Chapter 3

Transmission Lines

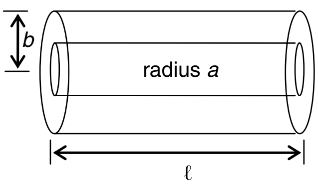
CHAPTER HIGHLIGHTS

Introduction

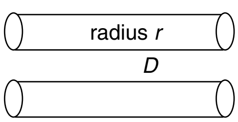
A transmission line is a means of transfer of energy or information from one body to another with maximum efficiency and minimum losses.

Transmission line must have forward path and return path.

The following are the examples of transmission lines.



(i) Coaxial cable



(ii) Parallel wires

Types of Transmission Lines

The various types of transmission lines are

1. Two-wire parallel lines
2. Coaxial lines
3. Twisted wires
4. Parallel plates or planar lines
5. Wire above conducting lines
6. Microstrip lines
7. Optical fibres

Construction of Transmission Lines

Radio frequency transmission lines are commonly in the following three forms:

Two-wire Line

This consists of two parallel wires whose uniform spacing, that is, is small when compared with the electrical wavelength.

Horizontal two-wire and four-wire lines are balanced to ground devices. If at a point of one conductor, there is some positive voltage with respect to the ground, then it will be balanced by an equal negative voltage at the corresponding point of the other conductor.

A two-wire line in vertical configuration is unbalanced structure because of which it gives rise to unbalanced currents. Further, these currents are undesirable as they increase energy losses. The characteristic impedance of such lines ranges from 100 Ω to 300 Ω.

Four-wire Line

This is constructed as follows:

Four wires placed at the corners of small square, diagonally opposite wires that are connected in parallel.

When compared to two-wire line, this is the weaker external field. Further, neighbouring four-wire lines have less mutual effect than similarly situated two-wire lines.

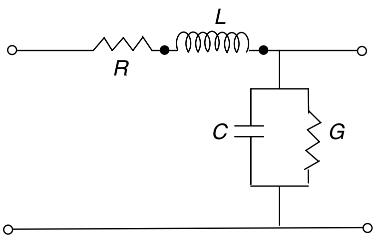
Coaxial Transmission Line

Coaxial line consists of a round conductor supported coaxially (by insulators) within a round tube that serves as the second conductor. Further, this type of line has practically no external field when used with shielded transmission.

Applications of Transmission Lines

1. They are used in transfer energy from one circuit to another circuit.
2. They are used as circuit elements.
3. They can be used as impedance matching devices.
4. They can be used as stubs.
5. Transmission lines can be used as measuring devices.
6. Coaxial cable are frequently used in laboratories and to connect television to TV antennas.
7. Twisted pairs and coaxial cables are used in computer networks such as Ethernet and internet.
8. Pair of parallel lines are used in telephony and power transmission.

EQUIVALENT CIRCUIT OF A PAIR OF TRANSMISSION LINES



The equivalent circuit of a transmission line is a distributed network. This consists of cascaded sections and each section consists of a series resistance R, series inductance L, shunt capacitance C, and shunt conductance G. The equivalent circuit is shown in the above mentioned figure. Here, R is expressed in ohm/unit length, L in H/unit length, C in F/length, and G in Mho/unit length.

Primary Constants

The primary constants are as follows:

1. Resistance R
2. Conductance G
3. Capacitance C
4. Inductance L

1. **Resistance R (Ω/km):**
   (a) Resistance per unit length (R/L)
   (b) It is due to the non-ideal conducting wire of the line.
   (c) It is distributed all along the line as $\rho l/A$.
   (d) It is a series aspect on the line that causes voltage attenuation or voltage losses.
2. **Conductance G** (mho/km)**:**
   (a) Conductance per unit length (G/l).
   (b) It is due to the non-ideal dielectric used between the lines.
   (c) It is distributed all along the line.
   (d) It is a shunt aspect that causes current leakage losses or current losses.
3. **Capacitance C** (F/km)
   (a) Capacitance per unit length (C/l)
   (b) It is due to electric field contained by the voltage on the line.
   (c) It is a shunt aspect across the two lines.
   (d) It is distributed all along the line.
4. **Inductance L** (H/km):
   (a) Inductance per unit length (L/l).
   (b) It is due to the magnetic fields contained by the currents on the line.
   (c) It is series aspect along the wire.
   (d) It is distributed all along the line.

Salient Features of Primary Constants

1. R is defined as loop resistance per unit length of line.
2. L is defined as loop inductance per unit line length.
3. C is defined as shunt capacitance between the two wires per unit length.
4. G is defined as the conductance per unit length due to dielectric medium separating the conduction.
5. R, L, C, and G are distributed along the length of the line.
6. For each line, the conductors are characterized by σ_c and $\mu_c = \mu_0$, $\varepsilon_c = \varepsilon_0$, and the dielectric medium, which is basically homogeneous, separating the conductors is characterized by σ_d, μ_d, and ε_d.
7. R, L, C, and G depend on the geometry of transmission line, characteristics of the dielectric material, and in some cases, on the frequency.

Two-wire Line

$$L = \frac{\mu_{0\ell}}{\pi}\left[\frac{1}{4} + \ell n\left[{}^{d}\!/\!_{r}\right]\right] \cong \frac{\mu_{o.}\ell}{\pi}\ell n\left[{}^{d}\!/\!_{r}\right]$$

$$C = \frac{\pi\varepsilon_{o.}\ell}{\ell n\left[{}^{d}\!/\!_{r}\right]}$$

where ℓn is $\log_e$

$$R = \frac{r\ell}{\delta_c} R_{dc}\ {}^{\Omega}\!/\!_{m}.$$

r is the radius of wire.

$$R_{dc} = \frac{1}{\sigma_c \pi r^2} \text{ and } \delta = \frac{1}{\sqrt{\pi f \mu_{c.} \sigma_c}}$$

$$G = \frac{\pi \sigma_d l}{\ell n\left(d/r\right)}$$

σ_d is the conductivity of dielectric involved in transmission line. $L/\ell, C/\ell, R/\ell$ and G/ℓ are distributed parameters.

Coaxial Line

$$L = \frac{\mu_o}{2\pi} \ell n\left(b/a\right) \; H/m$$

$$C = \frac{2\pi \in_0}{\ell n\left(b/a\right)} \; F/m$$

$$R = \frac{1}{2\pi \sigma_c \delta}\left[\frac{1}{b} + \frac{1}{a}\right] \Omega/m$$

$$G = \frac{2\pi\sigma_d}{\ell n\left(b/a\right)} \mho/\text{m}$$

For both, $LC = \mu_o \in_o$

Solved Examples

Example 1

If the inner and outer radii of coaxial cable are 2 cm and 4 cm, respectively, then the inductance and capacitance of coaxial cable per unit length are

(A) 0.15 nH/m, 0.08 nF/m
(B) 0.138 μH/m, 0.08 nF/m
(C) 0.15 μH/m, 0.08 μF/m
(D) 0.138 nH/m, 0.08 nF/m

Solution

$$L = \frac{\mu_o}{2\pi} \ell n\left(b/a\right)$$

$$= \frac{\mu_o}{2\pi} . \ell n(2)$$

$$= 2 \times 10^{-7} \ell n(2)$$

$$= 1.38 \times 10^{-7} \text{ H/m}$$

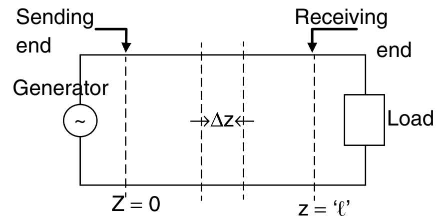
$$= 0.138 \; \mu\text{H/m}$$

$$C = \frac{2\pi \in_o}{\ell n\left(b/a\right)}$$

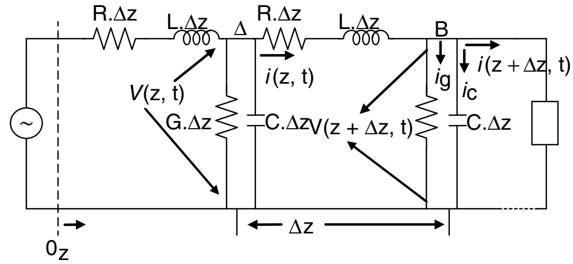
$$C = 79.95 \times 10^{-12} \text{ F/m}$$

$$= 0.07995 \text{ nF/m}$$

Transmission Line Equations



The distributed network of abovementioned transmission line block diagram is shown in the following figure.



If we solve the abovementioned distributed circuit, we get

$$\frac{d^2 v}{dz^2} = (R + j\omega L)(G + j\omega C)V$$

$$\frac{d^2 \text{I}}{dz^2} = (R + j\omega L)(G + j\omega C)\text{I}$$

where $z = R + j\omega L$

$y = G + j\omega C$

γ is the propagation constant.

$$\gamma = \sqrt{ZY} = \alpha + j\beta \quad (3.1)$$

Lossless Line

$$R = G = 0$$

$$\therefore \frac{dv}{d\,z} = -j\omega LI \quad (3.2)$$

$$\frac{dI}{dz} = -j\omega CV \quad (3.3)$$

$$\frac{d^2 V}{dz^2} = -\omega^2 LCV \quad (3.4)$$

$$\frac{d^2 I}{dz^2} = -\omega^2 LCI \quad (3.5)$$

Solution of Transmission Line Equation

$$\text{V} = \text{V}_+ \text{e}^{-\gamma z} + \text{V}_- \text{e}^{\gamma z} \quad (3.6)$$

$\downarrow$ Wave travelling from sending end to receiving end

$\downarrow$ Wave travelling from receiving end to sending end

From Equation (3.6)

$$\frac{dV}{dz} = -\gamma . V_+ . e^{-\gamma z} + \gamma . V_- e^{\gamma z}$$

$$I = \frac{-1}{z}\frac{dV}{dz} \text{ (from (9))}$$

$$I = \frac{\gamma}{2}\left[v_+ e^{-\gamma z} - v_- e^{\gamma z}\right]$$

$$\frac{\gamma}{2} = \frac{\sqrt{z\,y}}{z} = \sqrt{\frac{y}{z}} = \frac{1}{z_0} = y_0$$

where Z_o is the characteristic impedance and Y_o is the characteristic admittance.

$$I = Y_o\left[V_+ e^{-\gamma z} - V_- e^{\gamma z}\right] \tag{3.7}$$

This is the solution of transmission line in current form. In line theory, the current that flows from receiving end to sending end takes negative sign.

$$\gamma = \alpha + j\beta$$

$$\gamma z = \alpha z + j\beta z$$

$$e^{-\gamma z} = e^{-\alpha z} \,.\, e^{-j\beta z}$$

The change in phase per length ℓ is $\beta\ell$ (radius).

Propagation Constant

$$\gamma = \sqrt{ZY}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega c)} = \alpha + j\beta$$

where α is the rate of attenuation as the wave travels along the line and β is the phase constant.

Lossless Line

$$R = G = 0$$

$$\gamma = j\omega\sqrt{LC}$$

$$\alpha = 0,\ \beta = \omega\sqrt{LC}$$

Low-Loss Line or High Frequency Approximation

$$\omega L >> R \quad \omega C >> G$$

$$\gamma = \alpha + j\beta$$

$$= j\omega\sqrt{LC}.\sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega c}\right)}$$

$$= j\omega\sqrt{Lc}\left[\left(1 + \frac{R}{2\,j\omega c}\right)\left(1 + \frac{G}{2\,j\omega c}\right)\right]$$

$$(1+x)^{1/2} \cong 1 + \frac{x}{2} \text{ (if } x << 1)$$

$$\gamma \cong \frac{1}{2}\left[R\sqrt{\frac{c}{L}} + G\sqrt{\frac{L}{c}}\right] + j\omega\sqrt{LC}$$

$$\alpha = \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right]$$

$$\beta = \omega\sqrt{LC}$$

General Lossy Line

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega c)}$$

Solving the abovementioned equation:

$$\alpha = \frac{1}{\sqrt{2}}\left[\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} + (RG - \omega^2 LC)\right]^{1/2}$$

$$\beta = \frac{1}{\sqrt{2}}\left[\sqrt{(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)} - (RG - \omega^2 LC)\right]^{1/2}$$

Characteristic Impedance

It is the ratio of voltage to current of any one wave, that is, either forward or reverse travelling wave.

Lossless Line

$$R = G = 0$$

$$Z_o = \sqrt{\frac{L}{C}} = R_o + jx_o$$

$$R_o = \sqrt{\frac{L}{C}},\ x_o = 0$$

The characteristic impedance of loss line is purely resistive.

Low Lossy Line

$$\omega L >> R$$

$$\omega C >> G$$

$$Z_o = \sqrt{\frac{L}{C}}\left[1 + \frac{R}{j\omega L}\right]^{1/2}\left[1 + \frac{G}{j\omega c}\right]^{-1/2}$$

$$Z_o = \sqrt{\frac{L}{C}}\left[1 + \frac{R}{2\,j\omega c}\right]\left[1 - \frac{G}{2j\omega c}\right]$$

$$Z_o = \sqrt{\frac{L}{C}}\left[1 - \frac{j}{2\omega}\left[\frac{R}{L} - \frac{G}{C}\right]\right]$$

$$R_o = \sqrt{\frac{L}{C}}$$

$$X_o = \frac{-1}{2\omega}\sqrt{\frac{L}{C}}\left[\frac{R}{L} - \frac{G}{C}\right]$$

General Lossy Line

$$Z_o = R_o + jx_o = \sqrt{\frac{R + j\omega L}{G + j\omega c}}$$

Primary Constants in Terms of Secondary Constants

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega c)} = \alpha + j\beta$$

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega c}} = R_o + jx_o$$

$$\gamma z_0 = R + j\omega L = (\alpha + j\beta)(R_o + jx_o)$$

$$R = \alpha R_o - \beta x_o$$

$$L = \frac{\alpha x_o + \beta R_o}{\omega}$$

Similarly,

$$G = \frac{\alpha R_o + \beta x_o}{R_o^2 + x_o^2}$$

$$C = \frac{1}{\omega\left(R_o^2 + x_o^2\right)}(\beta R_o - \alpha x_o)$$

Example 2

A coaxial cable has the characteristic impedance of 75 Ω and a nominal capacitance of $1/9$ nF/m. The inductance of the cable is

(A) 0.625 nH/m (B) 0.625 μH/m
(C) 0.625 mH/n (D) 0.625 H/m

Solution

$$z_o = \sqrt{\frac{L}{c}}$$

$$L = z_o^2 c = 75 \times 75 \times \frac{1}{9} \times 10^{-9} = 0.625\ \mu\text{H/m}$$

Example 3

Airline has characteristic impedance of 70 Ω and phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter and the capacitance per meter of the line.

(A) 682 nH/m, 334 PF/m
(B) 682 PH/m, 334 PF/m
(C) 68.2 PF/m, 334.2 nH/m
(D) 334.2 nH/m, 68.2 PF/m

Solution

Airline can be regarded as lossless line

$$\alpha = 0, R = 0 = G$$

$$z_o = \sqrt{\frac{L}{C}} = 70\Omega \tag{1}$$

$$\beta = \omega\sqrt{LC} = 3 \tag{2}$$

$$\omega = 2\pi \times 10^8 \text{ rad/s}$$

Solving (1) and (2)

$$C = 68.2 \text{ PF/m}$$

$$L = 334.2 \text{ nH/m}$$

Condition for Distortionless Transmission

If the signal received is exact replica of the transmitted signal, then the signal is said to be distortionless.

The exact replica is obtained when

1. If all the frequency components are attenuated to same level.
2. Further, all frequency components travel with same velocity.

This means attenuation constant is independent of frequency and phase constant is a linear function of frequency

$$\beta \, \alpha f$$

Distortion is an effect that changes the phase of the signal and can be in two types:

1. **Frequency distortion**: If the attenuation α of a line is frequency dependent, then this distortion occurs.
2. **Delay distortion**: If the velocity of a line is frequency dependent, then this distortion occurs.

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega c)}$$

$$\gamma^2 = RG + j\omega(RC + LG) - \omega^2 LC.$$

$$\text{If } RC = LG = \sqrt{RCLG}$$

$$RC + LG = 2\sqrt{RCLG}$$

$$\gamma^2 = \left(\sqrt{RG} + j\omega\sqrt{LC}\right)^2$$

$$(\alpha + j\beta)^2 = \left(\sqrt{RG} + j\omega\sqrt{LC}\right)^2$$

$$\alpha = \sqrt{RG}$$

$$\beta = \omega\sqrt{LC}$$

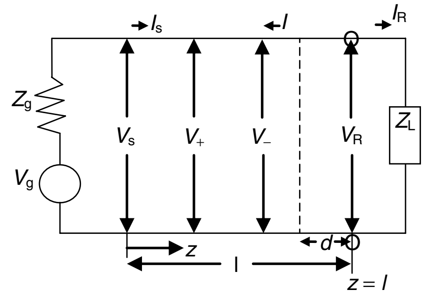
$$\theta_P = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$RC = LG$$

$$\boxed{\frac{R}{L} = \frac{G}{C}} \tag{3.8}$$

The abovementioned equation is called condition for distortionless transmission.

Reflection Coefficient



According to Equations (3.6) and (3.7)

$$V = V_+ e^{-\gamma z} + V_- e^{\gamma z}$$

$$I = \frac{V_+}{z_o} e^{-\gamma z} - \frac{V_-}{z_o} e^{\gamma z}$$

$$\text{at } z = \ell,\ v = v_R,\ I = I_R$$

where suffix R is the receiving end.

$$V_R = v_+ e^{-\gamma\ell} + v_- e^{\gamma\ell}$$

$$I_R = \frac{V_+}{z_o} e^{-\gamma\ell} - \frac{V_-}{z_o} e^{\gamma\ell}$$

$$Z_L = \frac{v_R}{I_R} = z_o\left(\frac{v_+ e^{-\gamma\ell} + v_- e^{\gamma\ell}}{v_+ e^{-\gamma\ell} - v_- e^{\gamma\ell}}\right)$$

$$\rho = \frac{\text{reflected voltage/current}}{\text{Incident voltage/current}}$$

$$\therefore\ \rho = \frac{V_{ref}}{V_{inc}} = \frac{-I_{ref}}{I_{inc}}$$

$$\rho_\ell = \frac{v_- e^{\gamma\ell}}{v_+ e^{-\gamma\ell}} = \frac{z_L - z_o}{z_L + z_o}$$

ρ_l is the reflection coefficient at $z = \ell$ (at load).

$$\rho = |\rho_l| .\, e^{j\theta_\ell}$$

θ_l is the phase angle between incident and reflected wave.

$$0 \le |\rho_\ell| \le 1$$

ρ at any point can be calculated as

$$\rho = \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}}$$

Let the point be at a distance 'd' from the receiving end. Then,

$$z = l - d$$

$$\therefore \quad \rho_d = \frac{v_- e^{\gamma(\ell - d)}}{v_+ e^{-\gamma(\ell - d)}}$$

$$= \frac{V_- e^{\gamma\ell} . e^{-\gamma d}}{V_+ e^{-\gamma\ell} . e^{\gamma d}}$$

$$\rho_d = \rho_\ell . e^{-2\gamma d}$$

$$\rho_d = \rho_\ell e^{-2\alpha d}\, e^{-2j\beta d}$$

$$\rho_d = |\rho_\ell| . e^{-2\alpha d} . e^{j(\theta}\, e^{j(\theta_\ell - 2\beta d)} \qquad (3.9)$$

Example 4

In a transmission line the reflection co-efficient at the load is given by $0.8e^{-j60}$. What is the reflection co-efficient at a distance of 0.1 wavelength towards the source?

(A) $0.8e^{j228^o}$ (B) $0.8e^{-j228^o}$
(C) $0.8e^{j132^o}$ (D) $0.8e^{-j72^o}$

Solution

$$\rho_l = 0.8e^{-j60^o}$$

$$\rho_d = \rho_l . e^{-j2\beta d}$$

$$2\beta d = 2 \times \frac{2\pi}{\lambda} . 0.1\lambda$$

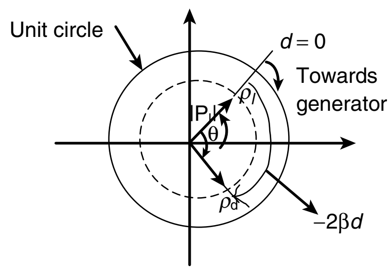
$$= 0.4\pi = 72^o$$

$$\rho_d = 0.8e^{-j132^o} = 0.8e^{j228^o}$$

Lossless Line

$$\alpha = 0$$

$$\rho_d = |\rho_\ell| .\, e^{j(\theta_\ell - 2\beta d)}$$

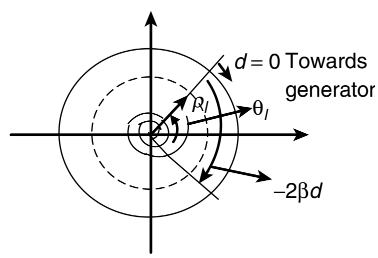


As d moves towards generator (i.e., towards sending end), 'd' increases positively.

On a lossless line, ρ varies circularly towards the generator.

Lossy Line

$$\rho_d = |\rho_l| . e^{-2\alpha d}\, e^{j(\theta_\ell - 2\beta d)}$$



ρ on a lossy transmission line varies spirally inwards.

If $\quad z_L = z_o,\ \rho_\ell = 0T$

$$\rho_d = 0$$

Matched Line

Reflected wave is absent on the line when $V_r = 0, I_r = 0, \Gamma = 0$, and hence, $Z_L = Z_0$ under this condition. The load is said to be matched to the line and all the incident power on the matched load is absorbed in it with no power reflected back.

Line Impedances

$$Z = \frac{v(z)}{I(z)}$$

$$v = v_+ e^{-\gamma z} + v_- e^{\gamma z}$$

$$I = \frac{1}{z_o}\left(v_+ e^{-\gamma z} - v_- e^{\gamma z}\right)$$

at $z = l,\ v = v_R = I_R z_L$

$$I_R z_L = v_+ e^{-\gamma l} + v_- e^{\gamma l}$$

$$I_R z_o = v_+ e^{-\gamma l} - v_- e^{\gamma l}$$

$$v_+ = \frac{I_R}{2}(z_L + z_o)e^{\gamma l}$$

$$V_- = \frac{I_R}{2}(z_L - z_o)e^{-\gamma l}$$

$$z = \ell - d$$

$$v = \frac{I_R}{2}\left[(z_L + z_o)e^{\gamma d} + (z_L - z_o)e^{-\gamma d}\right] \quad (3.10)$$

$$I = \frac{I_R}{2z_o}\left[(z_L + z_o)e^{\gamma d} - (z_L - z_o)e^{-\gamma d}\right] \quad (3.11)$$

$$Z = \frac{V}{I}$$

$$Z = Z_o\left[\frac{(z_L + z_o)e^{\gamma d} + z_L - z_o e^{-\gamma d}}{(z_L + z_o)e^{\gamma d} - (z_L - z_o)e^{-\gamma d}}\right]$$

$$e^{\pm\gamma d} = \cos h(\gamma d) \pm \sin h(\gamma d)$$

$$Z = Z_o\left[\frac{z_L + z_o \tan h(\gamma d)}{z_o + z_L \tan h(\gamma d)}\right]$$

Example 5

A 40-m long transmission line has $v_g = 15 \angle 0^o\ v_{rms}$ and $z_o = 30 + j40\Omega$ and $v_L = 5 \angle -48^o$. If the line is matched to load, then the input impedance is

(A) $30 + j40\Omega$ (B) $150 + j200\Omega$
(C) $30 - j40\Omega$ (D) $40 + j30\Omega$

Solution

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan\gamma_\ell}{Z_o + jZ_L + \tan\gamma_\ell}\right]$$

The line is matched

$$Z_L = Z_o$$

$$Z_{in} = Z_o$$

Lossless Line

$$\alpha = 0 \therefore \gamma = j\beta$$

$$\tan h(\gamma d) = j \tan \gamma d$$

$$= j \tan \beta d$$

$$\therefore \quad Z = Z_0\left[\frac{z_L + jz_0 \tan\beta d}{z_0 + jz_L \tan\beta d}\right]$$

at $d = l$

$z = z_s$(sending-end impedance)

$$z_s = z_0\left[\frac{z_L + jz_o \tan\beta l}{z_o + jz_L \tan\beta l}\right] \quad (3.12)$$

Impedance in terms of Reflection Co-efficient

$$Z = Z_o\left[\frac{1 + \left(\frac{z_L - z_o}{z_L + z_o}\right)e^{-2\gamma d}}{1 - \left(\frac{z_L - z_o}{z_L + z_o}\right)e^{-2\gamma d}}\right]$$

$$Z = Z_o\left[\frac{1 + \rho_\ell . e^{-2\gamma d}}{1 - \rho_\ell . e^{-2\gamma d}}\right]$$

$$Z = Z_o\left[\frac{1 + \rho_d}{1 - \rho_d}\right]$$

$$(\rho_d = \rho_l\, e^{-2\gamma d})$$

$$\rho_d = |\rho_d|.e^{j\phi}$$

$$\phi = \theta l - 2\beta d$$

$$|\rho_d| = |\rho_e|.e^{-2\alpha d}$$

Example 6

In Example 5, the relation reflection co-efficient at the load is

(A) 0 (B) 1 (C) −1 (D) ∞

Solution

If the line is matched, then the reflection co-efficient at the load is

$$\rho = \frac{Z_L - Z_O}{Z_L + Z_O}$$

$$\rho = 0\ (Z_L = Z_O)$$

Impedance Variation along a Lossless Line

$$Z(d) = z_o\left[\frac{1 + |\rho_d|.e^{j\phi}}{1 - |\rho_d|.e^{j\phi}}\right] \quad (3.13)$$

$$z\left(d + \frac{n\lambda}{2}\right) = z_o\left[\frac{1 + |\rho_d|.e^{j(\phi + \beta n\lambda)}}{1 - |\rho_d|.e^{j(\phi + \beta n\lambda)}}\right]$$

$$= z_o\left[\frac{1 + |\rho_d|.e^{j\phi}}{1 - |\rho_d|.e^{j\phi}}\right]$$

$$= z\,(d)$$

Line impedance on a lossless transmission line repeats for every half of wavelength.

Sending-end Impedance

If $\qquad Z_L = 0$

This means load is short-circuited

$$Z_{sc} = jz_0 \tan h(\gamma \ell)$$

$$Z_{sc} = jz_o \tan \gamma \ell \qquad (3.14)$$

Example 7

For a short-circuited lossless transmission line with length $\ell = \lambda/8$, the input impedance is (Z_o is the characteristic impedance)

(A) $-jZ_o$ (B) Z_o (C) jZ_o (D) 0

Solution

$$Z_{sc} = jZ_o \tan \beta \ell$$

$$Z_{sc} = jZ_o . \tan \frac{2\pi}{\lambda} . \lambda/8$$

$$Z_{sc} = jZ_o$$

(ii) If $\quad z_L = \infty$

i.e., If load is open circuited

$$\therefore \quad z_{oc} = z_o \cot h(\gamma \ell)$$

$$z_{oc} = -jz_0 \cot \gamma \ell \qquad (3.15)$$

Example 8

For open-circuited lossless transmission line with length $\lambda/4$, the input impedance is (Z_o is the characteristic impedance)

(A) $-jZ_o$ (B) Z_o (C) 0 (D) ∞

Solution

$$Z_{oc} = -jZ_o \cot \beta \ell$$

$$= -jZ_o . \cot \frac{2\pi}{\lambda} . \lambda/4 = -jZ_o . \cot \pi/2$$

$$Z_{oc} = 0$$

$$z_o = \sqrt{z_{sc} . z_{oc}} \qquad (3.16)$$

Example 9

If the short-circuited and open-circuited input impedances of a transmission line (loss less) are given by 25 Ω and 225 Ω, respectively, then the characteristic impedance of the transmission line is

(A) 50 Ω (B) 25 Ω (C) 75 Ω (D) 100 Ω

Solution

$$Z_o = \sqrt{Z_{sc} . Z_{oc}} \quad = \sqrt{25.225}$$

$$Z_o = 5.15 = 75\Omega$$

Example 10

The propagation constant in Example 9 is _____, if the length of the line is 10 m.

(A) 0.8435 (B) $j0.8435$
(C) $j(1.8435)$ (D) $-j(1.8435)$

Solution

$$\gamma = \frac{j}{\ell} \tan h^{-1}\left[\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right]$$

$$\gamma = \frac{j}{10} \tan^{-1}\left(\sqrt{1/9}\right)$$

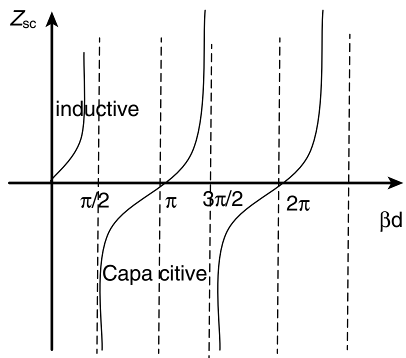
$$= \frac{j}{10} .18.435$$

$$= j1.8435$$

(iv) $\gamma = \frac{1}{\ell} \tan h^{-1}\left(\sqrt{\frac{z_{sc}}{z_{oc}}}\right) \qquad (3.17)$

(v) For lossless line:

(a) $z_{sc} = jz_0 \tan \beta d$



Example 11

If the length of the short-circuited transmission line is $3\lambda/8$, then the input impedance

(A) is inductive
(B) is capacitive
(C) is zero
(D) contains both real and imaginary parts

Solution

$$\beta d = \frac{2\pi}{\lambda} . \frac{3\lambda}{8}$$

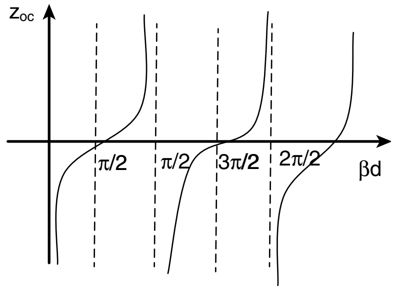
$$\beta d = \frac{3\pi}{4}$$

$$Z_{sc} = jZ_o \tan \beta l$$

$$= -jZ_o$$

$\therefore$ The impedance is capacitive.

(b) $Z_{oc} = -jz_o \cot \beta d$.



Standing Waves and Standing Wave Ratio

According to Equations (22) and (23)

$$V = v_R \cos \beta d + jI_R R_o \sin\beta d$$

$$I = I_R \cos\beta d + j.\frac{V_R}{R_o}\sin\beta d$$

Case (i) Open-circuited line:

$$z_L = \infty \therefore I_R = 0$$

$$v_{oc} = v_R \cos\beta d$$

$$I_{oc} = \frac{j\, v_R}{R_o}\sin\beta d$$

Case (ii) Short-circuited line:

$$z_L = 0 \therefore v_R = 0$$

$$v_{sc} = jI_R R_o \sin\frac{2\pi}{\lambda}.d$$

$$I_{sc} = I_R \cos\left(\frac{2\pi}{\lambda}.d\right)$$

In both the cases, voltage and current are in quadrature.

Case (iii) $z_L = R_o$ (matched line)

$$\rho\ell = 0$$

$$v = v_R.e^{j\beta d};$$

$$I = I_R.e^{j\beta d}$$

In this, maximum power is delivered to load.

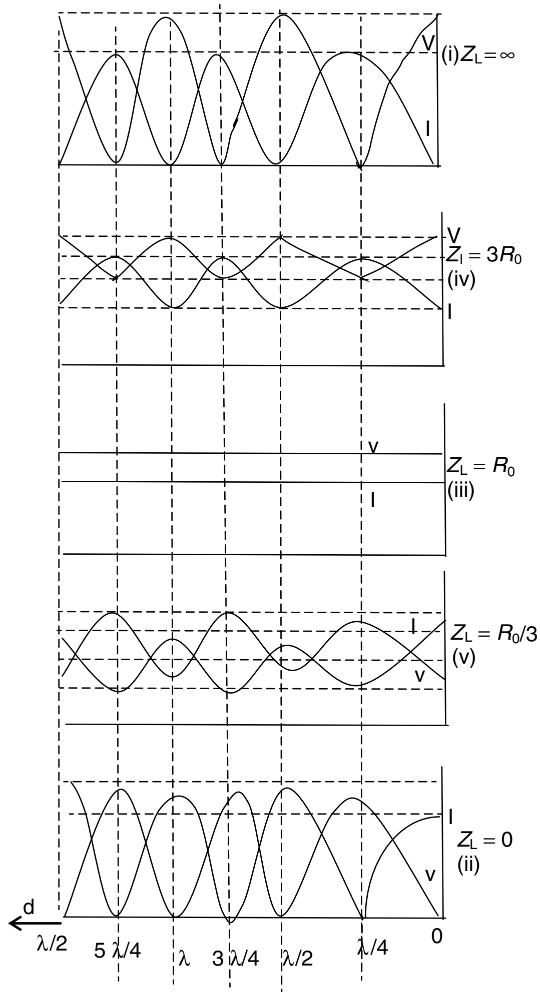
Case (iv): $z_L = R_L = 3R_o$

$$\therefore \qquad \rho\ell = \frac{z_L - z_o}{z_L + z_o} = \frac{3R_o - R_o}{3R_o + R_o} = \frac{1}{2}$$

The amplitude of reflected wave is $\frac{1}{2}$ of the incident wave, and these are in phase, since ρl has 'θ' phase angle.

Case (v): $z_L = R_L = \frac{R_o}{3}$ $\rho_\ell = \frac{-1}{2}$

$\therefore$ Phase angle of reflection co-efficient is 180°.

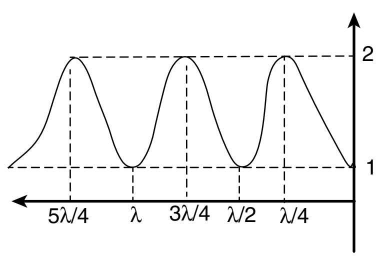


The standing wave shifted away from the load by $\theta_l/2$

$$S = v_{max}/v_{min} = I_{max}/I_{min}$$

Example 12

Voltage standing wave pattern in a lossless transmission line with characteristic impedance 50 Ω and a resistive load is shown in the figure.



Then VSWR is

(A) –2 (B) 2 (C) 0 (D) $\frac{1}{3}$

Solution

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{2}{1} = 2$$

Example 13

In Example 12, the reflection co-efficient is given by

(A) $1/3$ (B) 3 (C) $1/9$ (D) 0

Solution

$$\rho = \frac{S-1}{S+1} = \frac{2-1}{2+1} = 1/3$$

Maxima occurs at where incident and reflected waves are in phase

$$\beta d = n\pi$$

where $n = 0, 1, 2, 3\ldots.$

$$|v_{\max}| = \frac{v_R(z_L + z_o)}{2z_L}[1 + |\rho_\ell|]$$

Minima occurs at where incident and reflected waves are out of phase.

$$\beta d = (2n+1)\pi/2$$

$$n = 0, 1, 2, \ldots..$$

$$|v_{\min}| = \frac{v_R(z_L + z_o)}{2z_L}(1 - |\rho_\ell|)$$

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\rho_\ell|}{1-|\rho_\ell|}$$

$$|\rho_\ell| = \frac{s-1}{s+1} = \frac{|v_{\max}| - |v_{\min}|}{|v_{\max}| + |v_{\min}|}$$

VSWR Properties

1. **Matched line:** $\rho_l = 0$

   $$S = 1$$

2. VSWR cannot be measured on a lossy line since the wave is attenuating as it travels along the line from sending end to receiving end and vice versa.
3. **Low-loss line:** VSWR remains substantially constant over a section of line. Even for low-less line, '*s*' cannot be measured uniquely.
4. **Lossless line:** For a lossless line, *S* can be determined uniquely.

   Transmission line as an impedance transformer:

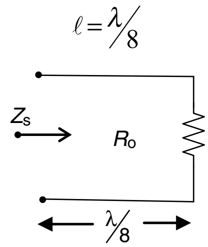
   $$z_s = z_o\left[\frac{z_L + jz_o \tan\beta\ell}{z_o + jz_L \tan\beta\ell}\right]$$

   where z_s is the sending-end impedance for a lossless line.

   $$z_o = R_o$$

   $$z_s = R_o\left[\frac{z_L + jR_o \tan\beta\ell}{R_o + jz_L \tan\beta\ell}\right]$$

One-eighth Wave Line

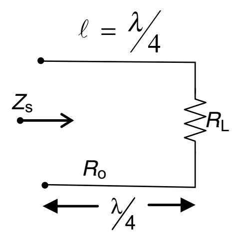


$$\beta l = \frac{2\pi}{\lambda}\cdot\lambda/8 = \pi/4$$

$$\therefore \quad z_s = R_o\left[\frac{z_L + jR_o}{R_o + jz_L}\right]$$

$$|z_s| = R_o$$

Quarter Wave Line Transformer



$$\beta l = \frac{2\pi}{\lambda}\cdot\lambda/4 = \frac{\pi}{2}$$

$$z_s = R_o\left[\frac{z_L + jR_o.(\infty)}{z_o + jz_L.(\infty)}\right]$$

$$z_s = \frac{R_o^2}{z_L}$$

$$R_o = \sqrt{z_s . z_L}$$

Example 14

In Example 12, the load resistance is

(A) 25 Ω (B) 50 Ω (C) 75 Ω (D) 100 Ω

Solution

$$\rho = \frac{Z_L - Z_o}{Z_L + Z_o} = 1/3$$

$$\frac{\frac{Z_L}{Z_o} - 1}{\frac{Z_L}{Z_o} + 1} = 1/3$$

$$\frac{Z_L}{Z_o}(2/3) = 4/3$$

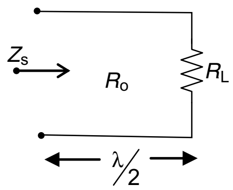
$$\frac{Z_L}{Z_o} = 2$$

$$Z_L = 100\ \Omega$$

Half Wave Line

$$\ell = \lambda/2$$

$$\beta\ell = \frac{2\pi}{\lambda}.\lambda/2 = \pi$$



$$z_o = R_o\left[\frac{z_L + jR_o(o)}{R_o + jz_L(o)}\right]$$

$$z_s = z_L$$

For every half wavelength, the load impedance repeats.

Smith Chart

Many of the computations required to solve transmission line problems involves the use of rather complicated equations. The solutions of such problems are tedious and difficult because the accurate manipulation of numerous equations is necessary. To simplify their solutions, we need a graphic method of arriving at a quick answer.

A number of impedance chart have been designed to facilitate the graphic solution of transmission line problems. Basically, all the charts are derived from the fundamental relationships expressed in the transmission equations, and the most popular chart was developed by Phillip M. Smith.

Construction:

$$z = z_o\left[\frac{1+|\rho_\ell|e^{j\phi}}{1-|\rho_\ell|e^{j\phi}}\right]$$

where z is impedance at any point P, which is at a distance 'd' from the receiving end; $\phi = \theta_1 - 2\beta d$ and θ_1 is the phase angle of reflection co-efficient.

Normalized impedance:

$$Z = \frac{z}{z_o} = \frac{1+|\rho_\ell|.e^{j\phi}}{1-|\rho_\ell|.e^{j\phi}}$$

$$Z = \frac{1+w}{1-w} \qquad (4)$$

$$W = |\rho_1|.e^{j\theta l}.e^{-j2\beta d} = u + jv$$

$$z = r + jx = \frac{1+u+jv}{1-u-jv}$$

where r is the normalized resistance of the line and x is the normalized reactance of the line.

$$r + jx = \frac{1-u^2-v^2}{(1-u)^2+\vartheta^2} + \frac{j.2v}{(1-u)^2+v^2}$$

$$r = \frac{1-u^2-v^2}{(1-u)^2+v^2}$$

$$x = \frac{2v}{(1-u)^2+v^2}$$

$$r(u^2 + v^2 - 2u) + r = 1 - u^2 - v^2$$

$$u^2(1+r) + v^2(1+r) - 2u\,r = 1 - r$$

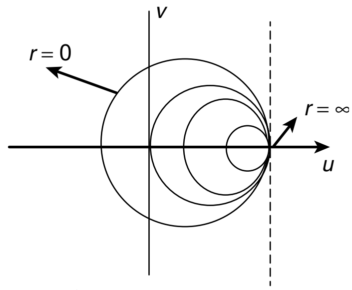
$$u^2 + v^2 - \frac{2ru}{1+r} = \frac{1-r}{1+r}$$

$$\left(u - \frac{r}{1+r}\right)^2 + v^2 = \left(\frac{1}{1+r}\right)^2 \qquad (3.18)$$

The abovementioned equation represents a circle on complex u, v plane with centre $(u, v) = \left(\frac{r}{1+r}, o\right)$ and radius $= \frac{1}{1+r}$

where r can take value from 0 to ∞.

r	radius $\left(\frac{1}{1+r}\right)$	Centre
0	1	(0, 0)
∞	0	(1, 0)
1	0.5	(0.5, 0)
½	0.67	(0.333, 0)



Constant resistance circles

$$x = \frac{2V}{(1-u)^2+v^2}$$

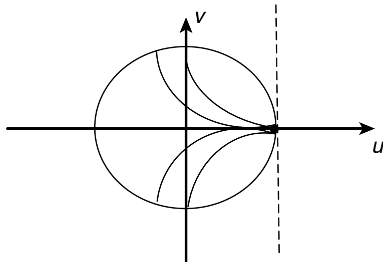
$$u^2 + v^2 - 24 - \frac{2v}{x} = -1$$

$$(u-1)^2 + \left(v - 1/x\right)^2 = \left(1/x\right)^2$$

$$\text{Radius} = 1/x$$

$$\text{Centre} = \left(1, 1/x\right)$$

x	radius $\left(1/x\right)$	centre $\left(1, 1/x\right)$
0	∞	(1, ∞)
∞	0	(1, 0)
1	1	(1, 1)
–1	–1	(1, –1)



Superimposing of constant resistance circles over constant reactance circles gives the Smith chart.

$$\phi = \theta_\ell - 2\beta d$$

1. The point of smith chart moves in clockwise direction towards the generator.
2. The point on smith chart moves in anticlockwise direction towards the load.
3. $Z_{\max} = \dfrac{v_{\max}}{I_{\min}} = z_o s$
4. $Z_{\min} = \dfrac{v_{\min}}{I_{\max}} = \dfrac{z_o}{s}$
5. $z_{\max} = \dfrac{Z_{\max}}{z_o} = s = r_{\max}$
6. $z_{\min} = \dfrac{Z_{\min}}{z_o} = \dfrac{1}{s} = r_{\min}$
7. $y = \dfrac{1}{z} = \dfrac{1}{z_o}\left[\dfrac{1-|\rho_\ell|.e^{j\phi}}{1+|\rho_\ell|.e^{j\phi}}\right]$

   $y = \dfrac{y}{Y_o} = \dfrac{1-|\rho_\ell|.e^{j\phi}}{1+|\rho_\ell|.e^{j\phi}}$

   $y = g + jb$ (normalized admittance)

Admittance diametrically opposite to impedance and constant resistant circles becomes constant conductance circles and constant reactance circles becomes constant susceptance circles.

The characteristics of Smith chart are summarized as follows:

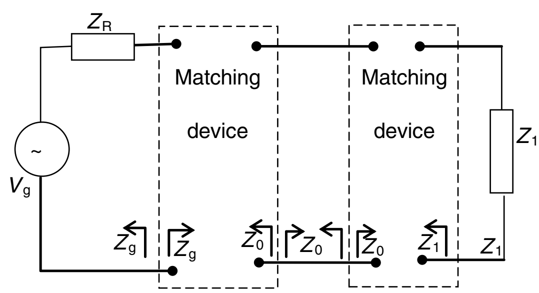
1. The constant r and constant x loci from two families of orthogonal circles in the chart.
2. The constant r and constant x circles are passed through the point ($T_r = 1$, $T_i = 0$)
3. The upper half of the diagram represents $+jx$.
4. The lower half of the diagram represents $-jx$.
5. For admittance, the constant r circles become constant g circles, and the constant x circle becomes constant susceptance b circles.
6. The distance around the Smith chart for one revolution is one-half wavelength $\left(\dfrac{\lambda}{2}\right)$.
7. At a point of $Z_{\min} = \dfrac{1}{S}$, there is a $V_{\min}$ on the line.
8. At a point of $Z_{\max} = S$, there is a $V_{\max}$ on the line.
9. The horizontal radius to the right of chart centre corresponds to $V_{\max}$, $I_{\min}$, $Z_{\max}$, and S (SWR).
10. The horizontal radius to the left of chart centre corresponds to $V_{\min}$, $I_{\max}$, $Z_{\min}$, and input.
11. Since the normalized admittance is a reciprocal of the normalized impedance Z, the corresponding quantities in the admittance chart are 180° out of phase with those in the impedance chart.
12. The normalized impedance of admittance is repeated for every half wavelength of distance.
13. The distance are given in wavelength towards the generator and also towards the load.

A Smith chart of slotted line can be used to measure a standing wave pattern directly and then the magnitudes of the reflection coefficient. Reflected power, transmitted power, and the load impedance can be calculated from it.

Impedance Matching

Impedance matching is very desirable with radio frequency (RF) transmission lines. A line terminated in its characteristic impedance has a standing wave ratio of unity and transmits a given power without reflection. Further, transmission efficiency is optimum where there is no reflected power. A 'flat' line is non-resonant; that is, its input impedance always remains at the same value Z_0 when the frequency changes.

In circuit theory, maximum power transfer requires the load impedance to be equal to the generator output impedance. This condition is sometimes referred to as a conjugate match. In transmission line problems, matching means simply terminating the line in its characteristic impedance. Matching devices are necessary to flatten the line. A complete matched transmission line system is shown in the figure.



Matched Transmission Line System

For a low-loss or lossless transmission line at radio frequency, the characteristic impedance Z_0 of the line is resistive. At every point, the impedances looking in opposite directions are conjugate.

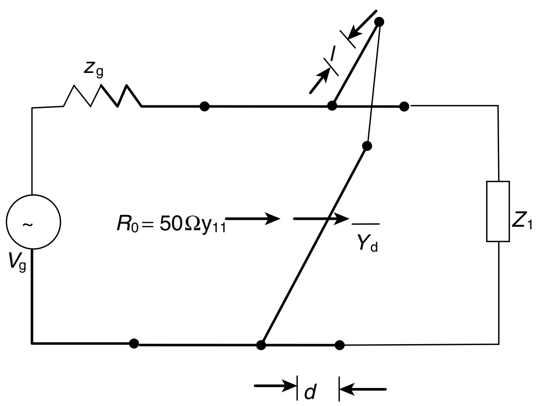
Matching can be tried first on the load side to flatten the line; then, adjustment may be made on the transmitter side to provide maximum power transfer. At audio frequencies, iron cored transformer is almost universally used as an impedance-matching device. Occasionally, an iron-cored transformer is also used at radio frequencies. In a practical transmission line system, the transmitter is ordinarily matched to the coaxial cable for maximum power transfer. However, because of the variable loads, an impedance matching technique is often required at the load side.

Since the matching problems involve parallel connections on the transmission line, it is necessary to work out the problems with admittances rather than impedances.

Single Stub Matching

Although single lumped inductors or capacitors can match the transmission line, it is more common to use the susceptive properties of short-circuited sections of transmission lines. Short-circuited sections are preferable to open circuited because a good short circuit is easier to obtain than a good open circuit.

For a lossless line with $Y_g = Y_0$, maximum power transfer requires $Y_{11} = Y_0$, where Y_{11} is the total admittance of the line and stub looking to the right at point 1–1. The stub must be located at that point on the line where the real part of the admittance looking towards the load is Y_0. In a normalized unit, Y_{11} must be in the form $Y_{11} = Y_d \pm Y_3 = 1$



Single stub matching

If the stub has the same characteristic impedance as that of the line, otherwise $Y_{11} = Y_d \pm Y_s = Y_0$

The stub length is then adjusted so that its susceptance just cancels out the susceptance of the line at the junction.

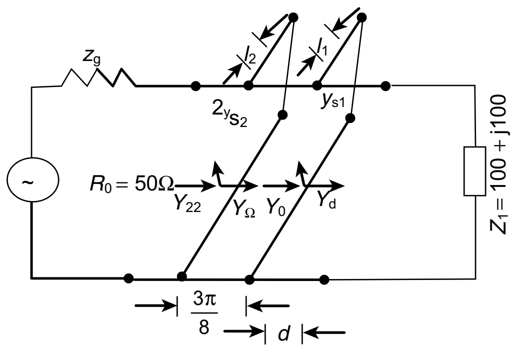
Double Stub Matching

Since single stub matching is sometimes impractical because the stub cannot be placed physically in the ideal location, double stub matching is needed. Double stub devices consist of two short-circuited stubs connected in parallel with a fixed length between them. The length of the fixed section is usually one-eighth, three-eighth, or five-eighth of a wavelength. The stub that is nearest to the load is used to adjust the susceptance and is located at a fixed wavelength from the constant conductance unity circle ($g = 1$) on an appropriate constant standing wave ratio circle, then the admittance of the line at the second stub as shown in the figure is

$$Y_{22} = Y_\Omega \pm Y_{s2} = 1$$

$$Y_{22} = Y_{d2} \pm Y_{s2} = Y_0$$

In these two equations, it is assumed that the stubs and the main line have the same characteristic admittance. If the positions and lengths of the stubs are chosen properly, there will be no standing wave on the line to the left of the second stub measured from the load.



Double stub matching

Quarter Wave Transformer

Characteristic impedance

$$R_o = \sqrt{z_s . z_L}$$

where z_s is the sending impedance and z_L = load.

This can be used for impedance transformation. However, as frequency changes, impedance changes; matching is required not for a single frequency, but for a range of frequencies over which communication is to be done.

1. If the transition line is terminated into an impedance z_o, standing wave formation takes place.
2. If, at any point between voltage minima and voltage maxima, the susceptable part is eliminated by matching stub, then from this point towards generator there is no standing wave.
3. If, at the point of E_{max}, the input conductance is $\frac{G_o}{S}$ and at the point of E_{min}, input impedance is $\frac{Z_o}{s}$.
4. There must be some point where the real part of input impedance is equal to R between E_{max} and E_{min}. However, there must be some susceptibility (since voltage and current are not in phase with each other, which can be eliminated by a matching stub that can be open ended or short circuited).
5. Now, if input impedance becomes equal to z_o of transmission line, then no standing wave formation takes place.

Practically, this can be achieved by adding a parallel wire that can be offered opposite susceptance to the transmission line, and hence, cancels it. Thus, only real components are left.

Power

If P_i is the incident power, then the reflected power at the load is $p_{ref} = |\rho_l|^2 \cdot p_{incident}$

P_t – power transmitted

$$P_t = P_{incident} - P_{ref}$$

$$P_t = P_{incident}(1 - |\rho_l|^2)$$

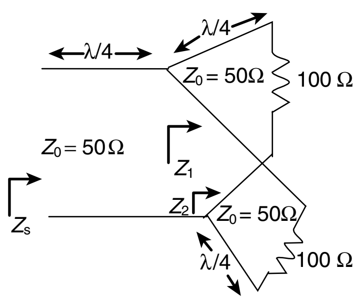
ρ_l = reflection co-efficient at load

NOTES

1. **z_{sc} = short-circuited T_x – line input impedance:**
   $Z_{sc} = jz_o \tan\beta\ell$
   (a) If $\ell = \lambda/4$, that is, $\beta\ell = \pi/2$
   $z_{sc} = \infty$
   If loading is short circuited, then input impedance is ∞ (i.e., open circuited)
   ∴ It is called impedance inverter.
   (b) If $\ell = \lambda/2$; $\beta\ell = \pi$
   $Z_{sc} = 0$
   As we already know that $\lambda/2$ line is a impedance repeater.
2. **z_{oc}: Open-circuited T_x – line input impedance:**
   $Z_{oc} = -jz_o \cot\beta\ell$
   (a) If $l = \lambda/4$
   $Z_{oc} = 0$
   if load is open circuited, then input impedance is 0 (short circuited).
   ∴ It is called impedance inverter.
   (b) $l = \lambda/2$
   $Z_{oc} = \infty$
   ∴ $\lambda/2$ line is a impedance repeater.

Example 15

The parallel branches of a two-wire transmission line are terminated in 100 Ω and 100 Ω resistors, as shown in the figure. The characteristic impedance of the line is Z_o = 50 Ω and each section has a length of $\lambda/4$, then the reflection co-efficient at the input is



(A) $3/5$ (B) $-3/5$ (C) $j3/5$ (D) $-j3/5$

Solution

$$Z_1 = \frac{Z_o^2}{Z_{L_1}} = \frac{2500}{100} = 25\Omega$$

$$Z_2 = \frac{Z_o^2}{Z_{L_2}} = \frac{2500}{100} = 25\Omega$$

$$Z_L = Z_1 \,||\, Z_2 = 12.5\Omega$$

$$Z_s = \frac{Z_o^2}{Z_L} = \frac{2500}{25/2} = 200\Omega$$

$$\rho = \frac{200-50}{200+50} = \frac{150}{250} = 3/5$$

Example 16

A quarter wave impedance transformer is terminated by a short circuit. What would be its impedance equal to?

(A) zero (B) infinity
(C) Z_o (characteristic impedance) (D) $\sqrt{Z_o}$

Solution

$$Z_{in} = \frac{Z_o^2}{Z_L} = \frac{Z_o^2}{0} = \infty$$

Example 17

In an impedance Smith chart, a clockwise movement along a constant resistance circle gives rise to

(A) a decrease in the value of reactance
(B) an increase in the value of reactance
(C) no change in reactance
(D) no change in impedance value

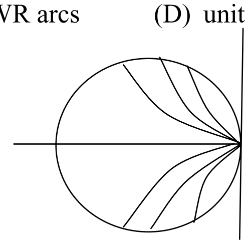
Solution

While moving along the constant resistant circle in Smith chart in clockwise direction, the reactance increases.

Example 18

Many circles arcs are drawn in a Smith chart used for transmission line calculations. The arcs (part of a circle) in the following figure represent

(A) constant reactance arcs (B) constant resistant arcs
(C) constant VSWR arcs (D) unit circles



Solution

The part of the circles in the Smith chart is constant reactance circle arcs.

Example 19

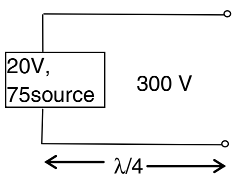
A $(50 - j25)\Omega$ load is connected to a coaxial cable of characteristic impedance 75 Ω at 12 GHz. In order to obtain the best matching, which one of the following will have to be connected?

(A) a short-circuited stub at load
(B) inductance at load
(C) a capacitance at a specific distance at load
(D) a short-circuited stub at some specific distance from load

Solution: (D)

Example 20

Consider a 300 Ω, quarter wave long line (at 1 GHz) transmission line, as shown in the figure. It is connected to a 20 v, 75 Ω source at the other end and is left open circuited at the other end. The magnitude of the voltage at the open-circuited end of the line is



(A) 10 v (B) 20 v (C) 40 v (D) 80 v

Solution

$$\frac{v_L}{v_{in}} = \frac{z_0}{z_{in}} \Rightarrow v_L = \frac{300 \times 20}{75} = 80 \text{ v}$$

Direction for questions 1 to 11: Select the correct alternative from the given choices.

Example 21

A quarter wave transformer is used for matching a load of 225 Ω connected to a transmission line of 256 Ω in order to reduce the SWR along the line to 11 the characteristic impedance (in ohms) of the transformer is

(A) 205 (B) 256 (C) 240 (D) 273

Solution

Z_0 (characteristic impedance) $= \sqrt{Z_L . Z_C} = \sqrt{225.256} = 240$

Example 22

A quarter wave transformer matching a 75 Ω source with a 300 Ω load should have a characteristic impedance of

(A) 50 Ω (B) 100 Ω (C) 150 Ω (D) 200 Ω

Solution

$$Z_C = \sqrt{Z_L . Z_S} = \sqrt{75.300} = 150\ \Omega$$

Example 23

Which of the following conditions will not guarantee a distortionless transmission line?

(A) $R = G = 0$
(B) $RC = GL$
(C) Very low frequency range ($R >> wL$, $G >> wC$)
(D) Very high frequency range ($R << wL$, $G << wC$)

Solution: (C)

Example 24

A lossless line having characteristic impedance Z_0 is terminated in a pure reactance of value $-jZ_0$. The VSWR of the line will be

(A) 10 (B) 2 (C) 1 (D) Infinite

Solution

$$T = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-jZ_0 - Z_0}{-jZ_0 + Z_0}$$

$$|T| = 1$$

$$\text{then VSWR} = \frac{1+|T|}{1-|T|} = \infty$$

Example 25

On a Smith chart, the concentric circle with $R = 0$ circle is

(A) R = constant circle (B) $X = 1$ circle
(C) $|\Gamma|$ = constant circle (D) none of these

Solution: (C)

Example 26

Which of the following circles will never intersect each other on a Smith chart?

(A) $R = 0$ circle and $X = 1$ circle
(B) $R = 1$ circle and $X = 0$ circle
(C) $R = \infty$ circle and $X = 0$ circle
(D) None of these

Solution

None of these

Example 27

An electro-magnetic wave is obliquely incident at the surface of a dielectric medium 2 (μ_0, ϵ_2) from dielectric medium $1(\mu_0, \epsilon_1)$. The angle of incidents is θ_i and θ_c is the critical angle. Then, the phenomenon of total reflection occurs when

(A) $\epsilon_1 > \epsilon_2$ and $\theta_i < \theta_c$ (B) $\epsilon_1 < \epsilon_2$ and $\theta_i > \theta_c$
(C) $\epsilon_1 < \epsilon_2$ and $\theta_i < \theta_c$ (D) $\epsilon_1 > \epsilon_2$ and $\theta_i > \theta_c$

Solution

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\mu_2 \in_2}{\mu_1 \in_1}} = \sqrt{\frac{\in_2}{\in_1}}$$

or $$\sin\theta_c = \sqrt{\frac{\in_2}{\in_1}} \quad [\text{for } \theta_t = 90°, \theta_i = \theta_c]$$

if $\theta_i > \theta_c$, total reflection occurs and or

$$\sqrt{\frac{\in_2}{\in_1}} > 1, \in_2 > \in_1$$

Example 28

A transmission line whose characteristic impedance is a pure resistance
(A) must be a lossless line
(B) must be a distortionless line
(C) may not be a lossless line
(D) may not be a distortionless line

Solution

$$Z_0 = \frac{R + jwL}{G + jwC}$$

With Z_0 as pure resistance, we have $Z_0 = R + j0$

(1) Lossless line:

$$R = G = 0$$

$$Z_0 = \sqrt{\frac{1}{C}}$$

(2) Distortionless line: $\frac{R}{G} = \frac{1}{cZ_0} = \sqrt{\frac{1}{C}}$

(3) $L = 0$

$C = 0$

$$Z = \sqrt{\frac{R}{G}}$$

Thus, transmission line may be lossless, distortionless or with $L = C = 0$

Example 29

The capacitance per unit length and the characteristic impedance of a lossless transmission line are C and Z_0, respectively. The velocity of a travelling wave on the transmission line is

(A) Z_oC (B) $\frac{1}{Z_oC}$ (C) $\frac{Z_0}{C}$ (D) $\frac{C}{Z_0}$

Solution

$$V_0 = \frac{1}{\sqrt{LC}}$$

and $$Z_0 = \sqrt{\frac{L}{C}}$$

or $$L = CZ_0^2$$

$$\therefore V_0 = \frac{1}{\sqrt{C \times CZ_0^2}} = \frac{1}{Z_0C}$$

Example 30

A transmission line of characteristic impedance $Z_0 = 50$ Ω, phase velocity $V_p = 2 \times 10^8$ m/s and length $l = 1$ m is terminated by a load Z_l $(30 - j40)$ Ω. The input impedance of the line for a frequency of 100 MHz will be

(A) $(30 + j40)$ Ω (B) $(30 - j40)$ Ω
(C) $(50 + j40)$ Ω (D) $(50 - j40)$ Ω

Solution

$$V_p = f\lambda$$

$$\therefore \lambda = \frac{V_p}{f} = 2 \times \frac{10^8}{100} \times 10^6 = 2 \text{ m}$$

Since length of the line is half wavelength, and therefore

$$\therefore Z_{in} = Z_L = (30 - j40)\ \Omega$$

Example 31

The reflection coefficient characteristic impedance and load impedance of a transmission line are connected together by the relationship

(A) $K_r = \frac{Z_L + Z_0}{Z_0 - Z_L}$ (B) $K_r = \frac{Z_0 Z_L}{Z_0 - Z_L}$
(C) $K_r = \frac{Z_L - Z_0}{Z_0 + Z_L}$ (D) $K_r = \frac{Z_L - Z_0}{Z_0 Z_L}$

Solution

$$K_r = \frac{Z_L - Z_0}{Z_0 + Z_L}$$

Exercises

Practice Problem I

Direction for questions 1 to 24: Select the correct alternative from the given choices.

1. A coaxial cable has the inductance of 0.25 μH/m and capacitance of 2.5 nF/m, then the characteristic impedance is
(A) 10 Ω (B) 50 Ω (C) 100 Ω (D) 75 Ω

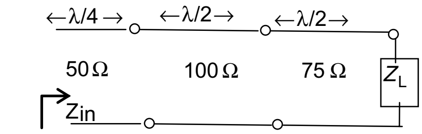
2. For a lossy transmission line, the characteristic impedance does not depend on
(A) frequency of operation on the line
(B) load of the line
(C) conductivity of the conductor
(D) inductance of the line

3. Which of the following satisfies condition for the distortionless transmission line?
(A) $RL = GC$ (B) $RC = LG$
(C) $RG = LC$ (D) $RC >> LG$

4. In a transmission line, the reflection coefficient at a distance 'd' from the load is $\rho d = 0.8e^{-j135^\circ}$ and the reflection co-efficient at the load is $\rho_l = 0.8e^{-j45^\circ}$, then the distance '$d$' is

(A) $\lambda/8$ (B) $\lambda/4$ (C) $3\lambda/8$ (D) $\lambda/2$

5. The three lossless lines are connected as in the following figure. The input impedance is



Where $Z_L = 100\ \Omega$

(A) 25 Ω (B) 50 Ω (C) 75 Ω (D) 100 Ω

6. Which of the following is not true about a lossless line?

(A) $Z_{in} = j\,Z_o$ for a short-circuited line with $\ell = \lambda/8$

(B) $Z_L = Z_o$ for a matched line

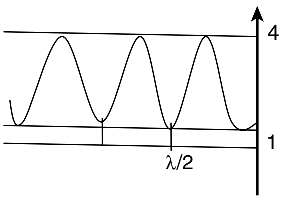
(C) $Z_{in} = +jZ_o$ for open-circuited line with $\ell = \lambda/8$

(D) $Z_{in} = j\infty$ for a shorted line with $\ell = \lambda/4$

7. In an airline, the adjacent maxima are found at 10 cm and 20 cm, then the operating frequency is

(A) 2 GHz (B) 3 GHz (C) 1.5 GHz (D) 4 GHz

8. VSWR pattern on a lossless line is shown in the following figure. If the characteristic impedance of the line is 75 Ω, then



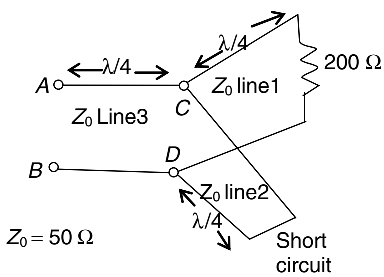
the load of the line is

(A) 75 Ω (B) 150 Ω (C) 300 Ω (D) 225 Ω

9. In problem 8, the reflection co-efficient is

(A) 0.6 (B) −0.6 (C) 0.2 (D) −0.2

10. Consider the following diagram



Z_{in} looking at line 1 is

(A) 12.5 (B) 25 Ω (C) 37.5 Ω (D) 0

11. In problem 10, Z_{in} looking at line 2 is

(A) zero (B) infinity (C) 50 Ω (D) 25 Ω

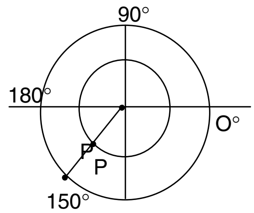
12. In problem 10, Z_{in} looking at line 3 is

(A) zero (B) infinity (C) 100 Ω (D) 200Ω

13. In problem 10, the VSWR at *CD* is

(A) 2 (B) 4 (C) 8 (D) 0

14. A 500 m long transmission line is terminated by a load that is located at *P* on the Smith chart. If λ = 150 m, how many voltage maxima exist at on the line?



(A) 4 (B) 5 (C) 6 (D) 7

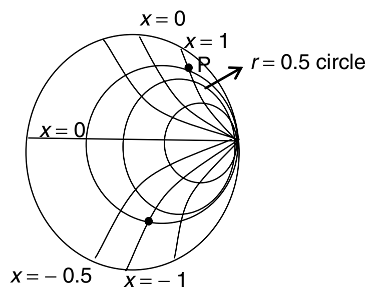
15. In problem 14, first minima occurs at what distance from the load?

(A) 6.25 m towards source
(B) 12.5 m towards source
(C) 6.25 m from source
(D) 12.5 m from source

16. A uniform plane EM wave incident normally on a plane surface of a dielectric material is reflected with a VSWR of 3. What is the percentage of incident power that is reflected?

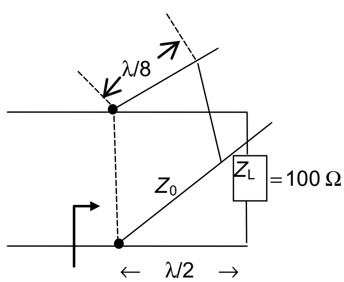
(A) 10% (B) 25% (C) 50% (D) 75%

17. Consider an impedance $Z = R + jx$ marked with point *P* in an impedance Smith chart as shown in the figure. The movement from point *P*, along a constant resistance circle in the clockwise direction by an angle 45° equivalent to



(A) adding an inductance in series with *Z*
(B) adding a capacitance in series with *Z*
(C) adding an inductance in shunt across *Z*
(D) adding a capacitance in shunt across *Z*

18. A short-circuited stub is shunt connected to a transmission line as shown in the figure. If $Z_o = 25\ \Omega$, the admittance *Y* seen at the junction of the stub and the transmission line is



(A) $(0.01 + j0.04)$ ℧ (B) $(0.01 - j0.04)$ ℧
(C) $(0.01 + j0.02)$ ℧ (D) $(0.01 - j0.02)$ ℧

19. A transmission line with a Z_o of 100 Ω is used to match a 50 Ω Ω section to a 200 Ω section. If the matching is done both at 300 MHz and 1 GHz, the length of the transmission line can be done approximately.
(A) 0.25 m (B) 0.3 m
(C) 0.075 m (D) 1.5 m

20. A transmission line of Z_o = 50 Ω is terminated by a 50 Ω load. When excited by a sinusoidal voltage source at 10 GHz, the phase difference and phase velocity are $\frac{\pi}{4}$ radian and 1.6×10^8 m/s, respectively. Then, minimum spacing between the two point with the above-mentioned phase difference.
(A) 2 mm (B) 4 mm
(C) 8 mm (D) 1.6 mm

21. The diameter of the inner conductor of a coaxial cable is 2 mm and that of the outer conductor is 6 mm and the dielectric constant of the insulation is 1.60. Calculate the characteristic capacitance is 60 pF/m. Find the value of inductance per meter at *RF*.
(A) 0.1625 μH/m (B) 0.1825 μH/m
(C) 1.2 μH (D) 0.9256 μHm

22. A transmission line of characteristic impedance Z_O = 50 Ω phase velocity $V_P = 2 \times 10^8$m/s and length ℓ = 1 m is terminated by a load Z_L = (30 – j40) Ω. The input impedance of the line for a frequency of 100 MHz will be
(A) (30 + j40) Ω (B) (30 – j40) Ω
(C) (50 + j40) Ω (D) (50 – j40) Ω

23. An airline has characteristic impedance of 70 Ω and phase constant of 3 rad/m at 100 MHz. Calculate the inductance per meter.
(A) 682 pF/m (B) 0.682 pF/m
(C) 68.2 pF/m (D) 6.82 pF/m

24. The VSWR on UHF transmission line, working at a frequency of 300 MHz is found to be 2. If the distance between load and voltage minimum is 0.8 m, find the value of load impedance.
(A) 17∠–6.4° (B) 1.7∠–6.4°
(C) 1.7∠6.4° (D) 17∠6.4°

Practice Problem 2

Direction for questions 1 to 24: Select the correct alternative from the given choices.

1. Which of the following statements are not true of the line parameters *R*, *L*, *G*, and *C*
(A) *R* and *L* are series elements
(B) *G* and *C* are shunt elements
(C) The parameters are distributed not lumped
(D) $G = {}^1\!/_R$

2. For a lossless transmission line with inductance 25 μH/m and characteristic impedance 50 Ω, then capacitance is
(A) 250 nF/m (B) 125 nF/m
(C) 0.5 μF/m (D) 0.25 μF/m

3. A 50 Ω coaxial cable feeds a 75 + j20Ω dipole antenna. Then, VSWR is
(A) 0.677 (B) 1.677 (C) 0.25 (D) 0.33

4. The reflection co-efficient on a lossy line at the load and at a distance '*d*' from the load are
(A) same
(B) same in magnitude differ in phase
(C) same in phase differ in magnitude
(D) different both in magnitude and phase

5. The power reflected along a matched transmission line with characteristic impedance Z_o is
(A) zero
(B) infinity
(C) depends on Z_o
(D) None of these

6. Match List-I with List-II and select the correct answer using the code given below.

List-I (Load)	List-II (Reflection co-efficient)
P. Short circuit	1. $\frac{1}{3}$
Q. Open circuit	2. 0
R. Z_o	3. –1
S. $2Z_o$	4. 1

(A) *P* – 1, *Q* – 2, *R* – 3, *S* – 4
(B) *P* – 4, *Q* – 3, *R* – 2, *S* – 1
(C) *P* – 3, *Q* – 4, *R* – 2, *S* – 1
(D) *P* – 3, *Q* – 4, *R* – 1, *S* – 2

7. A transmission line of length ${}^\lambda\!/_8$, with characteristic impedance Z_o, and load impedance Z_L, then the magnitude of sending-end impedance is
(A) Z_o (B) 0 (C) ∞ (D) $2Z_o$

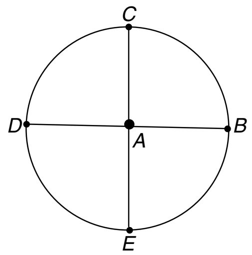
8. A quarter wave transformer is used for matching a load of 225 Ω connected to a transmission line of 256 Ω in order to reduce VSWR along the line to 1. Then, Z_o of transformer is
(A) 225 (B) 240 (C) 256 (D) 273

9. Which of the following statement is not true regarding the Smith chart?
1. A normalized Smith chart apply to a line of any characteristic resistance and serves as well for normalized admittance.
2. A polar co-ordinate Smith chart contains circles of constant |*z*| and circles of constant ∠*z*
3. In Smith chart, the distance towards the load is always measured in clockwise direction.

4. Upper half of Smith chart is with inductive reactance.

(A) 1 (B) 2 (C) 3 (D) 4

10. In the following Smith chart



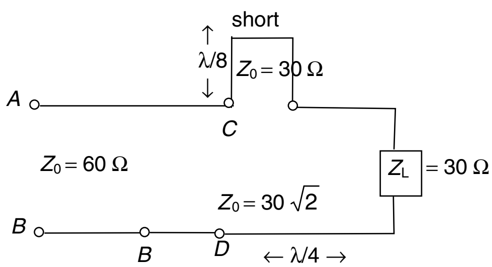
The normalized impedance at D is

(A) $1 + j0$ (B) $0 + j1$ (C) $0 + j0$ (D) $\infty + j\infty$

11. In problem 10, the impedance (normalized) at B is

(A) $1 + j0$ (B) $\infty + j\infty$ (C) $0 + j1$ (D) $1 + j1$

12. In the circuit shown in the following figure, all the transmission line sections are lossless, the VSWR on the 60 Ω line is



(A) 1.00 (B) 1.64 (C) 2.5 (D) 3.00

13. In problem 12, if $AC = \lambda/2$ then $Z_{AB} =$

(A) $60 + j30$ (B) $60 - j30$

(C) $\dfrac{3600}{60 + j30}$ (D) $\dfrac{60 + 30}{3600}$

14. A transmission line of Z_o = 50 Ω is terminated in a line with load impedance Z_L. The VSWR of the line is measured as 6 and the first of the voltage maxima is observed at a distance of $\lambda/4$ from the load. Then, Z_L is

(A) $25/3\ \Omega$ (B) $50/3\ \Omega$

(C) $25/6\ \Omega$ (D) $50/6\ \Omega$

15. A transmission line of characteristic impedance 400 Ω is to be matched to a load of 25 Ω through a quarter wavelength line. The quarter wave line characteristic impedance must be

(A) 40 Ω (B) 100 Ω (C) 400 Ω (D) 425 Ω

16. A lossless transmission line having 50 Ω characteristic impedance and length λ/4 is short-circuited at one end and connected to an ideal voltage source of 1 V at the other end. The current drawn from the voltage source is

(A) zero (B) 0.02 A

(C) infinite (D) None of the above

17. An open voice line having characteristic impedance 692 ∠–12° Ω is terminated in 200 Ω resistor. The line is 100 km long and is supplied power by a generator of 1.0 V at 100 Hz. Determined the voltage reflection coefficient

(A) 0.56∠172.4° (B) 0.56∠163.9°

(C) 0.56∠–163.9° (D) 0.56∠–172.4°

Direction for questions 18 and 19:

A 30 m long transmission line has no losses with Z_O = 50 Ω operating at 2 MHz is terminated with a load $Z_L = 60 + j40\ \Omega$.

18. Find reflection coefficient

(A) 0.3523 ∠56° (B) 0.323 ∠–56°

(C) 3.23 ∠56° (D) 3.23 ∠–56°

19. Find standing wave ratio

(A) 2.888 (B) –2.888

(C) 2.088 (D) 0.2088

20. The magnitude of the open circuit and short circuit input impedances of a transmission line are 100 Ω and 25 Ω, respectively. The characteristic impedance of the line is

(A) 25 Ω (B) 50 Ω (C) 75 Ω (D) 100 Ω

21. A transmission line is distortionless if

(A) $RL = \dfrac{1}{GC}$ (B) $RL = GC$

(C) $LG = RC$ (D) $RG = LC$

22. The inductance per unit length of a coaxial cable in terms of conductor size and spacing is

(A) $L = \dfrac{\mu}{2\pi}\log_e \dfrac{d}{D}$ H/m

(B) $L = \dfrac{\mu}{2\pi}\log_e \dfrac{D}{d}$ H/m

(C) $L = \dfrac{2\pi}{\mu}\log_e \dfrac{d}{D}$ H/m

(D) $L = \dfrac{2\pi}{\mu}\log_e \dfrac{D}{d}$ H/m

23. A 75 Ω transmission line is first short terminated and the minima locations are noted. When the short is replaced by a resistive load, R_L, the minima locations are not altered and the VSWR is measured to be 3. What is the value of R_L?

(A) 25 Ω (B) 50 Ω (C) 225 Ω (D) 250 Ω

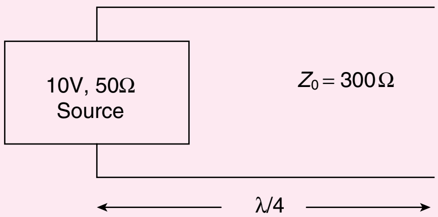
24. A certain transmission line operating at ω = 10^6rad/s has $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_o = 60 + j40\ \Omega$ and is 2 m long. If the line is connected to source of 10 ∠ 0V, Zg = 40 Ω and terminated by a load of 20 + j50Ω, determine input impedance Z_{in}.

(A) 6.025 + j 38.79 Ω (B) 60.25 + j38.79 Ω

(C) 60.25 + j3.879 Ω (D) 60.25 + 0.3879 Ω

Previous Years' Questions

1. Consider a 300 Ω quarter wave long (at 1 GHz) transmission line, as shown in the figure. It is connected to a 10 V, 50 Ω sources at one end and is left open circuited at the other end. The magnitude of the voltage at the open circuit end of the line is **[2004]**



(A) 10 V (B) 5 V

(C) 60 V (D) $\frac{60}{7}$ V

2. A plane electromagnetic wave propagating in free space is incident normally on a large slab of lossless, non-magnetic dielectric material with $\varepsilon > \varepsilon_0$. Maxima and minima are observed when the electric field is measured in front of the slab. The maximum electric field is found to be 5 times the minimum field. The intrinsic impedance of the medium should be **[2004]**

(A) 120 πΩ (B) 60 πΩ

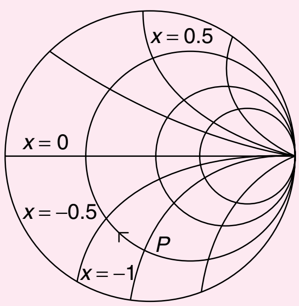
(C) 600 πΩ (D) 24 πΩ

3. A lossless transmission line is terminated in a load that reflects a part of the incident power. The measured VSWR is 2. The percentage of the power that is reflected back is **[2004]**

(A) 57.73 (B) 33.33

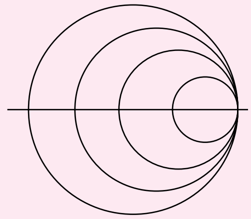
(C) 0.11 (D) 11.11

4. Consider an impedance $Z = R + jX$ marked with point P in an impedance Smith chart, as shown in Figure given in Q88. The movement from point P along a constant resistance circle in the clockwise direction by an angle 45° is equivalent to **[2004]**



(A) adding an inductance in series with Z
(B) adding a capacitance in series with Z
(C) adding an inductance in shunt across Z
(D) adding a capacitance in shunt across Z

5. Many circles are drawn in a Smith chart used for transmission line calculations. The circles shown in figure represent **[2005]**



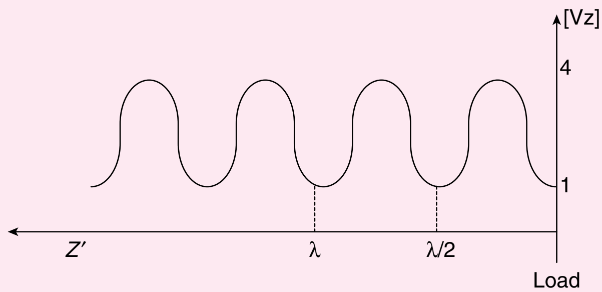
(A) unit circles
(B) constant resistance circles
(C) constant reactance circles
(D) constant reflection coefficient circles.

6. Characteristic impedance of a transmission line is 50 Ω. Input impedance of the open-circuited line is $Z_{oc} = 100 + j150\ \Omega$. When the transmission line is short circuited, then value of the input impedance will be **[2005]**

(A) 50 Ω (B) $100 + j150\ \Omega$

(C) $7.69 + j11.54\ \Omega$ (D) $7.69 - j11.54\ \Omega$

Direction for questions 7 and 8:



Voltage standing wave pattern in a lossless transmission line with characteristic impedance 50 Ω and a resistive load is shown in figure.

7. The value of the load resistance is **[2005]**

(A) 50 Ω (B) 200 Ω

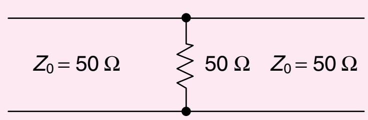
(C) 12.5 Ω (D) 0 Ω

8. The reflection coefficient is given by **[2005]**

(A) −0.6 (B) –1

(C) 0.6 (D) 0

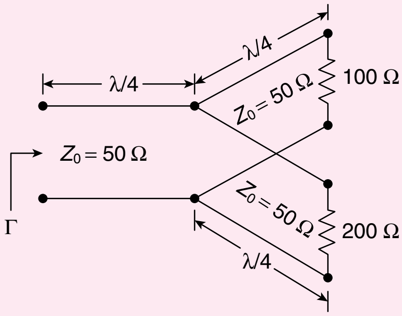
9. A load of 50 Ω is connected in shunt in a two-wire transmission line of $Z_0 = 50\ \Omega$, as shown in the figure. The two-port scattering parameter matrix (S-matrix) of the shunt element is **[2007]**



(A) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}$

10. The parallel branches of a two-wire transmission line are terminated in 100 Ω and 200 Ω resistors, as shown in the figure. The characteristic impedance of the line is $Z_0 = 50\ \Omega$ and each section has a length of $\frac{\lambda}{4}$. The voltage reflection coefficient Γ at the input is **[2007]**



(A) $-j\frac{7}{5}$ (B) $\frac{-5}{7}$

(C) $j\frac{5}{7}$ (D) $\frac{5}{7}$

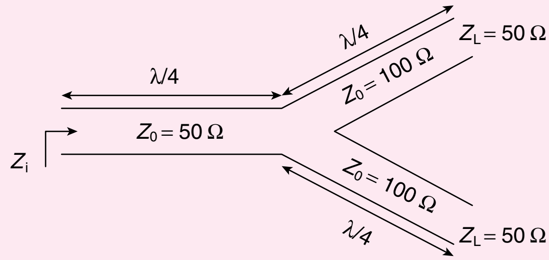
11. One of a lossless transmission line having the characteristic impedance of 75 Ω and length of 1 cm is short circuited. At 3 GHz, the input impedance at the other end of the transmission line is **[2008]**

(A) 0 (B) resistive
(C) capacitive (D) inductive

12. In the design of a single-mode step index optical fibre close to upper cut-off, the single mode operations is not preserved if **[2008]**

(A) radius as well as operating wavelength are halved
(B) radius as well as operating wavelength are doubled
(C) radius is halved and operating wavelength is doubled
(D) radius is doubled and operating wavelength is halved

13. A transmission line terminates in two branches, each of length λ/4, as shown. The branches are terminated by 50 Ω loads. The lines are lossless and have the characteristic impedances shown. Determine the impedance Z_i, as seen by the source **[2009]**

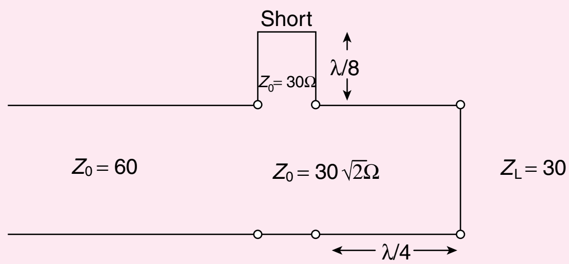


(A) 200 Ω (B) 100 Ω (C) 50 Ω (D) 25 Ω

14. A transmission line has a characteristic impedance of 50 Ω and a resistance of 0.1 Ω/m. If the line is distortionless, the attenuation constant (in Np/m) is **[2010]**

(A) 500 (B) 5
(C) 0.014 (D) 0.002

15. In the circuit shown, all the transmission line sections are lossless. The voltage standing wave ratio (VSWR) on the 60 Ω line is **[2010]**



(A) 1.00 (B) 1.64 (C) 2.50 (D) 3.00

16. A transmission line of characteristic impedance 50 Ω is terminated by a 50 Ω load. When excited by a sinusoidal voltage source at 10 GHz, the phase difference between two points spaced 2 mm apart on the line is found to be π/4 radians. The phase velocity of the wave along the line is **[2011]**

(A) 0.8×10^8 m/s (B) 1.2×10^8 m/s
(C) 1.6×10^8 m/s (D) 3×10^8 m/s

17. A transmission line of characteristic impedance 50 Ω is terminated in a load impedance Z_L. The VSWR of the line is measured as 5 and the first of the voltage maxima in the line is observed at a distance of λ/4 from the load. The value of Z_L is **[2011]**

(A) 10 Ω (B) 250 Ω
(C) (19.23 + j46.15) Ω (D) (19.23 – j46.15) Ω

18. A coaxial cable with a diameter of 1 mm and outer diameter of 2.4 mm is filled with a dielectric of relative permittivity 10.89. Given $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\varepsilon_0 = \frac{10^{-9}}{36\pi}$ F/m, the characteristic impedance of the cable is **[2012]**

(A) 330 Ω (B) 100 Ω
(C) 143.3 Ω (D) 43.4 Ω

19. A transmission line with a characteristic impedance of 100 Ω is used to match a 50 Ω section to a 200 Ω section. If the matching is to be done both at 429 MHz and 1 GHz, the length of the transmission line can be approximately. **[2012]**

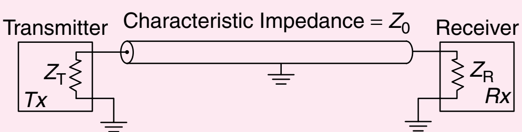
(A) 82.5 cm (B) 1.05 m
(C) 1.58 m (D) 1.75 m

20. For a parallel-plate transmission line, let v be the speed of propagation and Z be the characteristic impedance. Neglecting fringe effects, a reduction of the spacing between the plates by a factor of two results in **[2014]**

(A) halving of v and no change in Z
(B) no changes in v and halving of Z
(C) no change in both v and Z
(D) halving of both v and Z

21. The input impedance of a $\frac{\lambda}{8}$ section of a lossless transmission line of characteristic impedance 50 Ω is found to be real when the other end is terminated by a load $Z_L(= R + jX)$ Ω. If X is 30 Ω, the value of R(in Ω) is __________. **[2014]**

22. In the following figure, the transmitter T_X sends a wideband modulated RF signal via a coaxial cable to the receiver R_x. The output impedance Z_T of T_X, and the characteristic impedance Z_0 of the cable and the input impedance Z_R of R_X are all real

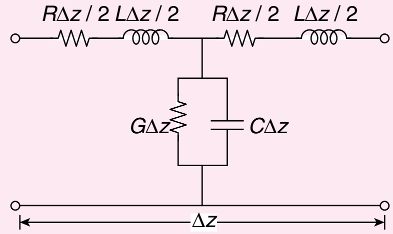


Which one of the following statements is true about the distortion of the received signal due to impedance mismatch? **[2014]**

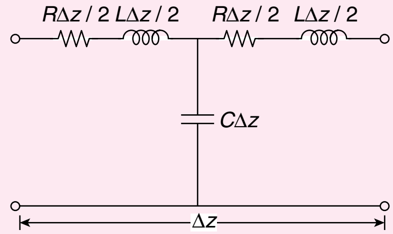
(A) The signal gets distorted if $Z_R \neq Z_0$, irrespective of the value of Z_T.
(B) The signal gets distorted if $Z_T \neq Z_0$, irrespective of the value of Z_R.
(C) Signal distortion implies impedance mismatch at both ends: $Z_T \neq Z_0$ and $Z_R \neq Z_0$.
(D) Impedance mismatches do not result in signal distortion but reduce power transfer efficiency.

23. A coaxial cable is made of two brass conductors. The spacing between the conductors is filled with Teflon ($\varepsilon_r = 2.1$, tan $\delta = 0$). Which one of the following circuits can represent the lumped element model of a small piece of this cable having length Δz? **[2015]**

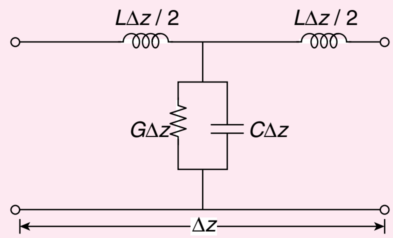
(a)



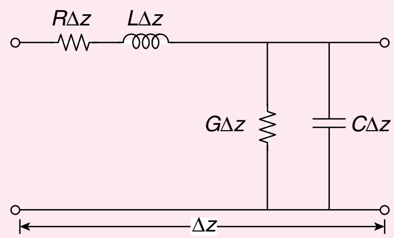
(b)



(c)



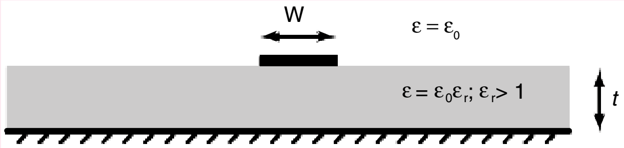
(d)



24. The propagation constant of a lossy transmission line is $(2 + j5)$ m^{-1} and its characteristic impedance is $(50 + j0)\Omega$ at $\omega = 10^6$ rad s^{-1}. The values of the line constants L, C, R, G are respectively. **[2016]**

(A) L = 200μH/m, C = 0.1μF/m, R = 50Ω/m, G = 0.02S/m
(B) L = 250μH/m, C = 0.1μF/m, R = 100Ω/m, G = 0.04S/m
(C) L = 200μH/m, C = 0.2μF/m, R = 100Ω/m, G = 0.02S/m
(D) L = 250μH/m, C = 0.2μF/m, R = 50Ω/m, G = 0.04S/m

25. A lossless micro strip transmission line consists of a trace of width w. II is drawn over a practically infinite ground plane and is separated by a dielectric slab of thickness t and relative permittivity $\varepsilon_r > 1$. The inductance per unit length and the characteristic impedance of this line are L and z_0, respectively. **[2016]**

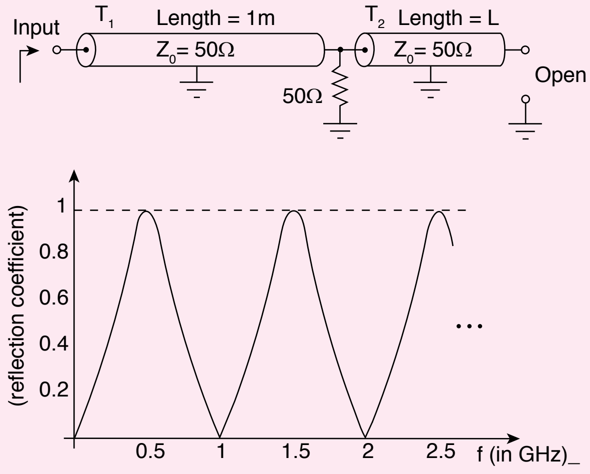


Which one of the following inequalities is always satisfied?

(A) $Z_0 > \sqrt{\dfrac{Lt}{\varepsilon_0 \varepsilon_r w}}$ (B) $Z_0 < \sqrt{\dfrac{Lt}{\varepsilon_0 \varepsilon_r w}}$

(C) $Z_0 > \sqrt{\dfrac{LW}{\varepsilon_0 \varepsilon_r t}}$ (D) $Z_0 < \sqrt{\dfrac{LW}{\varepsilon_0 \varepsilon_r t}}$

26. A microwave circuit consisting of lossless transmission lines T_1 and T_2 is shown in the figure. The plot shows the magnitude of the input reflection coefficient T_L as a function of frequency f. The phase velocity of the signal in the transmission lines is 2×10^8 m/s. **[2016]**



The length L (inn meters) of T_2 is ________.

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** B	**3.** B	**4.** A	**5.** A	**6.** C	**7.** C	**8.** C	**9.** A	**10.** A
11. B	**12.** D	**13.** B	**14.** D	**15.** A	**16.** B	**17.** B	**18.** B	**19.** C	**20.** A
21. A	**22.** B	**23.** C	**24.** B						

Practice Problems 2

1. D	**2.** A	**3.** B	**4.** D	**5.** A	**6.** C	**7.** A	**8.** A	**9.** C	**10.** C
11. B	**12.** B	**13.** A	**14.** A	**15.** B	**16.** A	**17.** A	**18.** A	**19.** C	**20.** B
21. C	**22.** B	**23.** C	**24.** B						

Previous Years' Questions

1. C	**2.** C	**3.** D	**4.** A	**5.** B	**6.** D	**7.** C	**8.** A	**9.** B	**10.** D
11. D	**12.** D	**13.** D	**14.** D	**15.** B	**16.** C	**17.** A	**18.** B	**19.** B	**20.** B
21. 39 to 41		**22.** C	**23.** B	**24.** B	**25.** B	**26.** 0.1			