Relations and Functions

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. **A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A?** [*CBSE Sample Paper (2021-22) (Term-1)*]

   (*a*) (1, 1) (*b*) (1, 2) (*c*) (2, 2) (*d*) (3, 3)

2. **Let the relation R in the set $A = \{x \in Z: 0 \le x \le 12\}$, given $R = \{(a, b): |a - b|$ is a multiple of 4$\}$. Then [1], the equivalence class containing 1, is** [*CBSE Sample Paper (2021-22) (Term-1)*]

   (*a*) $\{1, 5, 9\}$ (*b*) $\{0, 1, 2, 5\}$ (*c*) ϕ (*d*) A

3. **For real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is** [*NCERT Exemplar*]

   (*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) none of these

4. **Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b, then R is** [*NCERT Exemplar*]

   (*a*) symmetric but not transitive (*b*) transitive but not symmetric
   (*c*) neither symmetric nor transitive (*d*) both symmetric and transitive

5. **The maximum number of equivalence relation on the set $A = \{1, 2, 3\}$ are** [*NCERT Exemplar*]

   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5

6. **Let L denotes the set of all straight lines in a plane. Let a relation R be defined by lRm if and only if l is perpendicular to m $\forall$ $l, m \in L$. Then R is** [*NCERT Exemplar*]

   (*a*) reflexive (*b*) symmetric
   (*c*) transitive (*d*) none of these

7. **A relation R is defined on N. Which of the following is the reflexive relation?** [*CBSE 2021-22 (Term-1)*]

   (*a*) $R = \{(x, y) : x > y, x, y \in N\}$
   (*b*) $R = \{(x, y) : x + y = 10, x, y \in N\}$
   (*c*) $R = \{(x, y) : xy$ is the square number, $x, y \in N\}$
   (*d*) $R = \{(x, y) : x + 4y = 10; x, y \in N\}$

8. The number of equivalence relations in the set {1, 2, 3} containing the elements (1, 2) and (2, 1) is [CBSE 2021-22 (Term-1)]

(a) 0 (b) 1 (c) 2 (d) 3

9. A relation R is defined on Z as:

$a R b$ if and only if $a^2 - 7ab + 6b^2 = 0$. Then, R is [CBSE 2021-22 (Term-1)]

(a) reflexive and symmetric (b) symmetric but not reflexive
(c) transitive but not reflexive (d) reflexive but not symmetric

10. If a relation R on the set {1, 2, 3} be defined by $R = \{(1, 2)\}$, then R is [NCERT Exemplar]

(a) Reflexive (b) Transitive (c) Symmetric (d) None of these

11. Let R be a relation on $A = \{a, b, c\}$ such that $R = \{(a, a), (b, b), (c, c)\}$, then R is

(a) Reflexive (b) Symmetric only
(c) Non-transitive (d) Equivalence

12. Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$, then [CBSE Sample Paper 2021-22 (Term-1)]

(a) $(2, 4) \in R$ (b) $(3, 8) \in R$ (c) $(6, 8) \in R$ (d) $(8, 7) \in R$

13. Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is

(a) 2^{mn} (b) 2^{m+n} (c) mn (d) 0

14. Let $A = \{3, 5\}$. Then number of reflexive relations on A is [CBSE 2023 (65/5/1)]

(a) 2 (b) 4 (c) 0 (d) 8

15. The relation R in the set {1, 2, 3} given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is [CBSE 2020 (65/2/1)]

(a) symmetric and transitive, but not reflexive (b) reflexive and symmetric, but not transitive
(c) symmetric, but neither reflexive nor transitive (d) an equivalence relation

16. Let $A = \{1, 3, 5\}$. Then the number of equivalence relations in A containing (1, 3) is [CBSE 2020 (65/2/1)]

(a) 1 (b) 2 (c) 3 (d) 4

17. The function $f : R \to R$ defined as $f(x) = x^3$ is [CBSE Sample Paper 2021-22 (Term-1)]

(a) one-one but not onto (b) not one-one but onto
(c) neither one-one nor onto (d) one-one and onto

18. Set A has 3 elements and the set B has 4 elements. Then the number of injective mapping that can be defined from A to B is [NCERT Exemplar]

(a) 144 (b) 12 (c) 24 (d) 64

19. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Based on the given information, f is best defined as [CBSE Sample Paper 2021-22 (Term-1)]

(a) surjective function (b) injective function (c) bijective function (d) function

20. The number of functions defined from $\{1, 2, 3, 4, 5\} \longrightarrow \{a, b\}$ which are one-one is [CBSE Sample Paper 2021-22 (Term-1)]

(a) 5 (b) 3 (c) 2 (d) 0

21. The function $f : R \longrightarrow R$ defined by $f(x) = 4 + 3 \cos x$ is [CBSE 2021-22 (Term-1)]

(a) bijective (b) one-one but not onto
(c) onto but not one-one (d) neither one-one nor onto

22. Let $f: R \longrightarrow R$ be defined by $f(x) = \frac{1}{x}$, for all x, $x \in \mathbb{R}$. Then, f is [CBSE 2021-22 (Term-1)]

(a) one-one (b) onto (c) bijective (d) not defined

23. The function $f: N \longrightarrow N$ is defined by $f(n) = \begin{cases} \frac{n+1}{2}, \text{ if } n \text{ is odd} \\ \frac{n}{2}, \text{ if } n \text{ is even} \end{cases}$ [CBSE 2021-22 (Term-1)]

The function f is

(a) bijective (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

Answers

1. (b)	2. (a)	3. (a)	4. (b)	5. (d)	6. (b)	7. (c)
8. (c)	9. (d)	10. (b)	11. (d)	12. (c)	13. (a)	14. (b)
15. (a)	16. (b)	17. (d)	18. (c)	19. (b)	20. (d)	21. (d)
22. (d)	23. (c)					

Solutions of Selected Multiple Choice Questions

1. Given relation R on set $A = \{1, 2, 3\}$ is

$$R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$$

From the given relation R if we removed (1, 2) then

$R = \{(1, 1), (2, 2), (3, 3)\}$ is an equivalence relation in A.

Hence, ordered pair (1, 2) should be removed.

$\therefore$ Option (b) is correct.

2. $\because$ $[1] = \{x \in A : |x-1| \text{ is a multiple of } 4\} = \{1, 5, 9\}$

$\therefore$ Option (a) is correct.

4. Given, $aRb \Rightarrow a$ is brother of b

This does not mean that b is also a brother of a because b can be a sister of a.

Hence, R is not symmetric.

Again, $aRb \Rightarrow a$ is brother of b and $bRc \Rightarrow b$ is brother of c.

So, a is brother of c.

Hence, R is transitive.

$\therefore$ Option (b) is correct.

5. Number of equivalence relation in $A = B_3$

$$\because B_3 = B_{2+1} = \sum_{k=0}^{2} C_k B_k$$

$$= {}^2C_0 B_0 + {}^2C_1 B_1 + {}^2C_2 B_2 = B_0 + 2B_1 + B_2$$

$$= 1 + 2 + 2 = 5 \qquad [\because B_0 = 1 = B_1, B_2 = 2]$$

$\therefore$ Option (d) is correct.

6. For $l, m \in L$

if $(l, m) \in R \Rightarrow l \perp m \Rightarrow m \perp l \Rightarrow (m, l) \in R$

$\therefore$ R is symmetric.

$\therefore$ Option (b) is correct.

7. A relation R is defined on N by $R = \{(x, y) : xy$ is the square number, $x, y \in N\}$

Let $x \in N$

$\Rightarrow (x, x) : x \times x = x^2$ which is a square number $\in N$

$\Rightarrow (x, x) \in R$

$\therefore$ R is reflexive on N.

$\therefore$ Option (c) is correct.

8. We have total possible pairs = $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

The smallest equivalence relation R_1 containing $(1, 2)$ and $(2, 1)$ is $\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$.

Now, we are left with any 4 pairs namely $(2, 3)$, $(3, 2)$, $(1, 3)$ and $(3, 1)$.

If we add any one, say $(2, 3)$ to R_1 then for symmetry we must add $(3, 2)$ also and for transitivity we are forced to add $(1, 3)$ and $(3, 1)$. Thus the only equivalence relation bigger than R_1 is the universal relation. This shows that the total number of equivalence relations containing $(1, 2)$ and $(2, 1)$ is two.

$\therefore$ Option (c) is correct.

9. We have a relation R defined on Z as aRb if and only if $a^2 - 7ab + 6b^2 = 0$.

Reflexive: Let $a \in Z$

$$\because\ a^2 - 7a \times a + 6a^2 = 7a^2 - 7a^2 = 0 \Rightarrow aRa$$

It is reflexive.

Symmetric: Let $a, b \in Z$ and $(a, b) \in R$

$\therefore aRb \quad \Rightarrow a^2 - 7ba + 6b^2 = 0$

but $b \not R a$ because $b^2 - 7ba + 6a^2 \neq 0$ (may or may not be zero)

$\therefore$ It is not symmetric.

Hence, R is reflexive but not symmetric.

$\therefore$ Option (d) is correct.

10. $R = \{(1, 2)\}$, $A = \{1, 2, 3\}$

Clearly R is neither reflexive nor symmetric.

As $(1, 2) \in R$ but $\nexists\ (2, b) \in R$ for $b \in A$ such that $(1, b) \notin R$.

Hence R is a transitive relation on A.

$\therefore$ Option (b) is correct.

11. $R = \{(a, a), (b, b), (c, c)\}$

Reflexive: Let $(x, x) \in R\ \forall x \in A$

So, R is reflexive.

Symmetric: For $(x, y) \in R, x = y \Rightarrow (y, x) \in R \quad \forall x, y \in A$

So, R is symmetric.

Transitive: For $(x, y) \in R$ there is no $(y, z) \in R$ such that $(x, z) \notin R$ so R is transitive.

Hence, R is an equivalence relation.

$\therefore$ Option (d) is correct.

12. $a = b - 2$ and $b > 6$

$\Rightarrow \qquad (6, 8) \in R$

$\therefore$ Option (c) is correct.

14. If a set containing n elements then number of symmetric relations in $A = 2^{n^2 - n}$

Here $n(A) = 2$

$\therefore$ Number of symmetric relations $= 2^{2^2 - 2} = 2^2 = 4$

$\therefore$ Option (b) is correct.

15. Given relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1), (1, 1)\}$ is symmetric and transitive, but not reflexive because $(2, 2) \notin R$ and $(3, 3) \notin R$.

$\therefore$ Option (a) is correct.

16. Given set $A = \{1, 3, 5\}$

We have, smallest equivalence relation $R_1 = \{(1, 1), (3, 3), (5, 5), (1, 3), (3, 1)\}$

If we add (3, 5), then we have to add (5, 3) also, as it is symmetric.

$\therefore$ Second equivalence relation will be

$$R_2 = \{(1, 3), (3, 1), (1, 1), (3, 3), (5, 5), (3, 5), (5, 3), (1, 5), (5, 1)\}$$

Hence, number of equivalence relations containing (1, 3) is 2.

$\therefore$ Option (b) is correct.

17. Let $f(x_1) = f(x_2)\ \forall\ x_1, x_2 \in \mathbb{R}$ (domain)

$\Rightarrow \quad x_1^3 = x_2^3 \quad \Rightarrow \quad x_1 = x_2 \quad \Rightarrow \quad f$ is one-one.

Let $f(x) = x^3 = y\ \forall\ y \in \mathbb{R}$ (co-domain)

$\Rightarrow \quad x = y^{\frac{1}{3}} \in \mathbb{R}$ (domain)

Every image $y \in \mathbb{R}$ (co-domain) has a unique pre image in $\mathbb{R}$ (domain).

$\Rightarrow \quad f$ is onto.

Hence, f is one-one and onto.

$\therefore$ Option (d) is correct.

18. The total number of injective mappings from the set containing n elements into the set containing m elements is ${}^mP_n$. So here it is ${}^4P_3 = 4! = 24$.

19. As every pre-image $x \in A$ has a unique image $y \in B$.

$\Rightarrow \quad f$ is injective function.

$\therefore \quad$ Option (b) is correct.

20. Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{a, b\}$

$\therefore$ Number of functions defined from $f: X \to Y$ which are one-one is zero (0), because number of elements in set X is 5 and number of elements in set Y is 2 $\Rightarrow n(X) > n(Y)$

$\Rightarrow$ No one-one function possible.

$\therefore$ Option (d) is correct.

21. Given function $f: R \to R$ defined by $f(x) = 4 + 3 \cos x$

One-one : Let $x_1 = 0$ and $x_2 = 2\pi$ *i.e.* $x_1 \neq x_2$

$\Rightarrow \cos x_1 = \cos 0 = 1$ and $\cos x_2 = \cos 2\pi = 1$

$\therefore \cos x_1 = \cos x_2 \quad \Rightarrow 4 + 3\cos x_1 = 4 + 3 \cos x_2$

$\Rightarrow f(x_1) = f(x_2)$

It is not one-one.

Onto: As we know that for all $x \in R$, we have

$-1 \leq \cos x \leq 1 \Rightarrow -3 \leq 3\cos x \leq 3 \Rightarrow 1 \leq 4 + 3\cos x \leq 7$

$\Rightarrow -1 \leq f(x) \leq 7 \qquad \Rightarrow$ Range of $f = [1, 7]$

Clearly, Range of $f \neq$ co-domain of f

$\therefore f$ is not onto.

Hence, f is neither one-one nor onto.

$\therefore$ Option (d) is correct.

22. Given function $f: R \to R$ be defined by $f(x) = \frac{1}{x}$, for all $x \in R$

Clearly, when $x = 0 \in R$

$f(x) = \frac{1}{0} = (\infty)$ (not defined)

$\therefore$ Option (d) is correct.

23. Given function $f: N \to N$ is defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

One-one :

Let $n = 1$ (odd) $\Rightarrow f(1) = \frac{1+1}{2} = 1$

and, $n = 2$ (even) $\Rightarrow f(2) = \frac{2}{2} = 1$

Clearly, it is not one-one function.

Onto : For every value of n whether it is even or odd

$f(n) \in \mathbb{N}$

$\therefore$ Co-domain = Range

$\therefore$ It is onto.

Hence, function f is onto but not one-one.

$\therefore$ Option (c) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A) :** Let R be the relation on the set of integers Z given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.

   Reason (R) : A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

2. **Assertion (A) :** Let $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = x$, then f is a one-one function.

   Reason (R) : A function $g: A \to B$ is said to be onto function if for each $b \in B, \exists\, a \in A$ such that $g(a) = b$.

3. **Assertion (A) :** Let function $f: \{1, 2, 3\} \to \{1, 2, 3\}$ be an onto function. Then it must be one-one function.

Reason (R) : A one-one function $g : A \to B$, where A and B are finite set and having same number of elements, then it must be onto and vice-versa.

4. **Assertion (A):** Let $f: \mathbb{R} \to \mathbb{R}$ such that $f(x) = x^2$. The function f is an onto function.

   Reason (R) : A function $g : A \to B$ is said to be onto function if $g(A) = B$ *i.e.*, range of $g = B$.

5. **Assertion (A):** The number of all onto functions from the set $\{1, 2, 3, 4, 5\}$ to itself is $5!$.

   Reason (R) : Total number of all onto functions from the set $\{1, 2, 3, ..., n\}$ to itself is $n!$.

6. **Assertion (A):** Let $f: \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \frac{|x|}{x} = \begin{cases} 1 & \text{if} \quad x > 0 \\ 0 & \text{if} \quad x = 0 \\ -1 & \text{if} \quad x < 0 \end{cases} \text{ is a bijection.}$$

   Reason (R) : A function $g : A \to B$ is said to be bijection if it is one-one and onto.

7. **Assertion (A):** Let $A = \{1, 2, 3\}$ then define a relation on A as $R = \{(1, 2), (2, 1)\}$, R is not transitive relation.

   Reason (R) : A relation R defined on a non-empty set A is said to be transitive relation if (a, b), $(b, c) \in R \Rightarrow (a, c) \in R$.

Answers

1. (*a*) **2.** (*b*) **3.** (*a*) **4.** (*d*) **5.** (*a*) **6.** (*d*) **7.** (*a*)

Solutions of Assertion-Reason Questions

1. **Reflexivity:**

   Clearly $(a, a) \in R$ as $a - a = 0$ which is an even integer and is divisible by 2. So, it is reflexive.

   Symmetry:

   Let $(a, b) \in R \Rightarrow 2$ divides $(a - b)$.

   $\Rightarrow 2$ divides $-(a - b) \Rightarrow$ 2 divides $b - a \Rightarrow (b, a) \in R$

   So, it is symmetric.

   Transitivity:

   Let $(a, b) \in R$ and $(b, c) \in R$.

   $\Rightarrow 2$ divides $a - b$ and 2 divides $b - c \Rightarrow$ 2 divides $a - b + b - c = a - c \Rightarrow (a, c) \in R$

   So, it is transitive.

   $\Rightarrow$ Relation R is an equivalence relation. So A is true.

   Clearly R is also true and gives the correct explanation of A.

   Hence option (*a*) is correct.

2. Let $x_1, x_2 \in \mathbb{R}$ such that $f(x_1) = f(x_2)$

   $\Rightarrow x_1 = x_2 \Rightarrow f$ is a one-one function.

   Clearly A is true and R is also true. But R does not give correct explanation of A.

   Hence option (*b*) is correct.

3. Suppose f is not one-one function.

   i.e., $\exists$ two elements say 1 and 2 in the domain mapped to a single element of the co-domain.

   Then 3 can be mapped to any one of two remaining element.

   So, range set has only two elements.

$\Rightarrow$ $R(f) \neq \{1, 2, 3\}$ which contradict the fact that f is an onto function.

Thus f must be a one-one function.

So, A and R gives the correct explanation of A.

Hence option (*a*) is correct.

4. Clearly $R(f) = [0, \infty)$

Here $R(f) \neq \mathbb{R} = (-\infty, \infty)$

$\Rightarrow f$ is not an onto function.

So, A is false but R is true.

Hence option (*d*) is correct.

5. One-one 1 can map to any one of 1, 2, ..., 5 i.e., 5 ways.

After that 2 can be mapped to any four of the remaining 4 elements.

After that 4 can be mapped to remaining three elements.

Now 4 can be mapped to remaining two element.

And 5 can be mapped to remaining one element.

$\therefore$ Total number of one-one functions $= 5 \times 4 \times 3 \times 2 \times 1 = 5!$

We know that total number of onto map on a finite set = Total numbers of one-one map = 5!

$\therefore$ Total number of bijective map = 5!

So A is correct statement.

Also R is a correct and gives correct explanation of statement A.

$\therefore$ Option (*a*) is correct.

6. $\because$ $f: \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$\because$ For $1, 2 \in \mathbb{R}$ (domain)

$f(1) = 1 = f(2)$ but $1 \neq 2$

$\Rightarrow$ f is not one-one.

Also $R(f) = \{-1, 0, 1\} \neq \mathbb{R}$ (Co-domain)

$\Rightarrow$ f is not onto.

$\therefore$ f is neither one-one nor onto.

$\Rightarrow$ f is not a bijective map.

So A is not a true statement.

But R is the correct statement and R does not gives correct explanation of A.

$\therefore$ Option (*d*) is correct.

7. We have $A = \{1, 2, 3\}$,

$R = \{(1, 2), (2, 1)\}$

$\because$ $(1, 2), (2, 1) \in R$ but $(1, 1) \notin R$

$\Rightarrow$ R is not transitive.

So statement A is correct.

Also statement R is correct and gives correct explanation of statement A.

$\therefore$ Option (*a*) is correct.

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. Read the following passage and answer the following questions.

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever.

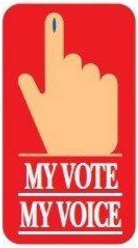
ONE – NATION

ONE – ELECTION

FESTIVAL OF

DEMOCRACY

GENERAL ELECTION– 2019



Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

$R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019$\}$.

(*i*) Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election – 2019. Is XRY? Give reason.

(*ii*) Mr. 'X' and his wife 'W' both exercised their voting right in general election -2019. Is it true that XRY and YRX? Give reason.

(*iii*) (*a*) Three friends F_1, F_2 and F_3 exercised their voting right in general election- 2019. Is it true that F_1RF_2, $F_2RF_3 \Rightarrow F_1RF_3$? Give reason.

OR

(*iii*) (*b*) Mr. Shyam exercised his voting right in General Election – 2019, then find the equivalence class of Mr. Shyam.

Sol. We have a relation 'R' is defined on I as follows:

$R = \{V_1, V_2\} : V_1, V_2 \in I$ and both use their voting right in general election – 2019$\}$.

(*i*) Two neighbours X and $Y \in I$. Since X exercised his voting right while Y did not cast her vote in general election – 2019.

Therefore, $(X, Y) \notin R$.

(*ii*) Since Mr. 'X' and his wife 'W' both exercised their voting right in general election – 2019.

$\therefore$ Both (X, W) and $(W, X) \in R$.

(*iii*) (*a*) Since three friends F_1, F_2 and F_3 exercised their voting right in general election – 2019, therefore

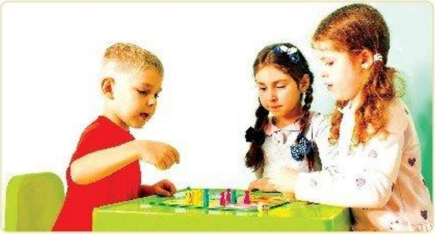
$$(F_1, F_2) \in R, (F_2, F_3) \in R \text{ and } (F_1, F_3) \in R.$$

OR

(*iii*) (*b*) Mr. Shyam exercised his voting right in General election – 2019, then Mr. Shyam is related to all those eligible voters who cast their votes.

2. **Read the following passage and answer the following questions.**

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1,2,3,4,5,6}. Let A be the set of players while B be the set of all possible outcomes.



$A = \{S, D\}, B = \{1,2,3,4,5,6\}$ [*CBSE Question Bank*]

(*i*) Let $R : B \rightarrow B$ be defined by $R = \{(x, y): y$ is divisible by $x\}$. Verify that whether R is reflexive, symmetric and transitive.

(*ii*) Raji wants to know the number of functions from A to B. Find the number of all possible functions.

(*iii*) (*a*) Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then R is which kind of relation?

OR

(*iii*) (*b*) Raji wants to know the number of relations possible from A to B. Find the number of possible relations.

Sol. (*i*) Given $R : B \rightarrow B$ be defined by $R = \{(x, y) : y$ is divisible by $x\}$.

Reflexive : Let $x \in B$, since x always divide x itself.

$\therefore (x, x) \in R$

It is reflexive.

Symmetric : Let $x, y \in B$ and let $(x, y) \in R$.

$\Rightarrow y$ is divisible by x.

$\Rightarrow \frac{y}{x} = k_1$, where k_1 is an integer.

$\Rightarrow \frac{x}{y} = \frac{1}{k_1} \neq$ integer.

$\therefore \ (y, x) \notin R$

It is not symmetric.

Transitive : Let $x, y, z \in B$ and

let $(x, y) \in R \ \Rightarrow \frac{y}{x} = k_1$, where k_1 is an integer.

and, $(y, z) \in R \Rightarrow \frac{z}{y} = k_2$, where k_2 is an integer.

$\therefore \ \frac{y}{x} \times \frac{z}{y} = k_1.k_2 = k$ (integer)

$\Rightarrow \frac{z}{x} = k \quad \Rightarrow (x, z) \in R$

It is transitive.

Hence, relation is reflexive and transitive but not symmetric.

(*ii*) We have,

$A = \{S, D\} \Rightarrow n(A) = 2$

and, $B = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(B) = 6$

$\therefore$ Number of functions from A to B is $6^2 = 36$.

(*iii*) (*a*) Given, R be a relation on B defined by

$R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$

R is not reflexive since $(1, 1), (3, 3), (4, 4) \notin R$

R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$

and, R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$

$\therefore$ R is neither reflexive nor symmetric nor transitive.

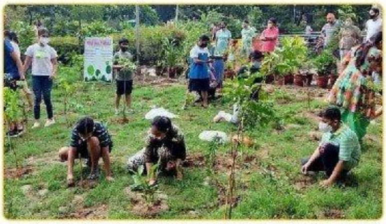
OR

(*iii*) (*b*) $\because n(A) = 2, n(B) = 6 \Rightarrow n(A \times B) = 12$

$\therefore$ Total number of possible relations from A to $B = 2^{12}$

3. Read the following passage and answer the following questions.

Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L. **[*CBSE Question Bank*]**



(*i*) Let relation R be defined by $R = \{(L_1, L_2): L_1 \parallel L_2$ where $L_1, L_2 \in L\}$. What is the type of relation R?

(*ii*) Let $R = \{(L_1, L_2): L_1 \perp L_2$ where $L_1, L_2 \in L\}$. What is the type of relation R?

(*iii*) (*a*) Check whether the function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x - 4$ is bijective or not.

OR

(*iii*) (*b*) Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x + 4$. Find the range of $f(x)$.

Sol. (*i*) Given relation R defined by

$R = \{(L_1, L_2) : L_1 \parallel L_2$ where $L_1, L_2 \in L\}$

Reflexive: Let $L_1 \in L \Rightarrow L_1 \parallel L_1 \Rightarrow (L_1, L_1) \in R$.

$\therefore$ It is reflexive.

Symmetric: Let $L_1, L_2 \in L$ and let $(L_1, L_2) \in R$.

$\Rightarrow L_1 \parallel L_2 \Rightarrow L_2 \parallel L_1 \Rightarrow (L_2, L_1) \in R$

$\therefore$ It is symmetric.

Transitive: Let $L_1, L_2, L_3 \in L$.

and, let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\therefore$ $L_1 \parallel L_2$ and $L_2 \parallel L_3$

$\Rightarrow$ $L_1 \parallel L_3 \Rightarrow (L_1, L_3) \in R$

$\therefore$ It is transitive.

Hence R is an equivalence relation.

(*ii*) Given relation R defined by $R = \{(L_1, L_2) : L_1 \perp L_2 \text{ where } L_1, L_2 \in L\}$

Reflexive : Since every line is not perpendicular to itself.

$\Rightarrow\ (L_1, L_1) \notin R$

$\therefore$ It is not reflexive.

Symmetric : Let $L_1, L_2 \in L$ and $(L_1, L_2) \in R$

$\Rightarrow\ L_1 \perp L_2 \quad \Rightarrow\ L_2 \perp L_1 \quad \Rightarrow (L_2, L_1) \in R$

$\therefore$ It is symmetric.

Transitive : Let $L_1, L_2, L_3 \in L$

and, let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\therefore\ L_1 \perp L_2$ and $L_2 \perp L_3$

$\Rightarrow\ L_1 \not\perp L_3 \quad \Rightarrow\ (L_1, L_3) \notin R$

$\therefore$ It is not transitive.

Hence relation R is symmetric but neither reflexive nor transitive.

(*iii*) (*a*) Given function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x - 4$

Injective : Let $x_1, x_2 \in R$ such that $x_1 \neq x_2$.

$\Rightarrow\ x_1 - 4 \neq x_2 - 4 \Rightarrow f(x_1) \neq f(x_2)$

$\therefore$ It is injective.

Surjective : Let $y = x - 4 \quad \Rightarrow\ x = y + 4$

For every $y \in R$ (co-domain) there exists $x = y + 4 \in R$ *(domain)*.

i.e., Co-domain = Range

$\therefore$ It is surjective.

Hence given function is bijective.

OR

(*iii*) (*b*) Given function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x + 4$

Let $y = f(x) \Rightarrow y = x + 4 \Rightarrow x = y - 4$

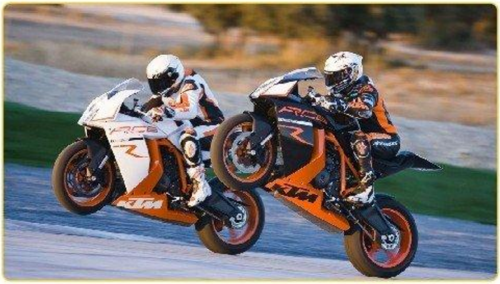
For $y \in \mathbb{R}$ (co-domain),

$\exists\ x = y - 4 \in \mathbb{R}$ (domain) such that $f(x) = \mathbb{R}$

$\therefore$ Range of $f(x)$ is $\mathbb{R}$ (Set of real numbers).

4. Read the following passage and answer the following questions.

An organization conducted bike race under two different categories- Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race. **[*CBSE 2023 (65/5/1)*]**



(*i*) **How many relations are possible from B to G?**

(*ii*) **Among all the possible relations from B to G, how many functions can be formed from B to G?**

(*iii*) (*a*) **Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation in B or not.**

OR

(*iii*) (*b*) **A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective, Justify your answer.**

Sol. $B = \{b_1, b_2, b_3\}, G = \{g_1, g_2\}$

$n(B) = 3, n(G) = 2$

$\because \quad n(B \times G) = n(B) \times n(G) = 3 \times 2 = 6$

(*i*) Number of relations from B to $G = 2^6$

(*ii*) Number of functions from B to $G = 2^{n(B \times G)}$

$= (n(G))^{n(B)} = 2^3 = 8$

(*iii*) (*a*) $R : B \longrightarrow B,\ R = \{(x, y) \mid x, y$ students of same sex$\}$

Reflexive:

$\because \quad (x, x) \in R \ \forall \ x \in B$

$\Rightarrow \quad R$ is reflexive.

Symmetric:

Let $(x, y) \in R$

$\Rightarrow \quad x$ and y are of same sex.

$\Rightarrow \quad y$ and x are of same sex.

$\Rightarrow \quad (y, x) \in R$

Transitive:

Let (x, y) and $(y, z) \in R$

$\Rightarrow \quad x$ and y are of same sex.

and y and z are of same sex.

$\Rightarrow \quad x$ and z are of same sex.

$\Rightarrow \quad (x, z) \in R \quad \Rightarrow \quad R$ is transitive.

Hence R is reflexive, symmetric and transitive.

$\therefore$ R is an equivalence relation in B.

OR

(*iii*) (*b*) $\quad f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

$\because \quad f(b_1) = g_1$ and $f(b_3) = g_1$

$\Rightarrow \quad f(b_1) = f(b_3)$ but $b_1 \neq b_3$

As b_1 and b_3 represents two different boys.

$\Rightarrow \quad f$ is not one-one.

$\Rightarrow \quad f$ is not a bijective map.

5. Read the following passage and answer the following questions.

Dhanush wants take a test of his son Amit is a student of class XII. Dhanush said to Amit, "Observe the two functions $f(x)$ and $g(x)$ carefully" $f : \mathbb{R} \to \mathbb{R}, g : \mathbb{R} \to \mathbb{R}$ such that

$$f(x) = x, \quad g(x) = x^2$$

The Dhanush asked some questions related to $f(x)$ and $g(x)$ and Amit answered correctly. Write the correct response given by Amit of the following questions.

(i) Check whether $f(x)$ is bijective or not.

(ii) Check whether $g(x)$ is bijective or not.

Sol. (i) $f : \mathbb{R} \to \mathbb{R}$ such that $f(x) = x$

One-one:

Let $x_1, x_2 \in \mathbb{R}$ (domain) such that

$f(x_1) = f(x_2) \quad \Rightarrow \quad x_1 = x_2$

$\Rightarrow$ f is one-one.

Onto:

Let $y \in \mathbb{R}$ (Co-domain) such that

$f(x) = y \quad \Rightarrow \quad x = y$

Now $f(x) = f(y) = y$

So for $y \in \mathbb{R}$ (Co-domain) $\exists\, x = y \in \mathbb{R}$ (domain)

such that $f(x) = y$

$\Rightarrow$ f is onto.

As f is one-one and onto. $\Rightarrow$ f is bijective.

(ii) We have $g : \mathbb{R} \to \mathbb{R}$ such that

$g(x) = x^2$

One-one:

$\because$ $1, -1 \in \mathbb{R}$ (domain) such that

$g(1) = 1, g(-1) = 1.$

i.e., $g(1) = g(-1)$ but $1 \neq -1$

$\Rightarrow$ g is not one-one.

Onto:

$\because$ $g(x) = x^2 \geq 0 \ \forall x \in \mathbb{R}$

$R(g) = [0, \infty) \neq \mathbb{R}$ (Co-domain)

$\Rightarrow$ g is not onto.

i.e., g is neither one-one nor onto.

CONCEPTUAL QUESTIONS

1. Let $A = \{1, 2, 3, 4\}$. Let R be the equivalence relation on $A \times A$ defined by $(a, b)\ R\ (c, d)$ iff $a + d = b + c$. Find the equivalence class $[(1, 3)]$. *[CBSE Sample Paper 2018]*

Sol. $[(1, 3)] = \{(x, y) \in A \times A : x + 3 = y + 1\} = \{(x, y) \in A \times A : y - x = 2\}$

$= \{(1, 3), (2, 4)\}$

2. If $R = \{(x, y) : x + 2y = 8\}$ is a relation on N, write the range of R. *[CBSE (AI) 2014]*

Sol. Given: $R = \{(x, y) : x + 2y = 8\}$

$\because$ $x + 2y = 8$

$\Rightarrow$ $y = \dfrac{8 - x}{2}$ $\Rightarrow$ when $x = 6, y = 1; x = 4, y = 2; x = 2, y = 3.$

$\therefore$ Range $= \{1, 2, 3\}$

3. **State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.** **[CBSE Delhi 2011]**

Sol. R is not transitive as $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.

[**Note:** A relation R in a set A is said to be transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R\ \forall\ a, b, c \in R$]

4. **Let $R = \{(a, a^3) : a$ is a prime number less than 5$\}$ be a relation. Find the range of R.** **[CBSE (F) 2014]**

Sol. Here $R = \{(a, a^3) : a$ is a prime number less than 5$\}$

$\Rightarrow \quad R = \{(2, 8), (3, 27)\}$

Hence range of $R = \{8, 27\}$.

5. **A relation R in the set of real numbers $\mathbb{R}$ defined as $R = \{(a, b) : \sqrt{a} = b\}$ is a function or not. Justify.** **[CBSE Sample Paper 2021]**

Sol. No. $R = \{(a, b) : \sqrt{a} = b\}$ not the function because $\sqrt{a}$ does not exist for all $a \in (-a, 0)$.

= Image of all elements of domain does not exist.

6. **An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$?** **[CBSE Sample Paper 2021]**

Sol. $A_1 \cup A_2 \cup A_3 = A$ and $A_1 \cap A_2 \cap A_3 = \phi$

7. **A relation R in $S = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which element(s) of relation R be removed to make R an equivalence relation?**

Sol. $(2, 1)$ because if $(1, 2) \in R$ then for being symmetric $(2, 1)$ should belong to R.

8. **How many reflexive relations are possible in a set A whose $n(A) = 3$.**

Sol. Number of reflexive relation $2^{n^2 - n} = 2^{3^2 - 3} = 2^6$ reflexive relations

9. **If X and Y are two sets having 2 and 3 elements respectively, then find the number of functions from X to Y.**

Sol. Number of functions from X to $Y = 3^2 = 9$.

Very Short Answer Questions

1. **Check if the relation R in the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : a < b\}$ is (*i*) symmetric, (*ii*) transitive** **[CBSE 2020 (65/5/1)]**

Sol. Given relation R the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : a < b\}$.

(*i*) **Symmetric:** Let $a, b \in \mathbb{R}$ (set of real numbers) if $(a, b) \in R$

$\Rightarrow \quad a < b$ then $b \not< a \quad \Rightarrow \quad (b, a) \notin R$

$\therefore$ It is not symmetric. 1

(*ii*) **Transitive:** Let $a, b, c \in \mathbb{R}$ (set of real number)

If $(a, b) \in R \quad \Rightarrow \quad a < b \quad ...(i)$

and $(b, c) \in R \quad \Rightarrow \quad b < c \quad ...(ii)$

From (*i*) and (*ii*), we have

$a < c$

$\Rightarrow \quad (a, c) \in R$

$\therefore$ It is transitive. 1

[CBSE Marking Scheme 2020 (65/5/1)]

2. **Write the inverse relation corresponding to the relation R given by $R = \{(x, y): x \in N, x < 5, y = 3\}$. Also write the domain and range of inverse relation.**

Sol. Given, $R = \{(x, y) : x \in N,\ x < 5, y = 3\}$

$\Rightarrow \quad R = \{(1, 3), (2, 3), (3, 3), (4, 3)\}$

Hence, required inverse relation is

$$R^{-1} = \{(3, 1), (3, 2), (3, 3), (3, 4)\}$$

$\therefore$ Domain of $R^{-1} = \{3\}$ and

Range of $R^{-1} = \{1, 2, 3, 4\}$

3. **A function $f: A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.** *[CBSE 2023 (65/1/1)]*

Sol. Given a function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto.

if $A = \{1, 2, 3, 4\}$

$\therefore f(1) = 2 \times 1 = 2, f(2) = 2 \times 2 = 4, f(3) = 2 \times 3 = 6$ and $f(4) = 2 \times 4 = 8$

$\therefore \quad B = \{2, 4, 6, 8\}$

4. **Consider $f : \mathbb{R}_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible.**

[CBSE (AI) 2013; (F) 2011]

Sol. One-one: Let $x_1, x_2 \in \mathbb{R}_+$ (Domain)

$f(x_1) = f(x_2) \quad \Rightarrow \quad {x_1}^2 + 4 = {x_2}^2 + 4$

$\Rightarrow \quad {x_1}^2 = {x_2}^2$

$\Rightarrow \quad x_1 = x_2 \quad$ [$\because x_1, x_2$ are +ve real number]

Hence, f is one-one function.

Onto: Let $y \in [4, \infty)$ such that

$y = f(x) \ \forall\, x \in \mathbb{R}_+ \quad$ [set of non-negative reals]

$\Rightarrow \quad y = x^2 + 4$

$\Rightarrow \quad x = \sqrt{y - 4} \quad$ [$\because$ x is + ve real number]

Obviously, $\forall\, y \in [4, \infty)$, x is real number $\in \mathbb{R}_+$ (domain)

i.e., all elements of codomain have pre image in domain.

$\Rightarrow$ f is onto.

Hence, f is invertible being one-one onto.

5. **Let f: $\mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \dfrac{4x}{3x + 4}$. Show that, in f: $\mathbb{R} - \left\{-\frac{4}{3}\right\} \to$ Range of f, f is one-one and onto.** [CBSE 2017(C)]

Sol. Let $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$

Now $f(x_1) = f(x_2) \quad \Rightarrow \quad \dfrac{4x_1}{3x_1 + 4} = \dfrac{4x_2}{3x_2 + 4}$

$\Rightarrow \quad 12\, x_1\, x_2 + 16\, x_1 = 12\, x_1\, x_2 + 16\, x_2 \quad \Rightarrow \quad 16\, x_1 = 16\, x_2 \quad \Rightarrow \quad x_1 = x_2$

Hence f is one-one function.

Since, co-domain f is range of f.

So, $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to$ Range of f is one-one and onto function.

6. **What is the range of the function $f(x) = \frac{|x-1|}{(x-1)}$?** [CBSE Delhi 2010]

Sol. Given $f(x) = \frac{|x-1|}{(x-1)}$

Obviously, $|x-1| = \begin{cases} (x-1) \text{ if } x-1>0 \text{ or } x>1 \\ -(x-1) \text{ if } x-1<0 \text{ or } x<1 \end{cases}$

Now, (*i*) $\forall\, x > 1$, $f(x) = \frac{(x-1)}{(x-1)} = 1$, (*ii*) $\forall\, x < 1$, $f(x) = \frac{-(x-1)}{(x-1)} = -1$,

$\therefore$ Range of $f(x) = \{-1, 1\}$.

7. **Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = \frac{1}{2-\cos x}$, $\forall\, x \in \mathbb{R}$. Then, find the range of f.** [NCERT Exemplar]

Sol. Given function, $f(x) = \frac{1}{2-\cos x}, \forall\, x \in \mathbb{R}$

$$y = \frac{1}{2-\cos x}$$

$\Rightarrow$ $2y - y\cos x = 1$ $\Rightarrow$ $y\cos x = 2y - 1$

$\Rightarrow$ $\cos x = \frac{2y-1}{y} = 2 - \frac{1}{y}$ $\Rightarrow$ $\cos x = 2 - \frac{1}{y}$

$\Rightarrow$ $-1 \le \cos x \le 1$ $\Rightarrow$ $-1 < 2 - \frac{1}{y} \le 1$

$\Rightarrow$ $-3 \le -\frac{1}{y} \le -1$ $\Rightarrow$ $1 \le \frac{1}{y} \le 3$

$\Rightarrow$ $\frac{1}{3} \le y \le 1$

So, range of y is $\left[\frac{1}{3}, 1\right]$.

8. **Prove that the function f is surjective, where $f: N \to N$ such that**

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

Is the function injective? Justify your answer. [CBSE Sample Paper 2023]

Sol. Let $y \in N$(codomain). Then $\exists\ 2y \in N$(domain) such that

$f(2y) = \frac{2y}{2} = y$. Hence, f is surjective. **1**

$1, 2 \in N$(domain) such that $f(1) = 1 = f(2)$

Hence, f is not injective. **1**

[CBSE Marking Scheme Sample Paper 2023]

Short Answer Questions

1. **Check whether the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.** [CBSE 2019 (65/2/1)]

Sol. $R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

For $1 \in A, (1, 1) \notin R \Rightarrow R$ is not reflexive **1**

For $1, 2 \in A, (1, 2) \in R$ but $(2, 1) \notin R \Rightarrow R$ is not symmetric **1½**

For $1, 2, 3 \in A, (1, 2), (2, 3) \in R$ but $(1, 3) \notin R \Rightarrow R$ is not transitive **1½**

[*CBSE Marking Scheme 2019 (65/2/1)*]

Detailed Solution:

Given relation R defined on the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$

Now,

Reflexivity: Let $a \in A$

We have, $a \neq a + 1 \quad \Rightarrow \quad (a, a) \notin R$

$\therefore$ It is not reflexive.

Symmetric: Let $a = 1$ and $b = 2$ *i.e.* $a, b \in A$

$\therefore \quad b = a + 1 \quad \Rightarrow \quad 2 = 1 + 1 \quad \Rightarrow \quad (a, b) \in R$

but $a \neq b + 1$ as $1 \neq 2 + 1 \quad \Rightarrow \quad (b, a) \notin R$

$\therefore$ It is not symmetric.

Transitive: Let $a, b, c \in A$

Now, if $(a, b) \in R \quad \Rightarrow \quad b = a + 1$ ...(*i*)

and $(b, c) \in R \quad \Rightarrow \quad c = b + 1$ ...(*ii*)

From (*i*) and (*ii*), we have

$$c = (a + 1) + 1 = a + 2$$

$$\Rightarrow \quad c = a + 2 \quad \Rightarrow \quad (a, c) \notin R$$

$\therefore$ Is is not transitive.

Hence, relation R is neither reflexive nor symmetric nor transitive.

2. Show that the relation R on the set $\mathbb{Z}$ of all integers, given by $R = \{(a, b) : 2$ divides $(a - b)\}$ is an equivalence relation. [*CBSE 2019 (65/3/1)*]

Sol. Given relation $R = \{(a, b) : 2$ divides $(a - b)\}$ on the set $\mathbb{Z}$ of all integers

Reflexive: Let $a \in \mathbb{Z}$

Since $(a - a) = 0$, which is divisible by 2 *i.e.*, $(a, a) \in R$

$\therefore \quad R$ is reflexive.

Symmetric: Let $a, b \in \mathbb{Z}$

such that $(a, b) \in R \quad \Rightarrow \quad (a - b)$ is divisible by 2.

$\Rightarrow \quad -(a - b)$ is also divisible by 2

$\Rightarrow \quad (b - a)$ is divisible by 2 $\quad \Rightarrow \quad (b, a) \in R$

i.e., $(a, b) \in R \quad \Rightarrow \quad (b, a) \in R$

$\therefore \quad R$ is symmetric.

Transitive: Let $a, b, c \in \mathbb{Z}$

such that $(a, b) \in R \quad \Rightarrow \quad (a - b)$ is divisible by 2.

Let $a - b = 2k_1$ where k_1 is an integer ...(*i*)

and $(b, c) \in R \quad \Rightarrow \quad (b - c)$ is divisible by 2 $\quad \Rightarrow \quad b - c = 2\,k_2$ where k_2 is an integer ...(*ii*)

Adding (*i*) and (*ii*), we have

$$(a - b) + (b - c) = 2\,(k_1 + k_2) \quad \Rightarrow \quad a - c = 2\,(k_1 + k_2) \quad \Rightarrow \quad (a - c) \text{ is divisible by } 2.$$

$\Rightarrow \quad (a, c) \in R$

$\therefore$ R is transitive.

Thus, R is reflexive, symmetric and transitive. Hence, given relation R is an equivalence relation.

3. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 3$\}$ is an equivalence relation. *[CBSE 2019 (65/4/1)]*

Sol. On the set $A = \{x \in Z : 0 \le x \le 12\}$ and relation S is given by

$$S = \{(a, b) : a, b \in Z,\ |a - b| \text{ is divisible by } 3\}$$

Reflexivity:

Let $a \in A$ Then

$(a, a) \Rightarrow |a - a| = 0$ which is divisible by 3.

$\Rightarrow (a, a) \in S$

$\therefore$ It is reflexive relation.

Symmetric:

Let $a, b \in A$ Then

$(a, b) \in S \Rightarrow |a - b|$ is divisible by 3. $\Rightarrow |b - a|$ is also divisible by 3.

$\Rightarrow (b, a) \in S$

$\therefore$ It is symmetric relation.

Transitive:

Let $a, b, c \in A$ Then

$(a, b) \in S \Rightarrow |a - b|$ is divisible by 3. $\Rightarrow a - b = \pm 3k_1$, where k_1 is an integer ...(*i*)

$(b, c) \in S \Rightarrow |b - c|$ is divisible by 3. $\Rightarrow b - c = \pm 3k_2$, where k_2 is an integer ...(*ii*)

$\therefore$ $a - c = a - b + b - c = \pm 3k_1 \pm 3k_2 = \pm 3(k_1 + k_2)$

$\Rightarrow$ $|a - c|$ is also divisible by 3.

$\Rightarrow$ $(a, c) \in S$

$\therefore$ It is transitive relation.

Hence, the relation S is an equivalence relation.

4. Show that the relation R on $\mathbb{R}$ defined as $R = \{a, b) : a \le b\}$, is reflexive and transitive but not symmetric. *[CBSE 2019 (65/1/1)]*

Sol.

$R = \{(a, b) : a \le b\}$

REFLEXIVE :

Every element $a \in \mathbb{R}$ is equal to itself

$\Rightarrow a = a$

$\Rightarrow a \le a$ is true

$\therefore$ $(a, a) \in R$ for all $a \in \mathbb{R}$ where $\mathbb{R}$ = set of Real nos

The relation is REFLEXIVE R → Relation

TRANSITIVE :

For all $(a, b) \in R$ and $(b, c) \in R$,

$a \le b$ and $b \le c$ where, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le c$

$\Rightarrow (a, c) \in R$

$\therefore$ The ~~set is tr~~ relation is TRANSITIVE

$\because$ for $(a, b), (b, c) \in R$, ~~$(a, c) \in R$~~ $(a, c) \in R$

$\therefore$ R is transitive.

Thus, R is reflexive, symmetric and transitive. Hence, given relation R is an equivalence relation.

3. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : a, b \in Z, |a - b|$ is divisible by 3$\}$ is an equivalence relation. *[CBSE 2019 (65/4/1)]*

Sol. On the set $A = \{x \in Z : 0 \le x \le 12\}$ and relation S is given by

$$S = \{(a, b) : a, b \in Z,\ |a - b| \text{ is divisible by } 3\}$$

Reflexivity:

Let $a \in A$ Then

$(a, a) \quad \Rightarrow |a - a| = 0$ which is divisible by 3.

$\Rightarrow (a, a) \in S$

$\therefore$ It is reflexive relation.

Symmetric:

Let $a, b \in A$ Then

$(a, b) \in S \Rightarrow |a - b|$ is divisible by 3. $\quad \Rightarrow |b - a|$ is also divisible by 3.

$\Rightarrow (b, a) \in S$

$\therefore$ It is symmetric relation.

Transitive:

Let $a, b, c \in A$ Then

$(a, b) \in S \Rightarrow |a - b|$ is divisible by 3. $\quad \Rightarrow a - b = \pm 3k_1$, where k_1 is an integer ...(*i*)

$(b, c) \in S \Rightarrow |b - c|$ is divisible by 3. $\quad \Rightarrow b - c = \pm 3k_2$, where k_2 is an integer ...(*ii*)

$\therefore$ $a - c = a - b + b - c = \pm 3k_1 \pm 3k_2 = \pm 3(k_1 + k_2)$

$\Rightarrow$ $|a - c|$ is also divisible by 3.

$\Rightarrow$ $(a, c) \in S$

$\therefore$ It is transitive relation.

Hence, the relation S is an equivalence relation.

4. Show that the relation R on $\mathbb{R}$ defined as $R = \{a, b) : a \le b\}$, is reflexive and transitive but not symmetric. *[CBSE 2019 (65/1/1)]*

Sol.

$R = \{(a, b) : a \le b\}$

REFLEXIVE :

Every element $a \in \mathbb{R}$ is equal to itself

$\Rightarrow a = a$

$\Rightarrow a \le a$ is true

$\therefore$ $(a, a) \in R$ for all $a \in \mathbb{R}$ where $\mathbb{R}$ = set of Real nos

The relation is REFLEXIVE $\quad$ R → Relation

TRANSITVE :

For all $(a, b) \in R$ and $(b, c) \in R$,

$a \le b$ and $b \le c$ $\quad$ where, $a, b, c \in \mathbb{R}$

$\Rightarrow a \le c$

$\Rightarrow (a, c) \in R$

$\therefore$ The ~~set is tr~~ relation is TRANSITVE

$\because$ for $(a, b), (b, c) \in R$, ~~(a, c) ∈ R~~ $(a, c) \in R$

SYMMETRIC : For relation to be symmetric,

for all $(a, b) \in R$, (b, a) should also exist in R

$\cancel{a \leq b \not\Rightarrow}$ $a \leq b$

$b \nleq a$ $\rightarrow$ This relation is true only $a = b = 1$

For eg: $\frac{1}{2} \leq 1 \Rightarrow \left(\frac{1}{2}, 1\right) \in R$

but $1 \nleq \frac{1}{2}$ $\therefore \left(1, \frac{1}{2}\right) \notin R$

$\therefore$ Relation is NOT SYMMETRIC

[Topper's Answer 2019]

5. Check whether the relation R in the set N of natural numbers given by

$$R = \{(a, b) : a \text{ is divisor of } b\}$$

is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

[CBSE 2020 (65/2/1)]

Sol. Given relation R on the set $\mathbb{N}$ of natural number given by

$$R = \{(a, b) : a \text{ is divisor of } b\}$$

Reflexive: Let $a \in N$

We have a is divisible by a itself

$\therefore \quad a\,R\,a$ i.e., $(a, a) \in R$

It is reflexive.

Symmetric: Let $a, b \in \mathbb{N}$

$\therefore$ $a\,R\,b \Rightarrow a$ is divisor of b.

$\Rightarrow b = ka$, where k is any positive integer.

$$\Rightarrow \frac{1}{k} = \frac{a}{b} \Rightarrow \frac{a}{b} = \frac{1}{k} \neq \text{any integer}$$

$\Rightarrow b$ is not divisor of a. $\Rightarrow \quad (b, a) \not\subset R$

$\therefore$ It is not symmetric.

Transitive: Let $a, b, c \notin R$

$a\,R\,b \Rightarrow a$ is divisor of b. $\Rightarrow \quad b = k_1 a \quad \ldots(i)$

where k_1 is any positive integer

and, $b\,R\,c \Rightarrow b$ is divisor of c.

$\Rightarrow c = k_2 b \quad \ldots(ii)$

where k_2 is positive integer

From (i) and (ii), we have

$$c = k_2 \times k_1 a \Rightarrow c = k_1 k_2 a$$

$$\Rightarrow \quad \frac{c}{a} = k_1 k_2 = \text{positive integer} \Rightarrow a \text{ is divisor of } c \quad \Rightarrow \quad (a, c) \in R$$

$\Rightarrow \quad a\,R\,c$

$\therefore$ It is transitive.

Hence, R is not an equivalence relation.

6. Let $A = \{1, 2, 3, \ldots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b)\,R\,(c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class $[(2, 5)]$.

[NCERT Exemplar]

Sol. Given that, $A = \{1, 2, 3, ..., 9\}$ and $(a, b) R(c, d)$ if $a + d = b + c$ for $(a, b) \in A \times A$ and $(c, d) \in A \times A$.

Reflexive: Since $(a, b) R(a, b)$ as

$$a + b = b + a \; \forall \; a, b \in A$$

So, R is reflexive.

Symmetric: Let $(a, b) R(c, d)$ then

$$a + d = b + c$$

$\Rightarrow \quad c + b = d + a \Rightarrow (c, d) R(a, b)$

So, R is symmetric.

Transitive: Let $(a, b) R(c, d)$ and $(c, d) R(e, f)$ then

$$a + d = b + c \text{ and } c + f = d + e$$

$\Rightarrow \quad a + d = b + c$ and $d + e = c + f$

$\therefore \quad (a + d) - (d + e) = (b + c) - (c + f)$

$\Rightarrow \quad (a - e) = b - f \quad \Rightarrow \quad a + f = b + e$

$\Rightarrow \quad (a, b) R(e, f)$

So, R is transitive.

Hence, R is an equivalence relation.

Now, equivalence class containing $[(2, 5)]$

$= \{(x, y) \mid 2 + y = 5 + x\}$

$= \{(x, y) \mid y - x = 3\}$

$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$.

7. Show that the modulus function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

Sol. $f(x) = |x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$

One-one: Let $x_1 = 1, x_2 = -1$ be two elements belongs to $\mathbb{R}$

$$f(x_1) = f(1) = |1| \text{ and } f(x_2) = f(-1) = -(-1) = 1$$

$\Rightarrow \quad f(x_1) = f(x_2)$ for $x_1 \neq x_2$

$\Rightarrow \quad f(x)$ is not one-one.

Onto: Let $f(x) = -1 \Rightarrow |x| = -1 \in \mathbb{R}$, which is not possible.

$\Rightarrow \quad f(x)$ is not onto.

Hence, f is neither one-one nor onto function.

8. Let $A = R - \{3\}, B = R - \{1\}$. If $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}, \forall \; x \in A$. Then, show that f is bijective. *[NCERT Exemplar]*

Sol. Given that, $\quad A = R - \{3\}, B = R - \{1\}$.

$f : A \to B$ is defined by $f(x) = \dfrac{x-2}{x-3}, \forall \; x \in A$

For injectivity:

Let $\quad f(x_1) = f(x_2) \Rightarrow \dfrac{x_1 - 2}{x_1 - 3} = \dfrac{x_2 - 2}{x_2 - 3}$

$\Rightarrow \quad (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow \quad x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 3x_2 - 2x_1 + 6$

$\Rightarrow \quad -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow \quad -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, $f(x)$ is an injective function.

For surjectivity:

Let $y = \frac{x-2}{x-3}$ $\Rightarrow$ $x - 2 = xy - 3y$

$\Rightarrow$ $x(1-y) = 2 - 3y$ $\Rightarrow$ $x = \frac{2-3y}{1-y}$

$\Rightarrow$ $x = \frac{3y-2}{y-1} \in A, \forall\, y \in B$ [codomain]

So, $f(x)$ is surjective function.

Hence, $f(x)$ is a bijective function.

9. **Let $A = R - \{2\}$ and $B = R - \{1\}$. If $f: A \to B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto.** [CBSE 2019 (65/4/1)]

Sol. Let $x_1, x_2 \in R - \{2\}$ such that $f(x_1) = f(x_2)$

$\Rightarrow$ $\frac{x_1 - 1}{x_1 - 2} = \frac{x_2 - 1}{x_2 - 2}$ ½

$\Rightarrow$ $x_1x_2 - 2x_1 - x_2 + 2 = x_1x_2 - 2x_2 - x_1 + 2$ 1

$\Rightarrow$ $x_1 = x_2$

So f is one-one. ½

For range let $f(x) = y$

$\frac{x-1}{x-2} = y$ ½

$x = \frac{2y-1}{y-1}$ 1

Range of $f = R - \{1\}$ = co-domain (B)

$\Rightarrow$ f is onto. ½

[CBSE Marking Scheme 2019 (65/4/1)]

Long Answer Questions

1. **Let N denotes the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)\, R\, (c, d)$ if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.**

[CBSE Delhi 2015] [CBSE 2023 (65/4/1)]

Sol. Here R is a relation defined as

$R = \{(a, b), (c, d)] : ad(b + c) = bc(a + d)\}$

Reflexivity: By commutative law under addition and multiplication

$b + a = a + b$ $\forall\, a, b \in \mathbb{N}$

$ab = ba$ $\forall\, a, b \in \mathbb{N}$

$\therefore$ $ab(b + a) = ba(a + b)$ $\forall\, a, b \in \mathbb{N}$

$\Rightarrow$ $(a, b)\, R\, (a, b)$

Hence, R is reflexive.

Symmetry: Let $(a, b)\, R\, (c, d)$

$(a, b)\, R\, (c, d) \Rightarrow ad(b + c) = bc(a + d)$

$\Rightarrow bc(a + d) = ad(b + c)$

$\Rightarrow cb(d + a) = da(c + b)$ [By commutative law under addition and multiplication]

$\Rightarrow (c, d)\ R\ (a, b)$

Hence, R is symmetric.

Transitivity: Let $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

Now, $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$

$\Rightarrow \quad ad(b + c) = bc\ (a + d)$ and $cf(d + e) = de(c + f)$

$\Rightarrow \quad \frac{b+c}{bc} = \frac{a+d}{ad}$ and $\frac{d+e}{de} = \frac{c+f}{cf}$

$\Rightarrow \quad \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a}$ and $\frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$

Adding both, we get

$\Rightarrow \quad \frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$

$\Rightarrow \quad \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \quad \Rightarrow \quad \frac{e+b}{be} = \frac{f+a}{af}$

$\Rightarrow \quad af(b + e) = be(a + f) \quad \Rightarrow \quad (a,b)R(e,f) \qquad [c, d \neq 0]$

Hence, R is transitive.

In this way, R is reflexive, symmetric and transitive.

Therefore, R is an equivalence relation.

2. Determine whether the relation R defined on the set $\mathbb{R}$ of all real numbers as $R = \{(a, b) : a, b \in \mathbb{R}$ and $a - b + \sqrt{3} \in S$, where S is the set of all irrational numbers$\}$, is reflexive, symmetric and transitive. **[*CBSE Ajmer 2015*]**

Sol. Here, relation R defined on the set R is given as

$R = \{(a, b) : a, b \in R$ and $a - b + \sqrt{3} \in S\}$

Reflexivity: Let $a \in R$ (set of real numbers)

Now, $(a, a) \in R$ as $a - a + \sqrt{3} = \sqrt{3} \in S$

i.e., R is reflexive.

Symmetry: Taking $a = \sqrt{3}$ and $b = 1$, we have

$(a, b) \in R$ as $a - b + \sqrt{3} = \sqrt{3} - 1 + \sqrt{3} = 2\sqrt{3} - 1 \in S$

But $b - a + \sqrt{3} = 1 - \sqrt{3} + \sqrt{3} = 1 \notin S \quad \Rightarrow \quad (b, a) \notin R$

As $(a, b) \in R$ but $(b, a) \notin R$.

$\therefore$ R is not symmetric.

Transitivity: Taking $a = 1, b = \sqrt{2}$ and $c = \sqrt{3}$

$(a, b) \in R$ as $a - b + \sqrt{3} = 1 - \sqrt{2} + \sqrt{3} \in S \quad \Rightarrow \quad (a, b) \in R$

$\therefore \quad b - c + \sqrt{3} = \sqrt{2} - \sqrt{3} + \sqrt{3} = \sqrt{2} \in S \quad \Rightarrow \quad (b, c) \in R$

But $\quad a - c + \sqrt{3} = 1 - \sqrt{3} + \sqrt{3} = 1 \notin S \quad \Rightarrow \quad (a, c) \notin R$

As (a, b) and (c, d) belongs to R but (a, c) does not belong to R.

$\therefore$ R is not transitive.

Hence, R is reflexive but neither symmetric nor transitive.

3. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by

(*i*) $R = \{(a, b) : |a - b|$ is a multiple of 4$\}$.

(*ii*) $R = \{(a, b) : a = b\}$ is an equivalence relation.

Find the set of all elements related to 1 in each case. **[*CBSE (AI) 2010*]**

Sol. $A = \{x \in Z : 0 \leq x \leq 12\}$

(*i*) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

Reflexive: Let $x \in A \Rightarrow |x - x| = 0$, which is a multiple of 4.

$\Rightarrow \quad (x, x) \in R \ \forall \ x \in A$

$\therefore \quad R$ is reflexive.

Symmetric: Let $x, y \in A$ and $(x, y) \in R$

$\Rightarrow \quad |x - y|$ is a multiple of 4.

or $\quad x - y = \pm 4p \quad \{p \text{ is any integer}\}$

$\Rightarrow \quad y - x = \mp 4p$

$\Rightarrow \quad |y - x|$ is a multiple of 4. $\quad \Rightarrow \quad (y, x) \in R$

$\Rightarrow \quad R$ is symmetric.

Transitive: Let $x, y, z \in A$, $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow \quad |x - y|$ is multiple of 4 and $|y - z|$ is multiple of 4.

$\Rightarrow \quad x - y$ is multiple of 4 and $y - z$ is multiple of 4.

$\Rightarrow \quad (x - y) + (y - z)$ is multiple of 4 $\Rightarrow (x - z)$ is multiple of 4.

$\Rightarrow \quad |x - z|$ is multiple of 4.

$\Rightarrow \quad (x, z) \in R \quad \Rightarrow \quad R$ is transitive.

So, R is an equivalence relation.

Let B be the set of elements related to 1.

$\therefore \quad B = \{a \in A : |a - 1| \text{ is multiple of } 4\}$

$\Rightarrow \quad B = \{1, 5, 9\} \quad [\text{as } |1 - 1| = 0, |5 - 1| = 4, |9 - 1| = 8]$

(*ii*) $R = \{(a, b) : a = b\}$

Reflexive: Let $x \in A$

as $\quad x = x \quad \Rightarrow \quad (x, x) \in R \quad \Rightarrow \quad R$ is reflexive.

Symmetric: Let $x, y \in A$ and $(x, y) \in R$

$\Rightarrow \quad x = y \quad \Rightarrow \quad y = x \quad \Rightarrow \quad (y, x) \quad \Rightarrow \quad R$

$\therefore$ R is symmetric.

Transitive: Let $x, y, z \in A$

and let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow \quad x = y$ and $y = z \quad \Rightarrow \quad x = z \quad \Rightarrow \quad (x, z) \in R \quad \Rightarrow \quad R$ is transitive.

$\therefore$ R is an equivalence relation.

Let C be the set of elements related to 1.

$\therefore \quad C = \{a \in A; a = 1\} = \{1\}$.

4. Check whether the relation R in $\mathbb{R}$ defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive. **[*CBSE (Delhi) 2010*]**

Sol. $R = \{(a, b) : a \leq b^3 \ \forall \ a, b \in \mathbb{R}\}$

Reflexivity: Here $\frac{1}{3} \in \mathbb{R}$ (Real number)

and $\quad \frac{1}{3} > \frac{1}{27}$ or $\frac{1}{3} > \left(\frac{1}{3}\right)^3$ or $\frac{1}{3} \not\leq \left(\frac{1}{3}\right)^3$

So, $\quad \left(\frac{1}{3}, \frac{1}{3}\right) \notin R$

$\therefore$ R is not reflexive.

Symmetry: $1, 2 \in \mathbb{R}$ (Real number)

and $\quad 1 \leq 8$ or $1 \leq 2^3$

So, $(1, 2) \in R$ but $(2, 1) \notin R$ $[\because 2 \geq 1 \text{ or } 2 \geq 1^3]$

$\therefore$ R is not symmetric.

Transitivity: Here $10, 3, 2 \in \mathbb{R}$ (Real number)

and $10 \leq 27$ or $10 \leq 3^3$

so, $(10, 3) \in R$ and

$3 \leq 8$ or $3 \leq 2^3$

so, $(3, 2) \in R$

But $10 \geq 8$ or $10 \geq 2^3$ or $10 \nleq 2^3$

So, $(10, 2) \notin R$

So, here $(10, 3) \in R$ and $(3, 2) \in R$ but $(10, 2) \notin R$

$\therefore$ R is not transitive.

5. Given a non-empty set X, define the relation R in $P(X)$ as follows:
For $A, B \in P(X)$, $(A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive and not symmetric.

[*CBSE Sample Paper 2023*]

Sol. **Reflexive:**

Let $A \in P(X)$. Then $A \subset A$

$\Rightarrow$ $(A, A) \in R$

Hence, R is reflexive.

Transitive:

Let $A, B, C \in P(X)$ such that

$(A, B), (B, C) \in R$

$\Rightarrow$ $A \subset B, B \subset C$ $\Rightarrow A \subset C \Rightarrow (A, C) \in R$

Hence, R is transitive.

Symmetry:

$\phi, X \in P(X)$ such that $\phi \subset X$. Hence, $(\phi, X) \in R$. But, $X \not\subset \phi$,

which implies that $(X, \phi) \notin R$.

Thus, R is not symmetric.

6. Let N be the set of natural numbers and R be the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$ iff $ad = bc$ for all $a, b, c, d \in$ N. Show that R is an equivalence relation.

[*CBSE Sample Paper 2023*] [*CBSE 2020 (65/1/1)*]

Sol.

$R: N \times N$

$(a,b) R (c,d)$ iff $ad = bc$.

$a, b, c, d \in N$.

for reflexive

let ~~(a,b)∈~~ $a, b \in N$

$(a,b) R (a,b)$

as $ab = ba$

$\therefore$ for all $a, b \in N$ ~~(a,b)∈R~~ $(a,b) R (a,b)$

$\therefore$ The relation is reflexive.

For symmetric

let $a, b, c, d \in N$

such that $(a,b) R (c,d)$

$\Rightarrow ad = bc$

$\Rightarrow cb = da$

$\Rightarrow (c,d) R (a,b)$

$\therefore$ The relation is symmetric

For transitive

let $a, b, c, d, e, f \in N$

such that $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow ad = bc$ — (i) $\Rightarrow cf = de$

$\Rightarrow c = \frac{de}{f}$ — (ii)

from equations (i) and (ii)

$ad = \frac{bde}{f}$

$\Rightarrow af = be$

$\cancel{\Rightarrow af = eb}$

$\therefore (a,b) R (e,f)$

$\therefore$ For all $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow (a,b) R (e,f)$

$\forall a, b, c, d, e, f \in N$

$\therefore$ The relation is transitive

As the relation is reflexive, symmetric and transitive, it is an equivalence relation.

Hence Proved

[*Topper's Answer 2020*]

7. ***A relation R is defined on a set of real numbers $\mathbb{R}$ as***

$$R = \{(x, y): x.\ y \text{ is an irrational number}\}.$$

Check whether R is reflexive, symmetric and transitive or not. [*CBSE 2023 (65/5/1)*]

Sol. A relation R is defined on a set of real numbers R as $R = \{(x, y) : x.y$ is an irrational number$\}$.

Reflexive: Let $x = \sqrt{3} \in R$

$\therefore\ \sqrt{3} \times \sqrt{3} = 3 \neq$ an irrational number

$\Rightarrow (\sqrt{3}, \sqrt{3}) \notin R$

It is not reflexive.

Symmetric: Let $x, y \in R$

If $(x, y) \in R$ $\Rightarrow x.y$ is an irrational number

$\Rightarrow y.x$ is also an irrational number

$\Rightarrow (y, x) \in R$

$\therefore$ It is symmetric.

Transitive: Let $x, y, z \in R$ such that $x = 1, y = \sqrt{2}, z = 3$

if $(x, y) \in R$ $\Rightarrow x.y$ is an irrational number *i.e.*, $1 \times \sqrt{2} = \sqrt{2}$ is an irrational.

and, $(y, z) \in R$ $\Rightarrow y.z = \sqrt{2} \times 3 = 3\sqrt{2}$ is an irrational.

Now, (x, z) $\Rightarrow x \times z = 1 \times 3 = 3$

$\therefore (x, z) \notin R$

$\therefore$ It is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

8. Consider $f : R_+ \to [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible.

[CBSE Allahabad 2015]

Sol. To prove f is invertible, it is sufficient to prove f is one–one onto.

Here, $f(x) = 5x^2 + 6x - 9$

One-one: Let $x_1, x_2 \in R_+$, then

$$f(x_1) = f(x_2) \Rightarrow 5x_1^2 + 6x_1 - 9 = 5x_2^2 + 6x_2 - 9$$

$$\Rightarrow 5x_1^2 + 6x_1 - 5x_2^2 - 6x_2 = 0 \Rightarrow 5(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$$

$$\Rightarrow 5(x_1 - x_2)(x_1 + x_2) + 6(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(5x_1 + 5x_2 + 6) = 0$$

$$\Rightarrow x_1 - x_2 = 0 \qquad [\because 5x_1 + 5x_2 + 6 \neq 0]$$

$$\Rightarrow x_1 = x_2$$

i.e., f is one-one function.

Onto: $\because$ $f(x) = 5x^2 + 6x - 9 = 5\left\{x^2 + 2 \times x \times \frac{3}{5} - \frac{9}{5}\right\}$

$$= 5\left\{x^2 + 2 \times x \times \frac{3}{5} + \left(\frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 - \frac{9}{5}\right\}$$

$$= 5\left\{\left(x + \frac{3}{5}\right)^2 - \frac{9}{25} - \frac{9}{5}\right\}$$

$$= 5\left\{\left(x + \frac{3}{5}\right)^2 - \frac{54}{25}\right\}$$

$$\because \left(x + \frac{3}{5}\right)^2 \geq \frac{9}{25} \Rightarrow \left(x + \frac{3}{5}\right)^2 - \frac{54}{25} \geq \frac{9}{25} - \frac{54}{25} = -\frac{9}{5}$$

$$\Rightarrow 5\left\{\left(x + \frac{3}{5}\right)^2 - \frac{54}{25}\right\} \geq -9 \Rightarrow f(x) \geq -9$$

$\Rightarrow$ $R(f) = [-9, \infty) =$ co-domain.

As f is both one-one and onto.

$\Rightarrow f$ is bijective.

Hence, f is one-one onto function, *i.e.*, invertible.

9. Consider $f : R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible.

[CBSE (AI) 2013; (F) 2011]

Sol. One-one: Let $x_1, x_2 \in R_+$ (Domain)

$f(x_1) = f(x_2)$

$\Rightarrow \quad x_1^2 + 4 = x_2^2 + 4$

$\Rightarrow \quad x_1^2 = x_2^2$

$\Rightarrow \quad x_1 = x_2$ [$\therefore$ x_1, x_2 are +ve real number]

Hence, f is one-one function.

Onto: Let $y \in [4, \infty)$ such that

$y = f(x) \ \forall \ x \in R_+$ [set of non-negative reals]

$\Rightarrow \quad y = x^2 + 4$

$\Rightarrow \quad x = \sqrt{y-4}$ [$\therefore$ x is + ve real number]

Obviously, $\forall \ y \in [4, \infty)$, x is real number $\in R_+$ (domain)

i.e., all elements of codomain have pre image in domain.

$\Rightarrow$ f is onto.

Hence, f is invertible being one-one onto.

10. Show that the function $f : (-\infty, 0) \to (-1, 0)$ defined by $f(x) = \dfrac{x}{1+|x|}$, $x \in (-\infty, 0)$ is one-one and onto. **[*CBSE 2020 (65/3/1)*]**

Sol. Let $x_1, x_2 \in (-\infty, 0)$ such that $f(x_1) = f(x_2)$.

i.e., $\dfrac{x_1}{1+|x_1|} = \dfrac{x_2}{1+|x_2|}$

$\Rightarrow \quad \dfrac{x_1}{1-x_1} = \dfrac{x_2}{1-x_2}$ **1**

$\Rightarrow \quad x_1 - x_1x_2 = x_2 - x_1x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one. **1**

Let $y \in (-1, 0)$, such that $y = \dfrac{x}{1+|x|}$

$\Rightarrow \quad y = \dfrac{x}{1-x} \quad \Rightarrow \quad x = \dfrac{y}{1+y}$ **1**

For each $y \in (-1, 0)$, there exists $x \in (-\infty, 0)$,

such that $f(x) = f\left(\dfrac{y}{1+y}\right) = \dfrac{\dfrac{y}{1+y}}{1+\left|\dfrac{y}{1+y}\right|}$

$= \dfrac{\dfrac{y}{1+y}}{1-\dfrac{y}{1+y}} = y$ **1**

Hence f is onto.

[*CBSE Marking Scheme 2020 (65/3/1)*]

11. Prove that the greatest integer function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x. **[*CBSE 2017(C)*]**

Sol. $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = [x]$

Injectivity: Let $x_1 = 2.5$ and $x_2 = 2$ be two elements of $\mathbb{R}$.

$$f(x_1) = f(2.5) = [\,2.5] = 2$$

$$f(x_2) = f(2) = [2] = 2$$

$\therefore \quad f(x_1) = f(x_2)$ for $x_1 \neq x_2$

$\Rightarrow \quad f(x) = [x]$ is not one-one *i.e.*, not injective.

Surjectivity: Let $y = 2.5 \in \mathbb{R}$ be any element.

$\therefore\ f(x) = 2.5 \quad \Rightarrow\ [x] = 2.5$

Which is not possible as $[x]$ is always an integer.

$\Rightarrow f(x) = [x]$ is not onto *i.e.*, not surjective.

12. Let $f : \mathbb{W} \to \mathbb{W}$ be defined as

$$f(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$$

Show that f is invertible. Here, $\mathbb{W}$ is the set of all whole numbers. **[*CBSE (Panchkula) 2015*]**

Sol.

$f : W \to W$

$$f(n) = \begin{cases} n-1 & \text{if } n \text{ is odd} \\ n+1 & \text{if } n \text{ is even} \end{cases}$$

prove that it is one-one:

let us suppose that $n_1, n_2 \in W$

$n_1 \neq n_2$ but $f(n_1) = f(n_2)$

(i) 1st case: if n_1 & n_2 are odd

$f(n_1) = f(n_2)$

$n_1 - \cancel{1} = n_2 - \cancel{1}$

$n_1 = n_2$ which is contradiction.

(ii) 2nd case: n_1, n_2 are even

$f(n_1) = f(n_2)$

$n_1 + \cancel{1} = n_2 + \cancel{1}$

$n_1 = n_2$

which is contradiction.

(iii) 3rd case if n_1 is even & n_2 is odd

$f(n_1) = f(n_2)$

$n_1 + 1 = n_2 - 1$

$\underset{\text{even}}{\underset{\downarrow}{n_1 + 2}} = \underset{\text{odd}}{\underset{\downarrow}{n_2}}$

if we add 2 in an even no. then we get an even no but n_2 is odd which is again contradiction

So from these 3 point we see that

$f(n_1) = f(n_2)$ only if $n_1 = n_2$

$\Rightarrow$ f is one-one.

To prove that f is onto :

Let $y \in W$
and y is even
then $y+1$ is odd and it belongs in whole no.
$f(y+1) = y+1-1$
$= y$

So for every $y \in W$ which is even there exist a preimage $y+1$ which is odd and $(y+1) \in W$

Let $y \in W$
y is odd
then $y-1$ is even and $y-1 \in W$
$f(y-1) = y-1+1$
$= y$

So for every $y \in W$ which is odd there exist a pre image $y-1 \in W$ which is even.
So ~~for all $y \in W$ there exist pre image~~ in whole no.
So f is onto.

f is one-one and onto so it is invertible

[Topper's Answer 2015]

13. Consider $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R} - \left\{\frac{4}{3}\right\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective.

[CBSE (AI) 2017]

Sol.

$f: R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ $\quad f(x) = \frac{4x+3}{3x+4}$

Let $f(x_1) = f(x_2)$

$\frac{4x_1+3}{3x_1+4} = \frac{4x_2+3}{3x_2+4}$

$(4x_1+3)(3x_2+4) = (4x_2+3)(3x_1+4)$

$\cancel{12x_1x_2} + 9x_2 + 16x_1 + \cancel{12} = \cancel{12x_1x_2} + 9x_1 + 16x_2 + \cancel{12}$

$7x_1 = 7x_2$

$x_1 = x_2$

Hence $f(x)$ is one to one.

Let $y = \frac{4x+3}{3x+4}$

$y(3x+4) = 4x+3$

$3xy + 4y - 4x = 3$

$x(3y-4) = 3-4y$.

$x = \dfrac{3-4y}{(3y-4)}$ Hence for every $y \in R - \left\{\frac{4}{3}\right\}$ there exists x such that $x \in R - \left\{\frac{-4}{3}\right\}$.

$\dfrac{-4}{3} = \dfrac{3-4y}{3y-4}$

$-12y + 16 = 9 - 12y$

$16 = 9$

which is not possible ; Hence x cannot be $-\frac{4}{3}$.

Hence $f(x)$ is onto function.

$\therefore$ $f(x)$ is a bijective.

[*Topper's Answer 2017*]

14. Let $f : \mathbb{R} - \left\{-\dfrac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \dfrac{4x}{3x+4}$. Show that f is one-one function. Also, check whether f is an onto function or not. **[*CBSE 2023 (65/4/1)*]**

Sol. Given function $f : \mathbb{R} - \left\{-\dfrac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \dfrac{4x}{3x+4}$.

One-one: Let $x_1, x_2 \in \mathbb{R} - \left\{-\dfrac{4}{3}\right\}$ such that $x_1 \neq x_2$.

$\therefore \quad 4x_1 \neq 4x_2$

and $\quad 3x_1 \neq 3x_2 \quad \Rightarrow \quad 3x_1 + 4 \neq 3x_2 + 4$

$\therefore \quad \dfrac{4x_1}{3x_1+4} \neq \dfrac{4x_2}{3x_2+4} \quad \Rightarrow \quad f(x_1) \neq f(x_2)$

$\therefore \quad f(x)$ is one-one function.

Onto: Let $y = f(x) = \dfrac{4x}{3x+4} \quad \Rightarrow \quad 3xy + 4y = 4x$

$\Rightarrow \quad x(3y-4) = -4y \quad \Rightarrow \quad x = \dfrac{-4y}{3y-4} = \dfrac{4y}{4-3y}$

$\Rightarrow \quad x = \dfrac{4y}{4-3y}$

Clearly when $y = \dfrac{4}{3}$, therefore does not exist x *i.e.*, $\dfrac{4}{3}$ has no pre image.

$\therefore$ It is not onto.

15. A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(x) = \sqrt{7}$. **[*CBSE 2023 (65/2/1)*]**

Sol. Given a function $f : [-4, 4] \to [0, 4]$ defined by $f(x) = \sqrt{16 - x^2}$.

Let $y = \sqrt{16 - x^2} \quad \Rightarrow \quad y^2 = 16 - x^2 \quad \Rightarrow \quad x^2 = 16 - y^2$

$\Rightarrow \quad x = \sqrt{16 - y^2}$

Clearly for x to be real and $x \in [-4, 4]$

$16 - y^2 \geq 0 \quad \Rightarrow \quad y^2 - 16 \leq 0 \quad \Rightarrow (y-4)(y+4) \leq 0$

$\Rightarrow \quad -4 \leq y \leq 4 \quad \quad ...(i)$

But $y = \sqrt{16 - x^2} \geq 0 \quad \quad (\because \sqrt{x} \geq 0)$

$\Rightarrow \quad y \geq 0 \quad ...(ii)$

From (*i*) and (*ii*), we have

$$0 \leq y \leq 4$$

Thus, for every value of $y \in [0, 4]$ there exists some $x \in [-4, 4]$.

$\therefore$ Given function is onto.

When $x = 4 \Rightarrow y = \sqrt{16-16} = 0$

$x = -4 \Rightarrow y = \sqrt{16-16} = 0$

Here, different value of x there is some y.

So it is not one-one.

Hence, given function is onto but not one-one.

Now, $f(a) = \sqrt{7}$ (given)

$\Rightarrow \quad \sqrt{16-a^2} = \sqrt{7} \quad \Rightarrow \quad 16 - a^2 = 7$

$\Rightarrow \quad a^2 = 16 - 7 = 9 \quad \Rightarrow \quad a = \{-3, 3\}$

16. Prove that a function $f : [0, \infty) \to [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$ is both one-one and onto. [*CBSE 2023 (65/3/2)*]

Sol. Given function $f : [0, \infty) \to [-5, \infty)$ defined as $f(x) = 4x^2 + 4x - 5$

One-one: Let $x_1, x_2 \in [0, \infty)$ such that $x_1 \neq x_2$

$\therefore \quad 4x_1 \neq 4x_2$

$\Rightarrow \quad 4x_1 - 5 \neq 4x_2 - 5$

$\Rightarrow \quad 4x_1^2 + 4x_1 - 5 \neq 4x_1^2 + 4x_2 - 5$

$\Rightarrow \quad f(x_1) \neq f(x_2)$

$\therefore$ Function is one-one.

Onto:

For $x \in [0, \infty)$

$\because f(x) = 4x^2 + 4x - 5$

$\Rightarrow f(x) \geq -5$

$\Rightarrow R(f) = [-5, \infty)$

$\therefore$ Given function is onto.

Hence, function is both one-one and onto.

Questions for Practice

Objective Type Questions

1. **Choose and write the correct option in each of the following questions.**

(*i*) Let R be a relation on the set $\mathbb{N}$ of natural numbers defined by nRm if n divides m. Then R is

(*a*) reflexive and symmetric (*b*) transitive and symmetric
(*c*) equivalence (*d*) reflexive, transitive but not symmetric

(*ii*) Let $A = \{1, 2, 3\}$ and consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then R is

(*a*) reflexive but not symmetric (*b*) reflexive but not transitive
(*c*) symmetric and transitive (*d*) neither symmetric nor transitive

(*iii*) If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is

(*a*) reflexive (*b*) transitive (*c*) symmetric (*d*) none of these

(iv) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{1}{x} \forall x \in \mathbb{R}$. Then f is

(a) one-one (b) onto (c) bijective (d) f is not defined

(v) If $A = \{1, 2, 3\}$, $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by 'x is greater than y'. Then range of R is

(a) $\{1, 4, 6, 9\}$ (b) $\{4, 6, 9\}$ (c) $\{1\}$ (d) none of these

(vi) If $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + y^2 \le 4\}$ is a relation in the set $\mathbb{Z}$, then the domain of R is **[CBSE 2021-22 (65/2/4) (Term-1)]**

(a) $\{0, 1, 2\}$ (b) $\{-2, -1, 0, 1, 2\}$ (c) $\{0, -1, -2\}$ (d) $\{-1, 0, 1\}$

(vii) Let $X = \{x^2 \mid x \in \mathbb{N}\}$ and the function $f : \mathbb{N} \to X$ is defined by $f(x) = x^2, x \in \mathbb{N}$. Then this function is **[CBSE 2021-22 (65/2/4) (Term-1)]**

(a) injective only (b) not bijective (c) surjective only (d) bijective

(viii) A function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2 + x^2$ is **[CBSE 2021-22 (65/2/4) (Term-1)]**

(a) not one-one (b) one-one

(c) not onto (d) neither one-one nor onto

Conceptual Questions

2. If $A = \{3, 5, 7\}$ and $B = \{2, 4, 9\}$ and R is a relation from A to B given by "is less than", then write R as a set of ordered pairs.
3. Check whether the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is transitive.
4. For the set $A = \{1, 2, 3\}$, define a relation R in the set A as follows $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Write the ordered pair to be added to R to make it the smallest equivalence relation.
5. A relation R in $S = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which element(s) of relation R be removed to make R an equivalence relation?
6. Check whether the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined as $f(x) = x^4$ is one-one onto or not.
7. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} 3x, & \text{if } x > 3 \\ x^2, & \text{if } 1 < x \le 3 \\ x, & \text{if } x \le 1 \end{cases}$

   then find $f(-2) + f(0) + f(2) + f(5)$.

Very Short Answer Questions

8. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined as $f(x) = 3x$. Then show that f is one-one onto.
9. Let the relation R be defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 8\}$. Then write the set R.
10. Let $A = \{0, 1, 2, 3\}$ and define a relation R on A as follows:

    $R\ \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$

    Is R reflexive? symmetric? transitive?
11. For real numbers x and y, a relation R is defined as xRy if $x - y + \sqrt{2}$ is an irrational number. Write whether R is reflexive, symmetric or transitive.
12. Let the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = 4x - 1$, $\forall x \in R$. Then show that f is one – one.
13. Let the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = 2x + \sin x$. Then show that f is onto function.
14. Let $R = \{(a, a^3) \mid a$ is a prime number less than $5\}$ be a relation. Find the domain and range of R.

Short Answer Questions

15. Show that the relation R in the set $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)R(c, d)$ iff $a^2 + d^2 = b^2 + c^2\ \forall\ a, b, c, d \in N$, is an equivalence relation. **[CBSE Sample Paper 2016]**

16. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b): a, b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric nor transitive. **[CBSE Delhi 2010]**

17. Prove that the relation R in the set $A = \{1, 2, 3, ..., 12\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 3\}$, is an equivalence relation. Find all elements related to the element 1. **[CBSE (F) 2013]**

18. Prove that the relation R on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ given by $R = \{(a, b) : |a - b| \text{ is even }\}$, is an equivalence relation. **[CBSE 2019 (65/4/2)]**

19. Let Z be the set of all integers and R be relation on $\mathbb{Z}$ defined as $R = \{(a, b) : a, b \in Z \text{ and } (a - b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation. **[CBSE Delhi 2010]**

20. Show that the function f in $A = \mathbb{R} - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto.

21. Prove that the function $f : N \to N$, defined by $f(x) = x^2 + x + 1$ is one-one but not onto. **[CBSE 2019 (65/1/1)]**

Long Answer Questions

22. Show that the function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2+1},\ \forall\ x \in R$ is neither one-one nor onto. **[CBSE 2018]**

23. Show that the relation R defined by $(a, b)\ R\ (c, d) \Leftrightarrow a + d = b + c$ on the $A \times A$, where $A = \{1, 2, 3, ..., 10\}$ is an equivalence relation. Hence write the equivalence class of $[(3, 4)]$; $a, b, c, d \in A$. **[CBSE (East) 2016]**

24. Check whether the relation R in the set N of natural numbers given by

$$R = \{(a, b) : a \text{ is divisor of } b\}$$

is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

25. Check if the relation R in the set $\mathbb{R}$ of real numbers defined as $R = \{(a, b) : a < b\}$ is (*i*) symmetric (*ii*) transitive.

26. Let $A = \mathbb{R} - \{3\}$, $B = \mathbb{R} - \{1\}$. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}\ \forall\ x \in A$. Show that f is bijective.

27. Let R be a relation on the set of natural numbers N as follows:
$R = \{(x, y) | x \in N, y \in N, 2x + y = 41\}$. Find the domain and range of the relation R. Also verify whether R is reflexive, symmetric and transitive.

28. Show that the relation S in the set $A = \{x \in Z; 0 \le x \le 12\}$ gives by $S = \{(a, b) : a, b \in Z, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

Answers

1. (*i*) (*d*) (*ii*) (*a*) (*iii*) (*b*) (*iv*) (*d*) (*v*) (*c*) (*vi*) (*b*)
(*vii*) (*d*) (*viii*) (*d*)

2. $R = \{(3, 4), (3, 9), (5, 9), (7, 9)\}$ 3. No, it is not transitive. 4. (3, 1) 5. (1, 2)

6. f is neither one-one nor onto. 7. 17

9. $\{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)\}$

10. Reflexive, Symmetric but not transitive.
11. Reflexive but neither symmetric nor transitive.
14. Domain = {2, 3}, Range = {8, 27}
17. 1, 4, 7, 10
23. {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10)}
24. Reflexive and transitive but not symmetric, No
25. (*i*) Not symmetric (*ii*) Transitive
27. Domain of R = {1, 2, 3, ... 20}.
    Range of R = {1, 3, 5, 7, 9, ..., 39}, R is neither reflexive, nor symmetric nor transitive.
28. [1] = {9, 5, 1}