Transmission Lines

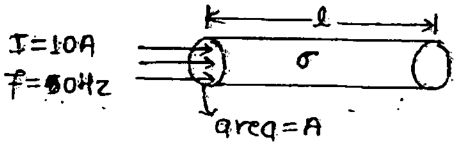
(1.) In ele. engg. 2 wire Tx line is used xfer energy at low freq. (50 Hz)

(2) In comm. engg 2 wire Tx line is used to xfer electromegnatic energy at very high freq. (>1 MHz).

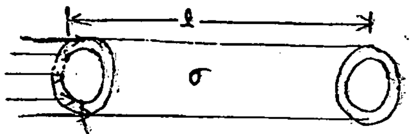
(3.) Tx line problems can be solve using EM theory method solving maxwells eqn (or) network theory method (KVL, KCL).

we prefer n/w theory method because it is simpler.

(4.) Tx lines parameters:-



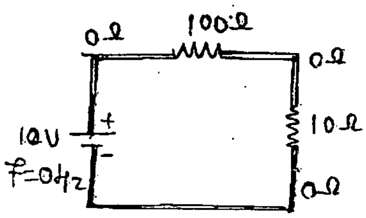
$R = \frac{l}{\sigma A}$

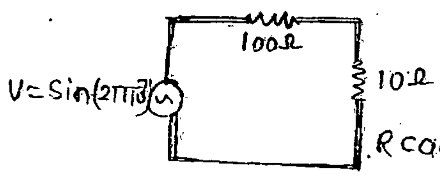


$I = 10\sin(2\pi 10^7 t)$ area decreased

$f = 10^7 Hz$

$\uparrow R_{AC} = \frac{l}{\sigma A\downarrow}$ (Resistance increas... every locn)





R can't be neglected

(i.) R→ whenever high freq. signal flowing through a condr cross sectional area decreases, so that resistance comes in to the picture of every location.

R→ series resistance of both condr per meter length $(\frac{\Omega}{m})$

* For a perfect condr $(\sigma_c = \infty)$

$$R = \frac{l}{\sigma_c A} = \frac{l}{\infty} = 0$$

(ii) L→ Whenever current goes through a condr inductance is present.

L→ series inductance of both condr per meter length (H/m)

(iii) G→ If the dielectric b/n the condr is imperfect then conductance exist.

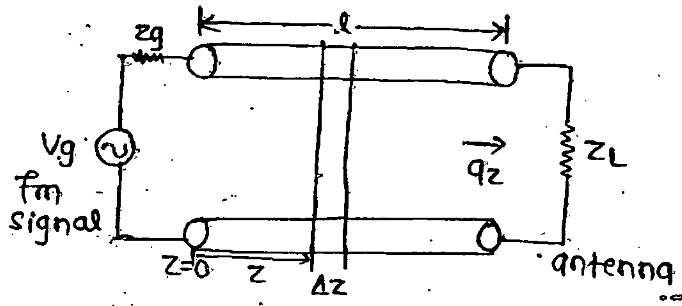
G→ shunt conductance per meter length $(\frac{\mho}{m})$

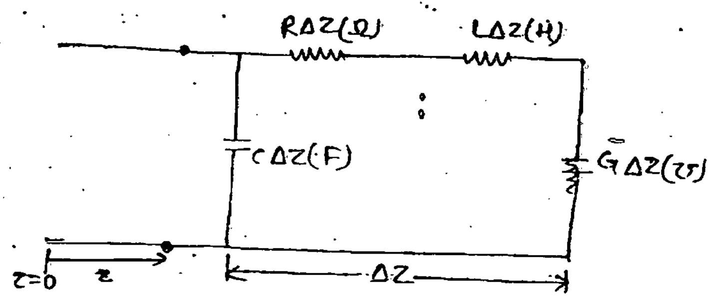
* If the dielectric is perfect $\sigma_m = 0$, $\epsilon_1 = \frac{\sigma_m A}{l} = 0$

(iv) C → Whenever 2 cond^r are seprated by dielectric then capacitance exist.

C → shunt capacitance per meter length $\left(\frac{F}{m}\right)$

Not req.
Tx lines eq. circuit →





Not Req.
Tx lines eq^n in phasor domain →

$$-\frac{\partial V_s}{\partial z} = (R + j\omega L)\, I_s$$

$$\frac{\partial V_s}{\partial z} = -Z I_s \text{ ——— (i)}$$

$$\boxed{Z = R + j\omega L \left(\frac{\Omega}{m}\right)}$$

$$-\frac{\partial I_s}{\partial z} = (\underset{\downarrow \left(\frac{℧}{m}\right)}{G} + j\omega \underset{\downarrow \left(\frac{F}{m}\right)}{C})\, V_s$$

$$\frac{\partial I_s}{\partial z} = -Y V_s \text{ ——— (ii)}$$

$$\boxed{Y = G + j\omega C \left(\frac{℧}{m}\right)}$$

$$\frac{\partial^2 V_s}{\partial z^2} = (R+j\omega L)(G+j\omega C)V_s$$

$$\frac{\partial^2 V_s}{\partial z^2} = \gamma^2 V_s \quad \text{———} (3) \qquad (\nabla^2 E_s = \gamma^2 E_s)$$

$$\boxed{\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}} \; (\alpha + j\beta)$$

$$\frac{\partial^2 I_s}{\partial z^2} = \gamma^2 I_s \quad \text{———} (4) \qquad (\nabla^2 H_s = \gamma^2 H_s)$$

$$Z_0 = \sqrt{\frac{R+j\omega L \left(\frac{\Omega}{m}\right)}{G+j\omega C \left(\frac{\mho}{m}\right)}} = (\Omega)^2 = \Omega \quad \text{(characterstic impedance)}$$

Note →

(i) Loss less transmission → $(\alpha = 0)(\beta \neq 0)$

- If the Tx lines ~~cond~~ ~~are~~ pefect then $(R=0)$,
  If the dielectric is perfect $(G=0)$

  Then $\gamma = \sqrt{(0+j\omega L)(0+j\omega C)} = j\omega\sqrt{LC} = \alpha + j\beta$

  compare; $\boxed{\alpha = 0, \beta = \omega\sqrt{LC}}$

(ii) Increase the freq so that $\omega L \gg R$; $\omega C \gg G$ then R, G can be neglected.

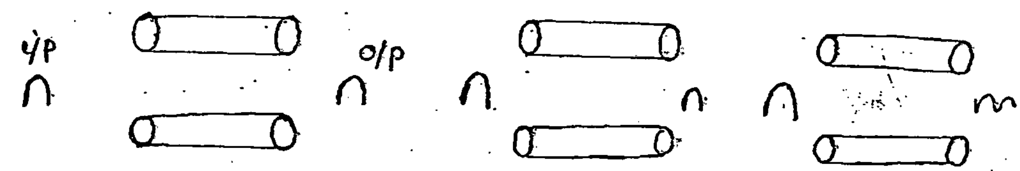
$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

R, G are neglected

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = \alpha + j\beta$$

$$\boxed{\alpha = 0, \; \beta = \omega\sqrt{LC}}$$

(iii) Distortionless transmission →



No loss $(\alpha=0)$
No distortion

loss is present $(\alpha \neq 0)$
No distortion

distortion is present

Defination → If the Tx line is distortionless then the α must be constant β must be linear f^n of freq.

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\gamma = \sqrt{R\left(1+j\omega\frac{L}{R}\right)G\left(1+\frac{j\omega C}{G}\right)}$$

If $\frac{L}{R} = \frac{C}{G}$ $\boxed{LG=RC}$ Cond^n to have a distortionless f^n.

$$\gamma = \sqrt{RG\left(1+j\omega\frac{L}{R}\right)^2}$$

$$\gamma = \sqrt{RG}\left(1+j\omega\frac{L}{R}\right)$$

$$\gamma = \sqrt{RG} + \sqrt{RG}\cdot\frac{L}{R}j\omega = \sqrt{RG} + j\omega\sqrt{\frac{G}{R}}\cdot L$$

$$\gamma = \sqrt{RG} + j\omega L\sqrt{\frac{C}{L}}$$

$$\gamma = \sqrt{RG} + j\sqrt{LC}\,\omega = \alpha + j\beta$$

$$\alpha = \sqrt{RG} = \text{constant}$$

$$\beta = \omega\sqrt{LC} \Rightarrow \beta \propto \omega \text{ linear } f^n \text{ of freq.}$$

Note →

	γ	Z_0
General	$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$	$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$
Lossless $(R=G=0)$	$\gamma = j\omega\sqrt{LC}$	$Z_0 = \sqrt{\frac{L}{C}}$
Distortion Less $\boxed{LG=RC}$	$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$	$Z_0 = \sqrt{\frac{L}{C}}$

Note →

(i) $R = G = 0$
(ii) $\omega L >> R$, $\omega C >> G$ } lossless; $\alpha = 0$; $\beta = \omega\sqrt{LC}$

(iii) $LG = RC$ } distortionless; $\alpha = \sqrt{RG}$, $\beta = \omega\sqrt{LC}$

} Distortionless

* Lossless tx line is distortionless but distortionless does not mean lossless.

Note →

(i) We know that for a coaxial cable:

$$C = \frac{2\pi\epsilon}{\ln\left(\frac{b}{a}\right)} \left(\frac{F}{m}\right) ; \quad L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) \left(\frac{H}{m}\right)$$

$$\boxed{LC = \mu\epsilon}$$

* for a lossless coaxial tx lines

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)}{\frac{2\pi\epsilon}{\ln(b/a)}}} = \frac{1}{2\pi} \ln\left(\frac{b}{a}\right) \sqrt{\frac{\mu}{\epsilon}}$$

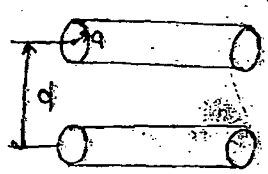
$$Z_0 = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{\frac{\mu_r}{\epsilon_r}} \ln\left(\frac{b}{a}\right)$$

$$= \frac{1}{2\pi} (120\pi) \frac{1}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right)$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}} = \frac{60}{\sqrt{\epsilon_r}} \ln\left(\frac{b}{a}\right) \Omega}$$

(ii) For a lossless 2 wire line:

$$\boxed{Z_0 = \frac{120}{\sqrt{\epsilon_r}} \ln\left(\frac{d}{a}\right)}$$



a → Radius of the cond^r

d → distance between the centers of cond^r

(3) βz is electrical length of T^n line

(4) $$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$$\boxed{V_p = \frac{c}{\sqrt{\mu_r\epsilon_r}}}$$

(5) $$\lambda = \frac{2\pi}{\beta} \qquad \boxed{\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r\mu_r}}}$$

(Not req)

Solution of T^n line eq^n →

$$\frac{\partial^2 V_s}{\partial z^2} = \gamma^2 V_s \quad \text{—— (i)}$$

$$\frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{xs}$$

↓ Solution

$$E_{xs} = E_{io}e^{-\gamma z} + E_{ro}e^{+\gamma z}$$

$$V_s = \underbrace{V^+e^{-\gamma z}}_{\text{Incident}} + \underbrace{V^-e^{+\gamma z}}_{\text{Reflected}} \quad \text{—— (ii)}$$

Similarly

$$\frac{\partial^2 I_s}{\partial z^2} = \gamma^2 I_s \quad \text{—— (iii)}$$

$$I_s = I^+e^{-\gamma z} + I^-e^{+\gamma z} \quad \text{—— (iv)}$$

$V^+ \rightarrow$ Initial value of voltage of incident wave

$I^+ \rightarrow$ Initial value of current of incident wave

$V^- \rightarrow$ Initial value of voltage of Reflected wave

$I^- \rightarrow$ Initial value of current of Reflected wave

Relation b/n V^+, I^+ & V^-, I^- →

$$\frac{\partial V_s}{\partial z} = -Z I_s \quad \text{—— (v)}$$

from (2), (4) in (5)

$$\frac{\partial}{\partial z}\left(V^+e^{-\gamma z} + V^-e^{+\gamma z}\right) = -Z\left(I^+e^{-\gamma z} + I^-e^{+\gamma z}\right)$$

$$V^+e^{-\gamma z}(-\gamma) + V^-e^{\gamma z}(\gamma) = -Z I^+e^{-\gamma z} - Z I^-e^{\gamma z}$$

Compare the cofficients →

$$V^{+}(-\gamma) = -Z(I^{+})$$

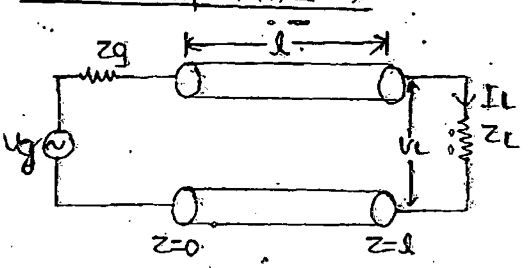
$$\left(\frac{V}{A}\right)\ \frac{V^{+}}{I^{+}} = \frac{Z}{\gamma} = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}\ (\Omega) = Z_0$$

$$\boxed{\frac{V^{+}}{I^{+}} = Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \text{defined as characterstic impedance}}$$

$$V^{-}(\gamma) = -ZI^{-} \Rightarrow \frac{V^{-}}{I^{-}} = \frac{-Z}{\gamma} = -Z_0$$

$$\boxed{\frac{V^{-}}{I^{-}} = -Z_0}$$

Load Impedance →



$$\left(Z_L = \frac{V_L}{I_L}\right)$$

$$Z_L = \frac{V_L}{I_L} = \frac{V^{+}e^{-\gamma l} + V^{-}e^{\gamma l}}{I^{+}e^{-\gamma l} + I^{-}e^{\gamma l}} = \frac{V^{+}e^{-\gamma l} + V^{-}e^{\gamma l}}{\frac{V^{+}}{Z_0}e^{-\gamma l} - \frac{V^{-}}{Z_0}e^{\gamma l}}$$

$$Z_L = Z_0 * \frac{\cancel{(V^{+}e^{-\gamma l})}\left[1 + \frac{V^{-}e^{\gamma l}}{V^{+}e^{-\gamma l}}\right]}{\cancel{(V^{+}e^{-\gamma l})}\left[1 - \frac{V^{-}e^{\gamma l}}{V^{+}e^{-\gamma l}}\right]}$$

$$\because\ \Gamma_l = \frac{V^{-}e^{\gamma l}}{V^{+}e^{-\gamma l}}$$

$$Z_L = Z_0 * \frac{1+\Gamma_l}{1-\Gamma_l} \Rightarrow \frac{Z_L}{Z_0} = \frac{1+\Gamma_l}{1-\Gamma_l}$$

$$\boxed{\Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0}} \qquad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Reflection coefficient →

Case(1) → Reflection coefficient at load $= \Gamma_l = \dfrac{\text{Reflected vol. at load}}{\text{incident vol. at load}}$

$$= \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}} \quad \text{---(i)}$$

$$\boxed{\Gamma_l = \underset{\text{(Complex)}}{\frac{Z_L - Z_0}{Z_L + Z_0}} = |\Gamma_l| e^{j\theta_l}}$$

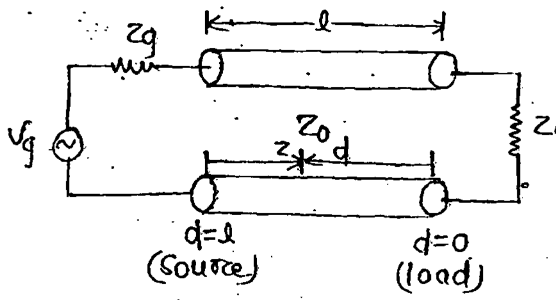
Case(2) → Reflection coefficient at a z from the source end.

$$\Gamma(z) = \frac{V^- e^{\gamma z}}{V^+ e^{-\gamma z}} \quad \text{---(ii)}$$

Case(3) → Reflection coefficient at a distance "d" from load end is

$$\boxed{\Gamma(d) = |\Gamma_l| e^{-2\alpha d} e^{j(\theta_l - 2\beta d)}}$$

Explanation →



$z + d = l$

In general mismatch occurs at load. So we measure from load.

Reflection coefficient at d distance is obtained by putting $z = l - d$ in eqn (2)

$$\Gamma(d) = \frac{V^- e^{\gamma(l-d)}}{V^+ e^{-\gamma(l-d)}} = \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}} \cdot e^{-2\gamma d} \quad \Big| \ \gamma = \alpha + j\beta$$

$$= \Gamma_l e^{-2(\alpha + j\beta)d}$$

$$= |\Gamma_l| e^{j\theta_l} e^{-2\alpha d} e^{-j2\beta d}$$

$$= |\Gamma_l| e^{-2\alpha d} e^{j(\theta_l - 2\beta d)}$$

Transmission Coefficient →

$$T_l = 1 + \Gamma_l$$

$$T_l = 1 + \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{2Z_L}{Z_L + Z_0} \quad \Big| \quad T = \frac{2\eta_2}{\eta_1 + \eta_2}$$

Standing Wave Ratio →

Voltage standing wave Ratio $= VSWR = \frac{|V|_{max}}{|V|_{min}}$

Current standing wave Ratio $= ISWR = \frac{|I|_{max}}{|I|_{min}}$

$$\boxed{VSWR = ISWR = SWR = \frac{1+|\Gamma|}{1-|\Gamma|}} \qquad \boxed{|\Gamma| = \frac{SWR-1}{SWR+1}}$$

Note →

(i) If load is SC → $Z_L = 0$; $\Gamma_l = \frac{0 - Z_0}{0 + Z_0} = -1$

$|\Gamma_l| = 1$

If load is OC → $Z_L = \infty$; $\Gamma_l = \frac{Z_L\left(1 - \frac{Z_0}{Z_L}\right)}{Z_L\left(1 + \frac{Z_0}{Z_L}\right)} = \frac{1 - \frac{Z_0}{\infty}}{1 + \frac{Z_0}{\infty}} = 1$

$|\Gamma_l| = 1$

In this case complete standing wave is present.

(2) If $Z_L = Z_0$; $|\Gamma_l| = 0$; $SWR = \frac{1+0}{1-0} = 1$

In this case no standing wave on a tx line. It is also called as flat tx line.

(3) If Z_L, Z_0 are real & $Z_L > Z_0$ then $SWR = \frac{Z_L}{Z_0}$

if Z_L, Z_0 are real & $Z_0 > Z_L$ then $SWR = \frac{Z_0}{Z_L}$

For 1st. $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1 + \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|}{1 - \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|} = \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0}}$

for 2^{nd} case;

$$SWR = \frac{1+\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|}{1-\left|\frac{Z_L-Z_0}{Z_L+Z_0}\right|} = \frac{1+\frac{Z_0-Z_L}{Z_0+Z_L}}{1-\frac{Z_0-Z_L}{Z_0+Z_L}}$$

(4.) Range :-

$$\begin{array}{ccccc} 0 & \le & |\Gamma| & \le & 1 \\ \downarrow & & & & \downarrow \\ 1 & \le & SWR & \le & \infty \end{array}$$

*** Tx line impedance at a distance "d" from the load end →

$$\boxed{Z(d) = Z_0 \frac{Z_L + Z_0 \tanh(\gamma d)}{Z_0 + Z_L \tanh(\gamma d)}} \text{ ——— (i)}$$

* For a lossless Tx lines $\alpha = 0$; $\gamma = j\beta$, $Z_0 = \sqrt{\frac{L}{C}} = R_0$ (Real)

$$\tanh(j\theta) = j\tan\theta$$

$$\tanh(j\beta d) = j\tan(\beta d)$$

$$\boxed{Z(d) = R_0 \cdot \frac{Z_L + jR_0\tan(\beta d)}{R_0 + jZ_L\tan(\beta d)}} \text{ ——— (ii)}$$

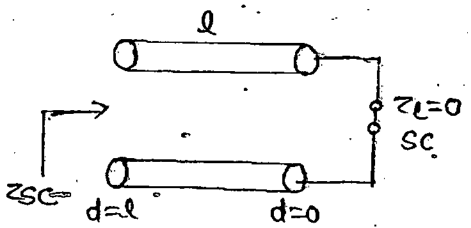
Tx line length is represented in terms of λ because $\tan(\beta d)$ can be calculated.

$$\beta d = \frac{2\pi}{\lambda} d$$

if d is given in terms of λ then βd comes in radian

Eqⁿ of γ, Z_0 →

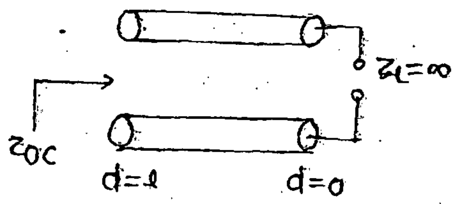
* Measure sending end impedance with sc at load



$$z(d=l) = z_0 \cdot \frac{0+z_0 \tanh(\gamma l)}{z_0+0}$$

$$Z_{sc} = Z_0 \tanh(\gamma l) \quad \text{——— (i)}$$

* Measure sending ~~end~~ impedance with oc at load.



$$Z(d=l) = \cancel{0+Z_0} \frac{1+0}{0+\tanh(\gamma l)}$$

$$Z_{oc} = Z_0 \coth(\gamma l) \quad \text{——— (ii)}$$

from (1) x (2)

$$Z_{sc} \cdot Z_{oc} = Z_0^2$$

$$\boxed{Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}} \quad \text{——— (iii)}$$

from (i) ÷ (2)

$$\gamma = \frac{1}{l} \tanh^{-1} \sqrt{\frac{Z_{sc}}{Z_{oc}}} \quad \text{——— (iv)}$$

Note → On a tx line all the parameters (z, I, v, etc) repeat. For a $\frac{\lambda}{2}$ distanc

V_{max} —— $\lambda/2$ —— V_{max} V_{min} —— $\lambda/2$ —— V_{min} V_{max} —— $\lambda/4$ —— V_{min}

Explanation →

* $$Z(d=0) = Z_0 \frac{Z_L + jZ_0 \tan(0)}{Z_0 + jZ_L \tan(0)}$$

$$= Z_0 \frac{Z_L+0}{Z_0+0} = Z_L \quad \text{——— (i)}$$

* ~~Z at λ/2~~ at $d = \pi/2$

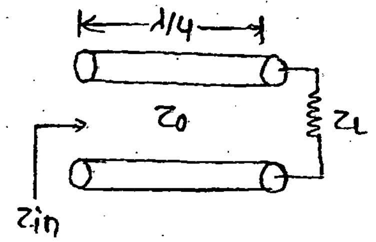
$$\beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$Z\left(d=\frac{\lambda}{2}\right) = Z_0 \frac{Z_L + jZ_0 \tan(\pi)}{Z_0 + jZ_L \tan(\pi)}$$

$$= Z_0 \frac{Z_L+0}{Z_0+0} = Z_L \quad \text{—— (2)}$$

<u>Note→</u> For a $\frac{\lambda}{4}$ length tx line {quater wavelength tx line}

$$\boxed{Z_{in} \cdot Z_L = Z_0^2}$$



<u>Explanation→</u>

$$d = \frac{\lambda}{4}, \quad \beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z\left(d=\frac{\lambda}{4}\right) = Z_0 \frac{\frac{Z_L}{\tan(\pi/2)} + jZ_0}{\frac{Z_0}{\tan(\pi/2)} + jZ_L}$$

$$= Z_0 \left(\frac{jZ_0}{jZ_L}\right)$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$Z_0^2 = Z_{in} Z_L$$

★ <u>Normalized impedance→</u> It is the ratio of impedance & Z_0, denoted by z.

<u>Example→</u> Normalised load impedance $= z_L = \frac{Z_L}{Z_0}$ (unit less)

★ max^m normalised impedance is present at the loc^n of vol. max^m (V_{max}) & is equal to SWR i.e $z_{max} = SWR$

$$z_{max} = \frac{Z_{max}}{Z_0}$$

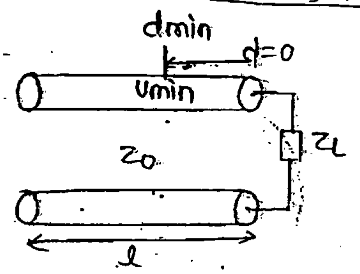
$$z_{max} = \frac{\frac{V_{max}}{I_{min}}}{Z_0} = \frac{V_{max}}{(Z_0) I_{min}} \times Z_0 = \frac{V_{max}}{V_{min}} = SWR$$

~~Minm normalised impedance is~~ present at the locn of V_{min} & ie equal to $\frac{1}{SWR}$ i.e. $z_{min} = \frac{1}{SWR}$.

$$z_{min} = \frac{Z_{min}}{Z_0} = \frac{\frac{V_{min}}{I_{max}}}{Z_0} = \frac{V_{min}}{I_{max} Z_0} = \frac{V_{min}}{V_{max}}$$

$$z_{min} = \frac{1}{\left(\frac{V_{max}}{V_{min}}\right)} = \frac{1}{SWR}$$

* Location of 1st voltage minm (d_{min}) from the load →



$$\Gamma(d) = |\Gamma_l| e^{-2\alpha d} \cdot e^{j(\theta_l - 2\beta d)}$$

d_{min} is obtained

$$\theta_l - 2\beta d_{min} = -\pi$$

$$\boxed{d_{min} = \frac{\lambda}{4\pi}(\pi + \theta_l)} \text{ ——— (i)} \quad \Gamma_l = |\Gamma_l| e^{j\theta_l}$$

$\boxed{\text{Location of 1st } V_{max} \text{ is } d_{max} = d_{min} \pm \frac{\lambda}{4}}$

Note →

(i) $\boxed{\text{If } Z_L, Z_0 \text{ real \& } Z_L > Z_0 \text{ then } V_{max} \text{ is at load}}$

Explanation →

$$\Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0} = +ve = e^{j0} \Rightarrow \theta_l = 0$$

$$d_{min} = \frac{\lambda}{4\pi}(\pi + 0) = \frac{\lambda}{4}$$

$$d_{max} = d_{min} \pm \frac{\lambda}{4}$$

$$= \frac{\lambda}{4} \pm \frac{\lambda}{4}$$

$$= 0, \frac{\lambda}{2}$$

(2) $\boxed{\text{If } Z_L, Z_0 \text{ are real } \& \ Z_0 > Z_L \text{ then } V_{min} \text{ is at load}}$

Explanation → $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -ve \Rightarrow e^{-j\pi} \Rightarrow \theta_L = -\pi$

$$d_{min} = \frac{\lambda}{4\pi}(\pi - \pi) = 0$$