Probability

An Experiment with a Die

When a single six-sided die is rolled, we get a single outcome every time. Every outcome can be either an odd number or an even number. In an experiment, a single six-sided die is rolled seven times. The seven outcomes are listed in the following table.

Trial	1	2	3	4	5	6	7
Outcomes	Even number	Odd number	Odd number	Even number	Even number	Even number	Odd number

What do you observe? Is there any pattern in the outcomes? There is no pattern. So, we cannot predict the outcome of rolling the die for an eighth time. However, we can calculate the probability of getting an odd or even number by observing the outcomes of the previous seven rolls of the die.

In this lesson, we will learn to calculate the probability of occurrence of an event in an experiment. We will also solve problems related to the same.

Theoretical Probability

If we divide the number of ways in which a favourable event can occur by the total number of outcomes, then we get the theoretical probability of occurrence of that particular event.

The formula of theoretical probability is as follows:

$$\text{Theoretical probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

For example, with respect to a single roll of a six-sided die, we have:

Number of favourable outcomes for getting 6 = 1

Total number of outcomes = 6(∵ Any number from 1−6 can be obtained)

$$\therefore \text{Theoretical probability of getting 6 on rolling a die} = \frac{1}{6}$$

Similarly,

Number of favourable outcomes for getting an even number = 3 (i.e., 2, 4 or 6)

Total number of outcomes = 6(i.e., 1, 2, 3, 4, 5 and 6)

$\therefore$ Theoretical probability of getting an even number on rolling a die $= \frac{3}{6} = \frac{1}{2}$

In this manner, we can find the theoretical probability of occurrence of any event.

Experimental probability

The probability of an event ascertained by observing the outcomes of an experiment is known as **empirical or experimental probability**. The formula for finding the experimental probability of an event (*E*) is as follows:

$$\mathbf{P}(E) = \frac{\textbf{Number of trials in which the event happened}}{\textbf{Total number of trials}}$$

Let us once again consider the observation table made at the beginning. In this experiment, we got an odd number 3 times and an even number 4 times on rolling the die a total of 7 times.

Let the probability of getting an odd number be $P(E_1)$ and that of getting an even number be $P(E_2)$. Then, we get:

$$P(E_1) = \frac{3}{7} \text{ and } P(E_2) = \frac{4}{7}$$

In this manner, we can find the experimental probability of any event in an experiment.

Did You Know?

- The probability of an event always lies between 0 and 1.
- The sum of the probabilities of all events in a single experiment is always 1, i.e., $P(E_1) + P(E_2) + \ldots + P(E_n) = 1$, where E_1, E_2, …., E_n are *n* events in a single experiment.

Roll the Die

Did You Know?

Applications of probability

1.Life expectancy: It is the prediction of the age of individuals on the basis of the ages of their ancestors or the ages of people belonging to similar groups in the past. This prediction is used as a guideline by financial advisers to help their clients in planning for their retirement years.

2.Casino games: Casino owners always consider the concept of probability to ensure that they don’t lose money in the business. The odds are always in favour of casino owners. Smart gamblers who know to use probability in casino games try to defy these odds.

Did You Know?

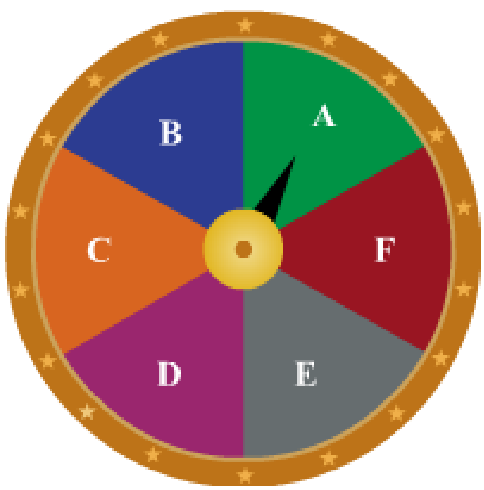
The concept of probability is applied in different contexts, for example, in risk assessment and in trading in financial markets. Governments apply methods of probability in environmental regulation. This is known as pathway analysis.

Solved Examples

Easy

Example 1:

The given figure shows a wheel with six letters of the English alphabet written in six sectors of equal area.



The follwing table shows the results of spinning the wheel ten times.

Trials	1	2	3	4	5	6	7	8	9	10
Outcomes	A	E	D	C	A	B	F	C	A	F

What is the probability of the most favorable outcome?

Solution:

It can be observed from the table that the most favorable outcome is ‘A’ as it is obtained the most number of times (3) in the experiment.

We know that the experimental probability of an event is given as:

$$\mathbf{P}(E) = \frac{\textbf{Number of trials in which the event happened}}{\textbf{Total number of trials}}$$

$$\therefore \text{Required probability} = \frac{\text{Number of trials in which 'A' is obtained}}{\text{Total number of trials}} = \frac{3}{10}$$

Thus, the probability of the most favourable outcome is 3/10.

Example 2:

In a pack of 5000 bulbs, 250 bulbs are defective. Find the probability that a bulb chosen at random will be non-defective.

Solution:

It is given that:

Total number of bulbs = 5000

Number of defective bulbs = 250

∴ Number of non-defective bulbs = 5000 − 250 = 4750

So, probability of choosing a non-defective bulb = $\frac{\text{Number of non-defective bulbs}}{\text{Total number of bulbs}}$

$$= \frac{4750}{5000}$$
$$= \frac{19}{20}$$

Medium

Example 1:

A die is thrown 100 times. The faces marked with numbers 1, 2, 3, 4, 5 and 6 are observed 20, 15, 25, 10, 10 and 20 times respectively. Find the probability of getting each of these events.

Solution:

Total number of trials = 100

Let E_1, E_2, E_3, E_4, E_5 and E_6 be the respective events of getting the faces marked with numbers 1, 2, 3, 4, 5 and 6.

Number of outcomes for the face marked '1' = 20

Number of outcomes for the face marked '2' = 15

Number of outcomes for the face marked '3' = 25

Number of outcomes for the face marked '4' = 10

Number of outcomes for the face marked '5' = 10

Number of outcomes for the face marked '6' = 20

We know that:

Probability of an event (E) = $\frac{\text{Number of outcomes of the event}}{\text{Number of trials}}$

So, we have:

$$P(E_1) = \frac{20}{100} = \frac{1}{5}$$

$$P(E_2) = \frac{15}{100} = \frac{3}{20}$$

$$P(E_3) = \frac{25}{100} = \frac{1}{4}$$

$$P(E_4) = \frac{10}{100} = \frac{1}{10}$$

$$P(E_5) = \frac{10}{100} = \frac{1}{10}$$

$$P(E_6) = \frac{20}{100} = \frac{1}{5}$$

Example 2:

In a survey conducted by a leading newspaper, 1000 families with two children were selected at random and the following data was recorded.

Number of boys in a family	2	1	0
Number of families	200	700	100

If we choose one of these families at random, then what is the probability that the chosen family has

i)1 boy.

ii) 2 boys.

iii) no boy.

Solution:

Let E_1, E_2 and E_3 denote the respective events of a family having one boy, two boys and no boy.

We know that:

Probability of an event (E) = $\frac{\text{Number of outcomes of the event}}{\text{Number of trials}}$

Using this formula, we can calculate the probabilities of events E_1, E_2 and E_3.

i) $P(E_1) = \frac{\text{Number of families having 1 boy}}{\text{Total number of families}}$

$$= \frac{700}{1000}$$
$$= \frac{7}{10}$$

ii) $P(E_2) = \frac{\text{Number of families having 2 boys}}{\text{Total number of families}}$

$$= \frac{200}{1000}$$
$$= \frac{1}{5}$$

iii) $P(E_3) = \frac{\text{Number of families having no boy}}{\text{Total number of families}}$

$$= \frac{100}{1000}$$
$$= \frac{1}{10}$$

Hard

Example 1:

The monthly salary ranges of 100 workers of a company are given in the following table.

Salary ranges (in Rs)	Number of workers in each range
0−1000	10
1000−2000	15
2000−3000	25
3000−4000	35
4000−5000	15

If a worker is chosen at random, then find the probability of selecting a worker who earns

i) above Rs 3000 per month.

ii) below Rs 2000 per month.

iii) Rs 24000 and Rs 48000 per annum.

Solution:

i)Total number of workers = 100

Number of workers who earn above Rs 3000 per month = 35 + 15 = 50

Thus, the probability of selecting a worker who earns above Rs 3000 per month is given as:

$$\frac{\text{Number of workers earning above Rs 3000 per month}}{\text{Total number of workers}}$$
$$= \frac{50}{100}$$
$$= \frac{1}{2}$$

ii) Number of workers who earn below Rs 2000 per month = 10 + 15 = 25

Thus, the probability of selecting a worker who earns below Rs 2000 per month is given as:

$$\frac{\text{Number of workers earning below Rs 2000 per month}}{\text{Total number of workers}}$$
$$= \frac{25}{100}$$
$$= \frac{1}{4}$$

iii)Annual salary of the workers = Salaries of 12 months

Now, the monthly salary of workers getting an annual salary of Rs 24000 is given as:

$$\frac{\text{Annual salary}}{12}$$
$$= \text{Rs}\left(\frac{24000}{12}\right)$$
$$= \text{Rs } 2000$$

Similarly, the monthly salary of workers getting an annual salary of Rs 48000 is given as:

$$\frac{\text{Annual salary}}{12}$$
$$= \text{Rs}\left(\frac{48000}{12}\right)$$
$$= \text{Rs } 4000$$

Number of workers earning between Rs 24000 and Rs 48000 per year

= Number of workers earning between Rs 2000 and Rs 4000 per month

= 25 + 35

= 60

Thus, the probability of selecting a worker who earns between Rs 24000 and Rs 48000 per annum is given as:

$$\frac{\text{Number of workers earning between Rs 2000 and Rs 4000 per month}}{\text{Total number of workers}}$$
$$= \frac{60}{100}$$
$$= \frac{3}{5}$$

Example 2:

There are 150 telephone numbers on each page of a telephone directory. The frequency distribution of the unit-place digits in the telephone numbers on a particular page is shown.

Digit:0123456789

Frequency:15221215171216151412

A number is chosen at random. Find the probability that the digit at the unit's place is

i) 6.

ii) a non-zero multiple of 3.

iii) a non-zero even number.

Solution:

i)Total number of telephone numbers = 150

It is given that the digit '6' occurs 16 times at the unit's place.

∴ Probability that the digit at the unit's place is 6 = $\frac{16}{150} = 0.1067$

ii) A non-zero multiple of 3 means 3, 6 or 9.

Number of telephone numbers in which the unit's digit is 3, 6 or 9 = 15 + 16 + 12 = 43

∴ Probability that the digit at the unit's place is a non-zero multiple of 3 = $\frac{43}{150} = 0.286$

iii)A non-zero even number means 2, 4, 6 or 8.

Number of telephone numbers in which the unit's digit is 2, 4, 6 or 8 = 12 + 17 + 16 + 14

= 59

∴ Probability that the digit at the unit's place is a non-zero even number = $\frac{59}{150} = 0.393$

Theoretical and Experimental Probability

Consider an experiment of tossing a coin. Before tossing a coin, we are not sure whether head or tail will come up. To measure this uncertainty, we will find the probability of getting a head and the probability of getting a tail.

A student tosses a coin 1000 times out of which 520 times head comes up and 480 times tail comes up.

The probability of getting a head is the ratio of the number of times head comes up to the total number of times he tosses the coin.

Probability of getting a head $= \frac{520}{1000}$

= 0.52

Similarly, probability of getting a tail = 480/1000

= 0.48

These are the probabilities obtained from the result of an experiment when we actually perform the experiment. The probabilities that we found above are called **experimental (or empirical) probabilities.**

On the other hand, the probability we find through the theoretical approach without actually performing the experiment is called theoretical probability.

The **theoretical probability (or classical probability)** of an event E, is denoted by P(E) and is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

Here, we assume that the outcomes of the experiment are equally likely.

When a coin is tossed, there are two possible outcomes. We can either get a head or a tail and these two outcomes are equally likely. The chance of getting a head or a tail is 1.

Thus, probability of getting a head P(E)

$$= \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}} = \frac{1}{2}$$

Similarly, probability of getting a tail = 1/2

Here, 1/2 (or 0.5) is the theoretical probability.

Relation between Experimental and Theoretical Probabilities:

There is a fact that the experimental probability may or may not be equal to the theoretical probability.

For example, if we take a coin and toss it by a particular number of times then the theoretical probability of getting a head or a tail will be $\frac{1}{2} = 0.5$ in each trial, but if we observe the outcomes of all the trials and calculate the experimental probability for head or tail then it will not be exactly equal to theoretical probability.

For approval, we can consider the theoretical and experimental probabilities of getting tail in the above experiment.

We have:

Theoretical probability of getting head = 0.52

Also, in our experiment we obtained:

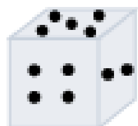
Experimental probability of getting tail = 0.5

It can be observed that experimental probability is not exactly equal to theoretical probability, but very close to it.

Also, experimental probability of the same event can vary according to the number of trials.

Finding Probability Using Complement of a Known Event

Consider the experiment of throwing a dice. Any of the numbers 1, 2, 3, 4, 5, or 6 can come up on the upper face of the dice. We can easily find the probability of getting a number 5 on the upper face of the dice?



Mathematically, probability of any event *E* can be defined as follows.

$$P(E) = \frac{\text{Number of outcomes favourable to event } E}{\text{Number of all possible outcomes}} = \frac{n(E)}{n(S)}$$

Here, *S* represents the sample space and *n*(*S*) represents the number of outcomes in the sample space.

For this experiment, we have

Sample space (*S*) = {1, 2, 3, 4, 5, 6}. Thus, *S* is a finite set.

So, we can say that the possible outcomes of this experiment are 1, 2, 3, 4, 5, and 6.

∴ Number of all possible outcomes = 6

Number of favourable outcomes of getting the number 5 = 1

∴ Probability (getting 5) = 1/6

Similarly, we can find the probability of getting other numbers also.

P (getting 1) = 1/6, *P* (getting 2) = 1/6, *P* (getting 3) = 1/6, *P* (getting 4) = 1/6 and

P (getting 6) = 1/6

Let us add the probability of each separate observation.

This will give us the sum of the probabilities of all possible outcomes.

P (getting 1) + P (getting 2) + P (getting 3) + P (getting 4) + P (getting 5) + P (getting 6)

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

∴ “**Sum of the probabilities of all elementary events is 1”.**

Now, let us find the probability of **not** getting 5 on the upper face.

The outcomes favourable to this event are 1, 2, 3, 4, and 6.

∴ Number of favourable outcomes = 5

∴ P (not getting 5) = 5/6

We can also see that P (getting 5) + P (not getting 5) $= \frac{1}{6} + \frac{5}{6} = 1$

∴ “**Sum of probabilities of occurrence and non occurrence of an event is 1”.**

i.e. **If *E* is the event, then *P* (*E*) + *P* (not *E*) = 1** … (1)

or we can write ***P(E) = 1 − P* (not *E*)**

Here, the events of getting a number 5 and not getting 5 are complements of each other as we cannot find an observation which is common to the two observations.

Thus, event **not *E*** is the complement of event *E.* Complement of event *E* is denoted by $\bar{E}$ or *E***'**.

Using equation (1), we can write

$$\boldsymbol{P(E) + P(\bar{E}) = 1}$$

Or

$$\boldsymbol{P(\bar{E}) = 1 - P(E)}$$

This is a very important property about the probability of complement of an event and it is stated as follows:

If *E* is an event of finite sample space S, then *P* ($\bar{E}$) = 1 – *P*(*E*) where $\bar{E}$ is the complement of event *E*.

Now, let us prove this property algebraically.

Proof:

We have,

$E \cup \bar{E} = S$ and $E \cap \bar{E} = \phi$

$\Rightarrow n(E \cup \bar{E}) = n(S)$ and $n(E \cap \bar{E}) = n(\phi)$

$\Rightarrow n(E \cup \bar{E}) = n(S)$ and $n(E \cap \bar{E}) = 0$ ...(1)

Now,

$n(E \cup \bar{E}) = n(S)$

$\Rightarrow n(E) + n(\bar{E}) - n(E \cap \bar{E}) = n(S)$

$\Rightarrow n(E) + n(\bar{E}) - 0 = n(S)$ [Using (1)]

$\Rightarrow n(\bar{E}) = n(S) - n(E)$

On dividing both sides by $n(S)$, we get

$$\frac{n(\bar{E})}{n(S)} = \frac{n(S)}{n(S)} - \frac{n(E)}{n(S)}$$

$\Rightarrow P(\bar{E}) = 1 - P(E)$

Hence proved.

Let us solve some examples based on this concept.

Example 1:

One card is drawn from a well shuffled deck. What is the probability that the card will be

(i) a king?

(ii) not a king?

Solution:

Let *E* be the event 'the card is a king' and *F* be the event 'the card is not a king'.

(i) Since there are 4 kings in a deck.

$\therefore$ Number of outcomes favourable to *E* = 4

Number of possible outcomes = 52

$\therefore P(E) = \frac{4}{52} = \frac{1}{13}$

2. Here, the events E and F are complements of each other.

$\therefore P(E) + P(F) = 1$

$P(F) = 1 - \frac{1}{13}$

$= \frac{12}{13}$

Example 2:

If the probability of an event A is 0.12 and B is 0.88 and they belong to the same set of observations, then show that A and B are complementary events.

Solution:

It is given that $P(A) = 0.12$ and $P(B) = 0.88$

Now, P(A) + P(B) = 0.12 + 0.88 = 1

$\therefore$ The events A and B are complementary events.

Example 3:

Savita and Babita are playing badminton. The probability of Savita winning the match is 0.52. What is the probability of Babita winning the match?

Solution:

Let E be the event ‘Savita winning the match’ and F be the event ‘Babita wining the match’.

It is given that $P(E) = 0.52$

Here, E and F are complementary events because if Babita wins the match, Savita will surely lose the match and vice versa.

$\therefore P(E) + P(F) = 1$

$0.52 + P(F) = 1$

$P(F) = 1 - 0.52 = 0.48$

Thus, the probability of Babita winning the match is 0.48.

Example 4:

In a box, there are 2 red, 5 blue, and 7 black marbles. One marble is drawn from the box at random. What is the probability that the marble drawn will be (i) red (ii) blue (iii) black (iv) not blue?

Solution:

Since the marble is drawn at random, all the marbles are equally likely to be drawn.

Total number of marbles = 2 + 5 + 7 = 14

Let A be the event ‘the marble is red’, B be the event ‘the marble is blue’ and C be the event ‘the marble is black.

(i) Number of outcomes favourable to event A = 2

$\therefore P(A) = \frac{2}{14} = \frac{1}{7}$

(ii) Number of outcomes favourable to event B = 5

$\therefore P(B) = 5/14$

(iii) Number of outcomes favourable to event C = 7

$\therefore P(C) = \frac{7}{14} = \frac{1}{2}$

(iv) We have, $P(B) = 5/14$

The event of drawing a marble which is not blue is the complement of event B.

$\therefore P(\overline{B}) = 1 - P(B) = 1 - \frac{5}{14} = \frac{9}{14}$

Thus, the probability of drawing a marble which is not blue is 9/14.