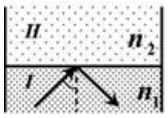
UPSEE 2017 - Solved Paper

Physics

Question 1

A light ray moving in medium-I (of refractive index n_1) is incident on interface of two media and it is totally internally reflected at the interface. Now refractive index n_2 of medium-II is decreased, then



Options:

A. ray will be totally transmitted in medium-II.

B. ray will move completely parallel to the interface .

C. ray will be still totally internally reflected at interface.

D. ray will be totally transmitted into medium-II only if angle of incidence is increased.

Answer: C

Solution:

Critical angle is defined as the angle of incidence that provides an angle of refraction of 90-degrees.
The critical angle is, $i_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$
where n_1, and n_2 is the refractive index of the medium.
So as the refractive index n_2 decreases, the critical angle is also decreased. So condition $i > i_c$ is still satisfied and there will be still total reflection at the interface.

If the angle of incidence is increased then ray will still totally internally reflected at the interface because $i > i_c$.

Question 2

A light beam consists of two types of photons. In one type each photon has energy 2eV and in other type each photon has energy 3eV. The light beam is incident on a photoelectric material of work function 1eV. The maximum kinetic energy of emitted photoelectron is:

Options:

A. 1eV

B. 2eV

C. 3eV

D. 4eV

Answer: B

Solution:

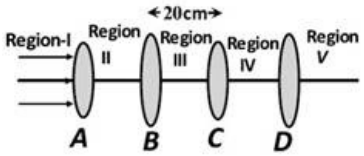
According to Einstein Photoelectric equation
$E = KE_{max} + W$
where E is the energy of the photon, W is the work function of the metal, and KE_{max} is the maximum kinetic energy of electrons.
So maximum kinetic energy of electrons is
$KE_{max} = E - W$
For the first type of photon,
$KE_{max} = 2 - 1 = 1$ eV

For the second type of photon
$KE_{max} = 3 - 1 = 2$ eV

Hence the maximum kinetic energy of the electron is 2 eV.

Question 3

A light beam parallel to axis is incident on the system of four convex lenses A, B, C and D. Focal lengths of A, B, C and D are 30cm,10cm, 30cm and 10cm respectively as shown. Here fixed distance BC = 20cm. What should be the distance between the lens A and lens D so that after refractions, rays will be parallel to axis in regions I, III and V?



Options:

A. 20 cm

B. 40 cm

C. 100 cm

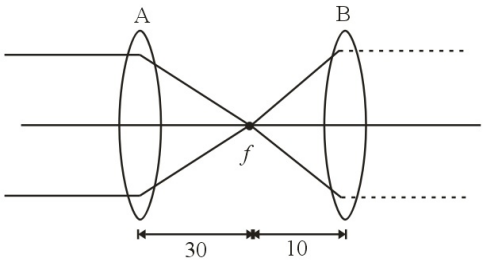
D. 80 cm

Answer: C

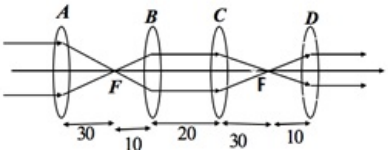
Solution:

For lens A the parallel beam will converge at the focus point f = 30 cm from the lens A, and if these rays are the focus the of

second lens then they will be rendered parallel.



Similarly for other lens



The ray will be passing as the above diagram suggests.

AD=30+10+20+30+10=100cm

Question 4

A long silver tea spoon is placed in a cup filled with hot tea. After some time, the exposed end (the end which is not dipped in tea) of the spoon becomes hot even without a direct contact with the tea. This phenomenon can be explained mainly by:

Options:

A. thermal expansion

B. conduction

C. reflection

D. radiation

Answer: B

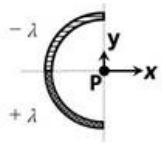
Solution:

Thermal expansion- Thermal Expansion is the phenomenon in which the dimension of a body increases due to the increase in the temperature of a body. Thermal expansion is present in solids, liquids, and gases.

Conduction- It is a mode of transfer in which heat is transmitted from one point to the other through the substance without the actual motion of the particles. So when the long silver teaspoon is placed in a cup filled with hot tea the atoms in the spoon vibrate about their equilibrium positions and transfer energy from one end to another end.

Question 5

Figure shows a nonconducting semicircular rod in xy plane. Top half (quarter circle) has uniform linear charge density − λ whereas remaining half has uniform linear charge density + λ. What is the direction of the net electric field at point P?



Options:

A. along +x axis

B. along +y axis

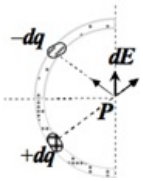
C. electric field is zero at point P, so direction cannot be determined.

D. along the bisector of x axis and y axis.

Answer: B

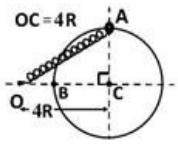
Solution:

Consider any two symmetric dipole elements ,net contribution due these elements at point P is along +y axis. Similarly by principle of superposition we can say that net electric field at P is directed along +y axis.



Question 6

A bead of mass m can slide without friction on a fixed circular horizontal ring of radius 3R having centre at the point C. The bead is attached to one of the ends of spring of spring constant k. Natural length of spring is R and the other end of the spring is fixed at point O as shown in figure. Bead is released from position A, what will be kinetic energy of the bead when it reaches at point B?



Options:

A. $12\,kR^2$

B. $\frac{25}{2}kR^2$

C. $\frac{9}{2}kR^2$

D. $8\,kR^2$

Answer: D

Solution:

According to the conservation of energy
Initial energy = final energy
$KE_i + PE_i = KF_f + PE_f$

$$0+\frac{1}{2}k(OA-R)^2=KE_f+\frac{1}{2}k(OB-R)^2$$

$$\frac{1}{2}k(5R-R)^2=KE_f+\frac{1}{2}k(R-R)^2$$

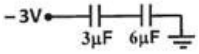
$$\frac{1}{2}k\times 16R^2=KE_f$$

$$KE_f=8kR^2$$

So kinetic energy of the bead when it is at the point B is, $KE_f=8kR^2$

Question 7

The total electrostatic energy stored in both the capacitors is :



Options:

A. 18 μJ

B. 9 μJ

C. 40.5 μJ

D. 13.5 μJ

Answer: B

Solution:

From the above figure equivalent capacitance of the capacitor is,

$$\frac{1}{C_{eq}}=\frac{1}{3}+\frac{1}{6}$$

$$\frac{1}{C_{eq}}=\frac{1}{2}$$

$$C_{eq}=2\mu F$$

Potential energy of capacitor is

$$U = \frac{1}{2} C_{eq} V^2$$

$$U = \frac{1}{2} \times 2 \times 10^{-6} \times (-3)^2$$

$$U = 9 \times 10^{-6}$$

$$U = 9\mu J$$

Question 8

Gravitational force acts on a particle due to fixed uniform solid sphere. Neglect other forces. Then particle:

Options:

A. experiences a force directed along the radial direction only.

B. always moves normal to the radial direction

C. always moves in the radial direction only.

D. always moves in circular orbit.

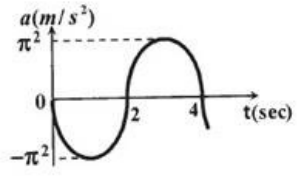
Answer: A

Solution:

Gravitational field due to uniform solid sphere is, $\vec{E} = -\frac{GM}{r^2}\hat{r}$ where M is the mass of sphere, r is the radius of the sphere, and -ve sign show that direction of field towards the centre of sphere. Similarly gravitational force acts on a particle due to fixed uniform solid sphere experiences a force directed along the radial direction only.

Question 9

A block performs simple harmonic motion with equilibrium point x = 0. Graph of acceleration of the block as a function of time is shown. Which of the following statement is correct about the block?



Options:

A. speed is maximum at t = 3s.

B. displacement from equilibrium is maximum at t = 4s.

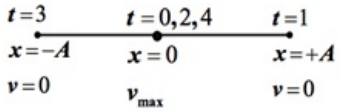
C. speed is maximum at t = 4s

D. speed is minimum at t = 2s.

Answer: C

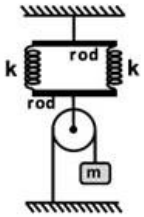
Solution:

Speed is maximum at t=4s.



Question 10

There are two identical springs each of spring constant k. Here springs, pulley and rods are massless and block has mass m. What is the extension of each spring at equilibrium?



Options:

A. $\frac{mg}{k}$

B. $\frac{2mg}{k}$

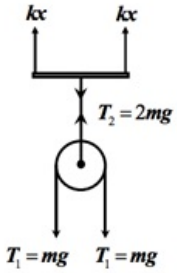
C. $\frac{mg}{2k}$

D. $\frac{3mg}{4k}$

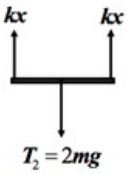
Answer: A

Solution:

Free body diagram of the spring-mass system is



Free body diagram of the lower rod is



Equation of motion

kx + kx=2mg

2kx = 2mg

x=mg/k

Question 11

Two tuning forks A and B produce 4 beats/sec. Forks B and C produce 5 beats/sec. Forks A and C may produce ……. beats/sec.

Options:

A. 2

B. 5

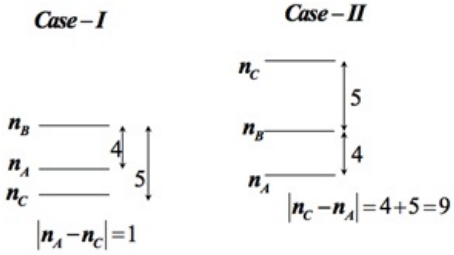
C. 9

D. 20

Answer: C

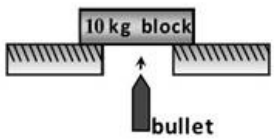
Solution:

We consider two possible cases:



Question 12

A 10gm bullet moving directly upward at 1000 m/s strikes and passes through the center of mass of a 10 kg block initially at rest .The bullet emerges from the block moving directly upward at 400 m/s. What will be velocity of the block just after the bullet comes out of it?



Options:

A. 0.6 m/s

B. 1 m/s

C. 0.4 m/s

D. 1.4 m/s

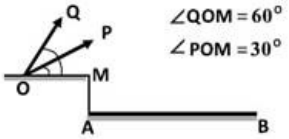
Answer: A

Solution:

According to the conservation of momentum
Initial momentum of the system = final momentum of the system
$M_1v_1=M_1v_2 + M_2v_2$
Where M_1 is the mass of bullet, M_2 is the mass of block
$(10 \times 10^{-3})1000=(10 \times 10^{-3})x400+10v_2$
$10 = 4 + 10v_2$
$10v_2 = 6$
$v_2 = 0.6$ m/s

Question 13

Two identical balls P and Q are projected with same speeds in vertical plane from same point O with making projection angles with horizontal 30° and 60° respectively and they fall directly on plane AB at points P' and Q' respectively. Which of the following statement is true about distances as given in options?



Options:

A. AP' = AQ' as there are complimentary projection angles.

B. AP' > AQ'

C. AP' < AQ'

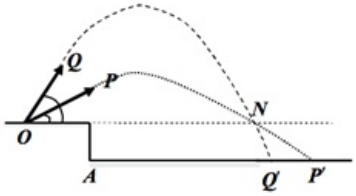
D. AP' ≤ AQ'

Answer: B

Solution:

for complimentary angles range is the same for P and Q as points O

and N are on the same horizontal plane.



From figure AP’ > AQ’

Question 14

A string has a length of 5m between fixed points and has fundamental frequency of 20 Hz. What is the frequency of the second overtone?

Options:

A. 30 Hz

B. 40 Hz

C. 50 Hz

D. 60 Hz

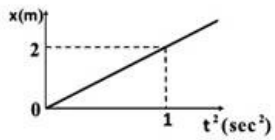
Answer: D

Solution:

Given that the length of the string is 5m and the fundamental frequency is 20 Hz.
for second overtone, n=3
Hence the frequency of the second overtone is, $f_3=3f_o$ = 60Hz.

Question 15

Displacement x versus t^2 graph is shown for a particle. The acceleration of the particle is:



Options:

A. $2m/s^2$

B. $4m/s^2$

C. $8m/s^2$

D. zero

Answer: B

Solution:

The slope of the curve is,

$slope = \frac{x}{t^2}$

$2 = \frac{x}{t^2}$

$x = 2t^2$

On taking differential w.r.t. t

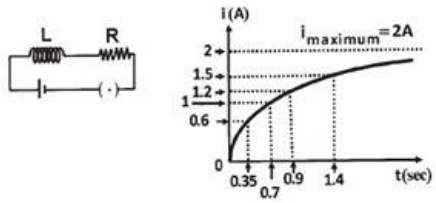
$\frac{dx}{dt} = 4t$

Again taking differential w.r.t. t

$\frac{d^2x}{dt^2} = 4\ ms^{-2}$

Question 16

For given LR circuit, growth of current as function of time t is shown in graph. Which of the following option represents value of time constant most closely for the circuit?



Options:

A. 0.4 s

B. 0.7 s

C. 1 s

D. 2.4 s

Answer: C

Solution:

In one time constant 63% change occurs in the value of current. The 63% of maximum current is 1.26A. It is obvious from graph that current 1.26A corresponds to time which is slightly greater than 0.9 sec. Hence option having 1s is most appropriate.

Question 17

Radii of two conducting circular loops are b and a respectively where b > > a. Centers of both loops coincide but planes of both loops are perpendicular to each other. The value of mutual inductance for these loops:

Options:

A. $\frac{\mu_0\pi^2}{2b}$

B. $\frac{\mu_0\pi b^2}{2a}$

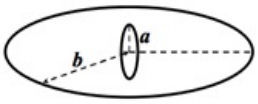
C. zero

D. $\frac{\mu_0\pi ab^2}{2(a+b)}$

Answer: C

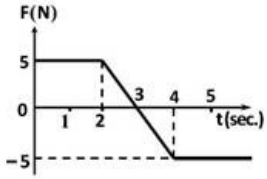
Solution:

At the position of the smaller loop, magnetic field due to larger loop is parallel to plane of the smaller loop.
Due to the larger loop, magnetic flux linked with the smaller loop is zero. Hence mutual induction is zero.
$Ø_1 = Mi_2 \Rightarrow 0 = Mi_2 \Rightarrow M = 0$



Question 18

A block of mass of 1 kg is moving on the x-axis. A force F acting on the block is shown. The velocity of the block at time t = 2s is −3m/s. What is the speed of the block at time t = 4s?



Options:

A. -3 m/s

B. 8 m/s

C. 2 m/s

D. 3 m/s

Answer: D

Solution:

Impulse during t = 2s to t = 4s is

$J = Area\ under\ the\ curve$

$J = \frac{1}{2}(5)\times(3-2)+\frac{1}{2}(-5)\times(4-3)$

$J = \frac{5}{2}-\frac{5}{2}=0$

Impulse = change in momentum

$J = Mv_f - Mv_i = 0$

$v_f = v_i$

$v_f = -3$

$v_f = 3$ m/s

As speed is required in the question, hence only the magnitude, no direction.

Question 19

Two particles P and Q are moving on a circle. At a certain instant of time both the particles are diametrically opposite and P has tangential acceleration 8 m/s^2 and centripetal acceleration 5 m/s^2 whereas Q has only centripetal acceleration of 1 m/s^2. At that instant acceleration (in m/s^2) of P with respect to Q is:

Options:

A. 12

B. 14

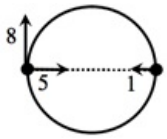
C. $\sqrt{80}$

D. 10

Answer: D

Solution:

Given that the tangential acceleration and centripetal acceleration of the particle P is respectively 8 m/s^2 and 5 m/s^2. And the centripetal acceleration of particle Q is 1 m/s^2.



Acceleration of particle Q in vector form is, a_Q = -i

Acceleration of particle P in vector form is, a_P= 5i+8j

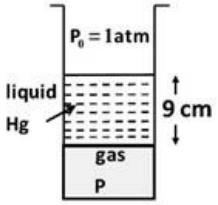
$a_{rel} = a_P - a_Q$

a_{rel} =6i+8j

Magnitude of acceleration is, $|a_{rel}|$=10 m/s^2

Question 20

In the given figure, atmospheric pressure P_0 = 1 atm and mercury column length is 9cm. Pressure P of the gas enclosed in the tube is:



Options:

A. pressure of 85cm of Hg

B. pressure of 67cm of Hg

C. pressure of 90cm of Hg

D. pressure of 78cm of Hg

Answer: A

Solution:

Given that the atmospheric pressure is, P_0 = 1 atm

Length of mercury column is, L = 9 cm

Pressure P of the gas enclosed in the tube is

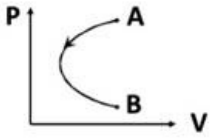
$P_{gas} = P_{atm} + P'$

P_{gas} =76cm of Hg + 9cm of Hg

P_{gas} =85cm of Hg

Question 21

PV diagram of an ideal gas is shown. The gas undergoes from initial state A to final state B such that initial and final volumes are same . Select the correct alternative for given process AB.



Options:

A. process is isochoric

B. work done by gas is positive

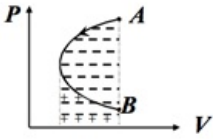
C. work done by gas is negative

D. temperature of gas increases continuously

Answer: C

Solution:

Volume does not remain constant throughout the process AB.
According to ideal gas equation, PV = nRT
Temperature is, T=PV/nR
Since the initial and final volume is the same, so the temperature decreases
Work done is, W = nRΔT, since temperature change is negative. So the net work is negative.



Question 22

A small object of mass of 100gm moves in a circular path. At a given instant velocity of the object is $10\hat{i}$ m/s and acceleration is $(20\hat{i}+10\hat{j})$ m/s^2. At this instant of time, rate of change of kinetic energy of the object is:

Options:

A. 20 kgm^2 s^{-3}

B. 200 kgm^2 s^{-3}

C. 300 kgm^2 s^{-3}

D. 10000 kgm^2 s^{-3}

Answer: A

Solution:

Given that the mass of the small object is, m = 100 gm
Velocity of the object is, v = $10\hat{i}$ m/s
Acceleration of the object is, a = $(20\hat{i}+10\hat{j})$ m/s^2
Kinetic energy is, KE = $\frac{1}{2}mv^2$
The rate of change of kinetic energy is

$$\frac{dKE}{dt} = \frac{1}{2}m\frac{d}{dt}(v^2)$$

$$\frac{dKE}{dt} = \frac{1}{2}m(2va)$$

$$\frac{dKE}{dt} = mav$$

$$\frac{dKE}{dt} = 100\times10^{-3}\times\left(20\hat{i}+10\hat{j}\right)\times\left(10\hat{i}\right)$$

$$\frac{dKE}{dt} = 10^{-1}\times200$$

$$\frac{dKE}{dt} = 20$$

Question 23

A time varying horizontal force (in Newton) $F = 8|\sin(4\pi t)|$ is acting on a stationary block of mass 2kg as shown. Friction coefficient between the block and ground is $\mu = 0.5$ and g = 10 m/s^2. Then resulting motion of the block will be:

Options:

A. It moves towards right

B. It will oscillate

C. It remains stationary

D. It moves towards left

Answer: C

Solution:

Maximum possible friction force, $f_{max}=0.5(2g)=10N$

Maximum applied force $F_{max}=8(1)=8N$

Since $F_{max}<f_{max}$, so the block remains stationary.

Question 24

Take Bulk modulus of water B = 2100MPa. What increase in pressure is required to decrease the volume of 200 liters of water by 0.004 percent?

Options:

A. 84 kPa

B. 210 kPa

C. 840 kPa

D. 8400 kPa

Answer: A

Solution:

Bulk Modulus-It is defined as the ratio of hydraulic stress to the volumetric strain.

Bulk modulus, $B = \frac{\text{Hydraulic Stress}}{\text{Volumetric Strain}}$

$$B = \frac{F/A}{\Delta V/V}$$

$$B = \frac{PV}{\Delta V}$$

Change in the pressure is,

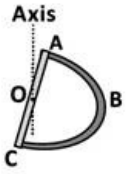
$$\triangle p = B\left(\frac{\triangle V}{V}\right)$$

$$\triangle p = 2100 \times 10^6 \times \left(\frac{0.004}{100}\right)$$

$$\triangle p = 84 \text{ kPa}$$

Question 25

Thin semicircular part ABC has mass m_1 and diameter AOC has mass m_2. Here axis passes through mid-point of diameter and the axis is perpendicular to plane ABC. Here AO = OC = R. The moment of inertia of this composite system about the axis is:



Options:

A. $m_1R^2 + \frac{m_2R^2}{12}$

B. $\frac{m_1R^2}{2} + \frac{m_2R^2}{3}$

C. $\frac{m_1R^2}{2} + \frac{m_2R^2}{6}$

D. $m_1R^2 + \frac{m_2R^2}{3}$

Answer: D

Solution:

Moment of inertia of the composite systems is

$I = I_{ABC} + I_{AOC}$

where (I_{ABC}) is the moment of inertia of part ABC and (I_{AOC}) is the

moment of inertia of part AOC.

Moment of inertia of part ABC is, $I_{ABC} = \int dmR^2$
$I_{ABC} = R^2 \int dm_1 = m_1R^2$
where R is the radius

Moment of inertia of part AOC is, $I_{AOC} = m_2(AC)^2/12$
$I_{AOC} = m_2(2R)^2/12$
$I_{AOC} = m_2R^2/3$

So the moment of inertia of the composite system is, I
$= m_1R^2 + m_2R^2/3$

Question 26

In Young's double slit experiment, the path difference between two interfering waves at a point on screen is 13.5 times the wavelength. The point is:

Options:

A. dark

B. bright but not central bright

C. neither bright nor dark

D. central bright

Answer: A

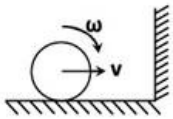
Solution:

Given that the path difference is, $\Delta S = 13.5\lambda$
where λ is the wavelength of the light source
$\Delta S = (13+1/2)\lambda$
$\Delta S = (n+1/2)\lambda$
so there will be minima.

Question 27

A ball having velocity v towards right and having angular velocity

clockwise approaches the wall. It collides elastically with wall and moves towards left. Ground and wall are frictionless. Select the correct statement about angular velocity of the ball after collision.



Options:

A. It will be clockwise

B. It will be anticlockwise

C. It becomes zero

D. Angular speed decreases

Answer: A

Solution:

As collision is elastic, so after collision ball moves towards left with speed v. As walls and ground are smooth, there is no tangential torque on the ball. Only normal forces and mg force pass through the centre of the ball, so their torque about centre are zero. Net torque on the ball about its center is zero.
Net torque is, $\tau = I\alpha = 0$
hence angular acceleration is zero and angular velocity does not change.

Question 28

Which of the following particle will describe the smallest circle when projected with same velocity perpendicular to magnetic field?

Options:

A. electron

B. proton

C. He^{+}

D. Li^{+}

Answer: A

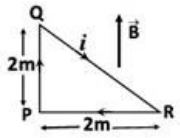
Solution:

The radius of the circle when a charged particle moves in the

magnetic field is, R=mv/Bq
where m is the mass of the charged particle, v is the velocity of the charged particle, q is the charge, and B is the magnitude of the magnetic field.
Since v,q and B are the same each given particle
Hence, R ∝ m
From the given particle mass of the electron is minimum so smallest circle will be made by the electron.

Question 29

A loop PQR carries a current of 2A as shown. A uniform magnetic field (B = 2T) is parallel to plane of the loop. The magnetic torque on the loop is:



Options:

A. 4 Nm

B. 16 Nm

C. 8 Nm

D. zero

Answer: C

Solution:

Torque on a current loop in magnetic field is, τ = iABsinθ
where A is the area of the loop, B is the magnetic field.
τ = iABsin90º
τ =2x(1/2)x2x2x2
τ = 8 Nm

Question 30

The sides of a rectangle are 7.01 m and 12 m. Taking the significant figures into account, the area of the rectangle is:

Options:

A. 84 m^2

B. 84.1 m^2

C. 84.00 m^2

D. 84.12 m^2

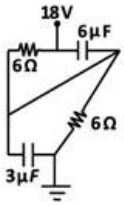
Answer: A

Solution:

According to the rule of the significant figure, the resultant number must have the least number of a significant figure.
Area of rectangle = Length × Bredth
Area of rectangle = 7.01 × 12 = 84.12 m^2
So the area of the rectangle should have only two significant numbers (same as 12m).
Area of rectangle = 84 m^2

Question 31

In steady state, charge on 3μF capacitor is:



Options:

A. 54 μC

B. 36 μC

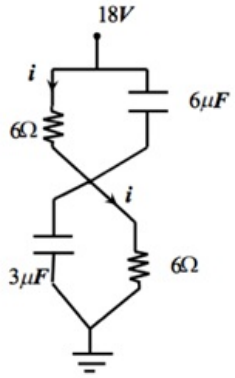
C. 27 μC

D. 18 μC

Answer: C

Solution:

Let the current i is flowing through the battery.



i = 18/(6+6) =1.5A

So the voltage across the 6 ohm resistance is, $V_2 = R_2 i = 6x1.5 = 9V$

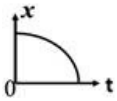
Since the capacitor and resistance are parallel to each other hence the voltage across the 3μF capacitor is 9 V

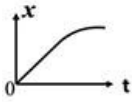
Charge on the 3μF capacitor is, $q_2=3(9)=27$ μC.

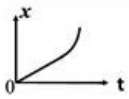
Question 32

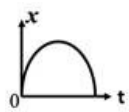
Consider one dimensional motion of a particle. Velocity v versus time t graph is shown. Which graph is most appropriate for displacement x versus time t?

Options:

A. 

B. 

C. 

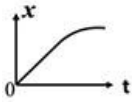
D. 

Answer: B

Solution:

Initially, the velocity is constant, so the slope of x-t graph is constant and finite.

Finally, the velocity becomes zero hence the slope of x-t graph becomes zero.
So the correct representation of the velocity-time graph in terms of the displacement time graph is



Question 33

An object of mass 26kg floats in air and it is in equilibrium state. Air density is 1.3 kg/m^3. The volume of the object is:

Options:

A. 26 m^3

B. 10 m^3

C. 20 m^3

D. 13 m^3

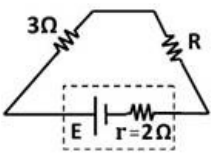
Answer: C

Solution:

Given that the mass of the object is, M = 26 kg
the density of air is, p_{air} = 1.3 kg/m^3
The weight of the mass in the air is, $Mg=p_{air}Vg$
where V is the volume of mass
26g = 1.3Vg
V = 20 m^3

Question 34

In the given circuit cell E has internal resistance of $r = 2\Omega$.What is the value of resistance R so that power delivered to resistor R is maximum?



Options:

A. 1Ω

B. 2Ω

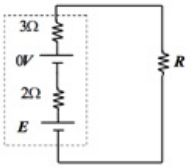
C. 3Ω

D. 5Ω

Answer: D

Solution:

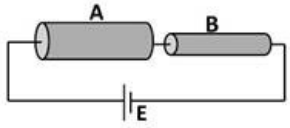
As we know that for maximum power, the internal resistance of the battery should be equal to the external resistance.



the internal resistance of the cell is, 2 + 3= 5Ω
Hence power delivered to R will be maximum if R = 5Ω.

Question 35

Two cylindrical rods A and B have same resistivities and same lengths. Diameter of rod A is twice the diameter of the rod B. Ratio of voltage drop across rod A to rod B is:



Options:

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 2

D. 4

Answer: A

Solution:

The resistance of the rod A is, $R_A=pl/\pi(2R)^2$,

The resistance of the rod B is, $R_B=pl/\pi(R)^2$

The ratio of the resistance of rod A to rod B is

$R_A/R_B=1/4$

$V_A/V_B=R_A/R_B$
$V_A/V_B = 1/4$

Question 36

Which of the following material is not ferromagnetic in nature?

Options:

A. Al

B. Fe

C. Co

D. Ni

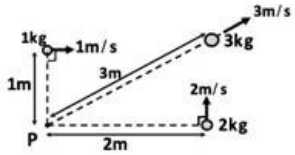
Answer: A

Solution:

Ferromagnetic material- A ferromagnetic substance contains permanent atomic magnetic dipoles that are spontaneously oriented. Ferromagnetism is a kind of magnetism that is associated with iron, cobalt, nickel, and some alloys or compounds containing one or more of these elements.

Question 37

Three small balls of masses 1kg, 2kg and 3kg are moving in a plane and their velocities are 1 m/s, 2 m/s and 3 m/s respectively as shown. The total angular momentum of the system of the three balls about point P at given instant of time is:



Options:

A. 7 kgm^2s^{-1}

B. 8 kgm^2s^{-1}

C. 9 kgm^2s^{-1}

D. 36 kgm^2s^{-1}

Answer: A

Solution:

Angular momentum- Angular momentum of a particle rotating about an axis is defined as the moment of linear momentum of the particle about that axis. It is measured as the product of linear momentum and the perpendicular distance of its line of action from the axis of rotation.

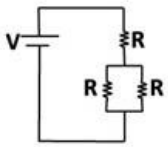
$$\vec{L} = \vec{r} \times \vec{p}$$

L=mvrsinθ
The total angular momentum of the system of the three balls about point P is

L = 2x2x2xsin90º + 3x3x3xsin0º - 1x1x1sin90º
L = 8-1
L =7 kgm^2s^{-1}

Question 38

Three identical resistors each of resistance R are connected to an ideal cell of voltage V as shown. Total power dissipated in all three resistors is:



Options:

A. $\frac{2V^2}{3R}$

B. $\frac{3V^2}{2R}$

C. $\frac{3V^2}{R}$

D. $\frac{V^2}{3R}$

Answer: A

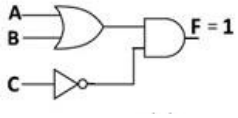
Solution:

Equivalent resistance of the circuit is, R_{eq} = (R+R/2)
R_{eq} = 3R/2
Total power dissipated in all three resistors is, P = V^2/Req
P = V^2/(3R/2)

$P = 2V^2/3R$

Question 39

For given logic diagram, output F = 1, then inputs are:



Options:

A. A = 0, B = 0, C = 1

B. A = 0, B = 0, C = 0

C. A = 0, B = 1, C = 0

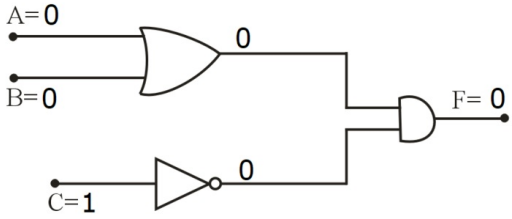
D. A = 1, B = 1, C = 1

Answer: C

Solution:

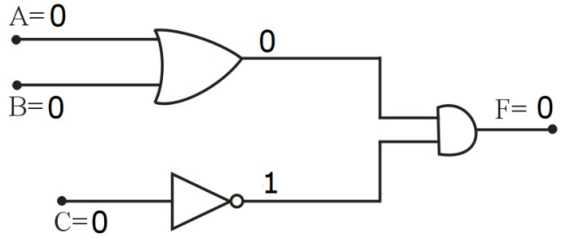
For the first option

Inputs are, A = 0, B = 0, C = 1



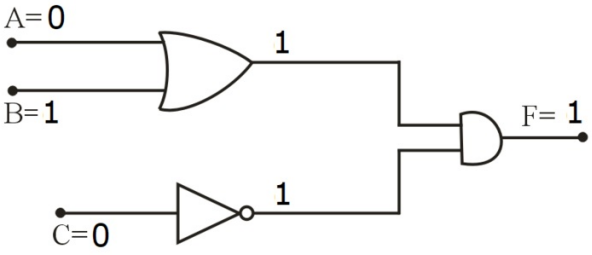
For the second option

Inputs are, A = 0, B = 0, C = 0



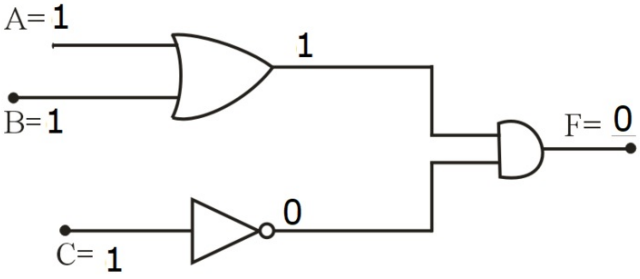
For the third option

Inputs are, A = 0, B = 1, C = 0



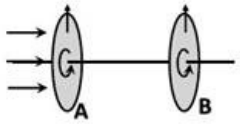
For the fourth option

Inputs are, A = 1, B = 1, C = 1



Question 40

Consider two polaroids A and B as shown. Unpolarized light is incident on polaroid A. Now both the polaroids are rotated simultaneously by 180 ° in same sense of rotation such that at every instant, their pass(transmission) axes always remain parallel to each other. During the rotation, intensity of transmitted light through polaroid B:



Options:

A. decreases continuously

B. increases continuously

C. first increases then decreases

D. remains same

Answer: D

Solution:

Intensity after passing through polaroid is $I_A=I_o/2$=intensity of unpolarised light/2

Intensity after passing through polaroid B is $I_B=I_A\cos^2ø$

Here ø is the angle between axes of A and B. Here their pass (

transmission) axes always remain parallel to each other i.e ø=0.
$I_B=I_A=I_o/2$ = constant
Hence during rotation intensity of transmitted light through polaroid B remains same.

Question 41

Activity of a radioactive substance becomes from 8000Bq to 1000Bq in 12 Days. What is the half-life of the radioactive substance?

Options:

A. 3 days

B. 4 days

C. 6 days

D. 2 days

Answer: B

Solution:

Given that the activity of a radioactive substance becomes from 8000Bq to 1000Bq in 12 Days.
So, 8000 → 4000 → 2000 → 1000
4days 4days 4days

Hence the half-life of the substance is $T_{1/2}$ = 4 days.

Question 42

The energy levels of a hypothetical one electron atom system are given by $E_n = -\frac{16}{n^2}eV$, where (n = 1, 2, 3,.....) The wavelength of emitted photon corresponding to transition from first excited level to ground level is about:

Options:

A. 690 A°

B. 1035 A°

C. 1220 A°

D. 3650 A°

Answer: B

Solution:

The energy of the electron at ground state is, E_1 = -16 eV

The energy of the electron at first excited state is, E_2 = -4 eV

The energy of a photon is, E = hf

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{12420}{\Delta E} = \frac{12420}{|E_2 - E_1|}$$

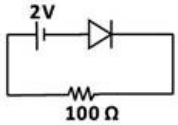
The wavelength of emitted photon corresponding to the transition from first excited level to ground level is

$$\lambda = \frac{12420}{|-4-(-16)|}\ A°$$

λ = 1035 A°

Question 43

What is the voltage across an ideal PN junction diode for shown circuit?



Options:

A. 0V

B. 0.7V

C. 1V

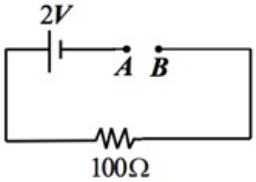
D. 2V

Answer: D

Solution:

In reverse bias it is equivalent to open circuit condition. From figure given below

$|V_{AB}|$=2V



Question 44

Power emitted by a black body at temperature 50°C is P. Now temperature is doubled i.e. temperature of black body becomes 100 °C. Now power emitted is:

Options:

A. 16 P

B. greater than P but less than 16 P

C. greater than 16 P

D. P

Answer: B

Solution:

The power emitted by the black body is directly proportional to the fourth power of temperature.

$P \propto T^4$

Let us assume that when the temperature is doubled the power emitted by the black body is

$P' \propto T'^4$

$here\ T = 273 + 50, T' = 273 + 100$

$P \propto (273+50)^4$ .....(1)

$P' \propto (273+100)^4$ .....(2)

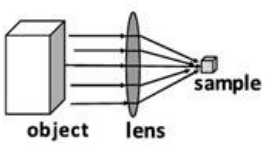
equation (2)/(1) gives

$P'/P=(273+100)^4/(273+50)^4$

$1<P'/P<2^4$

Question 45

An experimenter needs to heat a small sample to temperature 900K, but the only available large object has maximum temperature of 600K. Could the experimenter heat the sample to 900K by using a large lens to concentrate the radiation from the large object onto the sample as shown below?



Options:

A. Yes, if the volume of the large object is at least 1.5 times the volume of the sample.

B. Yes, if the front area of the large object is at least 1.5 times the area of the front of the sample.

C. Yes, if the sample is placed at the focal point of the lens.

D. It is not possible

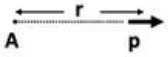
Answer: D

Solution:

It is not possible, because it will violate the second law of thermodynamics (Clausius statement) .If we consider imaginary case in which temperature of sample becomes more than 600K then it will radiated power is more than absorbed power. Hence it will correspond to decreasing temperature situation. So it is not possible to heat the sample to 900K.

Question 46

Consider a small electric dipole with magnitude of dipole moment p which is placed far away from point A as shown. The electric potential at the point A is:



Options:

A. exactly zero

B. $\frac{k\,p}{r^2}$

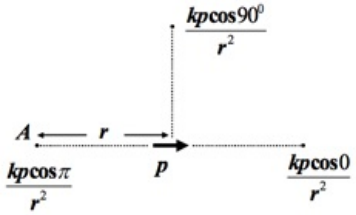
C. $\frac{-k\,p}{r^2}$

D. $\frac{k\,p}{r}$

Answer: C

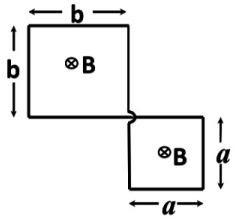
Solution:

Given that the distance between the dipole and the point A is r
So the potential at point A due to the dipole is, $V_A=(kp\cos\pi)/r^2$
$V_A = -kp/r^2$



Question 47

A conducting loop (as shown) has total resistance R. A uniform magnetic field B = γt is applied perpendicular to plane of the loop where γ is a constant and t is time. The induced current flowing through loop is:



Options:

A. $\frac{(b^2+a^2)\gamma}{R}$

B. $\frac{(b^2+a^2)\gamma t}{R}$

C. $\frac{(b^2-a^2)\gamma}{R}$

D. $\frac{(b^2-a^2)\gamma t}{R}$

Answer: C

Solution:

Flux passing through the loop is, $\Phi = B_1A_1 + B_2A_2$

$\Phi = Bb^2\cos 0^o + Ba^2\cos 180^o$

$\Phi = B(b^2-a^2)$

$\Phi = (b^2-a^2)\gamma t$

Induced current through the loop is, i = E/R

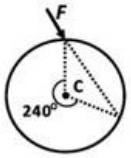
$$i = \frac{\frac{d\phi}{dt}}{R}$$

$$i = \frac{\frac{d(b^2-a^2)\gamma t}{dt}}{R}$$

$$i = \frac{(b^2-a^2)\gamma}{R}$$

Question 48

A uniform disc of mass M and radius R is hinged at its centre C. A force F is applied on the disc as shown. At this instant, angular acceleration of the disc is:



Options:

A. $\frac{F}{2MR}$

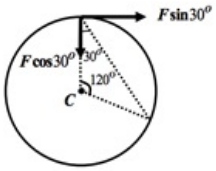
B. $\sqrt{3}\frac{F}{2MR}$

C. $\frac{F}{MR}$

D. $\frac{2}{\sqrt{3}}\frac{F}{MR}$

Answer: C

Solution:



$\tau = I\alpha$

$\Rightarrow$ Fsin30 ^{o}R=MR^2/2(α)

$\Rightarrow$ $\alpha = F/MR$

Question 49

The velocity of a particle is zero at time t = 2 , then

Options:

A. acceleration must be zero at t = 2

B. displacement must be zero in the interval t = 0 to t = 2.

C. acceleration may be zero at t = 2

D. velocity must be zero for t > 2

Answer: C

Solution:

Let us consider an example in which particle is projected vertically upward from ground at t=0 and it reaches highest point at t=2. Then at that instant(t=2) velocity v=0 but acceleration=−g. Here displacement is non zero for the duration t=0 to t=2. For t>2 the

ball again acquires velocity. In this example options(A),(B) and (D) are incorrect. Let us consider another example in which a particle is moving on horizontal plane and it comes to rest permanently at t=2 ,then this is the one of the special case in which acceleration of particle is zero at t=2

Question 50

A ball moving in xy plane, has velocity $(4\hat{i}-4\hat{j})$ m/s just before the collision with ground. Coefficient of restitution for collision is $e=\frac{1}{2}$. What will be velocity of the ball just after the collision with ground?

Options:

A. $(4\hat{i}-4\hat{j})\,m/s$

B. $(2\hat{i}-2\hat{j})\,m/s$

C. 4i+2j m/s

D. $(2\hat{i}-4\hat{j})\,m/s$

Answer: C

Solution:

As we know that the coefficient of restitution is, $e = v_{sep}/v_{app}$

$e = 1/2$

$$\frac{1}{2}=\frac{v_y-0}{4-0}$$

$$v_y=2$$

v_x remains the same, hence the final velocity of the ball is, $v_{final}=4i + 2j$

Chemistry

Question 51

The pair of metal carbonyl complexes that are isoelectronic is:

Options:

A. $[Co(CO)_4]^-$ and $Ni(CO)_4$

B. $Ni(CO)_4$ and $V(CO)_6$

C. $[Cr(CO)_6]$ and $V(CO)_6$

D. $[Fe(CO)_4]^-$ and $Cr(CO)_6$

Answer: A

Solution:

$[Co(CO)_4]^-$ has total electrons=3+2x4+1=12
$Ni(CO)_4$ has total electrons=4+2x4=12
Thus isoelectronic.

Question 52

Which one of the following has (have) octahedral geometry?
i. $SbCl_6^-$ **ii.** $SnCl_6^{2-}$
iii. XeF_6 **iv.** IO_6^{5-}

Options:

A. (i), (ii) & (iii)

B. (i), (ii) & (iv)

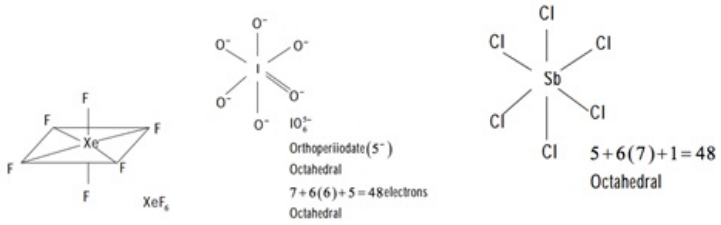
C. (ii), (iii) & (iv)

D. All of these

Answer: B

Solution:

XeF_6 has distortion and become pentagonal bipyramid with one lone pair.



$SnCl_6^{2-}$ = 4+6(7)+2=48
6 set of electrons-octahedral.

Question 53

In terms of polar character which one of the following orders is correct?

Options:

A. NH_3 < H2O < HF <H_2S

B. H_2S < NH_3 < H_2O < HF

C. H_2O < NH_3 < H_2S < HF

D. HF < H_2O < NH_3 <H_2S

Answer: B

Solution:

Polar character is developed when the electronegativity difference increases above a certain limit. We see that the max EN difference is between hydrogen and fluorine hence have the max polar character. We can predict the polar character of other compounds similarly.

Question 54

Among the following compounds of Boron, the species which also forms π – bond in addition to σ – bonds is:

Options:

A. BF_4^-

B. BH_3

C. B_2H_6

D. BF_3

Answer: D

Solution:

BF_3 can form π bond also in addition to sigma bond. As F is more electronegative and octate in B is not complete.

Question 55

Identify the Brönsted acid in the following equation:

$PO_4^{3-} + H_2O(1) \rightarrow HPO_4^{2-}(aq) + OH^-(aq)$

Options:

A. OH^-

B. PO_4^{3-}

C. HPO_4

D. H_2O

Answer: D

Solution:

H_2O will act as Bronsted acid as it provides H^+ ion.

Question 56

The number of grams/weight of NH_4Cl required to be added to 3 liters of 0.01M NH_3 to prepare the buffer of pH=9.45 at temperature 298K

(K_b for NH_3 is 1.85×10^{-5})

Options:

A. 3.53 gm

B. 0.354 gm

C. 4.55 gm

D. 0.455 gm

Answer: B

Solution:

$pOH=pK_b+\log[NH_4Cl]/[NH_3]$
$\Rightarrow pOH=-\log kb+\log[NH_4Cl]/[NH_3]$
$\Rightarrow pOH=\log[NH_4Cl]/[NH_3]\ K_b$
$14-9.45=\log[NH_4Cl]/[NH_3]\ K_b$
$\Rightarrow \log 3=0.47$
$\log 10^{14}-(\log 10^{9}+\log 3)\approx\log[NH_4Cl]/[NH_3]\ K_b$
On calculation we get,
$[NH_4Cl]=0.35$

Question 57

For the reaction $2HI(g)\rightleftharpoons H_2(g)+I_2(g)$ the degree of dissociation (α) of HI(g) is related to equilibrium constant K_p by the expression:

Options:

A. $\frac{1+2\sqrt{K_p}}{2}$

B. $\sqrt{\frac{1+2K_p}{2}}$

C. $\sqrt{\frac{2K_p}{1+2K_p}}$

D. $\frac{2\sqrt{K_p}}{1+2\sqrt{K_p}}$

Answer: D

Solution:

$2\sqrt{K_p}/1+2\sqrt{K_p}$

$2HI(g) \rightarrow H_2 + I_2$

$2(1-x) \quad x \quad x$

total moles at eql= 2-2x+x+x=2

here x= degree of dissociation

$K_P=(x/2)(x/2)P^2/[4(1-x)^2P^2/4]$

$=x^2/4(1-x)^2$

$=> 2\sqrt{K_p} = x/1-x$

$=> x= \frac{2\sqrt{K_p}}{1+2\sqrt{K_p}}$

Question 58

A 6% solution of sucrose $C_{22}H_{22}O_{11}$ is isotonic with 3% solution of an unknown organic substance. The molecular weight of unknown organic substance will be:

Options:

A. 342

B. 684

C. 171

D. 100

Answer: C

Solution:

A 6% solution of sucrose $C_{22}H_{22}O_{11}$ conc=6g/100ml = $0.06ml^{-1}$=$60/342molel^{-1}$

For unknown solution conc=3g per 100cc=30gl^{-1}=30/m $molel^{-1}$
For isotropic solution 60/342=30/m
=> m=30x342/60=171

Question 59

The enthalpy of the formation of CO_2 and H_2O are – 395 kJ and – 285 kJ respectively and the enthalpy of combustion of acetic acid is 869 kJ. The enthalpy of formation of acetic acid is:

Options:

A. 235 kJ

B. 340 kJ

C. 420 kj

D. 491 kJ

Answer: D

Solution:

$CH_3COOH + 2O_2 \rightarrow 2CO_2 + 2H_2O$
$-869 = 2x(-395) + 2x(-285) - \Delta H_{f(CH3COOH)}$
simplification gives,
$-2x434.5 = 2x(-395) + 2x(-285) - \Delta_S H_{(CH3COOH)}$
or $-434.5 = (-395) + (-285) - 1/2\Delta_f H$
=> $491 = \Delta H_f$

Question 60

Which of the following is a lyophobic colloid:

Options:

A. Gelatin

B. Sulphur

C. Starch

D. Gum Arabica

Answer: B

Solution:

Sulphur is lyophobic colloid which have less force of attraction between dispheresed phase and dispersion medium .

Question 61

For car battery which one is correct statement?

Options:

A. Cathode is Lead dioxide (PbO_2) and anode is Lead (Pb)

B. Cathode is Lead dioxide (PbO_2) and anode is Copper (Cu)

C. Cathode is Copper (Cu) and anode is Lead dioxide (PbO_2)

D. Cathode is Copper (Cu) and anode is Lead (Pb)

Answer: A

Solution:

Fact based question. Car battery is generally lead storage battery.

Question 62

Considering entropy(s) as a thermodynamic parameter, the criterion for the spontaneity of any process the change in entropy is:

Options:

A. $\left(\Delta S_{system} - \Delta S_{surrounding}\right) > 0$

B. $\Delta S_{system} > 0$ only

C. $\Delta S_{surrounding} > 0$ only

D. $\left(\Delta S_{system} + \Delta S_{surrouding}\right) > 0$

Answer: D

Solution:

For spontaneity, ΔG of system should be negative. For this to happen the net entropy of the universe should be increasing.

Question 63

At low pressure and high temperature, the Vander Waal’s equation is

finally reduced (simplified) to:

Options:

A. $PV_m = RT$

B. $\left(P + \frac{a}{V_m^2}\right)(V_m - b) = RT$

C. $P(V_m - b) = RT$

D. $\left(P + \frac{a}{V_m^2}\right)V_m = RT$

Answer: A

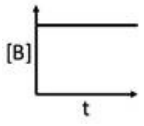
Solution:

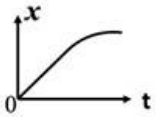
At low pressure and high pressure V is very high , thus a/V^2_m and b are negligible, finally reduced equation is $PV_m=RT$.

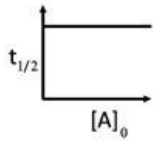
Question 64

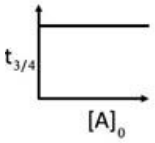
Which graph represents the zero order reaction $[A(g) \rightarrow B(g)]$

Options:

A. 

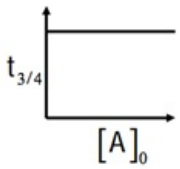
B. 

C. 

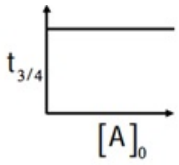
D. 

Answer: D

Solution:



If $t_{1/2}$ vs $[A]^0$ is constant, its radioactive decay hence is not of zero order. Thus answer will be



Question 65

Which of the following compounds is insoluble even in hot concentrated H_2SO_4?

Options:

A. Ethylene

B. Benzene

C. Hexane

D. Aniline

Answer: C

Solution:

Hexane is an alkane without any functional group which makes it a non polar compound where as concentrated H_2SO_4 is polar protic solvent. Since like dissolves like it makes it insoluble.

Question 66

The half-life of Th^{232} is 1.4×10^{10} years and that of its daughter element Ra^{238} is 7 years. What amount (most nearly) weight of Ra^{238} will be in equilibrium with 1gm of Th^{232}?

Options:

A. 5×10^{-10} gm

B. 5.0 gm

C. 1.95×10^{-9} gm

D. 2×10^{-10} gm

Answer: A

Solution:

Let W be the weight of Ra^{238} in equilibrium as

$Th^{232}/Ra^{238} = t_{1/2(Th)}/t_{1/2\ (Ra)}$

$=> 1xN_o/232/(WxN_o/238)$

$= 1.4x10^{10}/7$

N_o=Avagadro's Number

$W=238/232x2x10^{9}=5x10^{-10}$gm.

Question 67

Which of the following electron has minimum energy?

Options:

A. $n=3,\ l=2,\quad m=-2, s=+\frac{1}{2}$

B. $n=4,\ l=0,\quad m=0,\ s=+\frac{1}{2}$

C. $n=4,\ l=1,\quad m=+1, s=+\frac{1}{2}$

D. $n=5,\ l=0,\quad m=0, s=+\frac{1}{2}$

Answer: B

Solution:

The filling up of orbitals with electrons means building up of orbitals. According to Aufbau's principle, during the filling up of orbitals, the electrons first occupy the lowest energy orbital which is available to them and gradually enter into the higher energy orbitals after the filling up of the low energy orbitals.

The order of increasing energy of the orbitals in which the orbitals are filled is :
1s,2s,2p,3s,3p,4s,3d,4p,5s,4d,5p,.....

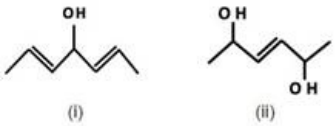
principal quantum number 'n' n=1,2,3,4...	azimuthal quantum number 'l' l=0 to (n-1) i.e. l=0; s orbital l=1; p orbital l=2; d orbital.....	magnetic quantum number'm' [m= -l to +l]	spin quantum number's' s= $+\frac{1}{2} or\ -\frac{1}{2}$	orbital

3	2	-2	$+\frac{1}{2}$	3d
4	0	0	$+\frac{1}{2}$	4s
4	1	+1	$+\frac{1}{2}$	4p
5	0	0	$+\frac{1}{2}$	5s

Thus, from the trend of increasing energy of orbitals shown above, the electron possessing the minimum energy is 4s.
Hence, option (2) is correct.

Question 68

Total number of stereoisomers of the following compounds are respectively:



Options:

A. 4, 6

B. 8, 0

C. 6, 6

D. 8, 8

Answer: A

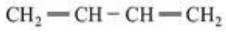
Solution:

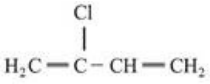
For calculation of stereoisomers of symmetrical compounds, we calculate by formula,
If n is even, $2^{n-1} + 2^{n/2 -1}$
If n is odd , 2^{n-1}
Where n is chiral center.
For (i) there are 3 chiral centre , n=3 hence we get $2^{3-1}=4$
For (ii) there are 2 chiral center, n=2 hence we get $2^{2-1} + 2^{2/2 -1} = 3$
Since it may have a different configuration about middle double bond hence no of stereo isomers = 2x3=6

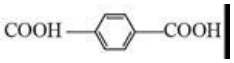
Question 69

Which of the following is a monomer of Dacron:

Options:

A. 

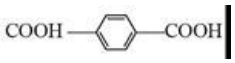
B. 

C. 

D. $HOH_2C - CH_2OH$

Answer: C

Solution:



Tere pthalic acid is a monomer of darcon.

Question 70

Which of the following is a meso compound?

Options:

A. trans–1, 4–dimethylcyclohexane

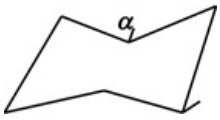
B. cis–1, 3–dimethylcyclohexane

C. trans–1, 3–dimethylcyclohexane

D. cis–1, 4–dimethylcyclohexane

Answer: B

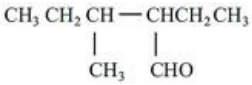
Solution:



two chiral center and plane of symmetry so it is meso compound.

Question 71

IUPAC name of the following is:



Options:

A. 2, 5 Butyl butenal

B. 2, 3 di ethyl butenal

C. 2 ethyl –3 methyl pentanal

D. 8 methyl – 2 ethyl pentanal

Answer: C

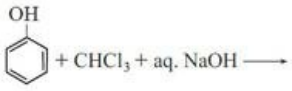
Solution:

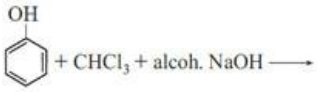
Fact based question. Nomenclature is done on the guidelines and rules of IUPAC nomenclature system.
We first first write the side chain as prefix and then the word root following with suffix which is functional group.

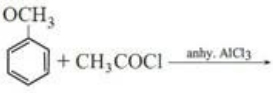
Question 72

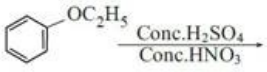
Which of the following is Reimer – Tieman reaction?

Options:

A. 

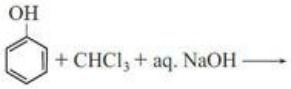
B. 

C. 

D. 

Answer: A

Solution:



in Riemer-Tieman reaction, phenol reacts with $CHCl_3$ and aq KOH/NaOH to give salicyldehyde.

Question 73

The increasing order of the first ionization enthalpies of the elements B, P, S and F is:

Options:

A. B < P < S < F

B. B < S < P < F

C. F < S < P < B

D. P < S < B < F

Answer: B

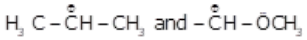
Solution:

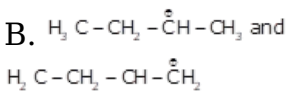
Ionization enthalpies in a period increase from left to right and decrease on going down the group. So fluorine will have the highest ionisation energy and boron will have the lowest. Between S and P, phosphorus has the higher ionisation energy due to its half-filled electronic configuration. Hence, the correct increasing order of ionisation enthalpy is B<S<P<F.

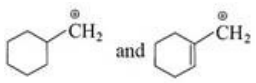
Question 74

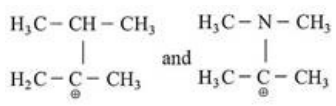
Some pairs of ions are given below. In which pair, first ion is more stable than second ion?

Options:

A. 

B. 

C. 

D. 

Answer: B

Solution:

2 º carbocation will be more stable than 1 º substituted carbocation will be more stable than simple.

Question 75

Which alkaline earth metal compound is volatile?

Options:

A. Be_3N_2

B. Mg_3N_2

C. Ca_3N_2

D. None of the options

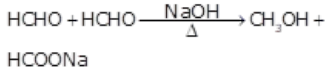
Answer: A

Solution:

Fact based question.
Be_3N_2 is volatile and gives purest N_2 gas.

Question 76

What is the name of the following reaction?



Options:

A. Hell–Volhard reaction

B. Clemmensen reaction

C. Cannizzaro reaction

D. None of the options

Answer: C

Solution:

It is an example of cannizzaro reaction which is a type of disproportionation reaction.

Question 77

Inorganic graphite is:

Options:

A. $B_3N_3H_6$

B. B_2H_6

C. BN

D. BF_3

Answer: C

Solution:

Inorganic graphite is BN . Boron Nitride (BN) is known as Inorganic graphite. The most stable crystalline form is the hexagonal one, also called h-BN, α-BN, or g-BN (graphitic BN). Its hexagonal form is the most stable and softest among BN polymorphs, and is therefore used as a lubricant and an additive to cosmetic products.

Question 78

Rank the following in decreasing order of basic strength:

i. $CH_3-CH_2-C\equiv C^-$

ii. $CH_3-CH_2-S^-$

iii. $CH_3-CH_2-CO_2^-$

iv. $CH_3-CH_2-O^-$

Options:

A. ii > i > iv > iii

B. iv > i > ii > iii

C. i > iv > ii > iii

D. i > iv > iii > ii

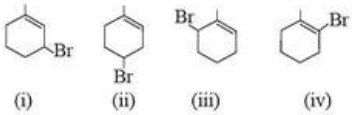
Answer: C

Solution:

Acidity is inversely proportional to basicity.
Basic strength here refers to the ability to donate lone pair, the one with most unstable negative charge is the most basic hence the order is i > iv > ii > iii.

Question 79

Among the given compound choose the two that yield same carbocation on ionization.



Options:

A. (i), (iii)

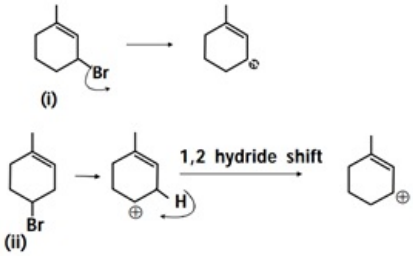
B. (ii), (iv)

C. (i), (ii)

D. (ii), (iii)

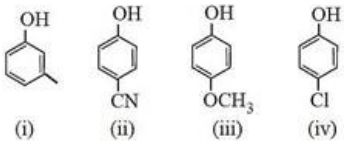
Answer: C

Solution:



Question 80

Increasing order of acidic strength of given compounds is:



Options:

A. iii < i < iv < ii

B. ii < i < iv < iii

C. i < iii < iv < ii

D. i < iii < ii < iv

Answer: A

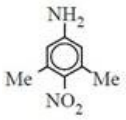
Solution:

Acidic strength is proportional to withdrawing group strength.

$$-R < -H - I < \frac{1}{+I} < \frac{1}{+H} < \frac{1}{+R}$$

Question 81

Which of the following effects of –NO_2 group operates on –NH_2 group in this molecule?



Options:

A. Only –I effect

B. Only +M effect

C. Only –M effect

D. Both –I and –M effect

Answer: A

Solution:

Due to Steric inhibition of resonance because of bulky groups on ortho position –NO_2 goes out of plane of the paper.

Question 82

Which of the following material is known as lunar caustic?

Options:

A. $NaNO_3$

B. AgC1

C. $AgNO_3$

D. NaOH

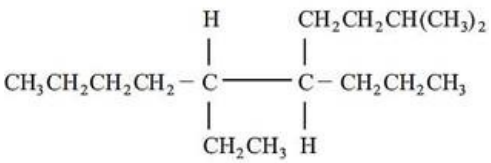
Answer: C

Solution:

Fact based question . $AgNO_3$ is commonly known as lunar caustic.

Question 83

Provide an acceptable name for the alkane shown below:



Options:

A. 6–ethyl–2–methyl–5–propyldecane

B. 5–ethyl–6–methyl–2–propyldecane

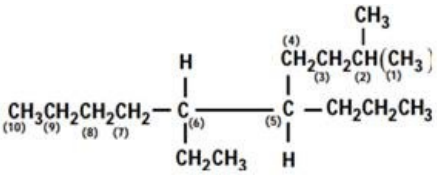
C. 2–ethyl–6–methyl–2–propyldecane

D. 2–ethyl–6–methyl–5–propyldecane

Answer: A

Solution:

An acceptable name for the alkane shown below is:



6–ethyl–2–methyl–5–propyldecane

Question 84

D – Mannose $\xrightleftharpoons{HO^-}$ D – glucose $\xrightleftharpoons{HO^-}$ (A) Product (A) of above reaction is:

Options:

A. D–glucose

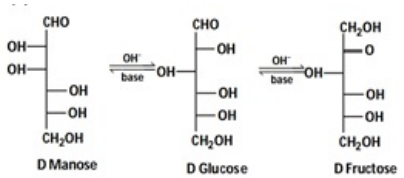
B. D–fructose

C. D–Talose

D. D–Idose

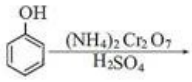
Answer: B

Solution:



Question 85

What is the product in the following reaction?



Options:

A. Benzoic Acid

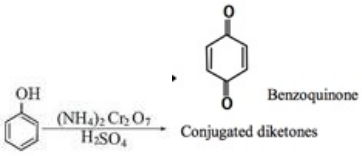
B. Benzoquionone

C. Cyclohexane-1-one

D. Benzoic sulphate

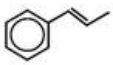
Answer: B

Solution:



Question 86

How many bonds are there in:



Options:

A. $14\sigma, 8\pi$

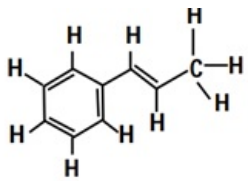
B. $18\sigma, 8\pi$

C. $19\sigma, 8\pi$

D. $14\sigma, 2\pi$

Answer: C

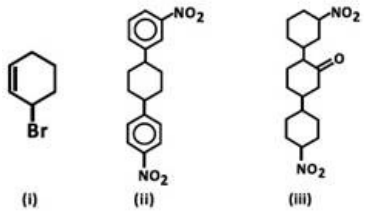
Solution:



$19\sigma, 8\pi$

Question 87

Which of the following molecules is optically active?



Options:

A. (i) and (ii)

B. (i) and (iii)

C. (ii) and (iii)

D. (i), (ii) and (iii)

Answer: B

Solution:

As a plane of symmetry exist in compound(ii),there is no chirality in it. Hence (i) &(iii) will be optically active.

Question 88

Which of the following statement is correct?

Options:

A. BCl_3 and $AlCl_3$ are both Lewis acids and BCl_3 is stronger than $AlCl_3$

B. BCl_3 and $AlCl_3$ are both Lewis acids and $AlCl_3$ is stronger than BCl_3

C. BCl_3 and $AlCl_3$ are both equally strong Lewis acid

D. Both BCl_3 and $AlCl_3$ are not Lewis acids

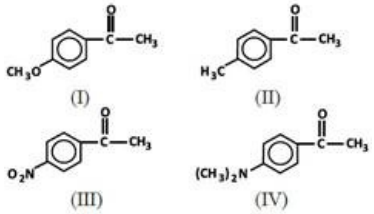
Answer: A

Solution:

BCl_3 and $AlCl_3$ both are Lewis acids but BCl_3 is more electron deficient than $AlCl_3$ so BCl_3 is stronger Lewis acid than AlCl3 due to high electron negativity.(E.N. B-2.0,Al 1.5)

Question 89

Consider the following compounds.



Friedel–Crafts acylation can be used to obtain:

Options:

A. I, III, IV

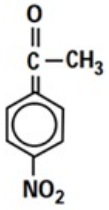
B. II, III, IV

C. I, II, IV

D. I, II, III

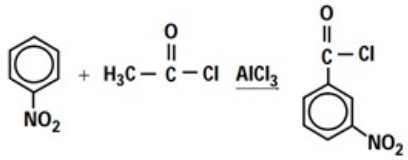
Answer: C

Solution:



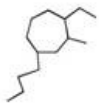
compound cannot be obtained by Friedel Craft acylation, In  ,

NO_2 group has –M or –R effect so it cannot be used in Friedel craft acylation, this is deactivating & meta directing group in ESR.



Question 90

Provide the systematic name of the compound shown:



Options:

A. 4 – butyl – 1 – ethyl – 2 – methylcycloheptane

B. 4 – butyl – 2 – ethyl – 1 – methylcycloheptane

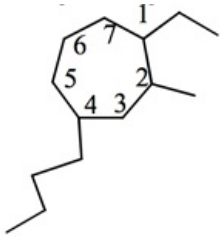
C. 1 – butyl – 4 – ethyl – 3 – methylcycloheptane

D. 2 – butyl – 4 – ethyl – 1 – methylcycloheptane

Answer: A

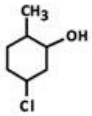
Solution:

Naming according to closet set of locant rule,



Question 91

Give the IUPAC name for the following structure:



Options:

A. 3 – chloro – 2 – methylcyclohexanol

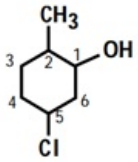
B. 2 – methyl – 5 – chlorocyclohexanol

C. 1 – chloro – 4 – methylcyclohexanol

D. 5 – chloro – 2 – methylcyclohexanol

Answer: D

Solution:



Naming according to closet set of locant rule.

Question 92

In aldol addition reaction product is always:

Options:

A. β – hydroxyaldehyde

B. β – hydroxyketone

C. α, β – unsaturated aldehyde

D. α, β – unsaturated ketone

Answer: A

Solution:

In aldol addition it is β hydroxyl aldehyde(aldol both alcohol and aldehyde group) But in aldol condensation the product is α,β unsaturated aldehyde

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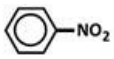
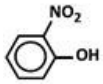
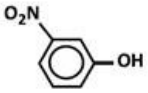
Question 93

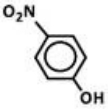
Which one of the following compounds will have the highest dipole moment?

Options:

A. 

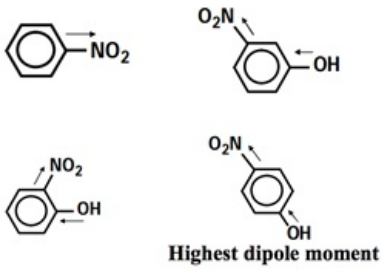
B. 

C. 

D. 

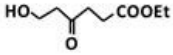
Answer: D

Solution:



Question 94

The number of moles of Grignard reagent consumed per mole of the compound:



Options:

A. 4

B. 2

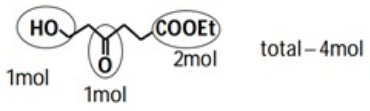
C. 3

D. 1

Answer: A

Solution:

ol.94. (A) 4 Grignard reagent will react on CO bond there are four C-O bond .These will be addition of Grignard reagent at four positions.



total – 4mol

Question 95

The paramagnetic species is:

Options:

A. KO_2

B. SiO_2

C. TiO_2

D. BaO_2

Answer: A

Solution:

$KO_2 \rightarrow K^+ + O_2^-$ =17e^- in valence shell so unpaired electron present and shows paramagnetism.
Rest all have O_2^{2-} dioxide ion in which all the electrons are paired and diamagnetic.

Question 96

Which one of the following has the highest Nucleophilicity?

Options:

A. F^-

B. OH^-

C. CH_3^-

D. NH_2^-

Answer: C

Solution:

Nucleophiles are nucleus loving species. Nucleophile donates electron pair to the species that loves electrons(electrophiles).
Nucleophilicity order is:

$CH_3^->NH_2^->OH^->F^-$

Option 3 is correct.

Question 97

In view of $\Delta_f G^0$ for the following reactions:

$PbO_2 + Pb \rightarrow 2PbO, \quad \Delta_f G^0 < 0$

$SnO_2 + Sn \rightarrow 2SnO, \quad \Delta_f G^0 > 0$

Which oxidation state is more characteristic for lead and tin?

Options:

A. For lead +4, for tin +2

B. For lead +2, for tin +2

C. For lead +4, for tin +4

D. For lead +2, for tin +4

Answer: D

Solution:

$PbO_2 + Pb \rightarrow 2PbO, \quad \Delta G^0 < 0$

i.e. 0 ΔG is negative so reaction is fisiable i.e. for Pb,+2 Oxidation state is more stable.

$SnO_2 + Sn \rightarrow 2SnO, \quad \Delta G^0 > 0$

i.e. 0 ΔG is positive so reaction is nonfisiable i.e. for Sn,+4 Oxidation state is more stable.
Correct answer is (D) For lead +2, for tin +4 For lead +2, for tin +4 Oxidation State are more characteristics.

Question 98

Which of the following compounds will exhibit geometrical isomerism?

Options:

A. 1–Phenyl–2–butane

B. 3–Phenyl–1–butane

C. 2–Phenyl–1–butane

D. 1,1–Diphenyl–1–propene

Answer: A

Solution:

$Ph-CH_2-CH=CH-CH_3$

we see there is unsymmetrical double bond hence geometrical isomer exist.

Question 99

At Critical Micell Concentration (CMC), the surfactant molecules:

Options:

A. decompose

B. dissociate

C. associate

D. become completely soluble

Answer: C

Solution:

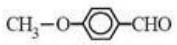
At Critical Micell Concentration they associate to form miscelles.

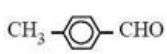
Question 100

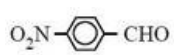
Which one of the following will be reactive for Perkin condensation?

Options:

A. C6H5 – CHO

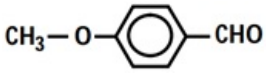
B. 

C. 

D. 

Answer: B

Solution:



will be most reactive. Hence most appropriate option is B

Maths

Question 101

The inverse of the function $y=2^x/(1+2^x)$ is

Options:

A. $x = \log_2 \frac{y}{1-y}$

B. $x = \log_2 \frac{y}{1-2^y}$

C. x = 2log y - y/2

D. x = log y - y

Answer: A

Solution:

Rewrite the equation as

$y(1+2^x)=2^x$

$y+y2^x=2^x$

$y=2^x(1-y)$

$2^x=y/(1-y)$

Taking $\log_2$ on both sides of the equation

$x=\log_2(y/(1-y))$

Question 102

The domain of the definition of the function $y = \frac{1}{\log_{10}(1-x)} + \sqrt{(x+2)}$ is

Options:

A. $-2 \le x < 1, x \ne 0$

B. x≥-2, x≠0

C. -3<x≤-2

D. -2≤x<0

Answer: A

Solution:

The input to the log function is always positive. So,
1-x>0
x<1

The input to the root function should be non-negative. So,
x+2≥0
x≥-2

Thus, the domain of the function is -2≤x<1.

But the function is not defined at x=0, so the domain excludes x=0.

Question 103

Let $f(x) = \begin{cases} -2\sin x & \text{if } x \le -\frac{\pi}{2} \\ A\sin x + B & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2}; \\ \cos x & \text{if } x \ge \frac{\pi}{2} \end{cases}$

For what values of A and B, the function f (x) is continuous throughout the real line?

Options:

A. A = 1, B = 1

B. A = − 1, B = 1

C. A = − 1, B = − 1

D. A = 1, B = − 1

Answer: B

Solution:

$f\left(-\frac{\pi}{2}-\right) = 2$
$f\left(-\frac{\pi}{2}+\right) = -A + B$

$f\left(\frac{\pi}{2}-\right)=A+B$
$f\left(\frac{\pi}{2}+\right)=0$
$A+B=0\ or\ A=-B$
$B-A=2$
$2B=2$
$B=1\ and\ A=-1$

Question 104

Let $f(x)=\begin{cases}a(x)\sin\frac{\pi x}{2} & \text{for } x\neq 0;\\ 1 & \text{for } x=0\end{cases}$
where $\alpha(x)$ **is such that** $\lim_{x\to 0}|a(x)|=\infty$
Then the function f(x) is continuous at x = 0 if $\alpha(x)$ **is chosen as:**

Options:

A. $\frac{1}{x}$

B. $\frac{2}{\pi x}$

C. $\frac{1}{x^2}$

D. $\frac{2}{\pi x^2}$

Answer: B

Solution:

$\lim_{x\to 0}\frac{a(x)\sin\frac{\pi x}{2}}{\frac{\pi x}{2}}*\frac{\pi x}{2}=1\ ;\ \lim_{x\to 0}a(x).\frac{\pi x}{2}=1$
$a(x)=\frac{2}{\pi x}$

Question 105

The $\lim_{y\to a}\left\{\left(\sin\frac{y-a}{2}\right).\left(\tan\frac{\pi y}{2a}\right)\right\}$ **is:**

Options:

A. $\frac{2}{2\pi}$

B. $\frac{2a}{\pi}$

C. $\frac{a}{\pi}$

D. $-\frac{a}{\pi}$

Answer: D

Solution:

$$\lim_{y\to a}\left\{\left(\sin\frac{y-a}{2}\right)\left(\tan\frac{\pi y}{2a}\right)\right\}$$

$$=\lim_{y\to a}\left\{\left(\frac{y-a}{2}\right)\frac{\left(\sin\frac{y-a}{2}\right)}{\left(\frac{y-a}{2}\right)}\left(\tan\frac{\pi y}{2a}\right)\right\}$$

$$=\lim_{y\to a}\left\{\left(\frac{y-a}{2}\right)\frac{\left(\sin\frac{y-a}{2}\right)}{\left(\frac{y-a}{2}\right)}\left(\tan\frac{\pi y}{2a}\right)\right\}$$

$$=\lim_{y\to a}\left\{\left(\frac{y-a}{2}\right)\left(\tan\frac{\pi y}{2a}\right)\right\}\times\lim_{y\to a}\left\{\frac{\left(\sin\frac{y-a}{2}\right)}{\left(\frac{y-a}{2}\right)}\right\}$$

$$=\lim_{y\to a}\left\{\frac{\left(\frac{y-a}{2}\right)}{\left(\cot\frac{\pi y}{2a}\right)}\right\}$$

The above limit is of 0/0 form.

So, use the L'hospital rule

$$\lim_{y\to a}\left\{\frac{\left(\frac{y-2a}{2}\right)}{\left(\cot\frac{\pi y}{2a}\right)}\right\}$$

$$=\lim_{y\to a}\left\{\frac{\left(\frac{1}{2}\right)}{\left(\frac{-\pi}{2a}\operatorname{cosec}^2\frac{\pi y}{2a}\right)}\right\}$$

$$=\lim_{y\to a}\left\{\frac{-a}{\pi}\sin^2\frac{\pi y}{2a}\right\}$$

$$=\frac{-a}{\pi}$$

Question 106

Let $\ell_n=\frac{2^n+(-2)^n}{2^n}$ **and** $L_n=\frac{2^n-(-2)^n}{3^n}$ **then as** $n\to\infty$

Options:

A. Both the sequences have limits

B. $\lim_{n\to\infty}\ell_n$ exists but $\lim_{n\to\infty}L_n$ does not exist

C. $\lim_{n\to\infty}\ell_n$ does not exist but $\lim_{n\to\infty}L_n$ exists

D. Both the sequences do not have limits.

Answer: C

Solution:

$ln = 1 + (-1)^n$ *This will oscillate between 0 and 2 depending on the value of n.*
So ln *does not exists*
$Ln = \left(\frac{2}{3}\right)^n + \left(-\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^n [1 + (-1)^n)]$
$\frac{2}{3} < 1$ *So as n tends to infinty this term tends to 0*
$1 + (-1)^n$ *oscillates but is finite.So t he product is 0*
Hence Ln exists

Question 107

For what interval of variation of x, the identity $\text{arc cos}\frac{1-x^2}{1+x^2} = -2\text{arc tanx}$ **is true?**

Options:

A. $0 \le x < \infty$

B. $-\infty < x \le 0$

C. $1 < x < \infty$

D. $0 \le x \le 1$

Answer: B

Solution:

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = -2\tan^{-1}x$$

Put $x = \tan\theta$

$$\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = -2\tan^{-1}(\tan\theta)$$

$$\cos^{-1}\left(\frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}\right) = -2\theta$$

$$\cos^{-1}\left(\frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}\right) = -2\theta$$

$$\cos^{-1}(\cos 2\theta) = -2\theta$$

There two possibilities of LHS, but only one satisfies the equation.
Thus, the interval of variation is from the negative values of θ.
$\theta \le 0$
$\tan\theta \le 0$
$x \le 0$

Question 108

The points of the curve $y = x^3 + x - 2$ at which its tangents are parallel to the straight line $y = 4x - 1$ are:

Options:

A. $(1, 0), (-1, -4)$

B. $(2, 7), (-2, -11)$

C. $(2, -2), \left(2^{\frac{1}{3}}, 2^{\frac{1}{3}}\right)$

D. $\left(-2^{\frac{1}{3}}, -2^{\frac{1}{3}}\right), (0, -4)$

Answer: A

Solution:

The slope of the tangent to the curve is given by

$$\frac{dy}{dx} = 3x^2 + 1$$

Since parallel lines have an equal slope,

$$3x^2 + 1 = 4$$
$$3x^2 = 3$$
$$x^2 = 1$$
$$x = \pm 1$$

Hence, the first option is the correct answer.

Question 109

If $\bar{a}, \bar{b}, \bar{c}$ are three vectors such that $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = 5$ then the value of $\left[\bar{a} \times \bar{b}, \bar{b} \times \bar{c}, \bar{c} \times \bar{a}\right]$ is:

Options:

A. 10

B. 15

C. 25

D. 20

Answer: C

Solution:

$$\left[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}\right]$$
$$=\left(\vec{a}\times\vec{b}\right)\cdot\left(\left(\vec{b}\times\vec{c}\ \right)\times\left(\vec{c}\times\vec{a}\right)\right)$$
$$=\left(\vec{a}\times\vec{b}\right)\cdot\left(\left[\ \vec{b}\ \vec{c}\ \vec{a}\right]\vec{c}-\left[\ \vec{b}\ \vec{c}\ \vec{c}\right]\vec{a}\right)$$
$$=\left(\vec{a}\times\vec{b}\right)\cdot\left(\left[\ \vec{b}\ \vec{c}\ \vec{a}\right]\vec{c}\right)$$
$$=\left[\ \vec{b}\ \vec{c}\ \vec{a}\right]\left(\vec{a}\times\vec{b}\right)\cdot\vec{c}$$
$$=\left[\ \vec{a}\ \vec{b}\ \vec{c}\right]^2$$
$$=25$$

Question 110

A chord of the parabola $y=x^2-2x+5$ joins the point with the abscissas 1 and 3 Then the equation of the tangent to the parabola parallel to the chord is:

Options:

A. $2x-y+\frac{5}{4}=0$

B. $2x-y+2=0$

C. $2x-y+1=0$

D. $2x+y+1=0$

Answer: C

Solution:

The point of contact of the chord for x=1
y=1-2+5=4
and
for x=3, y=9-6+5=8

The equation of chord joining the points (1,4) & (3,8) on the parabola is
(y-4)=((8-4)/(3-1))(x-1)
y-4=2(x-1)
y-4=2x-2
y=2x+2

Tangent parallel to this chord will have the same slope i.e. dy/dx=2

The general equation of the tangent at any point (x_1, y_1) is obtained by replacing

x with $(x+x_1)/2$

y with $(y+y_1)/2$

x^2 with xx_1

y^2 with yy_1

So, the equation of the tangent is

$(y+y_1)/2=xx_1-2((x+x_1)/2)+5$

$(y+y_1)/2=xx_1-x-x_1+5$

$(y+y_1)=2xx_1-2x-2x_1+10$

$y+y_1=(2x_1-2)x-2x_1+10$

$y=(2x_1-2)x-2x_1+10-y_1$

So, the slope of the tangent

$2x_1-2=2$

$x_1-1=1$

$x_1=2$

So, $y_1=x_1^2-2x_1+5$

$=4-4+5=5$

Thus, the equation tangent is

$y=2x-4+10-5=2x+1$

Hence, $2x-y+1=0$ is the equation of the tangent.

Question 111

The point of inflection of the function $y = \int_0^x (t^2 - 3t + 2)\,dt$ is:

Options:

A. $\left(\frac{1}{2}, \frac{3}{2}\right)$

B. $\left(\frac{3}{2}, \frac{3}{4}\right)$

C. $\left(-\frac{3}{2}, -\frac{3}{4}\right)$

D. $\left(-\frac{1}{2}, -\frac{3}{2}\right)$

Answer: B

Solution:

The point of inflection is the point where the second derivative of the function vanishes.
The function is
$y = \int_0^x (t^2 - 3t + 2)\,dt$
Differentiate the function w.r.t x
$\frac{dy}{dx} = x^2 - 3x + 2$
Again differentiate the function w.r.t x
$\frac{d^2y}{dx^2} = 2x - 3 = 0$
$\Rightarrow x = \frac{3}{2}$
Hence, the second option is correct.

Question 112

The $\lim_{x \to \frac{\pi}{2}} \left\{2x \tan x - \frac{\pi}{\cos x}\right\}$ is:

Options:

A. – 1

B. – 3

C. – 2

D. 0

Answer: C

Solution:

$$\lim_{x\to\frac{\pi}{2}}\left\{2x\tan x-\frac{\pi}{\cos x}\right\}$$

$$=\lim_{x\to\frac{\pi}{2}}\left\{\frac{2x\tan x\cos x-\pi}{\cos x}\right\}$$

$$=\lim_{x\to\frac{\pi}{2}}\left\{\frac{2x\sin x-\pi}{cos x}\right\}$$

The limit is of 0/0 form, so use L'Hospital rule

$$\lim_{x\to\frac{\pi}{2}}\left\{\frac{2\sin x+2x\cos x}{-\sin x}\right\}$$

$$=\frac{2+2\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2}}{-\sin\frac{\pi}{2}}$$

$$=-2$$

Question 113

The equation of the normal to the curve $y=-\sqrt{x}+2$ at the point of its intersection with the bisector of the first quadrant is:

Options:

A. 2x – y + 1 = 0

B. 4x – y + 16 = 0

C. 4x – y = 16

D. 2x – y – 1 = 0

Answer: D

Solution:

The bisector of the first quadrant is the line y=x
The curve is $y=-\sqrt{x}+2$
Differentiate the equation w.r.t x

$$\frac{dy}{dx}=\frac{-1}{2\sqrt{x}}$$

The point of intersection of the curve with the line is given by

$x+\sqrt{x}-2=0$

Put $x=t^2$

$t^2+t-2=0$

$t^2+2t-t-2=0$

$t(t+2)-1(t+2)=0$

$(t+2)(t-1)=0$

$t=1,-2(\text{not possible})$

Thus, $x=1$

and y=1

The slope of the normal is

$\frac{-dx}{dy}=2\sqrt{x}=2$ at $(1,1)$

Thus, the equation of normal to the curve is

y - 1=2(x-1)

2x - y=1

Question 114

Let the equation of a curve is given in implicit form as $y=\tan(x+y)$. Then $\frac{d^2y}{dx^2}$ in terms of y is:

Options:

A. $\frac{2(1+y^2)^2}{y^5}$

B. $\frac{2(1+y^2)^2}{y^6}$

C. $\frac{-2(1+y^2)^2}{y^6}$

D. $\frac{-2(1+y^2)^2}{y^5}$

Answer: D

Solution:

$x+y=\tan^{-1}y$

$\frac{dy}{dx}=-\frac{1+y^2}{y^2}$

$\frac{d^2y}{dx^2}=\frac{-\frac{2ydy}{dx}}{y^4}=-\frac{2(1+y^2)}{y^5}$

Question 115

Suppose the area of the ΔABC is $10\sqrt{3}$. Length of segments AC and AB be 5 and 8 respectively. Then the angle A is (are):

Options:

A. 45 ° or 135 °

B. 30 ° or 150 °

C. 90 °

D. 60 ° or 120 °

Answer: D

Solution:

$$Area = \frac{1}{2}bc\sin A 10\sqrt{3} = \frac{1}{2} * 5 * 8 * \sin A$$
$$\sin A = \frac{\sqrt{3}}{2}$$
$$A = 60 \text{ or } 120$$

Question 116

The angle at which the curve $y = x^2$ and the curve $x = \frac{5}{3}\cos t, y = \frac{5}{4}\sin t$ intersect is:

Options:

A. $2\tan^{-1}\frac{41}{2}$

B. $\tan^{-1}\frac{2}{41}$

C. $\tan^{-1}\frac{41}{2}$

D. $-\tan^{-1}\frac{2}{41}$

Answer: C

Solution:

$$Point\ of\ intersection\ is\ (1,1) Slope\ of\ tangent\ at\ (1,1) of\ both\ curves:$$
$$m1 = 2\ m2 = -\frac{9}{16}$$
$$\left|\frac{m1 - m2}{1 + m1m2}\right| = \left|\frac{2 + \frac{9}{16}}{1 - \frac{9}{8}}\right| = \tan^{-1}\frac{41}{2}$$

Question 117

The maximum value of the function $y = 2\tan x - \tan^2 x$ over $\left[0, \frac{\pi}{2}\right]$ is:

Options:

A. 2

B. ∞

C. 1

D. 3

Answer: C

Solution:

$y - 1 = -(1 - \tan x)^2$; $y = 1 - (1 - \tan x)^2$
This wil be maximum when $(1 - \tan x) = 0$
or $\tan x = 1$
So maximum value $= 1$

Question 118

Let $O = (0, 0)$, $A = (a, 11)$ and $B = (b, 37)$ are the vertices of an equilateral triangle OAB, then a and b satisfy the relation:

Options:

A. $(a^2 + b^2) - 3ab = 138$

B. $(a^2 + b^2) - 4ab = 138$

C. $(a^2 + b^2) - ab = 124$

D. $(a^2 + b^2) - 3ab = 130$

Answer: B

Solution:

$a^2 + 11^2 = b^2 + 37^2 a^2 - b^2 = 1248$
$(a - b)^2 + 26^2 = a^2 + 11^2$
$b^2 - 2ab + 555 = 0$
$2b^2 - 4ab + 1110 = 0$
Adding we get
$a^2 + b^2 - 4ab = 138$

Question 119

Let f be an odd function defined on the real numbers such that f(x) =3sinx + 4cosx, for, x≥0 then f(x) for x<0 is:

Options:

A. 3 sin x − 4 cos x

B. −3 sin x +4 cos x

C. − 3 sin x − 4 cos x

D. 3 sin x + 4 cos x

Answer: A

Solution:

For an odd function,
f(x)=-f(-x)
f(x) for x≥0 = - f(-x) for x≥0
similarly
f(x) for x<0 = - f(-x) for x<0
= -(3sin(-x) + 4cos(-x))
= 3sinx - 4cosx

Question 120

The function $f(x) = x \tan^{-1}\frac{1}{x}$ for $x \neq 0, f(0) = 0$ is:

Options:

A. continuous at x = 0 but not differentiable at x = 0

B. Differentiable at x = 0

C. Neither continuous at x = 0 nor differentiable at x = 0

D. Not continuous at x = 0

Answer: A

Solution:

$$\lim_{x\to 0} x tan^{-1}\left(\frac{1}{x}\right) = 0 = f(0)$$

$$\lim_{x\to 0}\frac{[f(x)-f(0)]}{x} = \lim_{x\to 0}\tan^{-1}\frac{1}{x}$$

This limit does not exist as the left hand limit is $-\frac{\pi}{2}$ *and the right hand limit is* $\frac{\pi}{2}$

Question 121

Let α and β be two numbers where α < β The geometric mean of these numbers exceeds the smaller number α by 12 and the arithmetic mean of the same number is smaller by 24 than the larger number β, then the value of $|\beta - \alpha|$ is:

Options:

A. 27

B. 48

C. 45

D. 44

Answer: B

Solution:

$\sqrt{\alpha\beta} = \alpha + 2\frac{\alpha+\beta}{2} + 24 = \beta$
$Solving\ \alpha = 6\ \beta = 54$
$\beta - \alpha = 48$

Question 122

The values of a and b for which the function $y=a\log_e x + bx^2 + x$, has extremum at the points $x_1=1$ and $x_2=2$ are:

Options:

A. $a=-\frac{1}{3}, b=-\frac{1}{6}$

B. $a=\frac{2}{3}, b=-\frac{1}{6}$

C. $a=-\frac{2}{3}, b=-\frac{1}{6}$

D. $a=\frac{2}{3}, b=\frac{1}{6}$

Answer: C

Solution:

Differentiating the equation w.r.t x
dy/dx =a/x + 2bx +1

At extremum points $x_1=1$

dy/dx =0

$a/x_1 + 2bx_1 + 1 = 0$

$a+2b+1=0$ .....(i)

Also, at $x_2=2$

$dy/dx = 0$

$a/x_2 + 2bx_2 + 1 = 0$

$a/2 + 4b+1=0$

$a+8b+2=0$ .....(ii)

Subtract (i) from (ii)

$6b+1=0$

$b=-1/6$

and $a=-(1+2b)=-(1+2(-1/6))=-(1-1/3)$

$=-2/3$

Question 123

A point P is selected randomly from the interior of the circle, then the probability that it is closer to the center of the circle rather than its boundary is:

Options:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{4}$

D. $\frac{3}{4}$

Answer: C

Solution:

The region close to the center of the circle is within the half of the radius of the circle.

The probability that the point is closer to the center of the circle than its boundary =
(the area close to the center of the circle)/ (Total area of the circle)
$=\pi(r/2)^2 / \pi r^2$
$=1/4$

Question 124

If the letters of the word ASHOKA are written down at randomly, then the chance that all A's are consecutive is:

Options:

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{2}{3}$

Answer: B

Solution:

$$Probabaility = \frac{\frac{5!}{2!}}{6!} = \frac{1}{3}$$

Question 125

In a triangle ΔABC 3sin A + 4cos B = 6 and 4 sin B + 3cos A = 1, then the angle C is:

Options:

A. 30 °

B. 150 °

C. 45 °

D. 60 °

Answer: A

Solution:

$3\sin A + 4\cos B = 6$ .....(i)
$4\cos B + 3\sin A = 1$ .....(ii)

Squaring equations (i) and (ii)

$9\sin^2A + 16\cos^2B + 24\sin A\cos B = 36$ ...(iii)
$16\sin^2B + 16\cos^2A + 24\cos A\sin B = 1$ ....(iv)

Adding equation (iii) and (iv) , we get

$9\sin^2A + 16\cos^2B + 24\sin A\cos B + 16\cos^2A + 16\sin^2B + 24\cos A\sin B = 36+1$
$9(\sin^2A+\cos^2A)+16(\cos^2B+\sin^2B)+24\sin A\cos B+24\cos A\sin B=37$
$25+24\sin(A+B)=37$
$24\sin(\pi-C)=12$ (Since sum of all angles of a triangle is 180°
$\sin C=1/2$
$C=30$ °

Question 126

The value of the integral $\int \frac{dx}{x\sqrt{x^2-a^2}}$ is equal to:

Options:

A. $c+\frac{1}{a}\sin^{-1}\frac{a}{|x|}$

B. $c-\frac{1}{a}\sin^{-1}\frac{a}{|x|}$

C. $c-\frac{1}{a}\cos^{-1}\frac{a}{|x|}$

D. $\sin^{-1}\frac{a}{|x|}+c$

Answer: B

Solution:

Let the value of integral be I

$$I = \int \frac{dx}{x\sqrt{x^2 - a^2}}$$

Substitute x=1/t

then on differentiating, dx=-(1/t^2) dt

$$I = \int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\left(\frac{1}{t}\right)^2 - a^2}}$$

$$= \int \frac{-dt}{t\sqrt{\left(\frac{1}{t}\right)^2 (1 - a^2t^2)}}$$

$$= \int \frac{-dt}{\sqrt{(1 - a^2t^2)}}$$

$$= -\frac{1}{a}\sin^{-1}(|at|) + c$$

$$= c - \frac{1}{a}\sin^{-1}\left(\frac{a}{|x|}\right)$$

Question 127

The function y specified implicitly by the relation $\int_0^y e^t\,dt + \int_0^x \cos t\,dt = 0$ satisfies the differential equations:

Options:

A. $e^y\left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = \sin x$

B. $e^{2y}\left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = \sin x$

C. $e^y\left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = \sin 2x$

D. $e^y\left(2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) = \sin x$

Answer: A

Solution:

Differentiating with respect to x

$$\frac{e^y dy}{dx} + \cos x = 0$$

Differentiating again

$$e^y \frac{d^2y}{dx^2} + e^y \left(\frac{dy}{dx}\right)^2 = \sin x$$

Question 128

Let a and b be real numbers such that $\sin a + \sin b = \frac{1}{\sqrt{2}}$ and $\cos a + \cos b = \frac{\sqrt{6}}{2}$ then the value of $\sin(a+b)$ is:

Options:

A. $\frac{1}{2\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

Answer: C

Solution:

$Squaring\ and\ adding 1+1+2\cos(a-b)=2$
$\cos(a-b)=0$
$a-b=90$
$a=90+b$
$\sin b+\cos b=\frac{1}{\sqrt{2}}$
$1+\sin 2b=\frac{1}{2}$
$\sin 2b=-\frac{1}{2}$
$2b=-30\ b=-15$
$a=90-15=75$
$a+b=75-15=60$
$\sin 60=\frac{\sqrt{3}}{2}$

Question 129

The tangent to the graph of a continuous function $y=f(x)$ at the point with abscissa $x=a$ forms with the x axis an angle of $\frac{\pi}{3}$ and at the point with abscissa $x=b$ and angle of $\frac{\pi}{4}$, then what is the value of the integral $\int_a^b e^x\{f'(x)+f''(x)\}dx$?
(where $f'(x)$ the derivative of f w.r.to x which is assumed to be continuous and similarly $f''(x)$ the double derivative of f w.r.to x)

Options:

A. $-e^b+\sqrt{3e^a}$

B. $e^b+\sqrt{3}e^a$

C. $e^b - \sqrt{3}e^a$

D. $e^b + \sqrt{3e^a}$

Answer: C

Solution:

$I = \int_a^b \frac{d}{dx}[e^x f'(x)]\,dx =$
$e^b f'(b) - e^a f'(a) = e^b - \sqrt{3}e^a$

Question 130

The system $\begin{pmatrix} 1 & -1 & 2 \\ 3 & 4 & -3 \\ 2 & 6 & a \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ b \\ 2 \end{pmatrix}$ **has no solution if**

Options:

A. $a \neq -5, b \neq 5$

B. $a = -5, b \neq 5$

C. $a = -5, b = 5$

D. $a \neq -5, b = 5$

Answer: B

Solution:

$[A|b] = \begin{pmatrix} 1 & -1 & 2:3 \\ 3 & 5 & -3:b \\ 2 & 6 & a:2 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2:3 \\ 0 & 8 & -9:b-9 \\ 0 & 0 & a+5:5-b \end{pmatrix}$
For no solution $a \neq 5$ *and* $b \neq 5$

Question 131

Let α **and** β **be the roots of** $x^2 + 3x + 5 = 0$ **then the equation whose roots are** $-\frac{1}{\alpha}$ **and** $-\frac{1}{\beta}$ **is**

Options:

A. $5x^2 - 3x + 1 = 0$

B. $5x^2 - 3x - 4 = 0$

C. $5x^2 - 3x + 4 = 0$

D. $5x^2 - 3x - 1 = 0$

Answer: A

Solution:

The sum of the roots of $x^2+3x+5=0$ is

$\alpha+\beta=-3$

and the products of the roots is

$\alpha\beta=5$

The equation whose roots are $-\frac{1}{\alpha}$ and $-\frac{1}{\beta}$ is

$$x^2+\left(-\left(-\frac{1}{\alpha}-\frac{1}{\beta}\right)\right)x+\left(-\frac{1}{\alpha}\right)\left(-\frac{1}{\beta}\right)=0$$

$$x^2+\left(\frac{\alpha+\beta}{\alpha\beta}\right)x+\frac{1}{\alpha\beta}=0$$

$$x^2+\left(\frac{-3}{5}\right)x+\frac{1}{5}=0$$

$$5x^2-3x+1=0$$

Question 132

A closed figure S is bounded by the hyperbola $x^2-y^2=a^2$ and the straight line $x=a+h;(h>0,a>0)$. This closed figure is rotated about the x-axis. Then the volume of the solid of revolution is:

Options:

A. $\frac{\pi h^2}{2}(3a+h)$

B. $\pi h^2(3a+h)$

C. $\frac{\pi h^2}{6}(3a+h)$

D. $\frac{\pi h^2}{3}(3a+h)$

Answer: D

Solution:

$$V=\pi\int_a^{a+h}y^2dx=\pi\int_a^{a+h}(x^2-a^2)dx=\frac{\pi h^2}{3}[3a+h]$$

Question 133

The general solution of the equation $\frac{dy}{dx}=\frac{y^2-x}{2y(x+1)}$ is:

Options:

A. $y^2=(1+x)\log\frac{c}{1+x}-1$

B. $y^2 = (1+x)\log(1+x) - c$

C. $y^2 = (1+x)\log\frac{c}{1-x} - 1$

D. $y^2 = (1-x)\log\frac{c}{1+x} - 1$

Answer: A

Solution:

$$Let\ y^2 = t\frac{dt}{dx} - \frac{t}{x+1} = -\frac{x}{x+1}$$
$$I.F: \frac{1}{x+1}$$
$$So\ y^2 = (1+x)\log\frac{c}{1+x} - 1$$

Question 134

The equation of displacement of a particle is $x(t) = 5t^2 - 7t + 3$. The acceleration at the moment when its velocity becomes 5m / sec is:

Options:

A. $8m/sec^2$

B. $3m/sec^2$

C. $7m/sec^2$

D. $10m/sec^2$

Answer: D

Solution:

$$v = 10t - 7$$
$$a = 10$$

Question 135

If $5p^2 - 7p - 3 = 0$ and $5q^2 - 7q - 3 = 0$, $p \neq q$, then the equation whose roots are 5p − 4q and 5q − 4p is:

Options:

A. $5x^2 + x - 439 = 0$

B. $5x^2 + 7x - 439 = 0$

C. $5x^2 - 7x - 439 = 0$

D. $5x^2 + 7x - 439 = 0$

Answer: C

Solution:

If $5p^2-7p-3=0$ and $5q^2-7q-3=0$, then p and q are the roots of the equation $5x^2-7x-3=0$

The sum of the roots of the equation is

$p+q=\frac{7}{5}$

and the product of the roots is

$pq=\frac{-3}{5}$

The equation whose roots are 5p – 4q and 5q – 4p is

$x^2-(5p-4q+5q-4p)x+(5p-4q)(5q-4p)=0$

$x^2-(p+q)x+\left(25pq-20p^2-20q^2+16pq\right)=0$

$x^2-(p+q)x+\left(41pq-20\left(p^2+q^2\right)\right)=0$

$x^2-(p+q)x+\left(41pq-20\left((p+q)^2-2pq\right)\right)=0$

$x^2-(p+q)x+\left(81pq-20(p+q)^2\right)=0$

$x^2-\frac{7}{5}x+\left(81\left(\frac{-3}{5}\right)-20\left(\frac{7}{5}\right)^2\right)=0$

$x^2-\frac{7}{5}x-\frac{439}{5}=0$

$5x^2-7x-439=0$

Question 136

The range of x for which the formula $3\sin^{-1}x=\sin^{-1}\left[x\left(3-4x^2\right)\right]$ hold is:

Options:

A. $-\frac{2}{3}\le x\le\frac{2}{3}$

B. $-\frac{1}{2}\le x\le\frac{1}{2}$

C. $-\frac{1}{4}\le x\le\frac{2}{3}$

D. $-\frac{1}{3}\le x\le 1$

Answer: B

Solution:

$Let\ \sin^{-1}x=a3a=\sin^{-1}[\sin a\ (3-\sin^2 a)]$

$$-\frac{\pi}{2} \le 3a \le \frac{\pi}{2} \; or \; -\frac{1}{2} \le x \le \frac{1}{2}$$

Question 137

The equation of the ellipse, whose focus is the point (–1, 1), whose directrix is the straight line x – y + 3 = 0 and whose eccentricity is 1/2 is:

Options:

A. $(x+1)^2+(y-1)^2=\frac{1}{2}(x-y+3)^2$

B. $(x+1)^2+(y-1)^2=\frac{1}{8}(x-y+3)^2$

C. $(x+1)^2+(y-1)^2=\frac{1}{8}(x-y+1)^2$

D. $(x+1)^2+(y-1)^2=\frac{1}{6}(x-y+3)^2$

Answer: B

Solution:

$$\sqrt{(x+1)^2+(y-1)^2}=\frac{\frac{1}{2}(x-y+3)}{\sqrt{2}}(x+1)^2+(y-1)^2=\frac{1}{8}(x-y+3)^2$$

Question 138

The mean value of the function $f(x)=\frac{2}{e^x+1}$ the interval $[0,2]$ is:

Options:

A. $-2+\log_e\left(\frac{2}{e^2+1}\right)$

B. $2-\log_e\left(\frac{2}{e^2+1}\right)$

C. $2+\log_e\left(\frac{2}{e^2+1}\right)$

D. $2+\log_e\left(\frac{2}{e^2-1}\right)$

Answer: C

Solution:

$$Mean=\frac{1}{2-0}\int_0^2\frac{2}{e^x+1}dx=$$
$$\int_0^2\frac{e^{-x}}{1+e^{-x}}dx=2+\ln\frac{2}{e^2+1}$$

Question 139

The general solution of the differential equation $\frac{dy}{dx}+\sin\frac{x+y}{2}=\sin\frac{x-y}{2}$ is:

Options:

A. $\log_e\left|\tan\frac{y}{4}\right|=-2\sin\frac{x}{2}+c$

B. $\log_e\left|\tan\frac{y}{2}\right|=-2\sin\frac{x}{2}+c$

C. $\log_e\left|\tan\frac{y}{4}\right|=2\sin\frac{x}{2}+c$

D. $\log_e\left|\tan\frac{y}{2}\right|=-\sin\frac{x}{2}+c$

Answer: A

Solution:

$\frac{dy}{dx}=-2\cos\frac{x}{2}\sin\frac{y}{2}\ln\left|\tan\frac{y}{4}\right|=-2\sin\frac{x}{2}+c$

Question 140

If 7/2 and 1 are the roots of the equation $\begin{vmatrix}2x & 3 & 7\\ 2 & 2x & 3\\ 7 & 6 & 2x\end{vmatrix}=0$ then the third root is:

Options:

A. − 5/2

B. − 7/2

C. − 9/2

D. − 3/2

Answer: C

Solution:

$$\begin{vmatrix} 2x & 3 & 7 \\ 2 & 2x & 3 \\ 7 & 6 & 2x \end{vmatrix} = \begin{vmatrix} 2x+9 & 2x+9 & 2x+9 \\ 2 & 2x & 3 \\ 7 & 6 & 2x \end{vmatrix} \quad (R_1 \to R_1 + R_2 + R_3)$$

$$= (2x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2x & 3 \\ 7 & 6 & 2x \end{vmatrix}$$

$$= (2x+9)\begin{vmatrix} 1 & 0 & 0 \\ 2 & 2x-2 & 3-2 \\ 7 & 6-7 & 2x-7 \end{vmatrix} \quad (C_2 \to C_2 - C_1) \text{ and } (C_3 \to C_3 - C_1)$$

$$= (2x+9)(2x-2)(2x-7)$$

Thus the roots of the equation are − 9/2, 1, 7/2.

Hence, the third root is − 9/2.

Question 141

If , $\cos\left(\log i^{-4i}\right) = a + ib$ then

Options:

A. $a = \cosh(2\pi)$, b = 2

B. a = 1, b = − 1

C. a =− 1, b = 1

D. $a = \cosh(2\pi)$, b = 0

Answer: D

Solution:

$$\cos\left(\log i^{-4i}\right) = \cos\left(-4i(\log i)\right)$$

$$= cos\left(-4i\left(loge^{i\frac{\pi}{2}}\right)\right)$$

$$= \cos\left(-4i\left(\frac{\pi}{2}\right)\right)$$

$$= \cos(i2\pi)$$

$$= \frac{e^{i(i2\pi)} + e^{-i(i2\pi)}}{2}$$

$$= \cosh(2\pi)$$

The function is purely real, so the imaginary part will be zero

b=0

and

$a = \cosh(2\pi)$

Question 142

The function $y = \sqrt{2x - x^2}$

Options:

A. increases in (0, 2)

B. increases in (0, 1) but decreases in (1, 2)

C. Decreases in (0, 2)

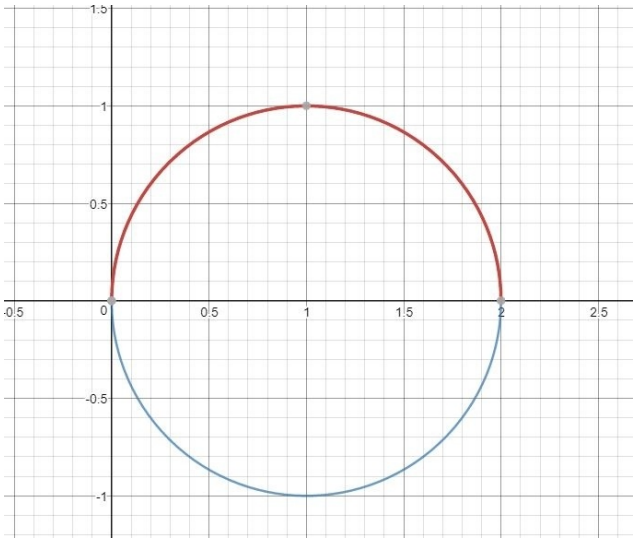
D. Increases in (1, 2) but decreases in (0, 1)

Answer: B

Solution:

Square both sides of the equation
$y^2=2x-x^2$
Add 1 to both sides of the equation
$x^2 + y^2 -2x +1=1$
$(x-1)^2 + y^2 =1$

The given function is equivalent to the equation of circle derived from it, in the common domain and range of the function.
The function represents the portion of the circle in the domain $x \in [0,2]$.
The circle has a center at (1,0) and radius 1 unit. Thus, the center is the mid-point of the required domain.
Hence, the function increases from $x \in (0,1)$ and decreases from (1,2).



Question 143

If the point (α,α) lies between the lines |2x+y|=5 then select one of the most appropriate options

Options:

A. |α|<5/2

B. |α|<5/3

C. |α|<7/2

D. |α|<11/3

Answer: B

Solution:

Given: |2x+y|=5

For a point to lie in the between the lines
|2x+y|-5<0

The given point is (a, a).
Thus,
|2α+α|-5<0
|3α|<5
|α|<5/3

Hence, the second option is correct.

Question 144

If $\log_{\sin\frac{\pi}{6}}\left\{\frac{|z-2|+3}{3|z-2|-1}\right\}>1$**, then**

Options:

A. $|z-2|>7$

B. $|z-2|<7$

C. $|z-2|<3$

D. $|z-2|<6$

Answer: A

Solution:

$\frac{|z-2|+3}{3|x-2|-1} < \frac{1}{2}|z-2| > 7$

Question 145

The n^{th} term of the series 1 + 4 + 13 + 40 + 121 + 364 + ..., is:

Options:

A. $3^n - 1$

B. $\frac{1}{2}(3^n + 1)$

C. $\frac{1}{2}(3^n - 1)$

D. $\left(\frac{2^n + 1}{2}\right)$

Answer: C

Solution:

$S=1+4+13+.....+T_{n-1}+T_n$ .....(1)

$S=0+1+4+13+.....+T_{n-2}+T_{n-1}+T_n$ .....(2)

Subtract (2) from (1)

$0=1+3+9+27+.....+T_{n-1} -T_{n-2} + T_n - T_{n-1}$

$T_n - T_{n-1}=$

$S = 1 + 4 + 3 + \cdots T(n-1) + Tn$

$0 = 1 + 3 + 3^2 + 3^3 + \cdots (Tn - T(n-1)) - Tn$

$Tn = \frac{3^n - 1}{2}$

Question 146

The interval in which the function y=x-2sinx; 0≤x≤2π increase throughout is:

Options:

A. $\left(0, \frac{\pi}{4}\right)$

B. $\left(\frac{5\pi}{3}, 2\right)$

C. $\left(0, \frac{\pi}{3}\right)$

D. $\left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

Answer: D

Solution:

y=x-2sinx

Differentiate the function w.r.t. x

$\frac{dy}{dx} = 1 - 2\cos x$

For increasing function

$\frac{dy}{dx} > 0$

$1 - 2\cos x > 0$

$1 > 2\cos x$

$\cos x < \frac{1}{2}$

$x \in \left(\frac{\pi}{3}, \frac{5\pi}{3}\right)$

Question 147

If the ratio of the seventh term from the beginning of the binomial expansion of $(2^{1/3} + 1/3^{1/3})^x$ to the seventh term from its end is 1/6, then the value of x is:

Options:

A. 14

B. 10

C. 22

D. 18

Answer: D

Solution:

The 7^{th} term from the beginning of $(2^{1/3} + 1/3^{1/3})^x$ is ${}^xC_6\ (2^{1/3})^{(x-6)}\ ((1/3)^{(1/3)})^6$

$= {}^xC_6\ (2^{(x-6)/3})((1/3)^2$

The 7^{th} term from the end of $(2^{1/3} + 1/3^{1/3})^x$ is ${}^xC_{x-6}\ (2^{1/3})^6((1/3)$

$(1/3)_)^{(x-6)}$

$= {}^{x}C_{x-6}\ (2^2)(1/3)^{(x-6)/3} = {}^{x}C_{6}\ (2^2)(1/3)^{(x-6)/3}$

So, the ratio is $\{{}^{x}C_{6}\ (2^{(x-6)/3})((1/3)^2\} : \{{}^{x}C_{6}\ (2^2)(1/3)^{(x-6)/3}\} = 2^{(x-12)/3}\ (1/3)^{(x-12)/3}$

$=(2/3)^{(x-12)/3} = 4/6 =$

$(x-12)/3 = 2$

$x-12=6$

$x=18$

Question 148

Let $A = \{u, v, w, z\}$ and $B = \{3, 5\}$, then the number relations from A to B is:

Options:

A. 64

B. 256

C. 1024

D. 512

Answer: B

Solution:

If cardinality of A = m & Cardinality of B is n ,

then total no. of relations from A to B is 2^{mn} .

Here $m = 4, n = 2 \ \therefore\ 2^8 = 256$

Question 149

Given $y = x^2$. As $x \to 2, y \to 4$ what must the value of δ be for which from $|x-2| < \delta$ it follows that $|y-4| < \epsilon = 0.001$?

Options:

A. $0 < \delta < 0.00025$

B. $0.03 < \delta < 0.05$

C. $0.2 < \delta < 0.25$

D. $0.4 < \delta < 0.5$

Answer: A

Solution:

$|x-2|^2 < \delta^2$

$x^2 + 4 - 4x < \delta^2$

$y + 4 - 4x + 4 - 4 < \delta^2$

$(y-4) - 4(x-2) < \delta^2$

$(y-4) < \delta^2 + 4(x-2)$

$|y-4| < \delta^2 + 4\delta$

which is less than ε
$\delta < \sqrt{\varepsilon + 4} - 2$
For $\varepsilon = 0.001\ \delta = 0.00025$

Question 150

Given that f(0)=0 and $\lim_{x\to 0} \frac{f(x)}{x}$ exists, say L. Here f'(0) denotes the derivative of f w.r.t. x at x=0. Then L is:

Options:

A. 0

B. $2f'(0) - 6$

C. $2f'(0) - 5$

D. $f'(0)$

Answer: D

Solution:

Since f(0)=0 and the limit exists then, we can apply the L' Hospital's rule.

$L = \lim_{x\to 0} \frac{f(x)}{x} = f'(0)$
