 + 2(ab + bc + ca)$

$\Rightarrow 2(ab + bc + ca) = 10$

$\Rightarrow ab + bc + ca = 5$

63. (a) Given, $x + y + z = 0$

$\therefore \frac{xyz}{(x+y)(y+z)(z+x)} = \frac{xyz}{(-z)(-x)(-y)} = \frac{xyz}{-xyz} = -1$

64. (c)

$$\begin{array}{r|l} x^2+2 & x^3 + 5x^2 + 10k \quad (x+5 \\ & \underline{x^3 + 2x} \\ & 5x^2 - 2x + 10k \\ & \underline{5x^2 + 10} \\ & -2x - 10 + 10k \end{array}$$

$-2x - 10 + 10k$ = Remainder

Given, remainder $= -2x$

$\therefore -2x - 10 + 10k = -2x$

$\Rightarrow 10k = 10$

$\Rightarrow k = 1$

65. (c) Given that, $x+\frac{1}{x}=a$

Then, $x^3+x^2+\frac{1}{x^3}+\frac{1}{x^2}=\left(x^3+\frac{1}{x^3}\right)+\left(x^2+\frac{1}{x^2}\right)$

$=\left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)+\left(x+\frac{1}{x}\right)^2-2$

$= a^3 - 3a + a^2 - 2 = a^3 + a^2 - 3a - 2$

66. (c) We know that,

$a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow (a+b+c)=\left(\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}\right)$ ... (i)

Given that,

$$\frac{\left[(2.247)^3+(1.730)^3+(1.023)^3-3\times 2.247\times 1.730\times 1.023\right]}{\left[(2.247)^2+(1.730)^2+(1.023)^2-(2.247\times 1.730)-(1.730\times 1.023)-(2.247\times 1.023)\right]}$$

$= (2.247 + 1.730 + 1.023)$ [from Eq. (i)]
$= 5.000 = 5$

67. (b) Given, $0.764\,y = 1.236\,x$

$\Rightarrow \frac{y}{x}=\frac{1.236}{0.764}$ ... (i)

Now,

$\frac{y-x}{y+x}=\frac{\frac{y}{x}-1}{\frac{y}{x}+1}$

$\Rightarrow \frac{\frac{1.236}{0.764}-1}{\frac{1.236}{0.764}+1}=\frac{1.236-0.764}{1.236+0.764}$

$=\frac{0.472}{2.000}=0.236$

68. (c) Given expression $= 5px - 10qy + 2rpx - 4qry$
$= (5px + 2rpx) - (10qy + 4qry)$
$= px(5 + 2r) - 2qy(5 + 2r)$
$= (5 + 2r)(px - 2qy)$

69. (c) $\frac{725\times 725\times 725+371\times 371\times 371}{725\times 725-725\times 371+371\times 371}=725+371$

$\left(\because \frac{a^3+b^3}{a^2-ab+b^2}=\frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)}=a+b\right)$

$= 1096$

70. (d) **Statement I :**

When $x = -3$ then $x^3 + 2x^2 + 3x + 8$
$= (-3)^3 + 2(-3)^2 + 3(-3) + 8$
$= -10 \neq 0$

Hence, $(x - 3)$ is not the factor of $x^3 + 2x^2 + 3x + 8$

Statement II :

Similarly,

When $x = 2$, then $x^3 + 2x^2 + 3x + 8$
$= (2)^3 + 2(2)^2 + 3(2) + 8$
$= 30 \neq 0$

Hence, $x - 2$ is also not the function of $x^3 + 2x^2 + 3x + 8$.

71. (b) $\frac{(x^2+y^2)(x-y)-(x-y)^3}{x^2y-xy^2}$

$=\frac{x^3+xy^2-x^2y-y^3-(x^3-y^3-3x^2y+3xy^2)}{x^2y-xy^2}$

$=\frac{x^3+xy^2-x^2y-y^3-x^3+y^3+3x^2y-3xy^2}{x^2y-xy^2}$

$=\frac{2x^2y-2xy^2}{x^2y-xy^2}=\frac{2(x^2y-xy^2)}{x^2y-xy^2}=2$

72. (c) $(1 - x)(1 + x^2) = 1 - x + x^2 - x^3$

$2x^3 - x^2 + x - 1$ is added to $1 - x + x^2 - x^3$ to obtain x^3.

73. (c) $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$

If divisible by (y – z), then $y - z = 0 \Rightarrow y = z$

On putting y = z, we get

$x(z^2 - z^2) + z(z^2 - x^2) + z(x^2 - z^2)$
$= z^3 - zx^2 + zx^2 - z^3 = 0$

Hence, $y - z$ is a factor, so it is divible by $(y - z)$.

Also, if $z - x$ is a factor, then

$z - x = 0 \Rightarrow z = x$

On putting $z = x$, we get

$x(y^2 - x^2) + y(x^2 - x^2) + x(x^2 - y^2)$
$= xy^2 - x^3 + x^3 - xy^2 = 0$

Hence, $(z - x)$ is also a factor, so it is also divisible by $(z - x)$.

74. (c) $x(x + a)(x + 2a)(x + 3a)$
$= (x^2 + ax)(x^2 + 5ax + 6a^2)$
$= x^4 + ax^3 + 5ax^3 + 5a^2x^2 + 6a^2x^2 + 6a^3x$
$= x^4 + ax(x^2 + 5x^2 + 5ax + 6ax + 6a^2)$
$= x^4 + ax(6x^2 + 11ax + 6a^2)$ ... (i)

So, for terms to be perfect square,

$= (x + y)^2 (x + y)^2 = (x^2 + 2xy + y^2)(x^2 + y^2 + 2xy)$
$= x^4 + 2x^3y + x^2y^2 + x^2y^2 + 2xy^3 + y^4 + 2x^3y + 4x^2y^2 + 2xy^3$
$= x^4 + xy(4x^2 + 6xy + 4y^2) + y^4$

On comparing equations. (i) and (ii), as $y = a$

a^4 must be added to make it a perfect square.

75. (b) Let $f(x) = 3x^4 - 2x^3 + 3x^2 - 2x + 3$

Remainder $= f\left(-\frac{2}{3}\right)$

$= 3\left(-\frac{2}{3}\right)^4 - 2\left(-\frac{2}{3}\right)^3 + 3\left(-\frac{2}{3}\right)^2 - 2\left(-\frac{2}{3}\right) + 3$

$= \frac{185}{27}$

76. (b) $\left(x^2 + \frac{1}{x^2}\right) = \frac{17}{4}$

$\Rightarrow x^2 + \frac{1}{x^2} + 2 - 2 = \frac{17}{4}$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = \frac{17}{4}$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{17}{4} - 2$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{9}{4}$

$\Rightarrow \left(x - \frac{1}{x}\right) = \frac{3}{2}$

On cubing both sides, we get

$\left(x - \frac{1}{x}\right)^3 = \left(\frac{3}{2}\right)^3$

$\Rightarrow x^3 - \frac{1}{x^3} - 3 \times \frac{1}{x}.x\left(x - \frac{1}{x}\right) = \frac{27}{8}$

$\Rightarrow x^3 - \frac{1}{x^3} = \frac{27}{8} + 3 \times \left(\frac{3}{2}\right)$

$\Rightarrow x^3 - \frac{1}{x^3} = \frac{27}{8} + \frac{9}{2}$

$\therefore x^3 - \frac{1}{x^3} = \frac{63}{8}$

77. (c) Let $f(x) = x^5 - 5x^2 + 125$

$\therefore$ Required remainder $= f(-5) = (-5)^2 - 5(-5)^2 + 125$

$= -3125 - 125 + 125 = -3125$

78. (c) 1. Given, $3abc + b^3 + c^3 - a^3$

$= -(a^3 - b^3 - c^3 - 3abc)$

$= -[a^3 + (-b)^3 + (-c)^3 - 3(a)(-b)(-c)]$

$= -(a - b - c)(a^2 + b^2 + c^2 + ab - bc + ac)$

Hence, $(a - b - c)$ is a factor of $3abc + b^3 + c^3 - a^3$

Therefore, Statement 1 is correct.

Given, $3bc + b^3 + c^3 - 1$

$= b^3 + c^3 - (1)^3 - 3bc(-1)$

$= (b + c - 1)[b^2 + c^2 + 1^2 - bc + c + b]$

Hence, $(b+c-1)$ is a factor of $3bc + b^3 + c^3 - 1$.

Therefore, Statement 2 is also correct.

79. (c) $x^3q^2 - x^3pt + 4x^2pt - 4x^2q^2 + 3xq^2 - 3xpt$

If we put $x = 1$

$= q^2 - pt + 4pt - 4q^2 + 3q^2 - 3pt = 0$

So, $(x - 1)$ is a factor of this function of we put $x = 3$

Checking $x = 3$ also,

$= 27q^2 - 27pt + 36pt - 36q^2 + 9q^2 - 9pt = 0$

So, $(x - 1)$ and $(x - 3)$ both are factors.

80. (c) Given $\frac{p}{x} + \frac{q}{y} = m$ and $\frac{q}{x} + \frac{p}{y} = n$

$\Rightarrow \frac{py + qx}{xy} = m$

$\Rightarrow \frac{qy + px}{xy} = n$

$\Rightarrow py + qx = mxy$ ....(1)

$\Rightarrow qy + px = nxy$ ...(2)

Dividing (1) by (2) we get

$\frac{py + qx}{qy + px} = \frac{m}{n}$

$\Rightarrow \frac{y\left[p + q\frac{x}{y}\right]}{y\left[q + p\frac{x}{y}\right]} = \frac{m}{n}$

$\Rightarrow n\left[p + q\frac{x}{y}\right] = m\left[q + p\frac{x}{y}\right]$

$\Rightarrow np + nq\frac{x}{y} = mq + mp\frac{x}{y}$

$\Rightarrow np - mq = (mp - nq) \times \frac{x}{y}$

$\Rightarrow \frac{x}{y} = \frac{np - mq}{mp - nq}$

$\therefore$ Option (c) is correct.

81. (c) Given $a^2 - by - cz = 0$ ...(1)

$ax - b^2 + cz = 0$ ...(2)

$ax + by - c^2 = 0$ ...(3)

Adding (1) and (3), we get

$a^2 + ax - cz - c^2 = 0$

$a(a + x) = c(c + z)$

$\Rightarrow \ a+x=\frac{c}{a}(c+z)$ ...(4)

Substracting (2) from (3), we get

$by+b^2-c^2-cz=0$

$b(b+y)=c(c+z) \Rightarrow b+y=\frac{c}{b}(c+z)$ ...(5)

Consider $\frac{x}{a+x}+\frac{y}{b+y}+\frac{z}{c+z}$

Using (4) and (5), we get

$\frac{ax}{c(c+z)}+\frac{by}{c(c+z)}+\frac{z}{c+z}$

$=\frac{ax+by}{c(c+z)}+\frac{z}{c+z}$

[Using (3) $ax+by=c^2$]

$=\frac{c^2}{c(c+z)}+\frac{z}{c+z}$

$=\frac{c}{c+z}+\frac{z}{c+z}=\frac{c+z}{c+z}$

= 1

∴ Option (c) is correct.

82. (b) 83. (c) 84. (b) 85. (c)

86. (a) Given $a^3=117+b^3$ ...(1)

and $a=3+b$

Putting the value of a in (1), we get

$(3+b)^3=117+b^3$

$27+b^3+9b^2+27b=117+b^3$

$9b^2+27b-90=0$

$b^2+3b-10=0$

$(b+5)(b-2)=0$

$b\neq-5 \ (\because b>0)$

$\Rightarrow \ b=2$

$\therefore \ a=3+b=3+2$

$\Rightarrow \ a=5$

Thus $a+b=5+2=7$

∴ Option (a) is correct.

87. (d) Given $(x+1)$ is a factor of x^3+kx^2-x+2 ...(1)

∴ $x=-1$ satisfies the equation (1), we get

$(-1)^3+k(-1)^2-(-1)+2=0$

$-1+k+1+2=0$

$k=-2$

∴ Option (d) is correct.

88. (c) Inequations are

$x+y\leq 4$ and $x-y\geq 2$

To check point P(5, – 1), we get

$5-1\leq 4$ and $5+1\geq 2$

$4\leq 4$ and $6\geq 2$

P is true.

To check Q(3, – 2), we get

$3-2\leq 4$ and $3+2\geq 2$

$1\leq 4$ and $5\geq 2$

Q is true.

To check R(1, 1) we get

$1+1\leq 4$ and $1-1\geq 2$

$2\leq 4$ and $0\geq 2$

R is not true.

∴ Option (c) is correct.

89. (c) If (x + 2) is a factor, x = –2 will satisfy the expression

$\Rightarrow \ x^4-6x^3+12x^2-24x+32$

$\Rightarrow (-2)^4-6(-2)^3+12(-2)^2-24(-2)+32$

$\Rightarrow \ 16+48+48+48+32\neq 0$

Again,

$\Rightarrow \ x^4+6x^3-12x^2+24x-32$

$\Rightarrow \ (-2)^4+6(-2)^3-12(-2)^2+24(-2)-32$

$\Rightarrow \ 16-48-48-48-32\neq 0$

Again if (x – 2) is a factor, x = 2 will satisfy the expression-

$\Rightarrow \ x^4-6x^3+12x^2-24x+32$

$\Rightarrow \ (2)^4-6(2)^3+12(2)^2-24(2)+32$

$\Rightarrow \ 16-48+48-48+32=0$

Again

$\Rightarrow \ x^4+6x^3-12x^2+24x-32$

$\Rightarrow \ (2)^4+6(2)^3-12(2)^2+24(2)-32$

$\Rightarrow \ 16+48-48+48-32\neq 0$

So, option (c) is correct.

90. (a) According to question.

$\Rightarrow \ 3x+2y=5k_1+2$ --------(i)

$\Rightarrow \ 2x+3y=5k_2+3$ --------(ii)

eq(i)-eq(ii)

$\Rightarrow \ x-y=5(k_1-k_2)-1$

so when $(x-y)$ is divided by 5 remainder will be

$5+(-1)=4$

So, option (a) is correct