# Chapter 12

# Introduction To Three Dimensional Geometry

## Exercise 12.2

Question 1: Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (–3, 7, 2) and (2, 4, –1)

(iii) (–1, 3, –4) and (1, –3, 4) (iv) (2, –1, 3) and (–2, 1, 3)

Answer 1:

The distance between points P ($x_1$, $y_1$, $z_1$) and P ($x_2$, $y_2$, $z_2$) is given by

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

= $\sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$

= $\sqrt{(2)^2 + (0)^2 + (-4)^2}$

= $\sqrt{4+16}$ = $\sqrt{20}$ = $2\sqrt{5}$

(ii) Distance between points (–3, 7, 2) and (2, 4, –1)

= $\sqrt{(2-(-3))^2 + (4-7)^2 + (-1-2)^2}$

= $\sqrt{(5)^2 + (-3)^2 + (-3)^2}$

= $\sqrt{25+9+9}$ = $\sqrt{43}$

(iii)Distance between points (–1, 3, –4) and (1, –3, 4)

= $\sqrt{(1-(-1))^2 + (-3-3)^2 + (4-(-4))^2}$

= $\sqrt{(2)^2 + (-6)^2 + (8)^2}$

$= \sqrt{4 + 36 + 64} = \sqrt{104} = 2\sqrt{26}$

(iv)Distance between points (2, –1, 3) and (–2, 1, 3)

$= \sqrt{(-2-2)^2 + (1-(-1))^2 + (3-3)^2}$

$= \sqrt{(4)^2 + (2)^2 + (0)^2}$

$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

Question 2: Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

Answer 2:

Let points (–2, 3, 5), (1, 2, 3), and (7, 0, –1) be denoted by P, Q, and R respectively.

Points P, Q, and R is collinear if they lie on a line.

$PQ = \sqrt{(1-(-2))^2 + (2-3)^2 + (3-5)^2}$

$= \sqrt{(3)^2 + (-1)^2 + (2)^2}$

$= \sqrt{9 + 1 + 4} = \sqrt{14}$

$QR = \sqrt{(7-1)^2 + (0-2)^2 + (-1-3)^2}$

$= \sqrt{(6)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{36 + 4 + 16} = \sqrt{56} = 2\sqrt{14}$

$PR = \sqrt{(7-(-2))^2 + (0-3)^2 + (-1-5)^2}$

$= \sqrt{(9)^2 + (-3)^2 + (-6)^2}$

$= \sqrt{81 + 9 + 36}$

$= \sqrt{126} = 3\sqrt{14}$

Here, PQ + QR $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$PR

Hence, points P (–2, 3, 5), Q (1, 2, 3), and R (7, 0, –1) are collinear.

Question 3: Verify the following:

(i) (0, 7, –10), (1, 6, –6) and (4, 9, –6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (–1, 6, 6) and (–4, 9, 6) are the vertices of a right angled triangle.

(iii) (–1, 2, 1), (1, –2, 5), (4, –7, 8) and (2, –3, 4) are the vertices of a parallelogram.

Answer 3:

(i) Let points (0, 7, –10), (1, 6, –6), and (4, 9, –6) be denoted by A, B, and C respectively.

AB = $\sqrt{(1-0)^2+(6-7)^2+(-6+10)^2}$

= $\sqrt{(1)^2+(-1)^2+(4)^2}$

= $\sqrt{1+1+8}$

= $\sqrt{10}$ = $3\sqrt{2}$

BC = $\sqrt{(4-1)^2+(9-6)^2+(-6+6)^2}$

= $\sqrt{(3)^2+(3)^2+(0)^2}$

= $\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

CA = $\sqrt{(0-4)^2+(7-9)^2+(-10+6)^2}$

= $\sqrt{(-4)^2+(-2)^2+(-4)^2}$

= $\sqrt{16+4+16}=\sqrt{36}$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle (ii)Let (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) be denoted by A, B, and C respectively.

AB = $\sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$

= $\sqrt{(-1)^2+(-1)^2+(-4)^2}$

= $\sqrt{1+1+16} = \sqrt{18}$

= $3\sqrt{2}$

BC = $\sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}$

= $\sqrt{(-3)^2+(3)^2+(0)^2}$

= $\sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CA = $\sqrt{(0+4)^2+(7-9)^2+(10-6)^2}$

= $\sqrt{(4)^2+(-2)^2+(4)^2}$

= $\sqrt{16+4+16} = \sqrt{36} = 6$

Now, $AB^2 + BC^2 = \left(3\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2 = 18 + 18 = 30 = AC^2$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) be denoted by A, B, C, and D respectively.

AB = $\sqrt{(1+1)^2+(-2-2)^2+(5-1)^2}$

= $\sqrt{4+16+16}$

= $\sqrt{36}$ = 6

BC = $\sqrt{(4-1)^2+(-7-2)^2+(8-5)^2}$

$= \sqrt{9 + 25 + 9} = \sqrt{43}$

CD = $\sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$

$= \sqrt{4 + 16 + 16}$

$= \sqrt{36}$

$= 6$

DA = $\sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$

$= \sqrt{9 + 25 + 9} = \sqrt{43}$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4: Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, –1).

Answer 4:

Let P (x, y, z) be the point that is equidistant from points A (1, 2, 3) and B (3, 2, –1).

Accordingly, PA = PB

$= PA^2 = PB^2$

= (x – 1)2 + (y – 2)2 + (z – 3)2 = (x – 3)2 + (y – 2)2 + (z – 1)2

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

Here, AB = CD = 6, BC = AD =

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question 5: Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (–4, 0, 0) is equal to 10.

Answer 5:

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (–4, 0, 0) respectively.

It is given that PA + PB = 10

$= \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$

$= \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$

On squaring both sides, we obtain

$= (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$

$= x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y2 + z2} + x^2 + 8x + 16 + y^2 + z^2$

$= 20\sqrt{x^2 + 8x + 16 + y2 + z2} = 100 + 16x$

$= 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2)$

$= 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.