Sample Question Paper - 19
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

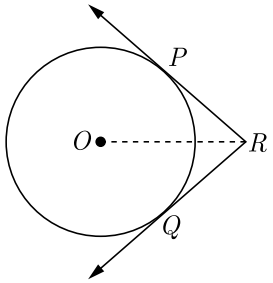
1. Find k so that the quadratic equation $(k+1)x^2 - 2(k+1)x + 1 = 0$ has equal roots.

OR

If one root of the quadratic equation $3x^2 + px + 4 = 0$is $\frac{2}{3}$, then find the value of p and the other root of the equation.

2. Find how many integers between 200 and 500 are divisible by 8.

3. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



4. A solid metallic cylinder of radius 3.5 cm and height 14 cm melted and recast into a number of small solid metallic ball, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

5. Find the mean of the following data :

Class	Frequency
0.5-5.5	13
5.5-10.5	16
10.5-15.5	22
15.5-20.5	18
20.5-25.5	11

6. Find the mean number of plants per house from the following data :

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	6	2	3

OR

Consider the following frequency distribution of the heights of 60 students of a class

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
Number of students	15	13	10	8	9	5

What is the upper limit of the median class in the given data?

Section B

7. If the ratio of the sums of first n terms of two AP's is $(7n+1):(4n+27)$, find the ratio of their m^{th} terms.

8. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole is 60° and the angle of depression from the top of the other pole of point P is 30°. Find the heights of the poles and the distance of the point P from the poles.

9. Let ABC be a right triangle in which $AB = 6$ cm, $BC = 8$ cm and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

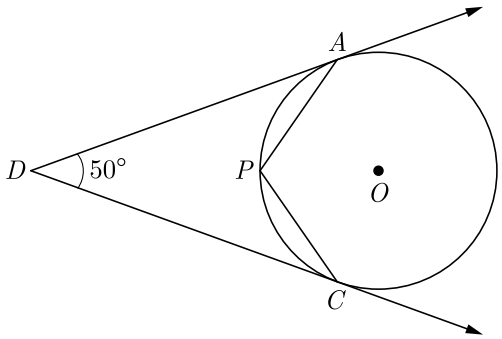
10. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

OR

A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe.

Section C

11. In the given figure, O is the centre of the circle. Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50^\circ$.



12. The median of the following data is 525. Find the values of x and y, if total frequency is 100 :

Class	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

OR

A survey regarding the heights in (cm) of 51 girls of class X of a school was conducted and the following data was obtained. Find the median height and the mean using the formulae.

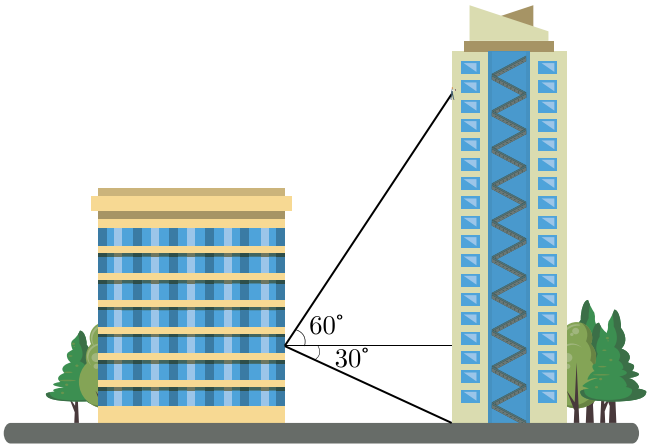
Height (in cm)	**Number of Girls**
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

13. A tour bus in Jaipur serves 400 customers a day. The charge is Rs 50 per person. The owner of the bus service estimates that the company would lose 10 passengers a day for each Rs 5 fare increase.

(i) How much should the fare be in order to maximize the income for the company?

(ii) What is the maximum income the company can expect to make?

14. From his hotel room window on the fourth floor, Ranjan notices some window washers high above him on the hotel across the street.



Curious as to their height above ground, he quickly estimates the buildings are 60 m apart, the angle of elevation to the workers is about 60°, and the angle of depression to the base of the hotel is about 30°.

(i) How high above ground is the window of Ranjan's hotel room?

(ii) How high above ground are the workers?

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

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SECTION A

1. Find k so that the quadratic equation $(k+1)x^2 - 2(k+1)x + 1 = 0$ has equal roots.

Ans :

We have $(k+1)x^2 - 2(k+1)x + 1 = 0$

Comparing with $Ax^2 + Bx + C = 0$ we get
$A = (k+1), B = -2(k+1), C = 1$
If roots are equal, then $D = 0$, i.e.

$$\begin{aligned} B^2 &= 4AC \\ 4(k+1)^2 &= 4(k+1) \\ k^2 + 2k + 1 &= k + 1 \\ k^2 + k &= 0 \\ k(k+1) &= 0 \\ k &= 0, -1 \end{aligned}$$

Since $k = -1$ does not satisfy the equation, thus $k = 0$

OR

If one root of the quadratic equation $3x^2 + px + 4 = 0$ is $\frac{2}{3}$, then find the value of p and the other root of the equation.

Ans :

Given, quadratic equation is $3x^2 + px + 4 = 0$.
Since $\frac{2}{3}$ is one root of the given quadratic equation,

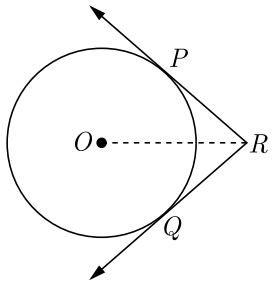
$$\begin{aligned} 3\left(\frac{2}{3}\right)^2 + p\left(\frac{2}{3}\right) + 4 &= 0 \\ \frac{4}{3} + \frac{2p}{3} + 4 &= 0 \\ \frac{4 + 2p + 12}{3} &= 0 \\ 2p + 16 &= 0 \\ p &= -8 \end{aligned}$$

Substituting $p = -8$ in given equation, we get

$$\begin{aligned} 3x^2 - 8x + 4 &= 0 \\ 3x^2 - 6x - 2x + 4 &= 0 \\ 3x(x-2) - 2(x-2) &= 0 \\ (x-2)(3x-2) &= 0 \Rightarrow x = 2, \frac{2}{3} \end{aligned}$$

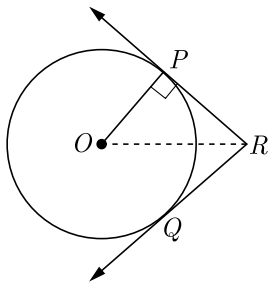
Hence, other root is 2.

2. In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120^\circ$, then prove that $OR = PR + RQ$.



Ans :

We redraw the given figure by joining O to P as shown below.



$\angle PRO = \frac{1}{2}\angle PRQ$

$$= \frac{120^\circ}{2} = 60^\circ$$

Here ΔOPR is right angle triangle, thus

$$\angle POR = 90^\circ - \angle PRO = 90^\circ - 60^\circ = 30^\circ$$

Now $$\frac{PR}{OR} = \sin 30^\circ = \frac{1}{2}$$

$$OR = 2PR = PR + PR$$

Since $PR = QR$,

$$OR = PR + QR \qquad \text{Hence Proved}$$

3. Find how many integers between 200 and 500 are divisible by 8.

Ans :

Number divisible by 8 are 208, 2016, 224, ... 496. It is an AP

Let the first term be a, common difference be d and nth term be a_n.

We have $a = 208, d = 8$ and $a_n = 496$

Now $a + (n-1)d = a_n$

$$208 + (n-1)d = 496$$

$$(n-1)8 = 496 - 208$$

$$n - 1 = \frac{288}{8} = 36$$

$$n = 36 + 1 = 37$$

Hence, required numbers divisible by 8 is 37.

4. A solid metallic cylinder of radius 3.5 cm and height 14 cm melted and recast into a number of small solid metallic ball, each of radius $\frac{7}{12}$ cm. Find the number of balls so formed.

Ans :

Let the number of recasted balls be N.

Radius of cylinder $R = 3.5$ cm

Height of cylinder $h = 14$cm

Radius of recasted ball $r = \frac{7}{12}$

$$\text{Volume of balls} = \text{Volume of cylinder}$$

$$n\frac{4}{3}\pi r^3 = \pi R^2 h$$

$$n \times \frac{4}{3} \times \frac{7}{12} \times \frac{7}{12} \times \frac{7}{12} = 3.5 \times 3.5 \times 14$$

$$n = \frac{3.5 \times 3.5 \times 14 \times 3 \times 12 \times 12 \times 12}{4 \times 7 \times 7 \times 7}$$

$$= 0.5 \times 0.5 \times 2 \times 3 \times 3 \times 12 \times 12$$

$$= 648$$

Hence, number of recasted balls is 648.

5. Find the mean of the following data :

Class	Frequency
0.5-5.5	13
5.5-10.5	16
10.5-15.5	22
15.5-20.5	18
20.5-25.5	11

Ans :

We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0.5-5.5	3	13	39
5.5-10.5	8	16	128
10.5-15.5	13	22	286
15.5-20.5	18	18	324
20.5-25.5	23	11	253
	Total	$\sum f_i = 80$	1,030

Mean $$\overline{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1,030}{80} = 12.9$$

6. Find the mean number of plants per house from the following data :

Number of plants	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Number of houses	1	2	1	5	6	2	3

Ans :

We prepare following table to find mean.

Class	$x_i = \frac{l_1 + l_2}{2}$	f_i	$f_i x_i$
0-2	1	1	1
2-4	3	2	6
4-6	5	1	5
6-8	7	5	35
8-10	9	6	54
10-12	11	2	22
12-14	13	3	39
	Total	20	162

Mean $$M = \frac{\sum f_i x_i}{\sum f_i} = \frac{162}{20} = 8.1$$

Mean number of plants per house is 8.1.

OR

Consider the following frequency distribution of the heights of 60 students of a class

Height (in cm)	150-155	155-160	160-165	165-170	170-175	175-180
Number of students	15	13	10	8	9	5

What is the upper limit of the median class in the given data?

Ans :

We prepare the following cumulative table

Height x (in cm)	Number of Students (f)	cf
150-155	15	15
155-160	13	28
160-165	10	38
165-170	08	46
170-175	09	55
175-180	08	63
	$N = 63$	

We have, $N = 63;\ \frac{N}{2} = \frac{63}{2} = 31.5$

The cumulative frequency just greater than $\frac{N}{2}$ is 38 and the corresponding class is 160-165. Thus upper limit is 165.

Section B

7. If the ratio of the sums of first n terms of two AP's is $(7n+1):(4n+27)$, find the ratio of their m^{th} terms.

Ans :

Let a, and A be the first term and d and D be the common difference of two AP's, then we have

$$\frac{S_n}{S_n'} = \frac{7n+1}{4n+27}$$

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2A+(n-1)D\right]} = \frac{7n+1}{4n+27}$$

$$\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$$

$$\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$$

Substituting $\frac{n-1}{2} = m-1$ or $n = 2m-1$ we get

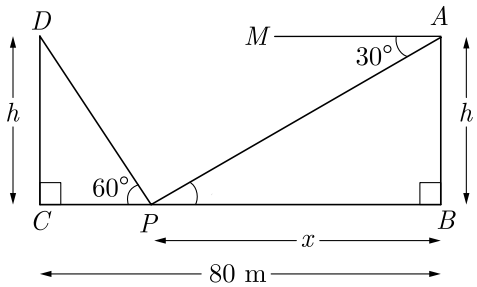
$$\frac{a+(m-1)d}{A+(m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27} = \frac{14m-6}{8m+23}$$

Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$

8. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole is 60° and the angle of depression from the top of the other pole of point P is 30°. Find the heights of the poles and the distance of the point P from the poles.

Ans :

Let the distance between pole AB and point P be x. As per given in question we have drawn figure below.



Here distance between pole CD and P is $80-x$.

In right angle triangle ΔABP, $\angle APB = 30^\circ$

$$\tan 30^\circ = \frac{h}{x}$$

$$h = \frac{x}{\sqrt{3}} \quad ...(1)$$

In angle triangle ΔCDP,

$$\tan 60^\circ = \frac{CD}{CP} = \frac{CD}{CB-PB}$$

$$\sqrt{3} = \frac{h}{80-x}$$

$$h = 80\sqrt{3} - x\sqrt{3} \quad ...(2)$$

Comparing (1) and (2) we have

$$\frac{x}{\sqrt{3}} = 80\sqrt{3} - x\sqrt{3}$$

$$x = 80 \times 3 - x \times 3$$

$$4x = 240$$

$$x = \frac{240}{4} = 60 \text{ m}$$

Substituting this value of x in (1) we have

$$h = \frac{60}{\sqrt{3}} = 20\sqrt{3} = 34.64 \text{ m}$$

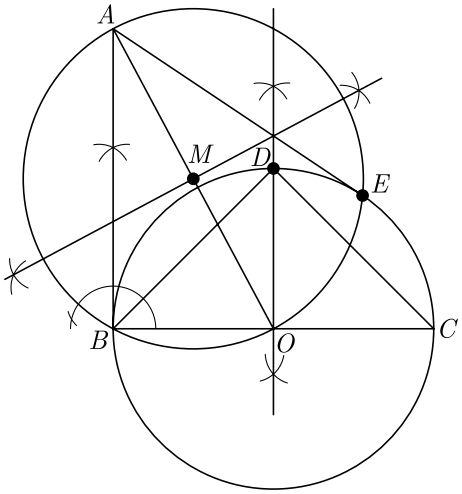
Hence, height of the pole AB and CD is 34.64 m
Distance of point P from pole AB is 20 m.
Distance of point P from pole CD is 60 m.

9. Let ABC be a right triangle in which $AB = 6\,\text{cm}$, $BC = 8\,\text{cm}$ and $\angle B = 90^\circ$. BD is the perpendicular from B on AC. The circle through B, C, D is drawn. Construct the tangents from A to this circle.

Ans :

Steps of Construction :

1. Draw a triangle ΔABC such that $AB = 6\,\text{cm}$, $BC = 8\,\text{cm}$ and $\angle B = 90^\circ$.
2. Draw $BD \perp AC$. Now bisect BC and let its mid-point be O.
   So, O is centre of the circle passing through B, C and D.



3. Join AO.
4. Bisect AO. Let M be the mid-point of AO.
5. Taking M as centre and MA as radius, draw a circle intersecting the given circle at B and E.
6. Join AB and AE. Thus, AB and AE are the required two tangents to the given circle from A.

Justification :

If we join OE, then

$$\angle AEO = 90^\circ$$

(Angle in a semi circle)

$$AE \perp OE$$

But OE is a radius of the given circle. Thus AE has to be a tangent to the circle. Similarly, AB is also a tangent to the given circle.

10. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weights, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth $\frac{8}{9}$ cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

Ans :

Volume of cylinder,

$$\pi r^2 h = \pi(3)^2 \times 5 = 45\pi \text{ cm}^3$$

Volume of conical hole,

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9} = \frac{2}{3}\pi \text{ cm}^3$$

Metal left in cylinder $= 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3}$

$$\frac{\text{Volume of metal left}}{\text{Volume of metal taken out}} = \frac{\frac{133}{3}\pi}{\frac{2}{3}\pi} = 133 : 2.$$

Hence required ratio is 133 : 2

OR

A hollow cylindrical pipe is made up of copper. It is 21 dm long. The outer and inner diameters of the pipe are 10 cm and 6 cm respectively. Find the volume of copper used in making the pipe.

Ans :

Volume of copper used in making the pipe is equal to the difference of volume of external cylinder and volume of internal cylinder.
Height of cylindrical pipe,

$$h = 21 \text{ dm} = 210 \text{ cm}$$

External Radius, $R = \frac{10}{2} = 5$ cm

Internal Radius, $r = \frac{6}{2} = 3$ cm

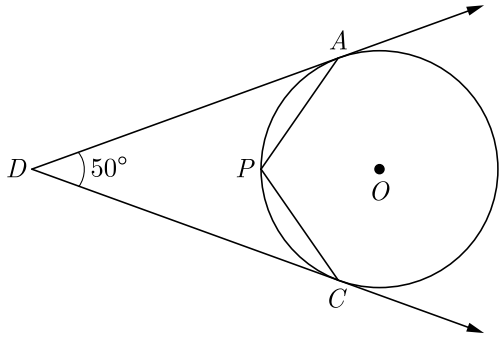
Volume of copper used in making the pipe

$$= (\text{Volume of External Cylinder}) - (\text{Volume of Internal Cylinder})$$
$$= \pi R^2 h - \pi r^2 h$$
$$= \pi h(R^2 - h^2)$$
$$= \frac{22}{7} \times 210 \times (5^2 - 3^2)$$
$$= \frac{22}{7} \times 210 \times (25 - 9)$$
$$= \frac{22}{7} \times 210 \times 16$$
$$= 10560 \text{ cm}^3.$$

Section C

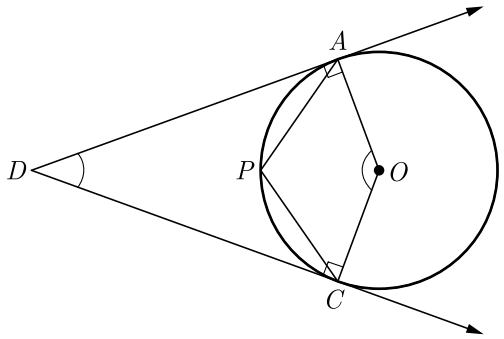
11. In the given figure, O is the centre of the circle.

Determine $\angle APC$, if DA and DC are tangents and $\angle ADC = 50^\circ$.



Ans :

We redraw the given figure by joining A and C to O as shown below.



Since DA and DC are tangents from point D to the circle with centre O, and radius is always perpendicular to tangent, thus

$$\angle DAO = \angle DCO = 90^\circ$$

and

$$\angle ADC + \angle DAO + \angle DCO + \angle AOC = 360^\circ$$
$$50^\circ + 90^\circ + 90^\circ + \angle AOC = 360^\circ$$
$$230^\circ + \angle AOC = 360^\circ$$
$$\angle AOC = 360^\circ - 230^\circ = 130^\circ$$

Now Reflex $\angle AOC = 360^\circ - 130^\circ = 230^\circ$

$$\angle APC = \frac{1}{2} \text{ reflex } \angle AOC$$
$$= \frac{1}{2} \times 230^\circ = 115^\circ$$

12. The median of the following data is 525. Find the values of x and y, if total frequency is 100 :

Class	Frequency
0-100	2
100-200	5
200-300	x
300-400	12
400-500	17
500-600	20
600-700	y
700-800	9
800-900	7
900-1000	4

Ans :

We prepare cumulative frequency table as given below.

Class Interval	Frequency (f)	Cum. freq. *c.f.*
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$
900-1000	4	$76 + x + y$
	$N = 100$	

From table we have

$$76 + x + y = 100$$
$$x + y = 100 - 76 = 24 \quad \ldots(1)$$

Here median is 525 which lies between class $500 - 600$. Thus median class is 500-600.

Median, M_d

$$= l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$
$$525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$$
$$25 = (50 - 36 - x)5$$
$$14 - x = \frac{25}{5} = 5$$
$$x = 14 - 5 = 9$$

Substituting the value of x is equation (1), we get

$$y = 24 - 9 = 15$$

Hence, $x = 9$ and $y = 15$

OR

A survey regarding the heights in (cm) of 51 girls of class X of a school was conducted and the following data was obtained. Find the median height and the mean using the formulae.

Height (in cm)	**Number of Girls**
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Ans :

To calculate the median height, we need to convert the given data in the form of intervals and their corresponding frequencies.

Given distribution is of less type and 140, 145, 150, 165 gives the upper limits of the corresponding class interval So, the classes should be below 140, 140-145, 145-150, 160-165.

Now, the frequency of class interval below 140 is 4, since then are 4 boys with height less than 140. For the frequency of class interval 140-145 subtract the number of boys having height less than 140 from the number of boys having height less than 145.

Thus, the frequency of class interval 140-145 is $11 - 4 = 7$ Similarly, we can calculate the frequencies of other class intervals and get the following table

Class interval	Frequency	Cumulative Frequency
Below 140	4	4
140-145	$11 - 4 = 7$	11
145-150	$29 - 11 = 18$	29
150-155	$40 - 29 = 11$	40
155-160	$46 - 40 = 6$	46
160-165	$51 - 46 = 5$	51

Here, $N = 51$

$$\frac{N}{2} = \frac{51}{2} = 25.5$$

Since, the cumulative frequency just greater than 25.5 is 29 and the corresponding class is 145-150. Thus 145-150 is median class,

Now, $l = 145,\ f = 18,\ F = 11$ and $h = 5$

Median, $M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$

$$= 145 + \left\{\frac{25.5 - 11}{18}\right\} \times 5$$

$$= 145 + \frac{72.5}{18}$$

$$= 145 + 4.03 = 149.03$$

For Mean we prepare following table.

Height (in cm)	f_i	x_i	$f_i x_i$
Below 140	4	137.5	550
140-145	7	142.5	997.5
145-150	18	147.5	2655
150-155	11	152.5	1617.5
155-160	6	157.5	945
160-165	5	162.5	812.5
	$N = \sum f_i = 51$		$\sum f_i x_i = 7637.5$

Now Mean $M = \frac{\sum f_i x_i}{N} = \frac{7637.5}{51} = 149.75$

13. A tour bus in Jaipur serves 400 customers a day. The charge is Rs 50 per person. The owner of the bus service estimates that the company would lose 10 passengers a day for each Rs 5 fare increase.

(i) How much should the fare be in order to maximize the income for the company?

(ii) What is the maximum income the company can expect to make?

Ans :

(i) Let x represent the number of Rs 5 fare increases. Then $50 + 5x$ is the price per passenger and $400 - 10x$ is the number of passengers.

The income is the number of passengers

multiplied by the price per ticket. Let $I(x)$ represent income as a function of x.

Now $I(x) = (400 - 10x)(50 + 5x)$

$= 10(40 - x)(5)(10 + x)$

$= 50(40 - x)(10 + x)$

$= 50(400 + 40x - 10x - x^2)$

$= 50(400 + 30x - x^2)$

$= -50(x^2 - 30x - 400)$

$= -50(x^2 - 30x + 15^2 - 15^2 - 400)$

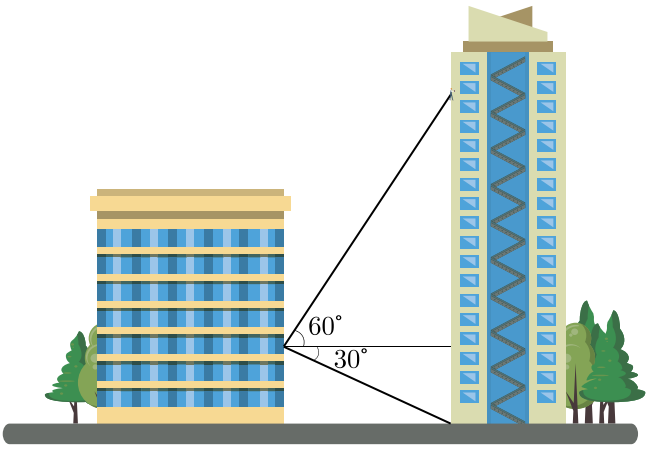
$= -50(x^2 - 30x + 15^2 - 625)$

$= -50(x^2 - 30x + 15^2) + 50 \times 625$

$= -50(x - 15)^2 + 31250$

(ii) From above equation it is clear that $I(x)$ is maximum at $x = 15$ and this maximum value is 31250. This means the company should make 15 fare increases of Rs 5 to maximize its income. Thus, the ticket price should be $50 + 5 \times 15 = 125$ Rs.

14. From his hotel room window on the fourth floor, Ranjan notices some window washers high above him on the hotel across the street.



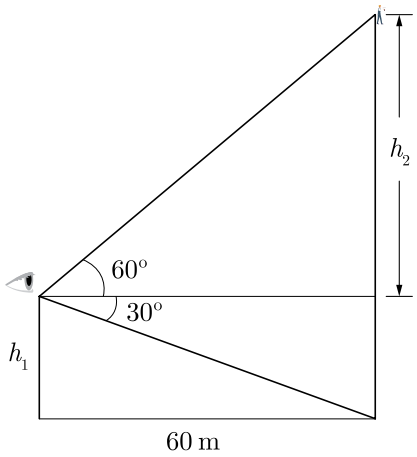
Curious as to their height above ground, he quickly estimates the buildings are 60 m apart, the angle of elevation to the workers is about $60°$, and the angle of depression to the base of the hotel is about $30°$.

(i) How high above ground is the window of Ranjan's hotel room?

(ii) How high above ground are the workers?

Ans :

Let h_1 be the height of Ranjan window from ground and h_2 be height of window washers from Ranjan. We draw a diagram of the situation as shown below.



Here $\tan 30° = \frac{h_1}{60}$

$\frac{1}{\sqrt{3}} = \frac{h_1}{60}$

$h_1 = \frac{60}{\sqrt{3}} = 20\sqrt{3}$

$= 20 \times 1.732 = 34.64$ m

Now $\tan 60° = \frac{h_2}{60}$

$\sqrt{3} = \frac{h_2}{60}$

$h_2 = 60\sqrt{3}$

$= 60 \times 1.732 = 103.92$ m

Height of workers from ground,

$h_1 + h_2 = 34.64 + 103.92$

$= 138.56$ m

(i) Window of Ranjan hotel is 34.64 meter above ground.

(ii) Workers are 138.64 meter above the ground.