DAY THIRTY ONE

Matter Waves

Learning & Revision for the Day

- de-Broglie Waves
- Davisson-Germer Experiment
- X-Rays
- Moseley's Law

de-Broglie Waves

Light is said to have dual character, i.e. it behaves like matter (particle) and wave both. Some properties like interference, diffraction can be explained on the basis of wave nature of light, while the phenomena like photoelectric effect, black body radiation, etc. can be explained on the basis of particle nature of light.

In 1942, Louis de-Broglie explained that like light, matter also show dual behaviour, there is a wave associated with moving particle, known as **matter waves or de-Broglie waves**.

de-Broglie Relation

According to quantum theory, energy of photon

$$E = h\nu \qquad \ldots\text{(i)}$$

If mass of the photon is taken as m, then as per Einstein's equation

$$E = mc^2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get, $h\nu = mc^2$

$$h\frac{c}{\lambda} = mc^2,$$

where, λ = wavelength of photon

$$\lambda = \frac{h}{mc}$$

de-Broglie asserted that the above equation is completely a general function and applies to photon as well as all other moving particles.

So, $$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

where, m is mass of particle and v is its velocity.

- **de-Broglie wavelength associated with charged particle**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

- **de-Broglie wavelength of a gas molecule**

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

where, T = absolute temperature

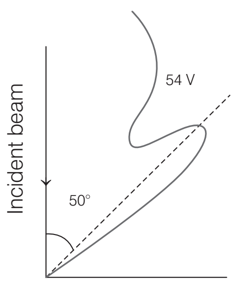
k = Boltzmann's constant $= 1.38 \times 10^{-23}$ J / K

- **Ratio of wavelength of photon and electron** The wavelength of photon of energy E is given by $\lambda_p = \frac{hc}{E}$ while the wavelength of an electron of kinetic energy K is given by $\lambda_c = \frac{h}{\sqrt{2mK}}$. Therefore for same energy, the ratio

$$\frac{\lambda_p}{\lambda_e} = \frac{c}{E}\sqrt{2mK} = \sqrt{\frac{2mc^2K}{E^2}}$$

Davisson-Germer Experiment

- The de-Broglie hypothesis was confirmed by Davisson-Germer experiment. It is used to study the scattering of electron from a solid or to verify the wave nature of electron.
- A beam of electrons emitted by electron gun is made to fall on nickel crystal cut along cubical axis at a particular angle. Ni crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.
- The diffracted beam of electrons is received by the detector which can be positioned at any angle by rotating it about the point of incidence.
- The energy of the incident beam of electrons can also be varied by changing the applied voltage to the electron gun.
- According to classical physics, the intensity of scattered beam of electrons at all scattering angle will be same but Davisson and Germer found that the intensity of scattered beam of electrons was not same but different at different angles of scattering.
- It is maximum for diffracting angle 50° at 54 V potential difference.
- If the de-Broglie waves exist for electrons, then these should be diffracted as X-rays.



Using the Bragg's formula $2d\sin\theta = n\lambda$,

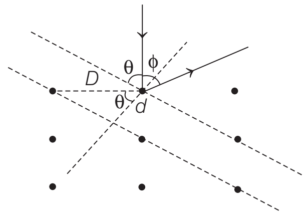
we can determine the wavelength of these waves,

where d = distance between diffracting planes,

$$\theta = \frac{180 - \phi}{2}$$

= glancing angle for incident beam = Bragg's angle.

Clearly from figure, we have $\theta + \phi + \theta = 180°$



X-Rays

X-rays were discovered by Roentgen. X-rays are produced when fast moving electrons strike a metal of high atomic weight and high melting point.

The phenomena of thermionic emission is used to produce electrons in coolidge tube. Intensity of X-rays is directly proportional to the square of the strength of the current in the filament which heats the cathode.

Quality of X-rays is measured in terms of their penetrating power and depends upon the potential difference applied to the X-ray tube. X-rays are diffracted by crystals in accordance with Bragg's law.

Properties of X-Rays

- X-rays are electromagnetic waves with wavelength range 0.1 Å to 100 Å.
- X-rays are invisible.
- X-rays carry no charge, so they are not deflected by electric and magnetic fields.
- They travel in straight line with speed $3 \times 10^8 \text{ ms}^{-1}$ through vacuum.
- They obey phenomenon of interference, diffraction and polarisation of light.
- They ionise gases.
- They effect photographic plate.
- They can pass through flesh and blood, but not through bones.
- They produce photoelectric effect and Compton effect.
- They are not used in RADAR as they are not reflected by the target. X-rays can be used to detect diseases and to cure them.

Types of X-Rays

X-rays are classified into two types on the basis of penetrating power

1. Soft X-rays

- These types of X-rays produced when the potential difference across the cathode and target is less than 20000 V.
- They have low penetrating power.
- These are used in the field of medicine.
- These are having large wavelengths.

2. Hard X-rays

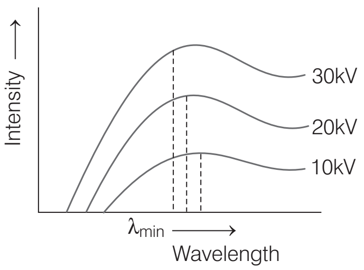
- These type of X-rays produced when the potential difference across the cathode and target is more than 30000 V.
- These rays are more penetrating than soft X-rays.
- They have low wavelength of the order of 1Å.
- These are used in the field of science and industry.

Typese of X-rays Spectra

X-rays spectra are of two types

1. Continuous X-Ray Spectrum

- X-rays of all wavelengths but having different intensities are emitted by the tube.
- As incident electron loses its energy continuously, due to collisions with atoms of the target, the loss of energy is found as X-rays. Hence, X-rays of all wavelengths are produced.



- These wavelength are of different intensity.
- The maximum frequency or minimum wavelength limit is due to the loss of the total energy of electron during a single collision.

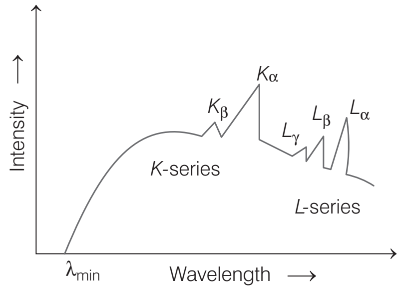
$$\frac{1}{2}mv^2 = h\nu_{\max} = eV$$

$$\Rightarrow \qquad \frac{hc}{\lambda_{\min}} = eV$$

$$\Rightarrow \qquad \lambda_{\min} = \frac{hc}{eV} = \frac{12400}{V} \text{ in Å}$$

2. Characteristics of X-Rays Spectrum

- Few of fast moving electrons having high velocity penetrate the surface atoms of the target material and knock out the tightly bound electrons even from the inner most shells of the atoms.
- If the electron striking the target eject an electron from *K*-shell of the atom, a vacancy is created in the *K*-shell. An electron from one of the outer shell say *L*-shell jumps to *K*-shell, emitting an X-ray photon of energy equal to the energy difference between the two shells.
- Similarly, if an electron from the *M*-shell jumps to the *K*-shell, X-ray photon of higher energy is emitted. The X-ray photons emission gives $K_\alpha, K_\beta, K_\gamma$ lines of the *K*-series of the spectrum.
- If the electron striking the target ejects an electron from *L*-shell of the target atom, an electron from the *M, N*,… shells jumps to the *L*-shell, so that X-rays photons of lesser energy are emitted. These photons form the *L*-series of spectrum.
- In similar way, the formation of *M*-series, *N*-series, etc. may be explained.
- Intensity wavelength graph is given by



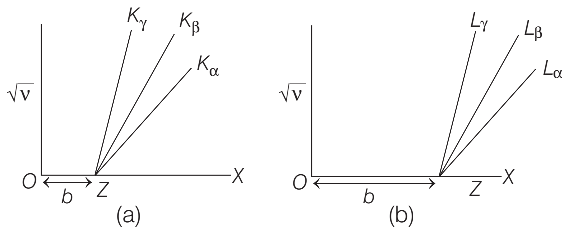
Moseley's Law

Moseley studied the characteristic of X-ray spectrum of a number of a heavy elements and concluded that the spectra of different elements are very similar and with increasing atomic number, the spectral lines merely shift towards higher frequency. If ν denotes the emitted frequency of a substance of atomic number *Z*, then

$$\sqrt{\nu} \propto Z \Rightarrow \nu = a(Z-b)^2$$

where, a and b are constants, a = proportionality constant, b = nuclear screening constant, values of (a) and (b) vary from one series to another series.

(i) For *K*-series, $b = 1$
(ii) For *L*-series, $b = 7.4$
(iii) For *M*-series, $b = 19.2$



The intercept along $+X$-axis denotes the constant b.

- Moseley's law is in accordance with Bohr's theory of spectral lines of atoms.

  Wavelength of characteristic spectrum,

$$\frac{1}{\lambda} = R(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

and energy of X-ray radiations,

$$\Delta E = h\nu = \frac{hc}{\lambda} = Rhc\,(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 If α, β and γ rays carry same momentum which has the longest wavelength?

(a) α-rays (b) β-rays (c) γ-rays (d) All have same wavelength

2 What is de-Broglie wavelength of a dust particle of mass 1×10^{-9} kg drifting with a speed of 2.2 m/s?

(a) 1.1×10^{-32} m (b) 3.01×10^{-25} m (c) 1.1×10^{-3} m (d) 3.01×10^{-2} m

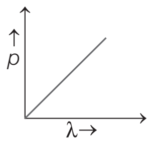
3 What will be the de-Broglie wavelength of a particle of rest mass m_0, if it moves with the speed of light?

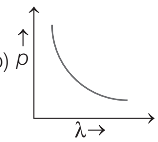
(a) $\frac{h}{m_0 c}$ (b) $\frac{2h}{m_0 c}$ (c) ∞ (d) 0

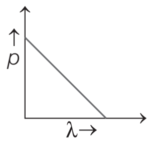
4 An α-particle and a deuteron are moving with the same kinetic energy. What will be the ratio of their de-Broglie wavelengths?

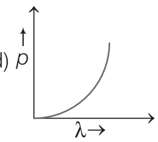
(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) 1 (d) $\sqrt{2}$

5 Which of the following figures represent the variation of particle momentum and the associated de-Broglie wavelength? ➜ CBSE AIPMT 2015

(a) 

(b) 

(c) 

(d) 

6 If the kinetic energy of the particle is increased to 16 times its previous value, the percentage change in the de-Broglie wavelength of the particle is ➜ CBSE AIPMT 2014

(a) 25 (b) 75 (c) 60 (d) 50

7 The following particles are moving with the same velocity, then maximum de-Broglie wavelength will be for ➜ CBSE AIPMT 2002

(a) proton (b) α-particle (c) neutron (d) β-particle

8 Nuclear radii may be measured by scattering high energy electrons from nuclei. What is the de-broglie wavelength for 200 MeV electrons?

(a) 8.28 fm (b) 7.98 fm (c) 6.45 fm (d) 6.20 fm

9 If an electron and a proton have the same de-Broglie wavelength, then the kinetic energy of the electron is

(a) zero
(b) less than that of proton
(c) more than that of a proton
(d) equal to that of proton

10 The de-Broglie wavelength of a proton (charge $= 1.6 \times 10^{-19}$ C, $m = 1.6 \times 10^{-27}$ kg) accelerated through a potential difference of 1 kV is

(a) 600 Å (b) 0.9×10^{-12} m (c) 7 Å (d) 0.9 nm

11 The de-Broglie wavelength of 1 MeV proton is

(a) 6.63×10^{-34} m (b) 3.33×10^{-30} m (c) 2.32×10^{-20} m (d) 2.86×10^{-4} m

12 The de-Broglie wavelength associated with an electron, accelerated through a potential difference of 100 V is

(a) 0.529 nm (b) 52.9 nm (c) 0.123 nm (d) 1.23 nm

13 A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to

(a) H (b) $H^{1/2}$ (c) H^0 (d) $H^{-1/2}$

14 Energy of neutron (in eV) whose de-Broglie wavelength is 1 Å

(a) 1.674×10^{-27} eV (b) 8.13×10^{-2} eV (c) 6.62×10^{-22} eV (d) 3.23×10^{-2} eV

15 In the Davisson-Germer experiment, if the incident beam consists of electrons, then the diffracted beam consists of

(a) protons (b) neutrons (c) α-particles (d) electrons

16 The X-ray tube is operated at 50 kV. The minimum wavelength produced, is

(a) 0.5 Å (b) 0.75 Å (c) 0.25 Å (d) 1.0 Å

17 What kV potential is to be applied on X-ray tube, so that minimum wavelength of emitted X-ray may be 1 Å (Take, $h = 6.6 \times 10^{-34}$ J-s)

(a) 12.42 kV (b) 12.84 kV (c) 11.98 kV (d) 10.78 kV

18 If f_1, f_2 and f_3 are the frequencies of corresponding K_α, K_β and L_α, X-rays of an electron, then

(a) $f_1 = f_2 = f_3$ (b) $f_1 - f_2 = f_3$ (c) $f_2 = f_1 + f_3$ (d) $f_2^2 = f_1 f_3$

19 According to the Moseley's law, the frequency of characteristic X-rays is related to the atom in number of target element as

(a) Z (b) Z^2
(c) Z^{-1} (d) Z^{-2}

20 Two elements A and B with atomic numbers Z_A and Z_B are used to produce characteristics X-rays with frequencies ν_A and ν_B, respectively. If $Z_A : Z_B = 1 : 2$, then $\nu_A : \nu_B$ will be

(a) $1 : \sqrt{2}$ (b) 1 : 8 (c) 4 : 1 (d) 1 : 4

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 A particle of mass 1 mg has the same wavelength as an electron moving with a velocity of 3×10^6 ms^{-1}. The velocity of the particle is (Take, mass of electron $= 9.1 \times 10^{-31}$ kg) ➜ CBSE AIPMT 2008

(a) 2.7×10^{-18} ms^{-1} (b) 9×10^{-2} ms^{-1}
(c) 3×10^{-31} ms^{-1} (d) 2.7×10^{-21} ms^{-1}

2 The de-Broglie wavelength of a bus moving with speed v is λ. Some passengers left the bus at a stoppage. Now, when the bus moves with twice its initial speed, its kinetic energy is found to be twice its initial value. The de-Broglie wavelength will now

(a) λ (b) 2λ (c) $\frac{\lambda}{2}$ (d) $\frac{\lambda}{4}$

3 X-rays of wavelength $\lambda_0 = 0.200$ nm are scattered from a block of material. The scattered X-rays are observed at an angle of 45° to the incident beam. Calculate their wavelength.

(a) 0.300700 nm (b) 0.100710 nm
(c) 0.200710 nm (d) 0.400710 nm

4 An electron is moving with an initial velocity $\mathbf{v} = v_0 \hat{\mathbf{i}}$ and is in a magnetic field $\mathbf{B} = B_0 \hat{\mathbf{j}}$. Then, it's de-Broglie wavelength

(a) remains constant (b) increases with time
(c) decreases with time
(d) increases and decreases periodically

5 A parallel beam of fast moving electrons is incident normally on a narrow slit. A fluorescent screen is placed at a large distance from the slit. If the speed of the electrons is increased, then which of the following statement(s) is/are correct? ➜ NEET 2013

(a) Diffraction pattern is not observed on the screen in the case of electrons
(b) The angular width of the central maximum of the diffraction pattern will increase
(c) The angular width of the central maximum will be decrease
(d) The angular width of the central maximum will be unaffected

6 The potential energy of a particle of mass m varies as

$$U(x) = \begin{cases} E_0 & \text{for } 0 \le x \le 1 \\ 0 & \text{for } x > 1 \end{cases}$$

The de-Broglie wavelength of the particle in the range $0 \le x \le 1$ is λ_1 and that in the range $x > 1$ is λ_2. The total energy of the particle is $2E_0$. Ratio $\frac{\lambda_1}{\lambda_2}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{2}$

7 The de-Broglie wavelength of a neutron in thermal equilibrium with heavy water at a temperature T (kelvin) and mass m, is ➜ NEET 2017

(a) $\frac{h}{\sqrt{mkT}}$ (b) $\frac{h}{\sqrt{3mkT}}$ (c) $\frac{2h}{\sqrt{3mkT}}$ (d) $\frac{2h}{\sqrt{mkT}}$

8 Electrons of mass m with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength (λ_0) of the emitted X-ray is ➜ NEET 2016

(a) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (b) $\lambda_0 = \frac{2h}{mc}$
(c) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (d) $\lambda_0 = \lambda$

9 The energy that should be added to an electron, to reduce its de-Broglie wavelength from 10^{-10} m to 0.5×10^{-10} m, will be

(a) four times the initial energy
(b) thrice the initial energy
(c) equal to the initial energy (d) twice the initial energy

10 An electron of mass m and a photon have same energy E. The ratio of de-Broglie wavelengths associated with them is (c being velocity of time) ➜ NEET 2016

(a) $\left(\frac{E}{2m}\right)^{\frac{1}{2}}$ (b) $c(2mE)^{\frac{1}{2}}$ (c) $\frac{1}{c}\left(\frac{2m}{E}\right)^{\frac{1}{2}}$ (d) $\frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$

11 An electron of mass m with a velocity $\mathbf{v} = v_0 \hat{\mathbf{i}}$ ($v_0 > 0$) enters an electric field $\mathbf{E} = -E_0 \hat{\mathbf{i}}$ (E_0 = constant > 0) at $t = 0$. If λ_0 is its de-Broglie wavelength initially, then its de-Broglie wavelength at time t is

(a) $\lambda_0 t$ (b) $\lambda_0 \left(1 + \frac{eE_0}{mv_0} t\right)$
(c) $\frac{\lambda_0}{\left(1 + \frac{eE_0}{mv_0} t\right)}$ (d) λ_0

ANSWERS

SESSION 1	**1** (d)	**2** (b)	**3** (d)	**4** (a)	**5** (b)	**6** (b)	**7** (d)	**8** (d)	**9** (c)	**10** (b)
	11 (d)	**12** (c)	**13** (d)	**14** (b)	**15** (d)	**16** (c)	**17** (a)	**18** (c)	**19** (b)	**20** (d)
SESSION 2	**1** (a)	**2** (a)	**3** (c)	**4** (a)	**5** (b)	**6** (d)	**7** (b)	**8** (a)	**9** (b)	**10** (d)
	11 (c)									

Hints and Explanations

SESSION 1

1 de-Broglie suggested that the dual nature is not only of light, but each moving material particle has the dual nature. The wavelength of wave, $\lambda = \frac{h}{p}$

where, p is momentum

It is given that, α, β and γ-rays carry same momentum, so they will have same wavelength. Hence, option (d) is true.

2 Given, $m = 1 \times 10^{-9}$ kg, $v = 2.2\ \text{ms}^{-1}$

$\therefore\ p = mv = 1 \times 10^{-9} \times 2.2$

$= 2.2 \times 10^{-9}\ \text{kgms}^{-1}$

$\therefore\ \lambda = \frac{h}{p} = \frac{6.62 \times 10^{-34}}{2.2 \times 10^{-9}}$

$\Rightarrow \lambda = 3.01 \times 10^{-25}$ m

3 The mass of particle will be infinite, hence $\lambda = \frac{h}{mc} = 0$.

4 $\lambda = \frac{h}{\sqrt{2mK}}$.

Hence, $\frac{\lambda_\alpha}{\lambda_d} = \sqrt{\frac{m_d}{m_\alpha}} = \frac{1}{\sqrt{2}}$

5 Since, $\lambda = \frac{h}{p}$ $\quad [\because p\lambda = h]$

Therefore, graph will be hyperbola.

6 From de-Broglie wavelength,

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$ $\quad [\because p = \sqrt{2mE}]$

$\therefore\ \lambda' = \frac{h}{\sqrt{2m(16E)}} = \frac{\lambda}{4} = 0.25\lambda$,

% change = 75%

7 de-Broglie wavelength is given by

$\lambda = \frac{h}{mv}$

For same velocity, $\lambda \propto \frac{1}{m}$

Out of the given particles, the mass of β-particle which is a fast moving electron is minimum. Thus, de-Broglie wavelength is maximum for β-particle.

8 The de-Broglie wavelength is given by

$\lambda = \frac{hc}{E} = \frac{1240}{200 \times 10^6} = 6.20$ fm

9 Given, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m\text{KE}}} \Rightarrow \text{KE} = \frac{1}{2m\lambda^2}$

As, λ is same for both electron and proton, KE $\propto \frac{1}{m}$.

Hence, kinetic energy will be maximum for particle with lesser mass, i.e. electron.

10 $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$\Rightarrow\ \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-19} \times 1000}}$

$\Rightarrow\ \lambda = \frac{6.6 \times 10^{-34}}{7.16 \times 10^{-22}} = 0.9 \times 10^{-12}$ m

11 $\because\ \lambda = \frac{0.286}{\sqrt{V}} = \frac{0.286}{\sqrt{10^6}} = 2.86 \times 10^{-4}$ m

12 Accelerating potential, $V = 100$ V

The de-Broglie wavelength,

$\lambda = \frac{1.227}{\sqrt{V}}$ nm,

$\lambda = \frac{1.227}{\sqrt{100}}$ nm $= 0.123$ nm

13 Velocity acquired by a particle, while falling from a height H is,

$v = \sqrt{2gH}$

$\therefore\ \lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}}$ or $\lambda \propto H^{-1/2}$

14 $\because\ \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}} \Rightarrow E = \frac{h^2}{2m\lambda^2}$

Given, $m = 1.674 \times 10^{-27}$ kg, $\lambda = 1$ Å,

$h = 6.62 \times 10^{-34}$ J-s

$= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.674 \times 10^{-27} \times (10^{-10})^2}$

$= 13.01 \times 10^{-21}$ J

$\therefore\ E = 8.13 \times 10^{-2}$ eV

15 In Davisson-Germer experiment working on the scattering basis, if the incident beam consists of electrons, then the diffracted beam consists of electrons.

16 $\therefore\ \lambda = \frac{hc}{eV} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 50 \times 10^3}$

$\Rightarrow \lambda = \frac{19.8 \times 10^{-26}}{80 \times 10^{-16}} = 0.25$ Å

17 $\therefore \lambda_{\text{min}} = \frac{12375}{V}$ Å $= \frac{12375}{1}$ Å

$= 12.375$ kV $= 12.42$ kV

18 For K_α, L-shell to K-shell, K_β,

For K_β, M-shell to K-shell

L_α, M-shell to L-shell

$E_M - E_K = (E_M - E_L) + (E_L - E_K)$

$\Rightarrow\ hf_2 = hf_3 + hf_1$

$\Rightarrow\ f_2 = f_1 + f_3$

19 According to the Moseley's law, the frequency of characteristic X-rays is related to the atom in number of target element as Z^2.

20 According to Moseley's law, $\sqrt{\nu} \propto Z$

$\therefore\ \frac{\nu_A}{\nu_B} = \left(\frac{Z_A}{Z_B}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

SESSION 2

1 According to de-Broglie relation, wavelength of a particle is given by

$\lambda = \frac{h}{p}$

where, h is Planck's constant and wavelength of an electron is given by

$\lambda_e = \frac{h}{p_e}$

But $\lambda = \lambda_e$, so $p = p_e$

or $m_e v_e = mv$

or $\quad v = \dfrac{m_e v_e}{m}$

Here, $m_e = 9.1 \times 10^{-31}$ kg,

$v_e = 3 \times 10^6 \text{ ms}^{-1}$

and $\quad m = 1 \text{ mg} = 1 \times 10^{-6}$ kg

$\therefore \quad v = \dfrac{9.1 \times 10^{-31} \times 3 \times 10^6}{1 \times 10^{-6}}$

$= 2.7 \times 10^{-18}$ m / s

2 Momentum, $p = mv = \dfrac{\frac{1}{2} mv \times v}{\frac{1}{2} \times v} = \dfrac{2\,\text{KE}}{v}$

If kinetic energy as well as speed are doubled, momentum p remains unchanged $\lambda = \dfrac{h}{p}$.

Hence, de-Broglie wavelength will be unchanged.

3 The shift in wavelength of the scattered X-rays is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

Substituting the values, we have

$$\lambda' - \lambda = \frac{6.626 \times 10^{-34}}{(9.11 \times 10^{-31})(3.00 \times 10^8)} (1 - \cos 45^\circ)$$

$= 7.10 \times 10^{-13}\text{ m} = 0.000710$ nm

$\therefore \lambda' = (0.200) + (0.000710)$

$= 0.200710$ nm

4 Here, $\mathbf{v} = v_0\,\hat{\mathbf{i}}$, $\mathbf{B} = B_0\,\hat{\mathbf{j}}$.

Force on moving electron due to magnetic field is

$\mathbf{F} = -e(\mathbf{v} \times \mathbf{B}) = -e[v_0\,\hat{\mathbf{i}} \times B_0\,\hat{\mathbf{j}}]$

$= -ev_0 B_0\,\hat{\mathbf{k}}$

As this force is perpendicular to **v** and **B**, so the magnitude of **v** will not change, i.e. momentum ($= mv$) will remain constant in magnitude. Hence, de-Broglie wavelength $\lambda = h/mv$ remains constant.

5 As the screen is placed at larger distance and the speed of electron increases, hence the angular width of the central maxima of the diffraction pattern will increase.

6 Total energy,

$2E_0$ = Kinetic energy + Potential energy

$= K + U(x)$

In the range $0 \le x \le 1, U(x) = E_0$ (given)

$\therefore \quad K = 2E_0 - U(x) = 2E_0 - E_0 = E_0$

de-Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda_1 = \frac{h}{\sqrt{2mE_0}}$$

In the range $x > 1$, $U(x) = 0$ (given)

$\therefore \quad K = 2E_0 - U(x) = 2E_0$

$$\lambda_2 = \frac{h}{\sqrt{2m(2E_0)}}$$

$\therefore \quad \dfrac{\lambda_1}{\lambda_2} = \sqrt{2}$

7 de-Broglie wavelength of the neutron,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m(\text{KE})}}$$

$$= \frac{h}{\sqrt{2m \times \frac{3}{2}kT}} = \frac{h}{\sqrt{3mkT}}$$

8 Given, mass of electrons $= m$

de-Broglie wavelength $= \lambda$

So, kinetic energy of electron $= \dfrac{p^2}{2m}$

$$= \frac{\left(\frac{h}{\lambda}\right)^2}{2m} = \frac{h^2}{2m\lambda^2}$$

Now, maximum energy of photon can be given by

$$E = \frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2}$$

$$\Rightarrow \quad \lambda_0 = \frac{hc \times 2\lambda^2 \cdot m}{h^2}$$

$$= \frac{2mc\lambda^2}{h}$$

9 The wavelength of electron of energy E is $\quad \lambda = \dfrac{h}{\sqrt{2mE}}$

$\Rightarrow \quad \lambda \propto \dfrac{1}{\sqrt{E}}$

$\Rightarrow \quad \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{E_2}{E_1}}$

$\Rightarrow \quad \dfrac{10^{-10}}{0.5 \times 10^{-10}} = \sqrt{\dfrac{E_2}{E_1}}$

$\Rightarrow \quad \dfrac{E_2}{E_1} = 4$

$\Rightarrow \quad E_2 = 4E_1$

Hence the required energy $= E_2 - E_1 = 3E_1$ is thrice of initial energy.

10 (d) Since, it is given that electron has mass m.

de-Broglie's wavelength for an electron will be given as

$$\lambda_e = \frac{h}{P} \quad \text{...(i)}$$

where, h = Planck's constant

P = Linear momentum of electron

As kinetic energy of electron, $E = \dfrac{P^2}{2m}$

$$\Rightarrow \quad P = \sqrt{2mE} \quad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\lambda_e = \frac{h}{\sqrt{2mE}} \quad \text{...(iii)}$$

Energy of a photon can be given as

$E = h\nu$

$\Rightarrow \quad E = \dfrac{hc}{\lambda_p}$

$$\Rightarrow \quad \lambda_p = \frac{hc}{E} \quad \text{...(iv)}$$

Hence, λ_p = de-Broglie's wavelength of photon.

Now, divide equation (iii) by (iv), we get

$$\frac{\lambda_e}{\lambda_p} = \frac{h}{\sqrt{2mE}} \cdot \frac{E}{hc}$$

$$\Rightarrow \quad \frac{\lambda_c}{\lambda_p} = \frac{1}{c} \cdot \sqrt{\frac{E}{2m}}$$

11 According to the question,

$\mathbf{v} = v_0\,\hat{\mathbf{i}}, \quad \mathbf{E} = -E_0\,\hat{\mathbf{i}}$

Thus, magnitude of force on the electron due to the electric field, $|\mathbf{F}| = q|\mathbf{E}|$

$\Rightarrow \quad F = eE_0$

From Newton's second law of motion,

$F = ma$

$\therefore \quad F = ma = eE_0$

$$\Rightarrow \quad a = \frac{eE_0}{m} \quad \text{...(i)}$$

or $\quad \mathbf{a} = \dfrac{(-e)(-E_0\,\hat{\mathbf{i}})}{m} = \dfrac{eE_0}{m}\,\hat{\mathbf{i}}$

From first equation of motion,

$v = u + at$

Here, u (initial velocity) $= v_0$

$$\Rightarrow \quad v = v_0 + \frac{eE_0}{m}t \quad \text{...(ii)}$$

(from Eq. (i))

Initial de-Broglie wavelength of the electron is given as

$$\lambda_0 = \frac{h}{mv_0} \Rightarrow h = \lambda m v_0 \quad \text{...(iii)}$$

After time t, de-Broglie wavelength is given as

$$\lambda = \frac{h}{mv}$$

Substituting the value of v from Eq. (ii), we get

$$\lambda = \frac{h}{m\left(v_0 + \frac{eE_0}{m}t\right)}$$

$$= \frac{h}{mv_0\left[1 + \frac{eE_0}{mv_0}t\right]}$$

$$= \frac{\lambda\, mv_0}{mv_0\left[1 + \frac{eE_0}{mv_0}t\right]} \quad \text{[from Eq. (iii)]}$$

$$= \frac{\lambda_0}{\left[1 + \frac{eE_0}{mv_0}t\right]}$$

$$\therefore \quad \lambda = \frac{\lambda_0}{1 + \frac{eE_0}{mv_0}t}$$