Talent & Olympiad

Pair if Linear Equations in Two Variables

- **Linear equation in two variables:** An equation of the form ax + by = c, where $a \neq 0, b \neq 0$ and a, b and c are real numbers is known as a linear equation in two variables x and y.
- A pair of linear equations in two variables: Two linear equations in the same two variables are called a pair of linear equations in two variables.
- General form of a pair of linear equations in two variables: The general form of a pair of linear equations in two variables is a₁x + b₁y + c₁ = 0 and a₂x + b₂y + c₂ = 0, where a₁, a₂, b₁, b₂, c₁ and c₂ are real numbers such that a₁² + b₁² ≠ 0 and a₂² + b₂² ≠ 0.

Note: A pair of linear equations in two variables is called a system of simultaneous linear equations.

 Solution of a pair of linear equations in two variables: A pair of values (x, y) that satisfies both the linear equations is called a solution of the system of simultaneous equations.

Note: (i) A pair of values (x, y) that satisfies an equation is called its solution. (ii) Every linear equation in two variables has an infinite number of solutions.

(iii) Every solution of a linear equation is a point on the line representing it.

 Methods of solving a pair of linear equations in two variables: A pair of linear equations in two variables can be solved by (i) Graphical method (ii) Algebraic method.

(i) **Graphical method:** The graph of a linear equation is a straight line. The graph of a pair of linear equations in two variables is represented by two lines.

(a) If the two lines intersect at a point, then the pair of linear equations has a unique solution (the point) and is said to be consistent.

(b) If the two lines coincide, then the pair of linear equations has infinitely many solutions (each point on the line being a solution), and is said to be dependent or consistent.

(c) If the two lines are parallel, then the pair of linear equations has no solution (no common point) and is said to be inconsistent.

In other words, there are three types of solutions of a pair of linear equations in two variables:

(a) Unique solution

- (b) Infinitely many solutions
- (c) No solution.

Note: Graphical method does not give an accurate answer as error is likely to occur in reading the coordinates of a point.

(ii) Algebraic methods: To obtain accurate result of solution of simultaneous linear equations, algebraic methods are used.

There are three algebraic methods to learn for this year.

- (a) Method of elimination by substitution.
- (b) Method of elimination by equating the coefficients.
- (c) Method of cross multiplication.
- Conditions for solvability of a pair of linear equations in two variables: Consider the pair of linear equations $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$

Conditions	Types of lines	No. of Solutions	Type of equation
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Intersecting lines	1 (Unique solution)	Consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many	Dependent (or) consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No Solution	Inconsistent

• Homogeneous equations: The system of equations $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$ is called the system of homogeneous equations, and has only one solution x = 0 and y = 0. When $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the system of homogeneous equations has only one solution and the system is consistent.

When $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ the system of homogeneous equations has infinitely many solutions and the system is

consistent.

 Some equations are not linear. But they can be reduced to linear form by making some suitable substitutions and solving them using any convenient method.