Ratio and Proportion

- **Percentages** are numerators of fractions with denominator 100. It is represented by the symbol % and means hundredths too, i.e., 25% $= \frac{25}{100} =$ 0.25
- Fractional numbers, whole numbers and decimals can be converted into percentages by multiplying them by 100%.

  Note: Percentages related to proper fractions are less than 100 whereas percentages related to improper fractions are more than 100.

  For example, $1\frac{1}{4} = \frac{5}{4} \times 100\% = 125\%$

- To convert ratio to percentage, we proceed as follows:

  Consider the ratio a:b.

  Sum of parts = $a + b$

  Percantage form = $\frac{a}{a+b} \times 100\%$

- Percentages can be converted into fractions or decimals by dividing them by 100.

  For example, 35% can be converted to decimals and fraction as follows:

  $35\% = \frac{35}{100} = 0.35$

  $35\% = \frac{35}{100} = \frac{7}{20}$

- To convert percentage to ratio, we have to find the ratio of the percentages of the two quantities.
- When added, all parts of a whole give whole or 100%.

- To express x as a percentage of y, percentage = $\left(\frac{x}{y}\times 100\right)\%$
- x % of a given quantity = $\frac{x}{100}\times$ **given quanity**
- If x % of a given quantity is y, then quantity = $\frac{y}{x}\times$ **100**

Example: In a bag there are 6 blue marbles, 4 red marbles and 5 green marbles. What percent of total marbles are blue?
Solution: Total number of marbles = 6 + 4 + 5 = 15
Number of blue marbles = 6
∴ Percentage of blue marbles = $\frac{6}{15}\times 100 = 40\%$

- The price at which an article is bought is called its **cost price** (CP).
- The price at which an article is sold is its **selling price** (SP).
    - Conditions of profit or loss:
        1. If CP < SP then profit is made and Profit = SP – CP
        2. If CP = SP then there is a no profit, no loss.
        3. If CP > SP then loss is incurred and Loss = CP – SP

For example, Suman bought a bottle for Rs 130 and sold it for Rs 142.

Here, SP = Rs 142, CP = Rs 130

As SP > CP, so profit is incurred.

Profit = SP – CP = Rs 142 – Rs 130 = Rs 12

- The formulae to calculate profit and loss are:
    - Profit % $= \frac{\text{Profit}}{\text{C.P}}\times 100$
    - Loss % $= \frac{\text{Loss}}{\text{C.P.}}\times 100$

Example:

A shopkeeper purchased 15 dozen cups for Rs 900. However, 9 cups cracked during transportation. The remaining cups were sold for Rs 9 each. Find the gain or loss percent.

Solution:

Cost price of 15 dozen i.e., 180 cups = Rs 900

9 cups were cracked. Therefore, number of cups left = 180 – 9 = 171

These 171 cups were sold at Rs 9 each.

∴ S.P. of 171 cups = Rs 9 × 171 = Rs 1539

⇒ Profit = SP – CP = Rs (1539 – 900) = Rs 639

$$\text{Profit\%} = \frac{\text{Profit}}{\text{C.P.}} \times 100 = \frac{639}{900} \times 100 = 71\%$$

- Discount is the reduction given on the Marked Price (M.P) of an article.
  Discount = Marked Price – Sale price

  Discount = Discount % of Marked Price

- If the successive discount %, d_1%, d_2%, d_3% … are given, then

$$\textbf{S.P.} = \text{M.P.} \times \left(\frac{\mathbf{100 - d_1}}{\mathbf{100}}\right) \times \left(\frac{\mathbf{100 - d_2}}{\mathbf{100}}\right) \times \left(\frac{\mathbf{100 - d_3}}{\mathbf{100}}\right) \times \ldots.$$

Example:

For the stock sale at the end of a season, a garment shop offers 50% and then 40% on the garments. What is the marked price of a shirt if the shop offers a total discount of Rs 840 after giving two successive discounts?

Solution:

Let the marked price of the shirt be Rs x.

In two successive discounts, d_1% = 50 and d_2% = 40.

We know that

$$\text{S.P.} = \left(\frac{100-d_1}{100}\right)\times\left(\frac{100-d_2}{100}\right)\times \text{M.P.}$$
$$= \left(\frac{100-50}{100}\right)\times\left(\frac{100-40}{100}\right)\times x$$
$$= \frac{50}{100}\times\frac{60}{100}\times x$$
$$= \frac{3x}{10}$$

We know that, discount = M.P. − S.P.

$$\Rightarrow 840 = x - \frac{3x}{10}$$
$$\Rightarrow \frac{7x}{10} = 840$$
$$\Rightarrow x = \frac{840\times 10}{7} = 1200$$

Hence, the marked price of the shirt is Rs 1200.

- The difference between the tax paid on the sale value and that paid on the purchase value is deposited with the government as **VAT.**

  For example, a retailer purchases an article from a manufacturer for Rs 1000. The rate of sales tax is 10%.

  Sales tax paid by the retailer = 10% of 1000 = Rs 100

  If he sells the article for Rs 1200 then tax recovered on the sale = 10% of Rs 1200 = Rs 120

  VAT = Tax recovered on sale − Tax paid on purchase

  = Rs 120 − Rs 100

  = Rs 20

  Here, the value added by the retailer is Rs 1200 − Rs 1000 = Rs 200

  Tax on the added value = 10% of Rs 200 = Rs 20

This is the reason why it is called value added tax (VAT).

- The difference between sales tax and value added tax is that sales tax is realized at single point only, while value added tax is realized at different stages.

- **Terminology related to simple interest:**
    1. The amount of money that is borrowed is known as principal and is denoted by P.
    2. The extra amount of money that one has to pay is known as interest and is denoted by I.
    3. The total amount of money, A that one pays back is equal to the sum of principal and interest.
    4. The simple interest (SI) on the principal (P) when borrowed for T years at R% rate of interest per year is given by the formula $SI = \frac{P \times T \times R}{100}$

Example:
Rashmi takes a loan of Rs 4000 from a bank at 8% rate of interest per year. Find the amount of money that Rashmi has to repay after 3 years.

Solution:
P = Rs 4000, R = 8% p.a., T = 3 years

$$SI = \frac{P \times R \times T}{100} \frac{4000 \times 8 \times 3}{100} = Rs.960$$

∴ Amount = P + I = Rs 4000 + Rs 960 = Rs 4960

Thus, Rashmi has to repay Rs 4960 after 3 years.

Note: Principal remains unchanged throughout the given time period while calculating the simple interest.

- Interest is the extra money paid by institutions such as banks or post offices on money deposited with them.
  It is also paid by people when they borrow money from these institutions.

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

Amount = Principal + Interest

- The interest calculated on the amount of the previous year (or duration at which interest is compounded) is known as **compound interest**. Compound interest allows the principal to grow faster than simple interest.

- Amount (A) when interest is compounded annually is $A = P\left(1 + \frac{R}{100}\right)^n$ here, P = Principal, R = Rate of interest, n = Time period. w

  For example, if Supriya invested Rs 75000 in a bank at the rate of 10% per annum for 2 years then the amount received by her can be calculated as follows:

  Here, P = Rs 75000, R = 10%, n = 2 years

  $$A = P\left(1 + \frac{R}{100}\right)^n$$
  $$= 75000\left(1 + \frac{10}{100}\right)^2$$
  $$= 75000 \times \frac{121}{100}$$

  = Rs 90750

  Thus, Supriya received Rs 90750 after 2 years.

- Amount when interest is compounded half yearly is given by, $A = P\left(1 + \frac{R}{200}\right)^{2n}$

  Where, $\frac{R}{2}$ = Half-yearly rate and $2n$ = Number of half years

- Amount when interest is compounded quarterly is given by, $A = P\left(1 + \frac{R}{400}\right)^{4n}$
  Where, $\frac{R}{4}$ = Quarterly rate and $4n$ = Number of quarters

- Two quantities, x and y, are said to be in **direct proportion**, if they increase (or decrease) together in such a manner that the ratio of their corresponding values remains constant. That is, $\frac{x}{y} = k$ where k is a positive number.

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  For example, price of wheat per kg and the weight of wheat that can be brought are in direct proportion as more the weight of wheat, more will be the cost.

- If y_1, y_2 are the values of y corresponding to the values x_1, x_2 of x respectively then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ is a case of direct proportion.

- Two variables x and y will be in direct proportion if $\frac{x}{y} = k$ or $x = ky$, where the constant k is known as constant of proportionality of the direct proportion. Thus, to check whether the variables x and y are in direct proportion, we need

to find the ratio $\frac{x}{y}$ for their corresponding values. If this ratio remains constant, then the variables are in direct proportion, otherwise they are not.

- Two quantities, x and y, are said to be in **inverse proportion**, if an increase in x causes a proportional decrease in y (and vice-versa) in such a manner that the product of their corresponding values remains constant. That is, $xy = k$, where k is a positive number.
- Two variables x and y will be in inverse proportion if $xy = k$, where the constant k is known as constant of proportionality of the inverse proportion. Thus, to check whether the two variables x and y of a given situation are in inverse proportion or not, we have to calculate the product of the value of variable x with its corresponding value of the variable y. If all these products are equal, then we can say that the variables x and y are in inverse proportion, otherwise not.

  For example, $x = 1$, $y = 20$ and $x = 5$, $y = 4$ are in inverse proportion.

  Here, $1 \times 20 = 20$

  $5 \times 4 = 20$

  It can be seen that $x \times y = 20$, which is constant for both observations.

  Therefore, x and y are in inverse proportion.

- The method in which we first find the value of one unit and then the value of the required number of units is known as **unitary method**.

  Example:

  If 15 men can do a piece of work in 10 days, then in how many days can 6 men do the same work?

  Solution:

This is the case of indirect variation since more the number of men, less will be the number of days required to finish the work.

It is given that 15 men can do the work in 10 days.

∴One man can do the work in (10 × 15) days.

Hence, 6 men can do the work in $\frac{10\times15}{6}$ days = 25 days

- Important formulae to solve problems related to time and work:

One day's work = $\frac{1}{\textbf{Number of days to complete the work}}$

Number of days to complete the work = $\frac{1}{\textbf{One day's work}}$

Example:

Nitika and Ruchika together can type 100 pages in 20 hours. Nitika alone can type 100 pages in 25 hours. How much time would Ruchika take to type 100 pages alone?

Solution:

Nitika and Ruchika together type 100 pages in 20 hours.

∴One hour's work done by Nitika and Ruchika together $=\frac{1}{20}$ hours

Now, Nitika alone can type 100 pages in 25 hours.

∴Nitika's one hour's work $=\frac{1}{25}$ hours

∴Ruchika one hour's work $=\frac{1}{20}-\frac{1}{25}$

$=\frac{5-4}{100}=\frac{1}{100}$

Thus, Ruchika alone would type 100 pages in 100 hours.