KINEMATICS

1. **Distance :**
   The length of the path is known as distance. Distance is a scalar.
   Displacement:
   The distance between two points with specific direction is known as displacement.
   Displacement is a vector quantity.
   Both distance and displaments are same units and dimensions.

2. **Speed :** The rate of distance traveled by a body is known as speed.

   $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

   Speed is a scalar quantity.

3. **Velocity :** The rate of displacement is known as velocity.

   $$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

   Velocity is vector quantity.
   Both speed and velocities have same units and dimensions.

4. Average speed

   $$= \frac{\text{Total distance travelled}}{\text{Total time}}$$

   Distance travelled = Average speed x time S
   = v x t

5. Average velocity

   $$= \frac{\text{Resultant displacement}}{\text{Time}}$$

   Resultant displacement = Average velocity X time
   If a body cover first half of the total displacement with a velocity v_1 and second half with velocity v_2.Then the average velocity is given by

   $$V_{av} = \frac{2v_1v_2}{v_1 + v_2} \quad \text{this is the harmonic mean of}$$

   v_1 and v_2

7. **Instantaneous velocity :** The velocity of a body at any particular instant in its journey is known as instantaneous velocity.

   $$V_{inst} = \frac{ds}{dt}$$

8. **Relative velocity :** velocity of one body with respect to that of another body is called relative velocity. The velocity of `A` with respect to that of `B` is given by

   $$\vec{V_{AB}} = \vec{V_A} - \vec{V_B}$$

   If 'A' and 'B' are moving with speed $V_A + V_B$ at an angle θ with one another.

   $$|\vec{V}_{AB}| = \sqrt{V_A^2 + V_B^2 - 2V_AV_B\cos\theta} \ .$$

   If α is the angle made $\vec{V}_{AB}$ with $\vec{V}_A$ there

   $$\text{Tan } \alpha = \frac{V_B}{V_A} \ .$$

10. **Acceleration :** the rate of change of velocity of a body is called acceleration .
    Acceleration

    $$= \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

    The direction of acceleration lie along the direction of change in velocity.

11. **Retardation :** The negative acceelration is known as retardation.
    Any body travelling with decreasing velocity possess retardation or deceleration.

12. **Equations of motion** (moving with uniform acceleration)
    u = intital velocity,   v = final velocity
    a = acceleartion ,   s = displacement ,
    t = time of travel

    1. V = u + at   2. $S = ut + \frac{1}{2}at^2$

    3. $v^2 - u^2 = 2as$ 4. $S_n = u + a\left(n - \frac{1}{2}\right)$

    Where S_n is the distance travelled in nth second of its journey.

13. **Equations of motion of a freely falling body** (take u = 0 and a = g)

    1. v = gt   2. $S = \frac{1}{2}gt^2$

    3. $v^2 = 2gs$ 4. $Sn = g\left(n - \frac{1}{2}\right)$

14. **Equations of motion for a body thrown vertically up** $(a = -g , s = h)$

1. $v = u - gt$ 2. $h = ut - \dfrac{1}{2}gt^2$

3. $v^2 = u^2 - 2gh$

4. $h_n = u - g\left(n - \dfrac{1}{2}\right)$

15. For a body thrown up vertically from the top of a tower (or) a body dropped from a balloon moving vertically up

1. $v = -u + gt$ 2. $h = -ut + \dfrac{1}{2}gt^2$

3. $v^2 = u^2 - 2gh$

4. $h_n = -u + g\left(n - \dfrac{1}{2}\right)$

16. **For a body thrown vertically up with velocity 'u'**

a. maximum height : Maximum vertical distance travelled is known as maximum height

$$H = \dfrac{u^2}{2g}$$

b. Time of ascent : The time taken by a body to the maximum height is known as time of ascent.

$$t_a = \dfrac{u}{g}$$

c. Time of descent
The time in which body comes down from maximum height is known as time of descent.

$$t_d = \dfrac{u}{g}$$

d. Time of flight
The total time for which a body remains in air before reaching the ground is known as time of flight.

$$T = t_a + t_d = \dfrac{u}{g} + \dfrac{u}{g} = \dfrac{2u}{g}$$

17. Time of ascent is always equal to time of descent if resistance forces due to air are neglected.
If we consider the force of buoyancy the time of ascent is less time of descent $(t_a < t_d)$.

18. For a body throws vertically up with velocity 'u' the maximum height reached is given by

$$h = \dfrac{u^2}{2g} \ (or\ )h \ \alpha \ u^2$$

19. If a body is thrown vertically up with a velocity 'u' if returns to ground with velocity 'v' which is equal is magnitude of 'u' but opposite in direction (ie) $\vec{v} = -\vec{u}$.

20. For a body falling freely through a height 'h'

The time of descent $t = \sqrt{\dfrac{2h}{g}}$

The velocity gained $V = \sqrt{2gh}$.

21. If a body is allowed to fall freely from the top of a tower of height 'h' and another is projected simultaneously from the foot of tower in the upward direction with velocity u

then they meet after time $t = \dfrac{h}{u}$.

22. **Projectile :** The body projected in to air with some velocity at an angle (other than 90^0) with the horizontal is called a projectile.

23. The path of a projectile is a parabola.

24. If a body is projected at an angle 'θ' to the horizontal with velocity 'u' its horizontal and vertical components of initial velocities are $u \cos \theta$ and $u \sin \theta$.

25. After a time 't' the horizantal displacement $= x = (u \cos \theta)t$
vertical displacement =

$$y = (u \sin \theta)t - \dfrac{1}{2}gt^2 .$$

26. The velocity of projectile is not zero at any point. Its value is minimum ($u \cos \theta$) at the highest point of its path. At this point only vertical component of velocity is zero.

27. At any instant of time 't' the horizontal component of velocity is $V_x = u \cos \theta$.
The vertical component of velocity is
$$v_y = u \sin \theta - gt$$
The resulant velocity is $V = \sqrt{v_x^2 + v_y^2}$.
The direction 'v' makes angle α with horizontal

Where Tan $\alpha = \dfrac{V_y}{V_x}$

28. Equation for the trajectory of the body
$$y = x \tan \theta - \left(\frac{g}{2u^2 \cos^2 \theta}\right)x^2$$

29. Time of ascent = $t_a = \dfrac{u \sin \theta}{g}$

30. Time of desent = $t_d = \dfrac{u \sin \theta}{g}$

31. Time of flight $T = t_a + t_d = \dfrac{2u \sin \theta}{g}$

32. Maximum height reached = H_{max} =
$$\frac{u^2 \sin^2 \theta}{2g}$$

33. horizontal range R = $\dfrac{u^2 \sin 2\theta}{g}$

34. Angle of projection for maximum range is $\theta = 45^0$

35. Two angles of projection for same range are 'θ' $and (90 - \theta)$

36. If the range and maximum height of a projectile are equal, the angle of projection $\theta = \tan^{-1}(4)$

37. The path of a body projected horizontally from the top of a tower of height 'h' is a parabola.

a. The equation of the trajectory is given by
$$Y = \left(\frac{g}{2u^2}\right)X^2.$$

b. The time of descent $t = \sqrt{\dfrac{2h}{g}}$

c. The horizontal range R = u x t = $u\sqrt{\dfrac{2h}{g}}$

d. At any instant 't' horizontal velocity is $v_x = u$, vertical velocity $v_y = gt$

e. The resultant velocity v = $\sqrt{v_x^2 + v_y^2}$

f. The angle made by V with horizontal is given by Tan $\alpha = \dfrac{v_y}{v_x}$.

38. Suppose a bomb is dropped from a plane moving horizontally with uniform velocity 'u'. the path of bomb as observed by a man stationary on ground is parabola and as observed by the pilot of the plane is a vertical straight line.