

CBSE Class11 Mathematics

Important Questions

Chapter 10

Straight Lines

1 Marks Questions

1. Find the slope of the lines passing through the point (3,-2) and (-1,4)

Ans. Slope of line through (3,-2) and (-1, 4)

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - (-2)}{-1 - 3} \\ &= \frac{6}{-4} = \frac{-3}{2} \end{aligned}$$

2. Three points $P(h, k)$, $Q(x_1, y_1)$ and $R(x_2, y_2)$ lie on a line. Show that

$$(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$$

Ans. Since P, Q, R are collinear

Slope of PQ = slope of QR

$$\begin{aligned} \frac{y_1 - k}{x_1 - h} &= \frac{y_2 - y_1}{x_2 - x_1} \\ \frac{-(k - y_1)}{-(h - x_1)} &= \frac{y_2 - y_1}{x_2 - x_1} \\ (h - x_1)(y_2 - y_1) &= (k - y_1)(x_2 - x_1) \end{aligned}$$

3. Write the equation of the line through the points $(1, -1)$ and $(3, 5)$

Ans. Req. eq. $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$

$$y + 1 = \frac{5 + 1}{2} (x - 1)$$

$$-3x + y + 4 = 0$$

4. Find the measure of the angle between the lines $x + y + 7 = 0$ and $x - y + 1 = 0$

Ans. $x + y + 7 = 0$

$$m_1 = \frac{-1}{1}$$

$$x - y + 1 = 0$$

$$m_2 = \frac{-1}{-1} = 1$$

Slopes of the two lines are 1 and -1 as product of these two slopes is -1, the lines are at right angles.

5. Find the equation of the line that has y-intercept 4 and is \perp to the line $y = 3x - 2$

Ans. $y = 3x - 2$

Slope (m) = $\frac{-3}{-1} = 3$, slope of any line \perp it is $-\frac{1}{3}$

$$C = 4$$

Req. eq. is $y = mx + c$

$$y = \frac{-1}{3}x + 4$$

6. Find the equation of the line, which makes intercepts -3 and 2 on the x and y -axis respectively.

Ans. Req. eq. $\frac{x}{a} + \frac{y}{b} = 1$

$$a = -3, b = 2$$

$$\therefore \frac{x}{-3} + \frac{y}{2} = 1$$

$$2x - 3y + 6 = 0$$

7. Equation of a line is $3x - 4y + 10 = 0$ find its slope.

Ans. $m = \frac{-\text{coeff. of } x}{\text{coeff. of } y}$

$$= \frac{-3}{-4} = \frac{3}{4}$$

8. Find the distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$

Ans. $A = 3, B = -4, C_1 = 7$ and $C_2 = 5$

$$d = \frac{|C_1 - C_2|}{\sqrt{a^2 + b^2}}$$

$$= \frac{|7 - 5|}{\sqrt{(3)^2 + (-4)^2}}$$

$$= \frac{2}{5}$$

9. Find the equation of a straight line parallel to y -axis and passing through the point

(4,-2)

Ans. Equation of line parallel to y -axis is $x = a \dots (i)$

Eq. (i) passing through (-4,2)

$$a = -4$$

So $x = -4$

$$x + 4 = 0$$

10. If $3x - by + 2 = 0$ and $9x + 3y + a = 0$ represent the same straight line, find the values of a and b .

Ans. ATQ

$$\frac{3}{9} = \frac{-b}{3} = \frac{2}{a}$$

$$b = -1$$

$$\Rightarrow a = 6$$

11. Find the distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$ when PQ is parallel to the y -axis.

Ans. When PQ is parallel to the y -axis,

Then $x_1 = x_2$

$$\therefore PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2}$$

$$= |y_2 - y_1|$$

12. Find the slope of the line, which makes an angle of 30° with the positive direction of

y -axis measured anticlockwise.

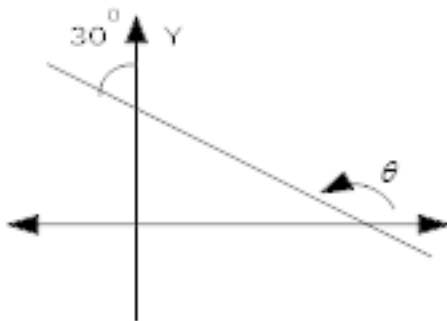
Ans. Let θ be the inclination of the line

$$\theta = 120^\circ$$

$$m = \tan 120^\circ$$

$$= \tan(90 + 30)$$

$$= -\sqrt{3}$$



13. Determine x so that the inclination of the line containing the points $(x, -3)$ and $(2, 5)$ is 135° .

Ans.

$$\frac{5 - (-3)}{2 - x} = \tan 135^\circ$$

$$\left[\begin{array}{l} \because m = \tan \theta \\ m = \frac{y_2 - y_1}{x_2 - x_1} \end{array} \right]$$

$$\frac{5 + 3}{2 - x} = -1$$

$$x = 10$$

14. Find the distance of the point $(4, 1)$ from the line $3x - 4y - 9 = 0$

Ans. Let d be the req. distance

$$d = \frac{|3 \cdot (4) - 4(1) - 9|}{\sqrt{(3)^2 + (-4)^2}}$$

$$= \frac{|-1|}{5} = \frac{1}{5}$$

15. Find the value of x for which the points $(x, -1)$, $(2, 1)$ and $(4, 5)$ are collinear.

Ans. Let $A(x, -1)$, $B(2, 1)$, $C(4, 5)$

Slope of AB = Slope of BC

$$\frac{1+1}{2-x} = \frac{5-1}{4-2}$$

$$\frac{2}{2-x} = \frac{4}{2}$$

$$2 - x = 1$$

$$-x = -1$$

$$x = 1$$

16. Find the angle between the x -axis and the line joining the points $(3, -1)$ and $(4, -2)$

Ans. $m_1 = 0$ [Slope of x -axis]

m_2 = slope of line joining points $(3, -1)$ and $(4, -2)$

$$= \frac{-2 - (-1)}{4 - 3} = -1$$

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right|$$

$$\tan \theta = \left| \frac{0 + 1}{1 + 0 \times (-1)} \right|$$

$$\tan \theta = 1$$

$$\theta = 45^\circ$$

17. Using slopes, find the value of x for which the points $(x, -1)$, $(2, 1)$ and $(4, 5)$ are collinear.

Ans. Since the given points are collinear slope of the line joining points $(x, -1)$ and $(2, 1)$
 = slope of the line joining points $(2, 1)$ and $(4, 5)$

$$\Rightarrow \frac{2}{2-x} = \frac{2}{1}$$

$$x = 1$$

18. Find the value of K so that the line $2x + ky - 9 = 0$ may be parallel to $3x - 4y + 7 = 0$

Ans. ATQ

Slope of 1st line = slope of 2nd line

$$\frac{-2}{k} = \frac{-3}{-4}$$

$$\Rightarrow k = \frac{-8}{3}$$

19. Find the value of K , given that the distance of the point $(4, 1)$ from the line $3x - 4y + K = 0$ is 4 units.

Ans. We are given that distance of $(4, 0)$ from the line $3x - 4y + k = 0$ is 4

$$\frac{|3(4) - 4(1) + k|}{\sqrt{(3)^2 + (-4)^2}} = 4$$

$$|k + 8| = 4 \times 5$$

$$k = 12, -28$$

20. Find the equation of the line through the intersection of $3x - 4y + 1 = 0$ $5x + y - 1 = 0$ which cuts off equal intercepts on the axes.

Ans. Slope of a line which makes equal intercept on the axes is -1 any line through the intersection of given lines is

$$(3x - 4y + 1) + K(5x - y - 1) = 0$$

$$(3 + 5K)x + y(K - 4) + 1 - K = 0$$

$$m = -\frac{(3 + 5K)}{K - 4} = -1$$

$$K = \frac{-7}{4}$$

21. Find the distance of the point $(2, 3)$ from the line $12x - 5y = 2$

Ans. $d = \frac{|12x - 5y - 2|}{\sqrt{(12)^2 + (-5)^2}}$

$$d = \frac{|12 \times 2 - 5 \times 3 - 2|}{\sqrt{169}}$$

$$= \frac{|-41|}{13} = \frac{41}{13}$$

22. Find the equation of a line whose perpendicular distance from the origin is 5 units and angle between the positive direction of the x -axis and the perpendicular is 30° .

Ans. $p = 5, \alpha = 30^\circ$

Req. eq. $x \cos \alpha + y \sin \alpha = p$

$$x \cos 30^\circ + y \sin 30^\circ = 5$$

$$\sqrt{3}x + y - 10 = 0$$

23. Write the equation of the lines for which $\tan \theta = \frac{1}{2}$, where θ is the inclination of the line and x intercept is 4.

Ans. $m = \tan \theta = \frac{1}{2}$ and $d = 4$

$$y = \frac{1}{2}(x - 4) \quad [\because y = m(x - d)]$$

$$2y - x + 4 = 0$$

24. Find the Angle between the x -axis and the line joining the points $(3, -1)$ and $(4, -2)$

Ans. Let $A(3, -1)$ $B(4, -2)$

Slope of $AB = \frac{-2 - (-1)}{4 - 3}$

$$= -1$$

$$\tan \theta = -1$$

$$\theta = 135 \left[\begin{array}{l} \text{where } \theta \text{ is the angle which AB makes} \\ \text{with positive direction of } x\text{-axis} \end{array} \right]$$

25. Find the equation of the line intersecting the x -axis at a distance of 3 unit to the left of origin with slope -2.

Ans. The line passing through (-3,0) and has slope = -2

Req. eq. is

$$y - 0 = -2(x + 3)$$

$$2x + y + 6 = 0$$

CBSE Class 12 Mathematics

Important Questions

Chapter 10

Straight Lines

4 Marks Questions

1. If p is the length of the \perp from the origin on the line whose intercepts on the axes are a and b . show that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Ans. Equation of the line is $\frac{x}{a} + \frac{y}{b} = 1$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} - 1 = 0$$

The distance of this line from the origin is P

$$\therefore P = \frac{\left| \frac{0}{a} + \frac{0}{b} - 1 \right|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}} \quad \left[d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}} \right]$$

$$\frac{P}{1} = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}$$

$$\frac{1}{P} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

Sq. both side

$$\frac{1}{P^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

2. Find the value of p so that the three lines $3x + y - 2 = 0$, $px + 2y - 3 = 0$ and $2x - y - 3 = 0$ may intersect at one point.

Ans. $3x + y - 2 = 0 \dots\dots (i)$

$$px + 2y - 3 = 0 \dots\dots (ii)$$

$$2x - y - 3 = 0 \dots\dots (iii)$$

On solving eq. (i) and (iii)

$$x = 1, \text{ And } y = -1$$

Put x, y in eq. (ii)

$$P(1) + 2(-1) - 3 = 0$$

$$p - 2 - 3 = 0$$

$$p = 5$$

3. Find the equation to the straight line which passes through the point (3,4) and has intercept on the axes equal in magnitude but opposite in sign.

Ans. Let intercept be a and -a the equation of the line is

$$\frac{x}{a} + \frac{y}{-a} = 1$$

$$\Rightarrow x - y = a \dots\dots (i)$$

Since it passes through the point (3, 4)

$$3 - 4 = a$$

$$a = -1$$

Put the value of a in eq. (i)

$$x - y = -1$$

$$x - y + 1 = 0$$

4. By using area of Δ . Show that the points $(a, b+c), (b, c+a)$ and $(c, a+b)$ are collinear.

$$\text{Ans. Area of } \Delta = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2} |a(c+a) - b(b+c) + b(a+b) - c(c+a) + c(b+c) - a(a+b)|$$

$$= \frac{1}{2} \cdot 0 = 0$$

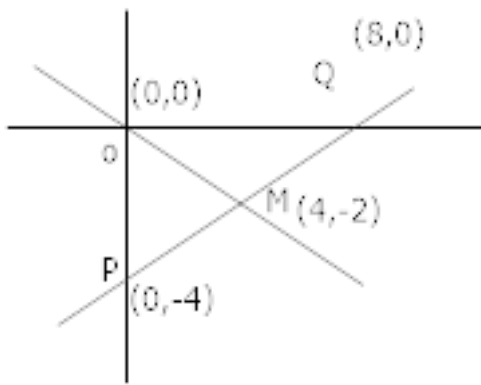
5. Find the slope of a line, which passes through the origin, and the midpoint of the line segment joining the point $P(0, -4)$ and $Q(8, 0)$

$$\text{Ans. Let } m \text{ be the midpoint of segment PQ then } M = \left(\frac{0+8}{2}, \frac{-4+0}{2} \right)$$

$$= (4, -2)$$

$$\text{Slope of } OM = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 0}{4 - 0} = \frac{-1}{2}$$



6. Find equation of the line passing through the point $(2, 2)$ and cutting off intercepts on the axes whose sum is 9

Ans. Req. eq. be $\frac{x}{a} + \frac{y}{b} = 1 \dots (i)$

$$a + b = 9$$

$$b = 9 - a$$

$$\Rightarrow \frac{x}{a} + \frac{y}{9-a} = 1$$

This line passes through $(2, 2)$

$$\therefore \frac{2}{a} + \frac{2}{9-a} = 1$$

$$a^2 - 9a + 18 = 0$$

$$a^2 - 6a - 3a + 18 = 0$$

$$a(a-6) - 3(a-6) = 0$$

$$(a-6)(a-3) = 0$$

$$a = 6, 3$$

$$a = 6 \quad a = 3$$

$$b = 3 \quad b = 6$$

$$\frac{x}{6} + \frac{y}{3} = 1 \quad \frac{x}{3} + \frac{y}{6} = 1$$

7. Reduce the equation $\sqrt{3}x + y - 8 = 0$ into normal form. Find the values p and ω .

Ans. $\sqrt{3}x + y - 8 = 0$

$$\sqrt{3}x + y = 8 \dots\dots (i)$$

$$\sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

Dividing (i) by 2

$$\frac{\sqrt{3}}{2}x + \frac{y}{2} = 4$$

$$x \cos 30^\circ + y \sin 30 = 4 \dots\dots (ii)$$

Comparing (ii) with

$$x \cos \omega + y \sin \omega = p$$

$$p = 4$$

$$\omega = 30^\circ$$

8. Without using the Pythagoras theorem show that the points $(4, 4)$, $(3, 5)$ and $(-1, -1)$ are the vertices of a right angled Δ .

Ans. The given points are $A(4, 4)$, $B(3, 5)$ and $C(-1, -1)$

$$\text{Slope of } AB = \frac{5-4}{3-4} = -1$$

$$\text{Slope of } BC = \frac{-1-5}{-1-3} = \frac{-6}{-4} = \frac{3}{2}$$

$$\text{Slope of } AC = \frac{-1-4}{-1-4} = +1$$

$$\text{Slope of } AB \times \text{slope of } AC = -1$$

$$\Rightarrow AB \perp AC$$

Hence $\triangle ABC$ is right angled at A.

9. The owner of a milk store finds that, he can sell 980 liters of milk each week at 14 liter and 1220 liter of milk each week at Rs 16 liter. Assuming a linear relationship between selling price and demand how many liters could he sell weekly at Rs 17 liter?

Ans. Assuming sell along x -axis and cost per litre along y -axis, we have two points $(980, 14)$ and $(1220, 16)$ in x y plane

$$y - 14 = \frac{16 - 14}{1220 - 980}(x - 980)$$

$$y - 14 = \frac{2}{120} (x - 980)$$

$$120y - 14 \times 120 = x - 980$$

$$120y - 1680 = x - 980$$

$$x - 120y = -700$$

When $y = 17$

$$x - 120 \times 17 = -700 \Rightarrow x = 1340 \text{ litres.}$$

10. The line through the points $(h, 3)$ and $(4, 1)$ intersects the line $7x - 9y - 19 = 0$ at right angle. Find the value of h .

Ans. Slope of line joining (h,3) and (4,1)

$$= \frac{1-3}{4-h} = \frac{-2}{4-h}$$

Given line is $7x - 9y - 19 = 0$

$$\text{Slope of this line} = \frac{-7}{-9}$$

ATQ

$$\left(\frac{-2}{4-h}\right) \times \left(\frac{7}{9}\right) = -1$$

$$\Rightarrow h = \frac{22}{9}$$

11. Find the equations of the lines, which cut off intercepts on the axes whose sum and product are 1 and -6 respectively.

Ans. ATQ $a + b = 1 \dots\dots (i)$

$$ab = -6 \dots\dots (ii)$$

$$b = 1 - a \quad [from (i)]$$

Put b in eq. (ii)

$$a(1 - a) = -6$$

$$a - a^2 = -6$$

$$a^2 - a - 6 = 0$$

$$(a - 3)(a + 2) = 0$$

$$a = 3, -2$$

When $a = 3$

$$b = -2$$

Eq. of the line is

$$\frac{x}{3} + \frac{y}{-2} = 1$$

$$2x - 3y - 6 = 0$$

When $a = -2$

$$b = 3$$

Eq. of the line is

$$\frac{x}{-2} + \frac{y}{3} = 1$$

$$3x - 2y + 6 = 0$$

12. The slope of a line is double of the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, find the slopes of the lines.

Ans. Let the slope of one line is m and other line is $2m$

$$\frac{1}{3} = \left| \frac{2m - m}{1 + (2m)(m)} \right|$$

$$\frac{1}{3} = \left| \frac{m}{1 + 2m^2} \right|$$

$$\pm \frac{1}{3} = \frac{m}{1 + 2m^2}$$

$$\frac{1}{3} = \frac{m}{1+2m^2}$$

$$2m^2 - 3m + 1 = 0$$

$$2m^2 - 2m - m + 1 = 0$$

$$2m(m-1) - 1(m-1) = 0$$

$$(m-1)(2m-1) = 0$$

$$m = 1, m = \frac{1}{2}$$

$$\frac{-1}{3} = \frac{m}{1+2m^2}$$

$$-1 - 2m^2 = 3m$$

$$2m^2 + 3m + 1 = 0$$

$$2m^2 + 2m + m + 1 = 0$$

$$2m(m+1) + 1(m+1) = 0$$

$$(m+1)(2m+1) = 0$$

$$m = -1$$

$$m = \frac{-1}{2}$$

13. Point $R(h, k)$ divides a line segment between the axes in the ratio 1:2. Find equation of the line.

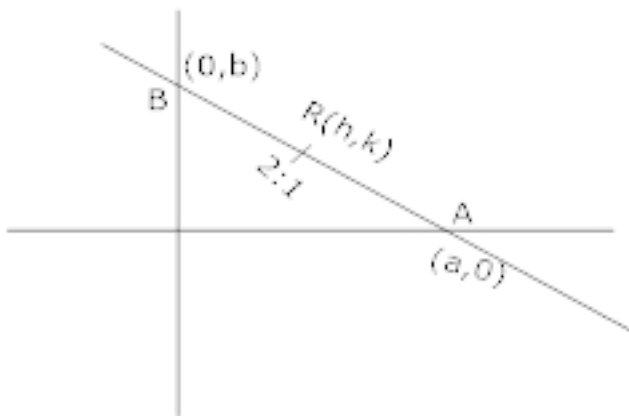
Ans. Let eq. be $\frac{x}{a} + \frac{y}{b} = 1 \dots\dots (i)$

It is given that $R(h, k)$ divides AB in the ratio 1:2

$$\therefore (h, k) = \left(\frac{2a}{3}, \frac{b}{3} \right)$$

$$\frac{2a}{3} = h$$

$$a = \frac{3h}{2}$$



$$k = \frac{b}{3}$$

$$b = 3k$$

Put a and b in eq..... (i)

$$\frac{\frac{x}{\frac{3h}{2}} + \frac{y}{3k}}{2} = 1$$

$$\frac{2x}{h} + \frac{y}{k} = 3$$

14. The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that K=273 when F=32 and that K= 373 when F=212 Express K in terms of F and find the value of F when K=0

Ans. Let F along *x*-axis and K along *y*-axis

$$K - 273 = \frac{373 - 273}{212 - 32} (F - 32) \left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \right]$$

$$K - 273 = \frac{100}{180} (F - 32)$$

$$K = \frac{5}{9} (F - 32) + 273$$

15. If three points $(h, 0)$, (a, b) and $(0, k)$ lie on a line, show that $\frac{a}{h} + \frac{b}{k} = 1$

Ans. Let $A(h, 0)$, $B(a, b)$ and $C(0, k)$

Slope of AB = slope of BC

$$\frac{b - 0}{a - h} = \frac{k - b}{0 - a}$$

$$\frac{b}{a - h} = \frac{h - b}{-a}$$

$$(a - h)(k - b) = -ab$$

$$ak - \cancel{ab} - hk + hb = -\cancel{ab}$$

$$ak + hb = hk$$

$$\frac{ak}{hk} + \frac{hb}{hk} = 1$$

$$\frac{a}{h} + \frac{b}{k} = 1$$

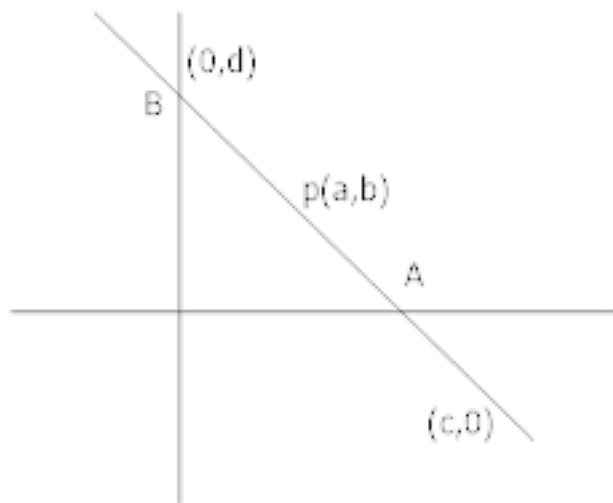
16. $p(a, b)$ is the mid point of a line segment between axes. Show that equation of the

line is $\frac{x}{a} + \frac{y}{b} = 2$

Ans. Req. eq. be

$$\frac{x}{c} + \frac{y}{d} = 1 \dots (i)$$

P is the mid point



Coordinate of $p = \left(\frac{c}{2}, \frac{d}{2} \right)$

$$(a, b) = \left(\frac{c}{2}, \frac{d}{2} \right)$$

$$\frac{a}{1} = \frac{c}{2}$$

$$c = 2a$$

$$\frac{b}{1} = \frac{d}{2}$$

$$d = 2b$$

Put the value of C and D in eq. (i)

$$\frac{x}{2a} + \frac{y}{2b} = 1$$

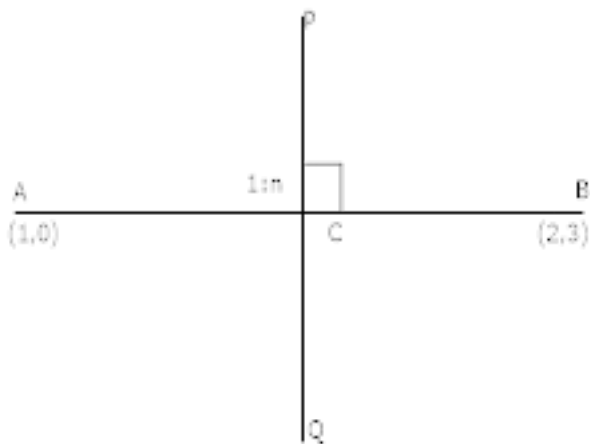
$$\frac{x}{a} + \frac{y}{b} = 2$$

17. The line \perp to the line segment joining the points $(1, 0)$ and $(2, 3)$ divides it in the ratio $1:n$ find the equation of the line.

Ans. Coordinate of $C\left(\frac{2+n}{1+n}, \frac{3}{1+n}\right)$

$$m_{AB} = 3$$

$$m_{PQ} = -\frac{1}{3}$$



Eq. of PQ is

$$\frac{y}{1} - \frac{3}{1+n} = -\frac{1}{3} \left(\frac{x}{1} - \frac{2+n}{1+n} \right)$$

$$(n+1)x + 3(n+1)y - (n+11) = 0$$

CBSE Class 12 Mathematics

Important Questions

Chapter 10

Straight Lines

6 Marks Questions

1. Find the values of k for the line $(k-3)x - (4-k^2)y + k^2 - 7k + 6 = 0$

(a). Parallel to the x -axis

(b). Parallel to y -axis

(c). Passing through the origin

Ans.

(a) The line parallel to x -axis if coeff. Of $x = 0$

$$k - 3 = 0$$

$$k = 3$$

(b) The line parallel to y -axis if coeff. Of $y = 0$

$$4 - k^2 = 0$$

$$k = \pm 2$$

(c) Given line passes through the origin if $(0, 0)$ lies on given eq.

$$(k-3).(0) - (4-k^2)(0) + k^2 - 7k + 6 = 0$$

$$(k-6)(k-1) = 0$$

$$k = 6, 1$$

2. If p and q are the lengths of \perp from the origin to the lines.

$x \cos \theta - y \sin \theta = k \cos 2\theta$, and $x \sec \theta + y \csc \theta = k$ respectively, prove that $p^2 + 4q^2 = k^2$

Ans.

$$P = \frac{|0 \cdot \cos \theta - 0 \sin \theta - k \cos 2\theta|}{\sqrt{(\cos \theta)^2 + (-\sin \theta)^2}} \left[\begin{array}{l} \perp \text{ from origin} \\ \because (0, 0) \end{array} \right]$$

$$P = K \cos 2\theta \dots (i)$$

$$q = \frac{|0 \cdot \sec \theta + 0 \csc \theta - k|}{\sqrt{\sec^2 \theta + \csc^2 \theta}}$$

$$= \frac{K}{\sqrt{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}}}$$

$$= \frac{k \cos \theta \cdot \sin \theta}{\sqrt{\sin^2 \theta + \cos^2 \theta}} = \frac{1}{2} k \cdot \sin \theta \cdot \cos \theta$$

$$2q = k \cdot \sin 2\theta \dots (ii)$$

Squaring (i) and (ii) and adding

$$P^2 + (2q)^2 = K^2 \cos^2 2\theta + K^2 \sin^2 2\theta$$

$$P^2 + 4q^2 = K^2 (\cos^2 2\theta + \sin^2 2\theta)$$

$$p^2 + 4q^2 = k^2$$

3. Prove that the product of the \perp drawn from the points $(\sqrt{a^2 - b^2}, 0)$ and $(-\sqrt{a^2 - b^2}, 0)$ to the line $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$ is b^2 .

Ans. Let

$$p_1 = \frac{\left| \frac{\sqrt{a^2 - b^2}}{a} \cdot \cos \theta - 1 \right|}{\sqrt{\left(\frac{\cos \theta}{a} \right)^2 + \left(\frac{\sin \theta}{b} \right)^2}} \left[\because \perp \text{ from the points } \sqrt{a^2 - b^2}, 0 \right]$$

Similarly p_2 be the distance from $\left(-\sqrt{a^2 - b^2}, 0 \right)$ to given line

$$p_2 = \frac{\left| -\frac{\sqrt{a^2 - b^2}}{a} \cos \theta - 1 \right|}{\sqrt{\left(\frac{\cos \theta}{a} \right)^2 + \left(\frac{\sin \theta}{b} \right)^2}}$$

$$p_1 p_2 = \frac{\left| \left(\frac{\sqrt{a^2 - b^2}}{a} \cos \theta - 1 \right) \left(-\frac{\sqrt{a^2 - b^2}}{a} \cos \theta - 1 \right) \right|}{\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2}}$$

$$= \frac{\left| \left(\frac{a^2 - b^2}{a^2} \right) \cdot \cos^2 \theta - 1 \right|}{\frac{b^2 \cos^2 \theta + a^2 \sin^2 \theta}{a^2 b^2}}$$

$$= \frac{\left| a^2 \cos^2 \theta - b^2 \cos^2 \theta - a^2 \right| a^2 b^2}{a^2 (a^2 \sin^2 \theta + b^2 \cos^2 \theta)}$$

$$= \frac{\left| -(a^2 \sin^2 \theta + b^2 \cos^2 \theta) \right| b^2}{a^2 \sin^2 \theta + b^2 \cos^2 \theta} \quad \left[\because a^2 \cos^2 \theta - a^2 = a^2 (\cos^2 \theta - 1) \right]$$

$$= \frac{(a^2 \sin^2 \theta + b^2 \cos^2 \theta) b^2}{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$$

$$= b^2$$

4. Find equation of the line mid way between the parallel lines $9x + 6y - 7 = 0$ and $3x + 2y + 6 = 0$

Ans. The equations are

$$9x + 6y - 7 = 0$$

$$3\left(3x + 2y - \frac{7}{3}\right) = 0$$

$$3x + 2y - \frac{7}{3} = 0 \dots\dots (i)$$

$$3x + 2y + 6 = 0 \dots\dots (ii)$$

Let the eq. of the line mid way between the parallel lines (i) and (ii) be

$$3x + 2y + k = 0 \dots\dots (iii)$$

ATQ

Distance between (i) and (iii) = distance between (ii) and (iii)

$$\left| \frac{K + \frac{7}{3}}{\sqrt{9 + 4}} \right| = \left| \frac{K - 6}{\sqrt{9 + 4}} \right| \quad \left[\because d = \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}} \right]$$

$$K + \frac{7}{3} = K - 6$$

$$K = \frac{11}{6}$$

Req. eq. is

$$3x + 2y + \frac{11}{6} = 0$$

5. Assuming that straight lines work as the plane mirror for a point, find the image of the point (1,2) in the line $x - 3y + 4 = 0$

Ans. Let $Q(h, k)$ is the image of the point $P(1, 2)$ in the line.

$$x - 3y + 4 = 0 \dots\dots (i)$$

$$\text{Coordinate of midpoint of } PQ = \left(\frac{h+1}{2}, \frac{k+2}{2} \right)$$

This point will satisfy the eq.(i)

$$\left(\frac{h+1}{2} \right) - 3 \left(\frac{k+2}{2} \right) + 4 = 0$$

$$h - 3k = -5 \dots\dots (i)$$

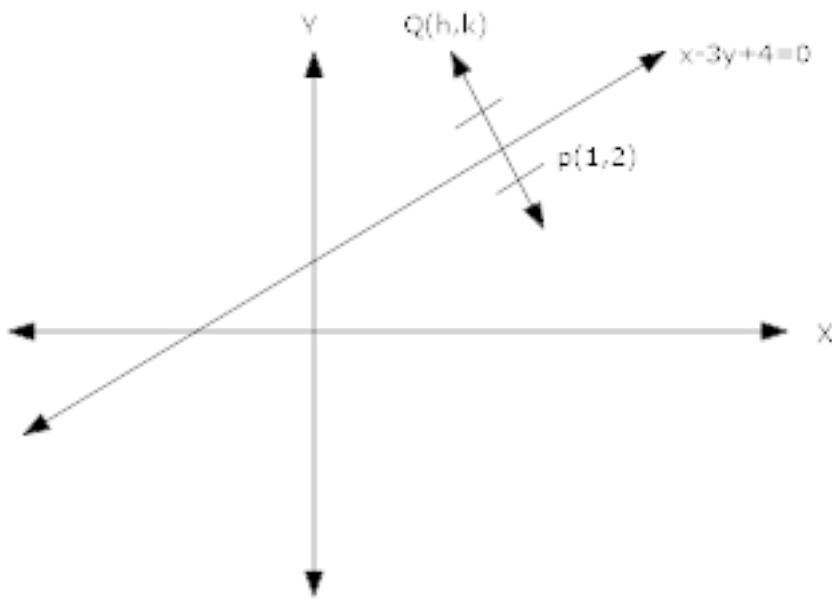
$$(\text{Slope of line PQ}) \times (\text{slope of line } x - 3y + 4 = 0) = -1$$

$$\left(\frac{k-2}{h-1} \right) \left(\frac{-1}{-3} \right) = -1$$

$$3h + k = 5 \dots\dots (ii)$$

On solving (i) and (ii)

$$h = \frac{6}{5} \text{ and } k = \frac{7}{5}$$



6. A person standing at the junction (crossing) of two straight paths represented by the equations $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ wants to reach the path whose equation is $6x - 7y + 8 = 0$ in the least time. Find equation of the path that he should follow.

Ans. $2x - 3y - 4 = 0 \dots (i)$

$3x + 4y - 5 = 0 \dots (ii)$

$6x - 7y + 8 = 0 \dots (iii)$

On solving eq. (i) and (ii)

We get $\left(\frac{31}{17}, \frac{-2}{17}\right)$

To reach the line (iii) in least time the man must move along the \perp from crossing point

$\left(\frac{31}{17}, \frac{-2}{17}\right)$ to (iii) line

Slope of (iii) line is $\frac{6}{7}$

Slope of required path $= \frac{-7}{6}$ [$\because m_1 \times m_2 = -1$]

$$y - \left(-\frac{2}{17}\right) = \frac{-7}{6} \left(x - \frac{31}{17}\right)$$

$$119x + 102y = 205$$

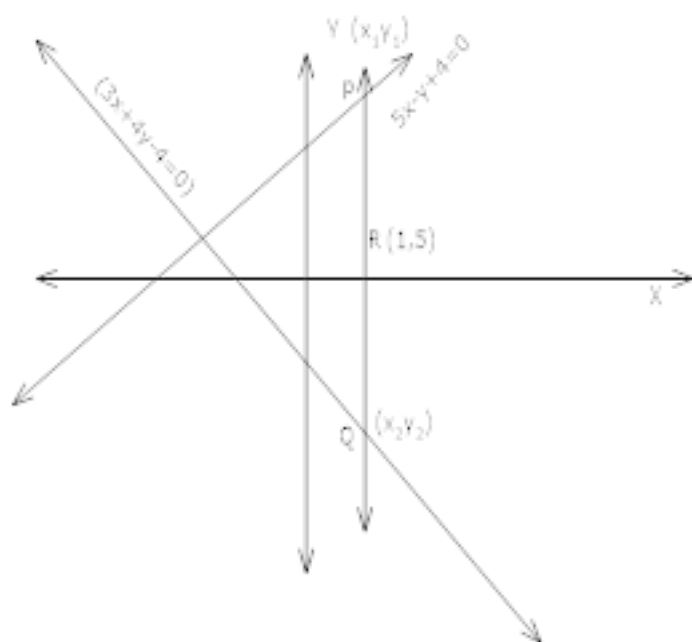
7. A line is such that its segment between the lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$ obtain its equation.

Ans. $P(x_1, y_1)$ lies on $5x - y + 4 = 0$

$$\Rightarrow 5x_1 - y_1 + 4 = 0$$

And $Q(x_2, y_2)$ lies on $3x + 4y - 4 = 0$

$$3x_2 + 4y_2 - 4 = 0$$



On solving

$$y_1 = 5x_1 + 4$$

$$y_2 = \frac{4-3x_2}{4}$$

Since R is the mid point of PQ

$$\frac{x_1 + x_2}{2} = 1, \frac{y_1 + y_2}{2} = 5$$

$$x_1 + x_2 = 2, y_1 + y_2 = 10$$

On solving

$$x_1 = \frac{26}{23}, x_2 = \frac{20}{23}$$

$$\text{And } y_1 = \frac{222}{23}, y_2 = \frac{8}{23}$$

Eq. of PQ

$$y - \frac{222}{23} = \frac{\frac{8}{23} - \frac{222}{23}}{\frac{20}{23} - \frac{26}{23}} \left(x - \frac{26}{23} \right)$$

$$107x - 3y - 92 = 0$$

8. Find the equations of the lines which pass through the point (4, 5) and make equal angles with the lines $5x - 12y + 6 = 0$ and $3x - 4y - 7 = 0$

Ans. The slopes of the given lines are $\frac{5}{12}$ and $\frac{3}{4}$

Let m be the slope of a required line

ATQ

$$\left| \frac{m - \frac{5}{12}}{1 + m \cdot \frac{5}{12}} \right| = \left| \frac{m - \frac{3}{4}}{1 + m \cdot \frac{3}{4}} \right|$$

$$\Rightarrow \left| \frac{12m - 5}{12 + 5m} \right| = \left| \frac{4m - 3}{4 + 3m} \right|$$

$$\frac{12m - 5}{12 + 5m} = \frac{4m - 3}{4 + 3m}$$

$$16m^2 = -16$$

$$m^2 = -1$$

Neglect

$$\frac{12m - 5}{12 + 5m} = -\frac{4m - 3}{4 + 3m}$$

$$m = \frac{4}{7}, \frac{-7}{4}$$

Req. eq. are

$$y - 5 = \frac{4}{7}(x - 4)$$

$$4x - 7y + 19 = 0$$

$$y - 5 = \frac{-7}{4}(x - 4)$$

$$7x + 4y - 48 = 0$$