Chapter – 8

Descriptive Statistics and Probability

Ex 8.1

Question 1.
Find the first quartile and third quartile for the given observations.
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22.

Solution:
Given data are arranged in ascending order 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22.
Here the number of observations is n = 11

$$Q_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{11+1}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 3^{rd} \text{ value} = 6$$

$$Q_3 = \text{size of } \left(\frac{3(N+1)}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{3\times12}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 9^{th} \text{ value} = 18$$

Question 2.
Find Q_1, Q_3, D_8, and P_{67} of the following data:

Size of Shares	4	4.5	5	5.5	6	6.5	7	7.5	8
Frequency	10	18	22	25	40	15	10	8	7

Solution:

X	f	cf
4	10	10
4.5	18	28
5	22	50
5.5	25	75
6	40	115
6.5	15	130
7	10	140
7.5	8	148
8	7	155
	N = 155	

$$Q_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{155+1}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 39^{th} \text{ value} = 5$$

$$Q_3 = \text{size of } \left[3\left(\frac{N+1}{4}\right)\right]^{th} \text{ value}$$

$$= \text{size of } \left(3 \times \frac{156}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 117^{th} \text{ value} = 6.5$$

$$D_8 = \text{size of } \left[8\left(\frac{N+1}{10}\right)\right]^{th} \text{ value}$$

$$= \text{size of } \left[8\left(\frac{155+1}{10}\right)\right]^{th} \text{ value}$$

$$= \text{size of } \left(8 \times \frac{156}{10}\right)^{th} \text{ value}$$

$$= \text{size of } 124.8^{th} \text{ value} = 6.5$$

$$P_{67} = \text{size of } \left[67\left(\frac{N+1}{100}\right)\right]^{th} \text{ value}$$

$$= \text{size of } \left(\frac{67 \times 156}{100}\right)^{th} \text{ value}$$

$$= \text{size of } 104.52^{th} \text{ value} = 6$$

Question 3.

Find lower quartile, upper quartile, 7th decile, 5th decile, and 60th percentile for the following frequency distribution.

Wages	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	1	3	11	21	43	32	9

Solution:

C.I	*f*	*cf*
10-20	1	1
20-30	3	4
30-40	11	15
40-50	21	36
50-60	43	79
60-70	32	111
70-80	9	120
	N = 120	

Lower quartile, Q_1 = size of $\left(\frac{N}{4}\right)^{th}$ value

$= \left(\frac{120}{4}\right)^{th}$

= size of 30th value

Q_1 lies in the class (40 – 50) and its corresponding values are L = 40, N/4 = 30, pcf = 15, f = 21 and C = 10

$$Q_1 = L + \left(\frac{\frac{N}{4} - pcf}{f}\right) \times C$$

$$= 40 + \left(\frac{30-15}{21}\right) \times 10$$

$$= 40 + \frac{15}{21} \times 10 = 40 + 7.14 = 47.14$$

$$Q_3 = \text{size of } \left(\frac{3N}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{3\times120}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 90^{th} \text{ value}$$

Q_3 lies in the class (60 – 70) and its corresponding values are L = 60, 3N/4 = 90, pcf = 79, f = 32 and C = 10.

$$Q_3 = L + \left(\frac{\frac{3N}{4} - pcf}{f}\right) \times C$$

$$= 60 + \left(\frac{90-79}{32}\right) \times 10$$

$$= 60 + \frac{11}{32} \times 10$$

$$= 60 + 3.4375 = 63.4375 = 63.44$$

$$7^{th}\ \text{decile} = D_7 = \text{size of } \left(\frac{7N}{10}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{7\times 120}{10}\right)^{th} \text{ value}$$

$$= \text{size of } 84^{th} \text{ value}$$

Thus D_7 lies in the class (60 – 70) and its corresponding values are L = 60, 7N/10 = 84, pcf = 79, f = 32 and C = 10.

$$D_7 = L + \left(\frac{\frac{7N}{10} - pcf}{f}\right) \times C$$

$$= 60 + \left(\frac{84-79}{32}\right) \times 10$$

$$= 60 + \frac{5}{32} \times 10$$

$$= 60 + \frac{50}{32} = 60 + 1.5625$$

$$= 60 + 1.56 = 61.56$$

$$5^{th}\ \text{decile} = D_5 = \text{size of } \left(\frac{5N}{10}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{5\times 120}{10}\right)^{th} \text{ value}$$

$$= \text{size of } 60^{th} \text{ value}$$

Thus D_5 lies in the class (50 – 60) and its corresponding values are L = 50, 5N/10 = 60, pcf = 36, f = 43 and C = 10.

$$D_5 = L + \left(\frac{\frac{5N}{10} - pcf}{f}\right) \times C$$

$$= 50 + \left(\frac{60-36}{43}\right) \times 10$$

$$= 50 + \frac{24}{43} \times 10$$

$$= 50 + \frac{240}{43} = 50 + 5.581$$

$$= 55.581 = 55.58$$

$$P_{60} = \text{size of } \left(\frac{60N}{100}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{60 \times 120}{100}\right)^{th} \text{ value}$$

$$= \text{size of } 72^{th} \text{ value}$$

Thus P_{60} lies in the class (50 – 60) and its corresponding values are L = 50, 60N/100 = 72, pcf = 36, f = 43 and C = 10.

$$P_{60} = L + \left(\frac{\frac{60N}{100} - pcf}{f}\right) \times C$$

$$= 50 + \left(\frac{72-36}{43}\right) \times 10$$

$$= 50 + \frac{36}{43} \times 10 = 50 + 8.37 = 58.37$$

Question 4.

Calculate GM for the following table gives the weight of 31 persons in the sample survey.

Weight (lbs):	130	135	140	145	146	148	149	150	157
Frequency	3	4	6	6	3	5	2	1	1

Solution:

X	f	log X	f(log X)
130	3	2.1139	6.3417
135	4	2.1303	8.5212
140	6	2.1461	12.8766
145	6	2.1614	12.9684
146	3	2.1644	6.4932
148	5	2.1703	10.8515
149	2	2.1732	4.3464
150	1	2.1761	2.1761
157	1	2.1959	2.1959
	N = 31		66.764

$$G.M = \text{Antilog}\left(\frac{\Sigma f \log X}{N}\right)$$

$$= \text{Antilog}\left(\frac{66.7764}{31}\right)$$

= Antilog (2.1540)
= 142.560
= 142.56 lbs

Question 5.
The price of a commodity increased by 5% from 2004 to 2005, 8% from 2005 to 2006, and 77% from 2006 to 2007. Calculate the average increase from 2004 to 2007?

Solution:
In averaging ratios and percentages, the geometric mean is more appropriate. Let us consider X represents prices at the end of the year.

Percentage rise	X	log X
5	105	2.0211
8	108	2.0334
77	177	2.2479
		$\Sigma \log X = 6.3024$

Here n = 3

$$GM = \text{Antilog}\left(\frac{\Sigma \log X}{n}\right)$$

$$= \text{Antilog}\left(\frac{6.3024}{3}\right)$$

= Antilog (2.1008)
= 126.1246
Average rate of increase of price = 126.1246 – 100
= 26.1246
= 26.1%

Question 6.
An aeroplane flies, along the four sides of a square at speeds of 160, 200, 300, and 400 kilometres per hour respectively. What is the average speed of the plane in its flight around the square?

Solution:
Harmonic mean would he suitable. Harmonic mean of n observations is

$$HM = \frac{n}{\Sigma \frac{1}{x}}$$

Here n = 4

$$\therefore HM = \frac{4}{\frac{1}{100}+\frac{1}{200}+\frac{1}{300}+\frac{1}{400}} = \frac{4}{\left(\frac{12+6+4+3}{1200}\right)}$$

$$= \frac{4}{\left(\frac{25}{1200}\right)} = \frac{4\times 1200}{25} = 4 \times 48 = 192 \text{ km/hr}$$

Question 7.
A man travelled by car for 3 days. He covered 480 km each day. On the first day, he drove for 10 hours at 48 km an hour. On the second day, he drove for 12 hours at 40 km an hour, and for the last day, he drove for 15 hours at 32 km. What is his average speed?

Solution:
Total distance covered 480 km.
The first-day distance covered 48 km.
The second-day distance covered 40 km.
Third-day distance covered 32 km.
Number of observations = 3
Average speed = HM

$$= \frac{3}{\Sigma \frac{1}{x}} = \frac{3}{\left(\frac{1}{48}+\frac{1}{40}+\frac{1}{32}\right)} = \frac{3}{\left(\frac{10+12+15}{480}\right)}$$

$$= \frac{3\times 480}{37} = 38.9189 = 38.92 \text{ km/hr}$$

Question 8.
The monthly incomes of 8 families in rupees in a certain locality are given below. Calculate the mean, the geometric mean, and the harmonic mean and confirm that the relations among them hold true. Verify their relationships among averages.

Family:	A	B	C	D	E	F	G	H
Income (Rs.):	70	10	50	75	8	25	8	42

Solution:

$$AM = \frac{70+10+50+75+8+25+8+42}{8} = \frac{288}{8} = 36$$

Now we will find the geometric mean. (GM)

X	log X
70	1.8451
10	1.0000
50	1.6990
75	1.8751
8	0.9031
25	1.3979
8	0.9031
42	1.6232
	11.2465

Here number of observations is n = 8

GM = Antilog [latex] $\left(\frac{\Sigma \log X}{n}\right)$ [/latex]

= Antilog $\left(\frac{11.2465}{8}\right)$

= Antilog 1.4058

GM = 25.4566

Now we will find Harmonic Mean (HM).

Here the number of observations is n = 8.

$$HM = \frac{n}{\Sigma \frac{1}{X}} = \frac{8}{\frac{1}{70}+\frac{1}{10}+\frac{1}{50}+\frac{1}{75}+\frac{1}{8}+\frac{1}{25}+\frac{1}{8}+\frac{1}{42}}$$

$$= \frac{8}{0.0143+0.1+0.02+0.0133+0.125+0.04+0.125+0.0238}$$

$$= \frac{8}{0.4614} = 17.3385$$

Thus AM = 36, GM = 25.466, HM = 17.3385

36 > 25.466 > 17.3385

Hence AM > GM > HM

Thus their relations among them is verified for the given data.

Question 9.
Calculate AM, GM, and HM and also verify their relations among them for the following data:

X	5	15	10	30	25	20	35	40
f	18	16	20	21	22	13	12	16

Solution:

X	f	fX	log X	f log X	$\frac{f}{X}$
5	18	90	0.6990	12.5820	3.6
15	16	240	1.1761	18.8176	1.0667
10	20	200	1.0000	20.0000	2.000
30	21	630	1.4771	31.0191	0.7000
25	22	550	1.3979	30.7538	0.8800
20	13	260	1.3010	16.9130	0.6500
35	12	420	1.5441	18.5292	0.3429
40	16	640	1.6021	25.6336	0.4000
	N = 138	$\Sigma fX = 3030$		$\Sigma f \log X = 174.2483$	$\frac{\Sigma f}{X} = 9.6396$

$$AM = \frac{\Sigma f X}{N} = \frac{3030}{138} = 21.9565$$
$$= 21.96$$

$$GM = \text{Antilog}\left(\frac{\Sigma f \log X}{N}\right)$$
$$= \text{Antilog}\left(\frac{174.2483}{138}\right)$$
$$= \text{Antilog}\,(1.26267)$$
$$= 18.3092 = 18.31$$

$$HM = \frac{N}{\Sigma \frac{f}{X}} = \frac{138}{9.6396} = 14.3159 = 14.32$$

Question 10.
Calculate AM, GM, and HM from the following data and also find its relationship:

Marks:	0-10	10-20	20-30	30-40	40-50	50-60
No. of students:	5	10	25	30	20	10

Solution:

X	f	fX	log X	f log X	$\frac{f}{X}$
5	5	25	0.6990	3.4950	1.0000
15	10	150	1.1761	11.761	0.6667
25	25	625	1.3979	34.9475	1
35	30	1050	1.5441	46.323	0.8571
45	20	900	1.6532	33.0664	0.4444
55	10	550	1.7404	17.404	0.1818
	N = 100	$\Sigma fX = 3300$		$\Sigma f \log X = 146.9969$	$\frac{\Sigma f}{X} = 4.15$

$$AM = \frac{\Sigma fX}{N} = \frac{3300}{100} = 33$$

$$GM = \text{Antilog}\left(\frac{\Sigma f \log X}{N}\right)$$

$$= \text{Antilog}\left(\frac{146.9969}{100}\right)$$

$$= \text{Antilog}\,(1.469969)$$

$$= \text{Antilog}\,(1.4700)$$

$$= 29.51$$

$$HM = \frac{N}{\Sigma \frac{f}{X}} = \frac{100}{4.15} = 24.096 = 24.10 \text{ (approx.)}$$

Question 11.
Calculate the quartile deviation and its coefficient from the following data:

Age in Years:	20	30	40	50	60	70	80
No. of Members:	13	61	47	15	10	18	36

Solution:

X	f	cf
20	13	13
30	61	74
40	47	121
50	15	136
60	10	146
70	18	164
80	36	200
	$N = \Sigma f = 200$	

$$Q_1 = \text{size of } \left(\frac{N+1}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{200+1}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{201}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 50.25^{th} \text{ value} = 30$$

$$Q_3 = \text{size of } \left(\frac{3(N+1)}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{3\times 202}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{606}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 151.5^{th} \text{ value} = 70$$

$$QD = \frac{1}{2}(Q_3 - Q_1)$$

$$= \frac{1}{2}(70 - 30) = \frac{1}{2} \times 40 = 20$$

$$\text{Coefficient of QD} = \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{70 - 30}{70 + 30} = \frac{40}{100} = 0.4$$

Question 12.

Calculate quartile deviation and its relative measure from the following data:

X	0-10	10-20	20-30	30-40	40-50	50-60
f	5	10	13	18	14	8

Solution:

X	*f*	*cf*
0-10	5	5
10-20	10	15
20-30	13	28
30-40	18	46
40-50	14	60
50-60	8	68
	N = 68	

$$Q_1 = \text{size of } \left(\frac{N}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{68}{4}\right)^{th} \text{ value}$$

$$= \text{size of } 17^{th} \text{ value}$$

Thus Q_1 lies in the class 20 – 30 and corresponding values are L = 20, N/4 = 17, pcf = 15, f = 13, C = 10.

$$Q_1 = L + \frac{\left(\frac{N}{4} - pcf\right)}{f} \times C$$

$$= 20 + \left(\frac{17-15}{13}\right) \times 10$$

$$= 20 + \frac{2}{13} \times 10$$

$$= 20 + \frac{20}{13} = 20 + 1.53846$$

$$= 20 + 1.5385 = 21.5385$$

$$Q_3 = \text{size of } \left(\frac{3N}{4}\right)^{th} \text{ value}$$

$$= \text{size of } \left(3 \times \frac{68}{4}\right)^{th} \text{ value}$$

$$= \text{size of } (3 \times 17)^{th} \text{ value}$$

$$= \text{size of } 51^{th} \text{ value}$$

Thus Q_3 lies in the class 40 – 50 and corresponding values are L = 40, 3N/4 = 51, pcf = 46, f = 14, C = 10

$$Q_3 = L + \frac{\left(\frac{3N}{4} - pcf\right)}{f} \times C$$

$$= 40 + \left(\frac{51-46}{14}\right) \times 10$$

$$= 40 + \frac{5}{14} \times 10$$

$$= 40 + \frac{50}{14} = 40 + 3.5714 = 43.5714$$

$$QD = \frac{1}{2}(Q_3 - Q_1)$$

$$= \frac{1}{2}(43.5714 - 21.5385) = \frac{1}{2}(22.0329)$$

$$= 11.01645 = 11.02$$

Relative measure, coefficient of QD

$$= \frac{Q_3 - Q_1}{Q_3 + Q_1} = \frac{43.5714 - 21.5385}{43.5714 + 21.5385}$$

$$= \frac{22.0329}{65.1099} = 0.33839 = 0.3384$$

Question 13.

Compute mean deviation about median from the following data:

Height in inches	No. of students
58	15
59	20
60	32
61	35
62	35
63	22
64	20
65	10
66	8

Solution:

X	f	cf
58	15	15
59	20	35
60	32	67
61	35	102
62	35	137
63	22	159
64	20	179
65	10	189
66	8	197
	N = 197	

$$\text{Median} = \text{size of } \left(\frac{N+1}{2}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{197+1}{2}\right)^{th} \text{ value}$$

$$= \text{size of } \left(\frac{198}{2}\right)^{th} \text{ value}$$

$$= \text{size of } 99^{th} \text{ value}$$

$$= 99^{th} \text{ item} = 61$$

$$\text{MD about median} = \frac{\Sigma f\,|X - \text{Median}|}{N} = \frac{\Sigma f\,|D|}{N}$$

Mean deviation about median:

X	f	$\lvert D\rvert = \lvert X - \bar{X}\rvert$ $= \lvert X - 61\rvert$	$f\lvert D\rvert$
58	15	3	45
59	20	2	40
60	32	1	32
61	35	0	0
62	35	1	35

63	22	2	44
64	20	3	60
65	10	4	40
66	8	5	40
	N = 197		$\Sigma f \lvert D \rvert = 336$

$$\text{MD about median} = \frac{\Sigma f \lvert D \rvert}{N} = \frac{336}{197} = 1.70558 = 1.71$$

Question 14.

Compute the mean deviation about mean from the following data:

Class Interval:	0-5	5-10	10-15	15-20	20-25
Frequency f	3	5	12	6	4

Solution:

C.I	X	f	fX	$\lvert D \rvert = \lvert X - \bar{X} \rvert = \lvert X - 13 \rvert$	$f\lvert D \rvert$
0-5	2.5	3	7.5	10.5	31.5
5-10	7.5	5	37.5	5.5	27.5
10-15	12.5	12	150.0	0.5	6.0
15-20	17.5	6	105.0	4.5	27.0
20-25	22.5	4	90.0	9.5	38
		$N = \Sigma f = 30$	$\Sigma fX = 390$		$\Sigma f \lvert D \rvert = 130$

$$\text{Mean} = \bar{X} = \frac{390}{30} = 13$$

$$\text{Mean deviation about Mean} = \frac{\Sigma f \lvert D \rvert}{N} = \frac{130}{30} = 4.33$$

Question 15.

Find out the coefficient of mean deviation about median in the following series:

Age in years	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons	8	12	16	20	37	25	19	13

Solution:

X	f	cf
0-10	8	8
10-20	12	20
20-30	16	36
30-40	20	56
40-50	37	93
50-60	25	118
60-70	19	137
70-80	13	150
	N = 150	

$$\frac{N}{2} = \frac{150}{2} = 75$$

The class interval corresponding to cumulative frequency 75 is (40 – 50).
So, the corresponding values from the median class are L = 40, pcf = 56, f = 37, C = 10, N = 75.

$$\text{Median} = L + \frac{\left(\frac{N}{2} - pcf\right)}{f} \times C$$

$$= 40 + \left(\frac{75 - 56}{37}\right) \times 10$$

$$= 40 + \frac{190}{37} = 40 + 5.1351$$

= 45.1351

= 45.14 (corrected to two decimals)

Now we calculate the mean deviation about the median 45.11

X	f	M	$\|D\| = \|M - 45.14\|$	$f\|D\|$
0-10	8	5	40.14	321.12
10-20	12	15	30.14	361.68
20-30	16	25	20.14	322.24
30-40	20	35	10.14	202.8
40-50	37	45	0.14	5.18
50-60	25	55	9.86	246.50
60-70	19	65	19.86	377.34
70-80	13	75	29.86	388.18
	N = 150			$\Sigma f\|D\| = 2225.04$

$$\text{Mean deviation about median} = \frac{\Sigma f|D|}{N} = \frac{2225.04}{150} = 14.8336$$

Ex 8.2

Question 1.

A family has two children. What is the probability that both the children are girls given that at least one of them is a girl?

Solution:

Let B denote a boy and G denote a girl.

Then the sample, S = {BG, GB, BB, GG}.

∴ n(S) = 4

Let E be the event that both children are girls.

Let F be the event that atleast one of them is a girl.

Then E = {GG}, n(E) = 1

F = {BG, GB, GG}, n(F) = 3

$P(F) = \frac{n(F)}{n(S)} = \frac{3}{4}$

$E \cap F = \{GG\}$, $n(E \cap F) = 1$

$P(E \cap F) = \frac{n(E \cap F)}{n(s)} = \frac{1}{4}$

Required Probability $P(F/E) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

Question 2.
A die is thrown twice and the sum of the number appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once?

Solution:
When a die is thrown twice, the sample is S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}
n(S) = 36

Let A be the event of getting the sum of the numbers is 6.
Let B be the event of the number 4 has appeared atleast once.
We have to find P(B/A)
A = {(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)}, n(A) = 5
∴ P(A) = 5/36

B = {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1),(4, 2), (4, 3), (4, 5), (4, 6)}
A ∩ B = {(2, 4), (4, 2)}, n(A ∩ B) = 2
P(A ∩ B) = 2/36

$$\therefore P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$$

Question 3.
An unbiased die is thrown twice. Let the event A be an odd number on the first throw and B the event odd number on the second throw. Check whether A and B events are independent.

Solution:
When a die is thrown twice, the sample space is S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6,2), (6, 3), (6, 4), (6, 5), (6, 6)}
n(S) = 36

The event A is odd number on the first throw
∴ A = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)}
n(A) = 18

$P(A) = \frac{18}{36} = \frac{1}{2}$

The event B is odd number on the second throw.

B = {(1, 1), (1, 3), (1, 5), (2, 1), (2, 3), (2, 5)

(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)

(5, 1), (5, 3), (5, 5), (6, 1), (6, 3), (6, 5)}

n(B) = 18

$P(B) = \frac{18}{36} = \frac{1}{2}$

A ∩ B = {(1, 1), (1, 3), (1, 5)

(3, 1), (3, 3), (3, 5)

(5, 1), (5, 3), (5, 5)}

n(A ∩ B) = 9

$P(A \cap B) = \frac{9}{36} = \frac{1}{4}$

Also $P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

Thus $P(A \cap B) = P(A) \cdot P(B)$

$\therefore$ A and B are independent events.

Question 4.

Probability of solving specific problem independently by A and B are 1/2 and 1/3 respectively. If both try to solve the problem independently, find the probability that the problem is (i) solved, (ii) exactly one of them solves the problem.

Solution:

Given $P(A) = 1/2$ and $P(B) = 1/3$

(i) The probability that problem is solved = The probability that atleast one solving the problem

$= 1 - P(\text{none of them solving the problem})$

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \cdot P(\bar{B})$

$= 1 - (1 - \frac{1}{2})(1 - \frac{1}{3})$

$= 1 - (\frac{1}{2})(\frac{2}{3})$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

(ii) P(exactly one of them solves the problem)

$= P(\bar{A} \cap B) + P(A \cap \bar{B})$

$= P(\bar{A}) \cdot P(B) + P(A) \cdot P(\bar{B})$

$= \left(1 - \frac{1}{2}\right)\left(\frac{1}{3}\right) + \frac{1}{2}\left(1 - \frac{1}{3}\right)$

$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{6} + \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

Question 5.
Suppose one person is selected at random from a group of 100 persons are given in the following:

Title	Psychologist	Socialist	Democrate	Total
Men	15	25	10	50
Women	20	15	15	50
Total	35	40	25	100

What is the probability that the man selected is a Psychologist?

Solution:
Total number of men = 50
Number of men psychologist = 15
The probability of selecting a psychologist given that a man has been already selected is

$$\frac{15}{50} = \frac{3}{10}$$

Question 6.
Two urns contain the set of balls as given in the following table.

Title	White	Red	Black
Urn 1	10	6	9
Urn 2	3	7	15

One ball is drawn from each urn and find the probability that (i) both balls are red, (ii) both balls are of the same colour.

Solution:
(i) Let A be the event of drawing a red from urn 1,
P(A) = 6/25
Let B be the event of selecting a red ball in urn 2.
P(B) = 7/25

∴ P(both balls are red) = P(A) . P(B) [∵ the events are independent]

$= \frac{6}{25} \times \frac{7}{25}$

$= \frac{42}{625}$

Let W_1, R_1, B_1 represents white, red, and black balls drawn from urn 1 and W_2, R_2, B_2 represents white, red, and black balls from urn 2.
P(both balls are of the same colour) = $P(W_1W_2) + P(R_1R_2) + P(B_1B_2)$ [∵ Events are mutually exclusive]

$= P(W_1)\, P(W_2) + P(R_1)\, P(R_2) + P(B_1)\, P(B_2)$ [∵ Events are independent]

$= \frac{10}{25} \times \frac{3}{25} + \frac{6}{25} \times \frac{7}{25} + \frac{9}{25} \times \frac{15}{25}$

$= \frac{1}{625}[30 + 42 + 135]$

$= \frac{207}{625}$

Question 7.
The bag I contain 3 Red and 4 Black balls while another Bag II contains 5 Red and 6 Black balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from Bag I.

Solution:

	Red	Black	Total
Urn I	3	4	7
Urn II	5	6	11

Let E_1 be the event of choosing the first bag, E_2 be the event of choosing the second bag.

Let A be the event of drawing a red ball.
Then $P(E_1) = 1/2$, $P(E_2) = 1/2$

Also $P(A/E_1)$ = P(Drawing a red ball from bag I) = 3/7
$P(A/E_2)$ = P(Drawing a red ball from bag II) = 5/11
The probability of drawing a ball from bag I, being given that it is red, is $P(E_1/A)$

$$P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{3}{7}}{\frac{1}{2}\times\frac{3}{7}+\frac{1}{2}\times\frac{5}{11}} = \frac{\frac{3}{7}}{\frac{3}{7}+\frac{5}{11}}$$

$$= \frac{\frac{3}{7}}{\left(\frac{33+35}{7\times11}\right)} = \frac{3}{7\times68}\times7\times11$$

$$= \frac{33}{68}$$

Question 8.

The first of three urns contains 7 White and 10 Black balls, the second contains 5 White and 12 Black balls, and the third "contains 17 White balls and no Blackball. A person chooses an urn at random and draws a ball from it. And the ball is found to be White. Find the probabilities that the ball comes from (i) the first urn, (ii) the second urn, (iii) the third urn.

Solution:

	White	Black	Total
Urn I	7	10	17
Urn II	5	12	17
Urn II	17	--	17

Let E_1, E_2, and E_3 be the event of choosing the first, second, and third urn.
Let A be the event of drawing a white ball.
then $P(E_1) = P(E_2) = P(E_3) = 1/3$

$P(A/E_1)$ = P(drawing a white ball from the first urn) = 7/17
$P(A/E_2)$ = P(drawing a white ball from the second urn) = 5/17
$P(A/E_3)$ = P(drawing a white ball from the third urn) = 17/17
(i) The probability of drawing a ball from the first urn, being given that it is

white, is $P(E_1/A)$,

$$P(E_1/A) = \frac{P(E_1)\times P(A/E_1)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{7}{17}}{\frac{1}{3}\times\frac{7}{17}+\frac{1}{3}\times\frac{5}{17}\times\frac{1}{3}\times\frac{17}{17}}$$

$$= \frac{\frac{1}{3}\times\frac{7}{17}}{\frac{1}{3}\left[\frac{7}{17}+\frac{5}{17}+\frac{17}{17}\right]}$$

$$= \frac{7/17}{29/17}$$

$$= \frac{7}{29}$$

(ii) the second urn

$$P(E_2/A) = \frac{P(E_2)\times P(A/E_2)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{5}{17}}{\frac{1}{3}\times\frac{7}{17}+\frac{1}{3}\times\frac{5}{17}+\frac{1}{3}\times\frac{17}{17}}$$

$$= \frac{\frac{5}{17}}{\frac{7}{17}+\frac{5}{17}+\frac{17}{17}} = \frac{5}{29}$$

(iii) the third urn

$$P(E_3/A) = \frac{P(E_3)\times P(A/E_3)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)+P(E_3)\times P(A/E_3)}$$

$$= \frac{\frac{1}{3}\times\frac{17}{17}}{\frac{1}{3}\times\frac{7}{17}+\frac{1}{3}\times\frac{5}{17}+\frac{1}{3}\times\frac{17}{17}}$$

$$= \frac{\frac{17}{17}}{\frac{7}{17}+\frac{5}{17}+\frac{17}{17}} = \frac{17}{29}$$

Question 9.
Three boxes B_1, B_2, B_3 contain lamp bulbs some of which are defective. The defective proportions in box B_1, box B_2 and box B_3 are respectively 1/2, 1/3 and 1/4. A box is selected at random and a bulb is drawn from it. If the selected bulb is found to be defective, what is the probability that box B_1 was selected?

Solution:
Given that B_1, B_2 and B_3 represent three boxes.
Then $P(B_1) = P(B_2) = P(B_3) = 1/3$
Let A be the event of selecting a defective bulb.

Then $P(A/B_1)$ = P(drawing a defective bulb from B_1) = 1/2
$P(A/B_2)$ = P(drawing a defective bulb from B_2) = 1/8
$P(A/B_3)$ = P(drawing a defective bulb from B_3) = 3/4
The probability of drawing a defective bulb from B_1, being given that it is defective, is $P(B_1/A)$.

$$P(B_1/A) = \frac{P(B_1)\times P(A/B_1)}{P(B_1)\times P(A/B_1)+P(B_2)\times P(A/B_2)+P(B_3)\times P(A/B_3)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{8}+\frac{1}{3}\times\frac{3}{4}}$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{8}+\frac{3}{4}} = \frac{\frac{1}{2}}{\left(\frac{4+1+6}{8}\right)} = \frac{1/2}{11/8}$$

$$= \frac{1}{2} \times \frac{8}{11} = \frac{4}{11}$$

Question 10.
Three horses A, B, C are in the race. A is twice as likely to win as B and B is twice as likely to win as C. What are their respective probabilities of winning?

Solution:
Given that A is twice as likely to win as B.
Therefore, A : B = 2 : 1
(or) A : B = 4 : 2 .....(1)
Also given that B is twice as likely to win as C.
Therefore, B : C = 2 : 1 .....(2)
From (1) and (2),
A : B : C = 4 : 2 : 1
∴ A = 4k, B = 2k, C = 1, where C is a constant of proportionality.

Probability of A wining is $\frac{4k}{7k} = \frac{4}{7}$
Probability of B wining is $\frac{2k}{7k} = \frac{2}{7}$
Probability of C wining is $\frac{k}{7k} = \frac{1}{7}$

Question 11.
A die is thrown. Find the probability of getting (i) a prime number, (ii) a number greater than or equal to 3.

Solution:
Let S be the sample when a die is thrown.
Then S = {1, 2, 3, 4, 5, 6}, n(S) = 6
Let A be the event of getting a prime number.
A = {2, 3, 5}, n(A) = 3
Let B be the event of getting a number greater than or equal to 3.

B = {3, 4, 5, 6}, n(B) = 4

(i) P(a prime number) = $\frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(ii) P(a number ≥ 3) = $\frac{n(B)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

Question 12.

Ten cards numbered 1 to 10 are placed in a box, mixed up thoroughly, and then one card is drawn randomly. If it is known that the number on the drawn card is more than 4. What is the probability that it is an even number?

Solution:

Let S = {1, 2, 3,..., 10}, n(S) = 10

Let A be the event of drawing a number greater than 4.

Then A= {5, 6, 7, 8, 9, 10}, n(A) = 6

$\therefore$ P(A) = $\frac{n(A)}{n(S)} = \frac{6}{10}$

Let B be the event of getting a even number.

Then B = {2, 4, 6, 8, 10}

Now A ∩ B = {6, 8, 10}, n(A ∩ B) = 3

P(A ∩ B) = 3/10

We have to find P (B/A)

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{3/10}{6/10} = \frac{3}{6} = \frac{1}{2}$$

Question 13.

There are 1000 students in a school out of which 450 are girls. It is known that out of 450, 20% of the girls studying in class XI. A student is randomly selected from 1000 students. What is the probability that the selected student is from class XI given that the selected student is a girl?

Solution:

Number of students = 1000 i.e., n(S) = 1000

Let A be the event of selecting a girl student from class XI.

Let B be the event of selecting a girl student

Number of girls = 450; n(B) = 450

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{450}{1000}$

Number df girls studying ih XI is 20% of 450 = $\frac{20}{100} \times 450 = 90$

i.e., $n(A \cap B) = 90$

$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{90}{1000}$

We have to find P(A/B)

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{90/1000}{450/1000} = \frac{90}{450} = \frac{1}{5} = 0.2$$

Question 14.
From a pack of 52 cards, two cards are drawn at random. Find the probability that one is a king and the other is a queen.

Solution:
Number of kings is 4
Number of queens is 4
Two cards are drawn.
Probability of one king and one queen is,

$$\frac{4C_1 \times 4C_1}{52C_2} = \frac{4 \times 4 \times 2}{52 \times 51} = \frac{8}{13 \times 51}$$

$$= \frac{8}{663} = 0.012$$

Question 15.
A card is drawn from a pack of playing cards and then another card is drawn without the first being replaced. What is the probability of drawing (i) two aces, (ii) two spades?

Solution:
A number of cards 52.
The number of aces 4.
Number of spades 13

(i) Probability of drawing two aces

$$\frac{{}^4C_2}{{}^{52}C_2} = \frac{\frac{4\times3}{2\times1}}{\frac{52\times51}{2\times1}} = \frac{4\times3}{52\times51} = \frac{12}{2652} = \frac{1}{221}$$

(ii) The probability of drawing two spades

$$\frac{{}^{13}C_2}{{}^{52}C_2} = \frac{13\times12/(2\times1)}{52\times51/(2\times1)} = \frac{13\times12}{52\times51} = \frac{156}{2652} = \frac{1}{17}$$

Question 16.
A company has three machines A, B, C which produces 20%, 30%, and 50% of the product respectively. Their respective defective percentages are 7, 3, and 5. From these products, one is chosen and inspected. If it is defective what is the probability that it has been made by machine C?

Solution:
The probability of the product produced by machine A is $P(A) = 20/100$
The probability of the product produced by machine B is $P(B) = 30/100$
The probability of the product produced by the machine C is $P(C) = 50/100$
Let D be the event of selecting a defective product.

Then $P(D/A)$ = The probability of selecting a defective product produced by the machine $A = 7/100$

$P(D/B)$ = the probability of selecting a defective product produced by the machine $B = 3/100$
and $P(D/C)$ = The probability of selecting a detective product produced by the machine $C = 5100$

$$P(C/D) = \frac{P(C).P(D/C)}{P(A)P(D/A)+P(B)P(D/B)+P(C)P(D/C)}$$

$$= \frac{\frac{50}{100}\times\frac{5}{100}}{\frac{20}{100}\times\frac{7}{100}+\frac{30}{100}\times\frac{3}{100}+\frac{50}{100}\times\frac{5}{100}}$$

$$= \frac{50\times5}{20\times7+30\times3+50\times5}$$

$$= \frac{5\times5}{2\times7+3\times3+5\times5}$$

$$= \frac{25}{14+9+25} = \frac{25}{48}$$

$$= 0.5208$$

Ex 8.3

Choose the correct answer.

Question 1.
Which of the following is a positional measure?
(a) Range
(b) Mode
(c) Mean deviation
(d) Percentiles

Answer:
(d) Percentiles

Question 2.
When calculating the average growth of the economy, the correct mean to use is?
(a) Weighted mean
(b) Arithmetic mean
(c) Geometric mean
(d) Harmonic mean

Answer:
(c) Geometric mean

Question 3.
When an observation in the data is zero, then its geometric mean is:
(a) Negative
(b) Positive
(c) Zero
(d) Cannot be calculated

Answer:
(c) Zero

Question 4.
The best measure of central tendency is:
(a) Arithmetic mean
(b) Harmonic mean
(c) Geometric mean
(d) Median

Answer:
(a) Arithmetic mean

Question 5.
The harmonic mean of the numbers 2, 3, 4 is:
(a) 12/13
(b) 12
(c) 36/13
(d) 13/36

Answer:
(c) 36/13
Hint:

$$HM = \frac{n}{\frac{1}{X_1} + \frac{1}{X_2} + \frac{1}{X_3} + \ldots + \frac{1}{X_n}}$$

Here n = 3

$$\therefore \text{HM} = \frac{3}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = \frac{3}{\frac{6+4+3}{12}}$$

$$= \frac{3}{13} \times 12 = \frac{36}{13}$$

Question 6.
The geometric mean of two numbers 8 and 18 shall be:
(a) 12
(b) 13
(c) 15
(d) 11.08

Answer:
(a) 12
Hint:

$$\text{GM} = \sqrt[n]{X_1 . X_2 . X_3 .... X_n}$$

$$= \sqrt[2]{8 \times 18} = \sqrt[2]{4 \times 2 \times 9 \times 2}$$

$$= \sqrt{(2 \times 2) \times (2 \times 2) \times (3 \times 3)}$$

$$= 2 \times 2 \times 3 = 12$$

Question 7.
The correct relationship among A.M., G.M. and H.M. is:
(a) A.M. < G.M. < H.M.
(b) G.M. > A.M. > H.M.
(c) H.M. > G.M. > A.M.
(d) A.M. > G.M. > H.M.

Answer:
(d) A.M. > G.M. > H.M.

Question 8.
Harmonic mean is the reciprocal of:
(a) Median of the values.

(b) Geometric mean of the values.
(c) Arithmetic mean of the reciprocal of the values.
(d) Quartiles of the values.

Answer:
(c) Arithmetic mean of the reciprocal of the values.

Question 9.
Median is same as:
(a) Q_1
(b) Q_2
(c) Q_3
(d) D_2

Answer:
(b) Q_2

Question 10.
The median of 10, 14, 11, 9, 8, 12, 6 is:
(a) 10
(b) 12
(c) 14
(d) 9

Answer:
(a) 10
Hint:
The ascending order of 10, 14, 11, 9, 8, 12, 6 is 6, 8, 9, 10, 11, 12, 14.
In this order middle number is 10.

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ value}$$

$$= \left(\frac{7+1}{2}\right)^{th}$$

$$= 10$$

$\therefore$ Median 10.

Question 11.
The mean of the values 11, 12, 13, 14 and 15 is:
(a) 15
(b) 11
(c) 12.5
(d) 13

Answer:
(d) 13
Hint: The values are in ascending, order.
∴ The mean is the middle value.

Question 12.
If the mean of 1, 2, 3, ..., n is 6n/11, then the value of n is:
(a) 10
(b) 12
(c) 11
(d) 13

Answer:
(c) 11
Hint:

The mean of 1, 2, 3,...., n is $\frac{6n}{11}$

i.e., $\frac{1+2+3+4...+n}{n} = \frac{6n}{11}$

$\frac{\frac{n(n+1)}{2}}{n} = \frac{6n}{11}$

$\frac{n+1}{2} = \frac{6n}{11}$

11(n + 1) = 2 × 6

11n + 11 = 12n

∴ n = 11

Question 13.
The harmonic mean is better than other means if the data are for:
(a) Speed or rates.
(b) Heights or lengths.
(c) Binary values like 0 and 1.

(d) Ratios or proportions.

Answer:
(a) Speed or rates

Question 14.
The first quartile is also known as:
(a) median
(b) lower quartile
(c) mode
(d) third decile

Answer:
(b) lower quartile

Question 15.
If $Q_1 = 30$ and $Q_3 = 50$, the coefficient of quartile deviation is:
(a) 20
(b) 40
(c) 10
(d) 0.25

Answer:
(d) 0.25
Hint:

Coefficient of quartile deviation is $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{50-30}{50+30}$

$= \frac{20}{40}$

$= 0.25$

Question 16.
If median = 45 and its coefficient is 0.25, then the mean deviation about median is:
(a) 11.25
(b) 180
(c) 0.0056

(d) 45

Answer:
(a) 11.25
Hint:
Coefficient of M.D = MD/Median
MD = Coefficient of MD × Median
= 0.25 × 45
= 11.25

Question 17.
The two events A and B are mutually exclusive if:
(a) $P(A \cap B) = 0$
(b) $P(A \cap B) = 1$
(c) $P(A \cup B) = 0$
(d) $P(A \cup B) = 1$

Answer:
(a) $P(A \cap B) = 0$

Question 18.
The events A and B are independent if:
(a) $P(A \cap B) = 0$
(b) $P(A \cap B) = P(A) \times P(B)$
(c) $P(A \cap B) = P(A) + P(B)$
(d) $P(A \cup B) = P(A) \times P(B)$

Answer:
(b) $P(A \cap B) = P(A) \times P(B)$

Question 19.
If two events A and B are dependent then the conditional probability of P(B/A) is:
(a) P(A) P(B/A)
(b) $\frac{P(A \cap B)}{P(B)}$
(c) $\frac{P(A \cap B)}{P(A)}$
(d) P(A) P(A/B)

Answer:

(c) $\frac{P(A \cap B)}{P(A)}$

Question 20.

The probability of drawing a spade from a pack of card is:
(a) 1/52
(b) 1/13
(c) 4/13
(d) 1/4

Answer:

(d) 1/4
Hint:
Number of spade cards is 13.
Total number of cards in pack = 52
Probability of drawing a spade card is = 13/52 = 1/4

Question 21.

If the outcome of one event does 'not influence another event then the two events are:
(a) Mutually exclusive
(b) Dependent
(c) Not disjoint
(d) Independent

Answer:

(d) Independent

Question 22.

Let a sample space of an experiment be S = {E_1, E_2, ..., E_n} then $\sum_{i=1}^{n} P(E_i)$ is equal to:

(a) 0
(b) 1
(c) 1/2
(d) 1/3

Answer:
(b) 1

Hint:
Sum of probabilities is 1
i.e., $\sum_{i=1}^{n} \mathrm{P}\left(\mathrm{E}_i\right) = 1$

Question 23.
The probability of obtaining an even prime number on each die, when a pair of dice is rolled is:
(a) 1/36
(b) 0
(c) 1/3
(d) 1/6

Answer:
(a) 1/36

Hint:
When a pair of dice is rolled number of elements in the sample space is 36.
2 is the only even prime number. (2, 2) is the only event of even prime number on both dice.
Required probabilities 1/36.

Question 24.
Probability of an impossible event is:
(a) 1
(b) 0
(c) 0.2
(d) 0.5

Answer:
(b) 0

Question 25.
The probability that at least one of the events A, B occur is:
(a) P(A ∪ B)
(b) P(A ∩ B)
(c) P(A/B)

(d) $(A \cup B)$

Answer:
(a) $P(A \cup B)$