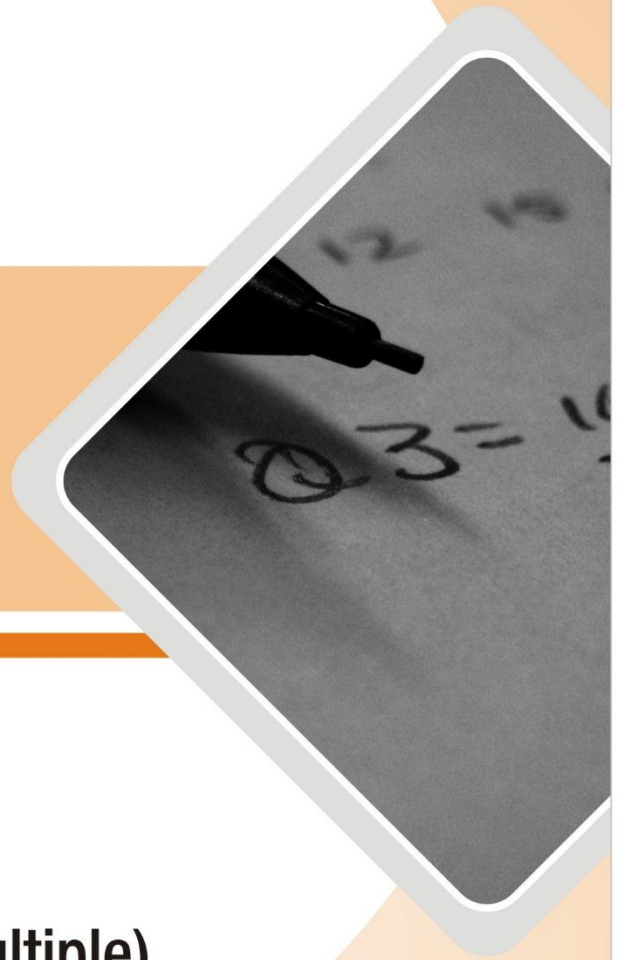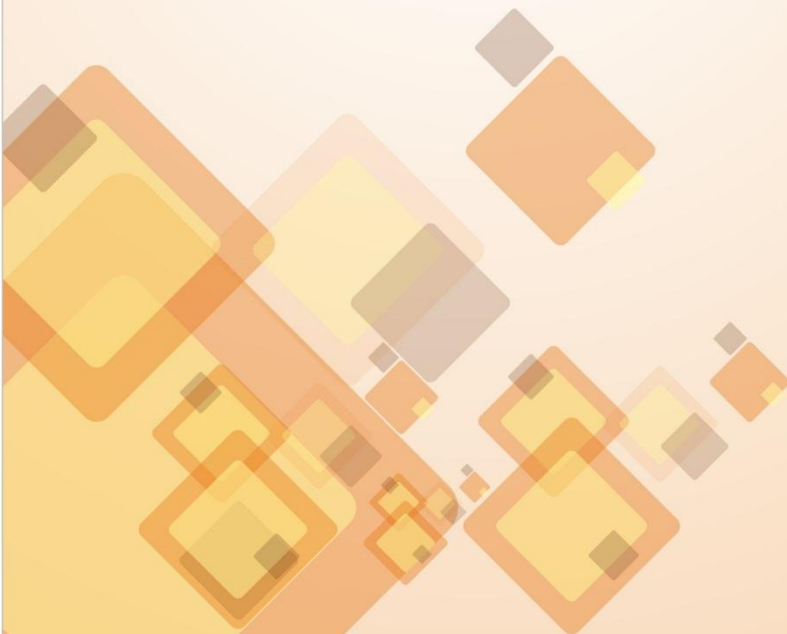# Chapter

LCM and HCF

Learning Objectives

- ❖ Introduction
- ❖ LCM (Least Common Multiple)
- ❖ HCF (Highest Common Factor)

◈ Introduction

LCM of two or more numbers is their least common multiple. LCM of 4 and 6 is 12, it means, 12 is the common multiple of 4 and 6, therefore, 12 is exactly divisible by each 4 and 6.

The least common multiple or LCM of 4 and 6 is not other than 12. HCF or highest

Common factor of two or more numbers is obtained by factorization of the numbers.

Therefore, the factors of 12 and 14 are 1, 2, 3, 4, 6, 12 and 1, 2, 7, 14 respectively, but the HCF or highest common factor is only 2.

◈ LCM (Least Common Multiple)

In arithmetic and number theory, the least common multiple (also called the lowest common multiple or smallest common multiple) of two integers a and b usually is denoted by LCM (a, b). LCM is the smallest positive integer which is a multiple of both a and b. The LCM of 12 and 80 is obtained by the product of greatest power of the prime factors.

Therefore, the prime factors of $12 = 2 \times 2 \times 3$ and prime factors of $80 = 2 \times 2 \times 2 \times 2 \times 5 = 80$. Now, LCM is obtained by multiplying factors those having greatest power occurred in either numbers. 12 has one 3, and 80 has four 2's and one 5, so we multiply 2 four times, 3 once and five once. This gives us 240, the smallest number that can be exactly divided by both 12 and 80.

Therefore, the LCM of two or more numbers is obtained by its prime factorization and common division method.

✳ LCM by Prime Factorization Method

LCM of two or more numbers is obtained by the following:

Step 1: Prime factorization of every number.

Step 2: Product of highest power of prime factors is the LCM of the numbers.

Illustrative EXAMPLE ◈

Find the LCM of 24 and 46 by prime factorization method.

(a) 564   (b) 546

(c) 552   (d) All of these

(e) None of these

Answer: (c)

$$
\begin{array}{c|c}
2 & 24 \\
\hline
2 & 12 \\
\hline
2 & 6 \\
\hline
 & 3 \\
\end{array}
$$

Prime factors of 24 = $= 2 \times 2 \times 2 \times 3 = 24$

$$
\begin{array}{c|c}
2 & 46 \\
\hline
 & 23 \\
\end{array}
$$

Prime factors of 46 = $= 2 \times 23 = 46$

In prime factors of 24, 2 occurred three times but in prime factors of 46, 2 occurred only one time, therefore, the maximum power of 2 is 3 .

Factor 3 is only one time in the prime factors of 24 but in the prime factors of 46, 3 is not a prime factor therefore, it has maximum power 1.

23 is also a prime factor of 46 and it has maximum power 1.

Hence, LCM of 24 and $46 = 2 \times 2 \times 2 \times 3 \times 23 = 552$. Therefore, option (c) is correct and rest of the options is incorrect.

✸ LCM by Division Method

The following steps are used to determine the LCM of two or more numbers by division method:

Step 1: Numbers are arranged or separated in a row by commas.

Step 2: Find the number which divides exactly at feast two of the given numbers.

Step 3 : Follow step 2 tiff there are no (at (east two) numbers divisible by any number.

Step 4: LCM is the product of all divisor and indivisible numbers.

$$
\begin{array}{c|l}
3 & 15,45,65 \\
\hline
5 & 5,15,65 \\
\hline
 & 1,3,13 \\
\end{array}
$$

The l C M of 15,45 and 65 = $\quad = 3 \times 5 \times 3 \times 13 = 585$

In the above division, 3 and 5 are divisor and 3 and 13 are indivisible numbers.

Therefore, LCM is the product of divisors and indivisible numbers.

Illustrative EXAMPLE

Find the LCM of 8, 24, 28 by division method.

(a) 164   (b) 168

(c) 52   (d) All of these

(e) None of these

Answer: (b)

Explanation

$$
\begin{array}{c|l}
2 & 8,24,28 \\
\hline
2 & 4,12,14 \\
\hline
2 & 2,6,7 \\
\hline
 & 1,3,7 \\
\end{array}
$$

$= 2 \times 2 \times 2 \times 3 \times 7 = 168$

✸ Applications of LCM

Illustrative EXAMPLE

The least number which is exactly divisible by 120, 130 and 160 is.

(a) 6250   (b) 6245

(c) 6240   (d) All of these

(e) None of these

Answer: (c)

Explanation

2	120,130,160	
2	60,65,80	
2	30,65,40	$= 2 \times 2 \times 2 \times 5 \times 3 \times 13 \times 4 = 6240$
5	15,65,20	
	3,13,4	

Illustrative EXAMPLE

Find the nearest numbers t0 5000 which Is exactly divisible by each of 2, 3, 4 5, 6, 7, 8, and 9.
(a) 2520,5040   (b) 2530,5050
(c) 2540,5060   (d) All of these
(e) None of these

Answer: (a)

Illustrative EXAMPLE

The nearest number to 5000 which is exactly divisible by each one of the given numbers = 5000 − {Remainder of division 5000 − LCM of all divisors} − 5000 − {Remainder of division 5000 −2520} = 5000 − 24 80 = 2520 Another Nearest number to which is exactly divisible by each one of the = 5000 + {2520 − 3480} = 5000 + 40 = 5040.

HCF (highest Common Factor)

Highest Common Factor is also called as Greatest Common Measure (GCM) or Greatest Common Divisor (GCD). H.C.F of two or more numbers is the greatest number which exactly divides each of the number. Therefore, HCF of two or more numbers is its highest common factors.
Let us consider the HCF of 4, 8, 16 and 32.
Factorsof $4 = 2 \times 2$, $8 = 2 \times 2 \times 2$, $16 = 2 \times 2 \times 2 \times 2$, $32 = 2 \times 2 \times 2 \times 2 \times 2$
Therefore, $2 \times 2 = 4$ is the highest common factor which can exactly divide the numbers, 4, 8, 16, and 32. so, 4 is the HCF of 4, 8, 16 and 32.
H.C.F is calculated by prime factorization and continued division methods.

✳ HCF by Prime Factorization Method

The HCF of two or more numbers is obtained by the following steps:
Step 1: Find the prime factors of each one of the given number.
Step 2 : Find the common prime factors from prime factors of all the given numbers.
Step 4: The product of the common prime factors is the HCF of given numbers.

Let us consider the Greatest Common Measure of 36, 90, 72.
Step 1: The prime factors of
$36 = 2 \times 2 \times 3 \times 3, 90 = 2 \times 5 \times 3 \times 3$ and $72 = 2 \times 2 \times 2 \times 3 \times 3$
Step 2 : The common prime factors from all the prime factors $= 2 \times 3 \times 3$
Step 3 : Therefore, HCF of 36, 90 and $72 = 2 \times 3 \times 3 = 18$

Illustrative EXAMPLE

Find the HCF of 101, 573, and 1079 by prime factorization method?
(a) 4   (b) 1
(c) 2   (d) All of these
(e) None of these

Answer: (b)
Explanation
The prime factors of 101 = 1 and 101,573 = 1,3,191 but the common factor is 1. Therefore, the HCF of 101, 573 and 1079 = 1.

✸ HCF by Continued Division Method

The HCF of two or more numbers is obtained by continued division. The greatest number is considered as dividend and smallest number is divisor.

Follow the following steps to perform the HCF of the given numbers:
Step 1: Divide the greatest number by smallest.
Step 2: If remainder is zero, then divisor is the HCF of the given number.
Step 3: If remainder is not zero then, divide again by considering divisor as ne dividend and remainder as new divisor till remainder becomes zero.
Step 4: The HCF of the numbers is last divisor which gives zero remainder.

Illustrative
EXAMPLE

The HCF of 45, 76 = ?
(a) 1   (b) 2
(c) 3   (d) 0
(e) None of thes

Answer: (a)

```
45)76(1
   45
   31)45(1
      14)31(2
         28
          3)14(4
            12
            2)3(1
            2
            1)2(2
            2
            0
```

(45 is smaller than 76, therefore, 75 is divided by 45)
(31 is the remainder therefore, considered as the new divisor)
(45 is the first divisor therefore, it is consederd as new dividend)
(Divisor 1, gives zero remainder therefore, required HCF is 1)

Illustrative
EXAMPLE

Find the HCF of 78 and 1786 by continued division method?
(a) 2   (b) 5
(c) 1   (d) All of these
(e) None of these

Answer: (a)

Explanation

```
78)1786(2
   156
    22)78(3
       66
       12)22(1
          12
          10)12(1
             10
              2)10(5
                10
                 0
```

	HCF of 78 and 1786 = 2

✹ **HCF of more than Two Numbers**

The HCF of more than two numbers is the HCF of resulting HCF of two numbers with third one number. Therefore, HCF of more than two numbers is obtained by finding the HCF of two numbers with third, fourth and fifth numbers.

Illustrative
EXAMPLE

Find the HCF of 56, 98 and 123?
(a) 2   (b) 1
(c) 3   (d) All of these
(e) None of these

Answer: (b)
Explanation

Step 1: The HCF of 98 and 56

```
56)98(1
   56
   42)56(1
      42
      14)42(3
         42
          0
```

The HCF of 98 and 56=14, therefore, the required HCF of 56, 98 and 123 the HCF of 14 and 123

```
14)123(1
   112
   11)14(1
      11
      3)11(3
        9
        2)3(1
          2
          1)2(2
            2
            0          HCF of 56, 98 and 123=1
```

✸ HCF of Larger Numbers

the HCF of samaler number (one or two digit numbers) is simply obtained by division but division of larger numbers take more time, therefore, the shortes method for finding the HCF larger mumbers is performed by the following method.

Step: 1 divide, all the given numbers by the common divisor which divided which divide all numbers exactly till last.

Step: 2 divide the numbers which are obtained in step 1 by another divisor if divisible.

Step: 3 the required HCF is the product of common divisors.

Illustrative
EXAMPLE

Find the HCF of 3264 and 57384?

(a) 23   (b) 52
(c) 24   (b) all of these
(e) None of these

Answer: (c)

Explanation

2	3264,57384
2	1632,28692
2	816,14346
3	408,7173
	136,2991

The product of common divisors $= 2 \times 2 \times 2 \times 3 = 24,$ hence the required HCF is 24.

✸ Properties of HCF and LCM

1. The product of HCF and LCM of two numbers a and b is always equal to their product, therefore,

(a) LCM of numbers, a and b $\times$ HCF of a and $b = a \times b$

(b) LCM $\text{LCM} = \dfrac{a \times b}{\text{HCF}}$

(c) HCF of numbers a and $b = \dfrac{a \times b}{\text{LCM}}$

2. HCF of two or more numbers is not greater than any of the numbers.

3. LCM of two or more numbers is not less than any of the numbers.

4. HCF of two co - prime numbers is 1.

5. LCM of co - primes is equal to their product.

6. The HCF of two or more numbers is always a factor of their LCM.

7. If x and y are whole numbers and q is the quotient of their division and r is the remainder then,

(a) The least subtracted number from x so that y divides the difference exactly = r

(b) The least added number to x so that y divides the sum exactly = y − r.

✳ Applications on HCF

Illustrative EXAMPLE

Two containers contain 500 litres and 1000 litres of water. The maximui capacity of pot which can measure the water of either tanker in exact numb of times is?

(a) 250 litres   (b) 500 litres

(c) 1000 litres   (d) All of these

(e) None of these

Answer: (b)

Explanation

The maximum capacity of pot can be used to measure the water in exact number of times = The HCF of 500 and 1000 = 500.

Illustrative EXAMPLE

The measurement of a rectangular shaped iron piece is 3 m, 15 cm long, 2 m, cm wide and 1 m, 5 cm height. Find the longest tape which can measure the dimensions of the iron piece in exact number of times?

(a) 6cm   (b) 10cm

(c) 5cm   (d) All of these

(e) None of these

Answer: (c)

Explanation

The longest tape that can measure dimension of the iron piece in exact number of times = The HCF of the given dimension = 5.

Commonly Asked QUESTIONS

The HCF of two numbers is 28 and their LCM is 336. If one of them is 112. Find the other number?

(a) 78

(b) 68

(c) 84

(d) All of these

(e) None of these

Answer: (c)

Explanation

Product of two numbers = Product of their LCM and HCF.

Therefore, $112 \times$ unknown number $= 336 \times 28$, therefore, unknown number

$$= \frac{336 \times 28}{112} = 84.$$

The HCF of two numbers is 24, if their product is 4032 then their LCM is?
(a) 166   (b) 168
(c) 162   (d) All of these
(e) None of these

Answer: (b)
Explanation
The LCM of the numbers

$$= \frac{\text{Product of numbers}}{\text{HCF of the numbers}} = \frac{4032}{24} = 168$$

The greatest number which exactly divides the following numbers, 38, 95 and 171 is?
(a) 29   (b) 14
(c) 19   (d) All of these
(e) None of these

Answer: (c)
Explanation
The greatest number = HCF of the numbers = 19.

The HCF of an even and an odd number is?
(a) 0   (b) 2
(c) 1   (d) All of these
(e) None of these

Answer: (c)
Explanation
There is no common factor other than 1 for the HCF of an even and an odd number.

Find the least number which is exactly divisible by 4, 6 and 8?
(a) 24   (b) 94
(c) 20   (d) All of these
(e) None of these

Answer: (a)
Explanation
The least number which is exactly divisible by 4, 6 and 8 is their LCM = 24

Find the smallest number of five digits which is exactly divisible by 60, 90 and 80?
(a) 10080   .   (b) 10090
(c) 10100   (d) All of these
(e) None of these
Answer: (a)
Explanation

The LCM of 60, 80 and 90 is 720. The smallest five digit number which is exactly divisible by 60, 80 and 90 must be divisible their LCM. Division of 10000 by 720 leaves 640 as remainder, therefore, smallest five digit number which is exactly divisible by 60, 80 and 90 = 10000 + (720 − 640) = 10080.

Find the smallest number which when divided by 6, 8, 12, 15 and 20 leaves the remainder 5?
(a) 1075
(b) 1085
(c) 1065
(d) All of these
(e) None of these

Answer: (b)
Explanation
The LCM of 6, 8, 12, 15 and 20 is 120. The smallest number is more than three digits, therefore, the required number = 1000 + (120 − 40) + 5 = 1085.

Four traffic light bulbs blink after every 5, 10, 15 and 20 seconds. If they start blinking simultaneously at 5 a.m. then after what period of time will they blink again simultaneously?
(a) 80 seconds
(b) 60 seconds
(c) 50 seconds
(d) All of these
(e) None of these

Answer: (b)
Explanation
All four traffic lights will blink simultaneously after 5 a.m is the LCM of the given period of time

$$= \frac{\begin{array}{c} 2\overline{|5,10,15,20} \\ 5\overline{|5,5,15,10} \\ \overline{1,1,3,2} \end{array}}{} = 2 \times 2 \times 3 \times 5 \times = 60 \text{ seconds.}$$

In a factory three vessels produce sound simultaneously at 10. a.m. if the vessels produce sound after every 60, 120 and 180 minutes then at what period of time will they again produce sound simultaneously?
(a) 7 hours
(b) 8 hours
(c) 6 hours
(d) All of these
(e) None of these

Answer: (c)
Explanation
The vessels will produce sound again simultaneously = LCM of the given time period = 360 minutes = 6 hours.

You Must KNOW

- ❖ G.C.M and G.C.D are the common name of H.C.F.
- ❖ If a number a divides exactly to the another number b then a is a factor of b.
- ❖ Common division method is the short cut method for finding the LCM of the given numbers.
- ❖ LCM is used for comparison of the fractions.
- ❖ Prime factors of a number is always in prime numbers.

SUMMARY

- ❖ The HCF of the numbers is the largest number that divides each one of the given numbers.
- ❖ The LCM of the numbers is the smallest number that is divisible by each one of the given numbers.
- ❖ The HCF of numbers is a factor of their LCM.
- ❖ The LCM of numbers is always equal or greater than the numbers.
- ❖ The product of HCF and LCM of two numbers is always equal to the product of the numbers.

Self Evaluation TEST

Duration 10 Minutes

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1. Read the given options and choose the correct one?
(a) HCF of two consecutive prime numbers is always greater than 1
(b) 2 is not the smallest prime number
(c) If two numbers and their HCF and LCM are represented on the number line, then LCM would be nearest to the origin
(d) Product of two numbers is not equal to the product of their LCM and HCF.
(e) None of these

---

2. Find the HCF of two and four digit smallest number and check is it three digits smallest number?
(a) Yes   (b) No
(c) Data is insufficient   (d) All of these the
(e) None of these

---

3. HCF of three and four digit greatest number is:
(a) One digit smallest number   (b) One digit greatest number
(c) Lowest prime number   (d) Lowest composite number
(e) None of these

---

4. Find the lowest number that is exactly divisible by the lowest prime number, lowest composite number and lowest two digit number?
(a) 10   (b) 20
(c) 24   (d) 30
(e) None of these

---

5. Find the LCM of 20, 30 and 50.
(a) 300   (b) 150
(c) 160   (d) 400
(e) None of these

---

6. The LCM of two numbers is 24 their HCF is 4 and one of the numbers is 8. The other number is?
(a) 30
(b) 12
(c) 16
(d) 15
(e) None of these

7. **Consider the following two statements,**
   Statement 1. Factors of a number exactly divide the number.
   Statement 2. A number is not exactly divisible by their factors.
   Which one of the following options is correct about the above statements?
   (a) 1 is false and 2 is true   (b) 1 is true and 2 is false
   (c) Both are false statements   (d) Both are true statements
   (e) None of these

8. **The factors of 36 have been given in the options below. Which one of the following options is correct?**
   (a) 1, 2, 3, 4, 6, 9, 12, 18, 36   (b) 1, 3, 4, 6, 9, 18, 36
   (c) 1, 2, 3, 6, 9, 12, 36   (d) All of these
   (e) None of these

9. **What is the HCF of any two consecutive odd numbers?**
   (a) 1   (b) 0
   (c) 6   (d) All of these
   (e) None of these

10. **The product of two prime numbers and the smallest natural number are multiplied with each other and their product is 77. If one of the prime numbers is 11, then find the other prime number?**
    (a) 2   (b) 5
    (c) 11   (d) 7
    (e) None of these

Answers – Self Evaluation Test									
1. C	**2.** A	**3.** B	**4.** B	**5.** A	**6.** B	**7.** B	**8.** A	**9.** A	**10.** D

2. HCF of 1000 and 100 is 100.

3. HCF of 999 and 9999 is 9 which is one digits greatest number.

4. Lowest prime number is 2 and lowest composite number is 4. Tow digit lowest number is 10. Hence, the least common multiple which is exactly divisible by each of the given number is the LCM of the numbers = 20.

5. LCM of 20, 30 and 50 = 300

6. The product of LCM and HCF is equal to the product of the numbers. Thus, another number $= \dfrac{24 \times 4}{8} = 12$

7. Factors of a number exactly divide the number.

8. Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

9. The HCF of every two consecutive odd numbers is always 1.

10. The product of two prime numbers with the smallest natural number (1) is the number itself. If one number is 11 then the other number will be $\dfrac{77}{11} = 7$