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All elements consist of very small invisible particles are called **atoms**. Atoms of same element are same and atoms of different elements are different.
Every atom is a sphere of radius of the order 10^{-10} m, in which entire mass is uniformly distributed and negative charged electrons revolve around the nucleus.

ATOMS

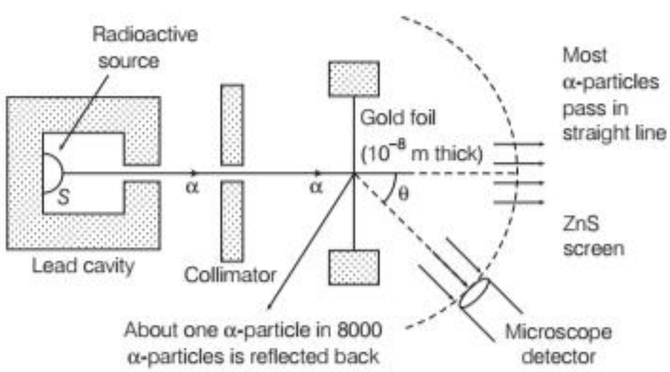
The first model of atom was proposed by JJ Thomson in 1898 called **plum pudding model** of the atom. Later, Rutherford worked on it and named this model as Rutherford's planetary model of atom in 1911. In 1913, Niels Bohr worked on the model named as Bohr model of H-atom.

CHAPTER CHECKLIST

- α-Particles Scattering Experiment by Rutherford
- Rutherford's Model of Atom
- Electron Orbits
- Bohr's Model of Hydrogen Atom
- Hydrogen Spectrum or Line Spectra of Hydrogen Atom

α-PARTICLES SCATTERING EXPERIMENT BY RUTHERFORD

This experiment was suggested by Rutherford in 1911 as given in the figure below



Experimental arrangement for Rutherford's theory

In this experiment, H Geiger and E Marsden took $^{214}_{83}Bi$ as a source for α-particles. A collimated beam of α-particles of energy 5.5 MeV was allowed to fall on 2.1×10^{-7} m thick gold foil. The α-particles were observed through a rotatable detector consisting of a zinc sulphide screen and microscope and it was found that α-particles got scattered. These scattered α-particles produced scintillations on the zinc sulphide screen. Now, these scintillations were counted at different angles from the direction of incident beam.

Observations

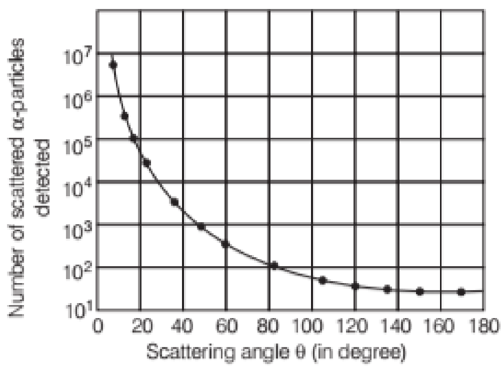
Rutherford made the following observations from his experiment that are given below

(i) Most of the α-particles passed through the gold foil without any appreciable deflection.

(ii) Only about 0.14% of the incident α-particles scattered by more than 1°.

(iii) About one α-particle in every 8000 α-particles deflected by more than 90°.

The total number of α-particles (N) scattered through an angle (θ) is as shown in the below figure



Experimental data points (shown by dots) on scattering of α-particles by a thin foil at different angles

(iv) The number of α-particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\sin^4 \theta/2$.

$$N(\theta) \propto \frac{1}{\sin^4 \theta/2}$$

(v) The force between α-particles and nucleus is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2e)(Ze)}{r^2}$$

where, r is the distance between the α-particles and the nucleus. This force is directed along the line joining the α-particle and the nucleus. The magnitude and direction of this force on α-particle continuously changes as it approaches the nucleus and recedes away from it.

Conclusions

On the basis of his experiment, Rutherford concluded that

(i) Atom has a lot of empty space and practically the entire mass of the atom is confined to an extremely small central core called **nucleus**, whose size is of the order from 10^{-15} m to 10^{-14} m.

(ii) Scattering of α-particles (positively charged) is due to the Coulomb's law for electrostatic force of repulsion between the positive charge of nucleus and α-particles.

(iii) Distance between electron and nucleus is from 10^4 to 10^5 times the size of the nucleus itself.

(iv) More is the distance of the velocity vector of an α-particle from the central line of the nucleus, lesser is the angle of scattering.

EXAMPLE |1| The number of α-particles scattered at an angle of 90° is 100 per minute. What will be the number of α-particles, when it is scattered at an angle of 60°?

Sol. Number of α-particles scattered at an angle of θ is given by

$$N \propto \frac{1}{\sin^4 \theta/2} \Rightarrow \frac{N_1}{N_2} = \left(\frac{\sin \frac{\theta_2}{2}}{\sin \frac{\theta_1}{2}}\right)^4$$

$$\Rightarrow \frac{100}{N_2} = \left(\frac{\sin 30°}{\sin 45°}\right)^4 \Rightarrow \frac{100}{N_2} = \frac{4}{16} \Rightarrow N_2 = 400$$

RUTHERFORD'S MODEL OF ATOM

The essential features of Rutherford's nuclear model of the atom or planetary model of the atom are as follows

(i) Every atom consists of a central core, called the **atomic nucleus**, in which the entire positive charge and almost entire mass of the atom is concentrated.

(ii) The size of nucleus is of the order of 10^{-15} m, which is very small as compared to the size of the atom which is of the order of 10^{-10} m.

(iii) The atomic nucleus is surrounded by certain number of electrons. As atom on the whole is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.

(iv) These electrons revolve around the nucleus in various circular orbits as the planets do around the sun.

The centripetal force required by electrons for revolution is provided by the electrostatic force of attraction between the electrons and nucleus.

Distance of Closest Approach

As the α-particle approaches the nucleus, the electrostatic force of repulsion due to nucleus increases and the kinetic energy of α-particle goes on converting into the electrostatic potential energy.

At a certain distance r_0 from the nucleus, whole of the KE of α-particle converts into electrostatic potential energy and α-particles cannot go further close to nucleus, this distance (r_0) is called **distance of closest approach**.

At distance of closest approach,

KE of α-particle = Electrostatic potential energy

$$K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze)(2e)}{r_0}$$

[∵ charge on α-particle is $+2e$ and charge on nucleus is Ze, where Z is atomic number]

$$\therefore \quad r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K} \text{ or } \boxed{r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{\left(\frac{1}{2}mv^2\right)}}$$

where, m = mass of α-particle and v = initial velocity of α-particle.

From the formula, it is clear that distance of closest approach of α-particle to the nucleus depends on the kinetic energy of α-particle.

EXAMPLE |2| In a head on collision between an α-particle and gold nucleus, the closest distance of approach is 4×10^{-14} m. Calculate the initial kinetic energy of α-particle.

Sol. Here, closest distance of approach, $r_0 = 4 \times 10^{-14}$ m,

atomic number, $Z = 79$, $KE_i = ?$

$$\therefore \ KE_i \text{ of } \alpha\text{-particle} = \frac{Ze(2e)}{4\pi\varepsilon_0 r_0} = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$$

$$= \frac{2 \times 79 \times (1.6 \times 10^{-19})^2 \times 9 \times 10^9}{4 \times 10^{-14}}$$

$$= 9.1 \times 10^{-13} \text{ J}$$

Angle of Scattering (θ)

Angle by which α-particle gets deviated from its original path around the nucleus is called **angle of scattering**.

Impact Parameter (b)

Perpendicular distance of the velocity vector of α-particle from the central line of the nucleus of the atom is called **impact parameter**. Mathematically, it is expressed as

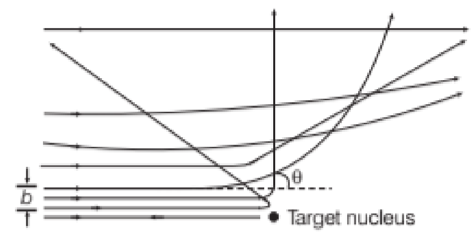
$$\boxed{b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{KE}}$$

where, b = impact parameter

θ = angle of scattering

KE = kinetic energy of α-particle $= \frac{1}{2}mv^2$.

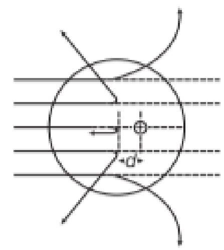
In case of head on collision, the impact parameter is minimum and the α-particle rebounds back ($\theta = \pi$). For a large impact parameter, the α-particle goes nearly undeviated and has a small deflection ($\theta = 0°$).



Trajectory of α-particles in the Coulombic field of a target nucleus. The impact parameter b and scattering angle θ are also depicted

Alpha-Particle Trajectory

The α-particles which pass through the atom at a large distance from the nucleus experience a small electrostatic force of repulsion due to the nucleus and hence, undergo a very small deflection. The α-particles which pass through the atom at a close distance from the nucleus suffer a large deflection. The α-particles which travel towards the nucleus directly, slow down and ultimately comes to rest and then after being deflected through 180° retrace their path.



Trajectory of α-particles close to an atom

ELECTRON ORBITS

The Rutherford nuclear model of the atom pictures the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits.

The electrostatic force of attraction F_e between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to keep them in their orbits. Thus, for a dynamically stable orbit in a H-atom,

$$F_c = F_e$$

$$\Rightarrow \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} \qquad [\because Z = 1]$$

Thus, the relation between the orbit radius and the electron velocity is

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

The kinetic energy (K) and electrostatic potential energy (U) of the electron in H-atom are

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \qquad \left[\because mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}\right]$$

and $$U = -\frac{e^2}{4\pi\varepsilon_0 r}$$

(the negative sign in U signifies that the electrostatic force is in the $-r$ direction or attractive in nature.)

Thus, the total mechanical energy E of the electron in a H-atom is

$$E = K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{E = -\frac{e^2}{8\pi\varepsilon_0 r}}$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If E is positive, then an electron will not follow a closed orbit around the nucleus and it would leave the atom.

EXAMPLE |3| It is found experimentally that 13.6 eV energy is required to separate a H-atom into a proton and an electron. Compute the orbital radius and velocity of the electron in a H-atom.

Sol. Total energy of the electron in H-atom,

$$\text{TE} = -13.6 \text{ eV} = -13.6 \times 1.6 \times 10^{-19} \text{ J}$$
$$= -2.2 \times 10^{-18} \text{ J}$$

Total energy is

$$\text{TE} = \frac{-e^2}{8\pi\varepsilon_0 r} \Rightarrow r = \frac{-e^2}{8\pi\varepsilon_0 \text{TE}}$$
$$= \frac{-9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times (-2.2 \times 10^{-18})}$$
$$= 5.3 \times 10^{-11} \text{ m}$$

$\therefore$ Velocity of the revolving electron, $v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$

$$= \frac{1.6 \times 10^{-19}}{\sqrt{4 \times 3.14 \times 8.85 \times 10^{-12} \times 9.1 \times 10^{-31} \times 5.3 \times 10^{-11}}}$$
$$= 2.2 \times 10^6 \text{ m/s}$$

Drawbacks of Rutherford's Model

Rutherford's model suffers two major drawbacks

Regarding Stability of Atom

Electrons revolving around the nucleus have centripetal acceleration. According to classical electromagnetic theory, the electrons must radiate energy in the form of electromagnetic wave.

Due to this continuous loss of energy of the electrons, the radii of their orbits should be continuously decreasing and ultimately the electrons should fall in the nucleus. Thus, atom cannot remain stable.

Regarding Explanation of Line Spectrum

Due to continuous decrease in radii of electron's orbit, the frequency of revolution of electron will also change. According to classical theory of electromagnetism, frequency of EM wave emitted by electron is equal to frequency of revolution of electron.

So, due to continuous change in frequency of revolution of electron, it will radiate EM waves of all frequencies, i.e. the spectrum of these waves will be continuous in nature. But, this is not the case, experimentally we get line spectrum. Rutherford model was unable to explain line spectrum.

BOHR'S MODEL OF HYDROGEN ATOM

Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates

These three postulates are as follows

(i) **Bohr's first postulate** was that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, contrary to the predictions of electromagnetic theory. According to this postulate, each atom has certain definite stable states in which it can exist and each possible state has definite total energy. These are called the **stationary states** of the atom.

(ii) **Bohr's second postulate** states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$, where h is the Planck's constant $(= 6.63 \times 10^{-34} \text{ J-s})$.

Thus, the angular momentum (L) of the orbiting electron is quantised,

i.e. $$L = \frac{nh}{2\pi}$$

As, angular momentum of electron $= mvr$

$\therefore$ For any permitted (stationary) orbit

$$\boxed{mvr = \frac{nh}{2\pi}}$$

where, n = any positive integer 1, 2, 3, ...

It is also called **principal quantum number**.

(iii) **Bohr's third postulate** states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states.

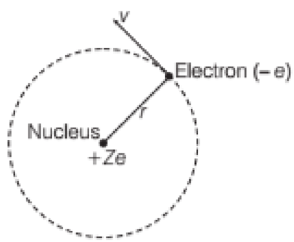
The frequency of the emitted photon is given by

$$h\nu = E_i - E_f$$

where, E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

Bohr's Theory

Bohr's model is valid for all one-electron atoms or ions which consists of a tiny positively charged nucleus and an electron revolving in a stable circular orbit around the nucleus. These one-electron atoms or ions can be called hydrogen like atoms. For example, singly ionised helium (He^+) and doubly ionised lithium (Li^{2+})



Let *e*, *m* and *v* be respectively the charge, mass and velocity of the electron and *r* be the radius of the orbit. The positive charge on the nucleus is *Ze*, where *Z* is the atomic number (in case of H-atom, $Z = 1$). As, the centripetal force is provided by the electrostatic force of attraction, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze) \times e}{r^2}$$

$$\Rightarrow \quad mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r} \quad \text{...(i)}$$

From the second postulate, the angular momentum of the electron is

$$mvr = n\frac{h}{2\pi} \quad \text{...(ii)}$$

where, n (= 1, 2, 3, ...) is **principal quantum number**.

From Eqs. (i) and (ii), we get

$$\boxed{r = n^2 \frac{h^2\varepsilon_0}{\pi m Z e^2}} \quad \text{...(iii)}$$

This is the equation for the radii of the permitted orbits. According to this equation,

$$r_n \propto n^2$$

Since, $n = 1, 2, 3, ...$ it follows that the radii of the permitted orbits increase in the ratio 1 : 4 : 9 : 16 : ..., from the first orbit. Clearly, the stationary orbits are not equally spaced.

Bohr Radius

The radius of the first orbit ($n = 1$) of H-atom ($Z = 1$) will be

$$\boxed{r_1 = \frac{h^2\varepsilon_0}{\pi m e^2}}$$

This is called **Bohr radius** and its value is 0.53 Å. Since, $r \propto n^2$, the radius of the second orbit of H-atom will be (4×0.53) Å and that of the third orbit (9×0.53) Å.

Velocity of Electron in Stationary Orbits

We can obtain formula for the velocity of electron in permitted orbits. From Eq. (ii), we have

$$v = n\frac{h}{2\pi m r}$$

Putting the value of r from Eq. (iii), we get

$$v = \frac{Ze^2}{2h\varepsilon_0} \cdot \frac{1}{n}$$

where, principal quantum number, $n = 1, 2, 3, ...$

Thus, $$v \propto \frac{1}{n}$$

This shows that the velocity of electron is maximum in the lowest orbit ($n = 1$) and goes on decreasing in higher orbits. The velocity of electron in the first orbit ($n = 1$) of H-atom ($Z = 1$) is

$$\boxed{v_1 = \frac{e^2}{2h\varepsilon_0} = \frac{c}{137}} \quad [\because c = 3 \times 10^8 \text{ m/s}]$$

Frequency of Electron in a Stationary Orbit

It is the number of revolutions completed per second by the electron in a stationary orbit around the nucleus.

It is represented by ν.

From $$v = r\omega = r(2\pi\nu) \quad [\because \omega = 2\pi\nu]$$

$$\therefore \quad \nu = \frac{v}{2\pi r}$$

Putting the values of v and r in above equation, we get

$$\nu = \frac{1}{2\pi r} \cdot \frac{Ze^2}{2h\varepsilon_0} \cdot \frac{1}{n}$$

$$\Rightarrow \quad \nu = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{nhr}$$

$$\boxed{\nu = \frac{kZe^2}{nhr}} \quad \left[\because k = \frac{1}{4\pi\varepsilon_0}\right]$$

Energy of Electron in Stationary Orbits

The energy E of an electron in an orbit is the sum of kinetic and potential energies.

Using Eq. (i) in Bohr's theory $mv^2 = \dfrac{Ze^2}{4\pi\varepsilon_0 r}$

The kinetic energy of the electron is

$$\text{KE} = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

Substituting for r from Eq. (iii), we get kinetic energy of the electron in the nth orbit

$$\boxed{\text{KE} = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)}$$

In terms of Rydberg constant R, its simplified form is

$$\text{KE} = \frac{Rhc}{n^2} \qquad \left[\because R = \frac{me^4}{8\varepsilon_0^2 ch^3}\right]$$

The potential energy of the electron in an orbit of radius r due to the electrostatic attraction by the nucleus is given by

$$\text{PE} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Ze)(-e)}{r} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r}$$

In terms of Rydberg constant R, its simplified form is

$$\text{PE} = -\frac{2Rhc}{n^2}$$

The total energy of the electron is

$$E = \text{KE} + \text{PE} = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$= -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{Rhc}{n^2}$$

Substituting for r from Eq. (iii), we get

$$\boxed{E = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)}$$

where, $n = 1, 2, 3, \ldots$. This is the expression for the energy of the electron in the nth orbit.

For hydrogen atom $Z = 1$, substituting the standard values, we get $E_n = \dfrac{-13.6}{n^2}$ eV. Negative energy of the electron shows that the electron is bound to the nucleus and is not free to leave it.

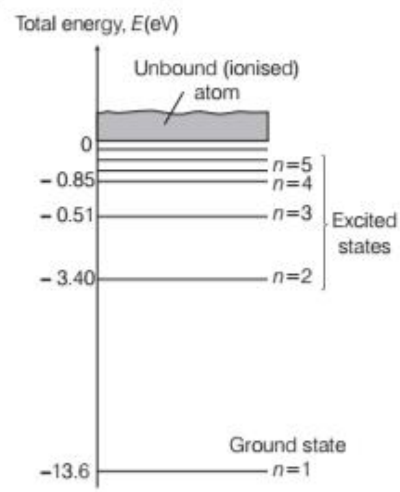
This topic is not included into the syllabus but essential to understand line spectrum of hydrogen atom.

Energy Levels

The energy of an atom is the least, when its electron is revolving in an orbit closest to the nucleus, i.e. for which $n = 1$. For $n = 2, 3, \ldots$ the absolute value of energy E is smaller, so the energy is progressively larger in outer orbits.

The lowest state of the atom is called the **ground state**, this state has lowest energy. The energy of this state is -13.6 eV. Therefore, the minimum energy required to free the electron from the ground state of the H-atom is -13.6 eV. It is called **ionisation energy** of the H-atom.

At room temperature, most of the H-atoms are in ground state. When an atom receives some energy (i.e. by electron collisions), the atom may acquire sufficient energy to raise electron to higher energy state. In this condition, the atom is said to be in **excited state**. From the excited state, the electron can fall back to a state of lower energy, emitting a photon equal to the energy difference of the orbit.



Energy level diagram for hydrogen atom

Suppose in the excited atom, an electron jumps from some higher energy state n_2 to a lower energy state n_1.

The energy difference between these states is

$$E_2 - E_1 = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

According to Bohr's third postulate, the frequency ν of the emitted electromagnetic wave (photon) is

$$\nu = \frac{E_2 - E_1}{h} = \frac{mZ^2e^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

The corresponding wavelength λ of the emitted radiation is given by

$$\frac{1}{\lambda}=\frac{\nu}{c}=\frac{mZ^2e^4}{8\varepsilon_0{}^2ch^3}\left(\frac{1}{n_1{}^2}-\frac{1}{n_2{}^2}\right)$$

$\frac{1}{\lambda}$ is called **wave number** (number of waves per unit length).

In the last equation, the quantity $\frac{me^4}{8\varepsilon_0{}^2ch^3}$ is a constant known as **Rydberg constant** R and its value is $1.097\times10^7\,m^{-1}$.

i.e. $$\frac{me^4}{8\varepsilon_0{}^2ch^3}=R.$$

Thus, $$\frac{1}{\lambda}=Z^2R\left(\frac{1}{n_1{}^2}-\frac{1}{n_2{}^2}\right)$$

This is Bohr's formula for hydrogen and hydrogen like atoms ($He^+, Li^{2+}, \ldots$).

For hydrogen atom ($Z = 1$), we have

$$\frac{1}{\lambda}=R\left(\frac{1}{n_1{}^2}-\frac{1}{n_2{}^2}\right)$$

EXAMPLE |4|

(i) The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. Calculate its radius in $n = 2$ orbit.

(ii) The total energy of an electron in the second excited state of the hydrogen atom is −1.51 eV. Find out its

(a) kinetic energy and

(b) potential energy in this state. **Delhi 2014**

Sol. (i) Given, Bohr radius, $r_1 = 5.3\times10^{-11}$ m

We know that, $r_n = n^2 r_1$

Let r_2 be radius of the orbit for $n = 2$

$\therefore\quad r_2 = (2)^2\times5.3\times10^{-11}$

$= 2.12\times10^{-10}$ m

(ii) Given, total energy of an electron in second excited state,

$$E = -1.51\text{ eV}$$

(a) Kinetic energy of electron is equal to negative of the total energy,

$\therefore\quad K = -E = -(-1.51)$

$= 1.51$ eV

(b) Potential energy of electron is equal to negative of twice of its kinetic energy.

$$U = -2K = -2\times1.51 = -3.02\text{ eV}$$

Hydrogen Spectrum or Line Spectra of Hydrogen Atom

Hydrogen spectrum consists of discrete bright lines in a dark background and it is specifically known as **hydrogen emission spectrum**. There is one more type of hydrogen spectrum that exists where we get dark lines on the bright background, it is known as **absorption spectrum**.

Balmer found an empirical formula by the observation of a small part of this spectrum and it is represented by

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right),\text{ where } n=3,4,5,\ldots$$

where, R is a constant called **Rydberg constant** and its value is $1.097\times10^7\,m^{-1}$.

So, $\frac{1}{\lambda}=1.522\times10^6\,m^{-1}=656.3$ nm for $n=3$

Other series of spectra for hydrogen were subsequently discovered and known by the name of their discoverers. The lines of Balmer series are found in the visible part of the spectrum. Other series were found in the invisible parts of the spectrum.

e.g. **Lyman series** in the **ultraviolet region** and **Paschen, Brackett** and **Pfund** in the **infrared region**.

The wavelengths of line in these series can be expressed by the following formulae

(i) **For Lyman series**

$$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right),\text{ where } n=2,3,4,\ldots$$

(ii) **For Balmer series**

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right),\text{ where } n=3,4,5,\ldots$$

(iii) **For Paschen series**

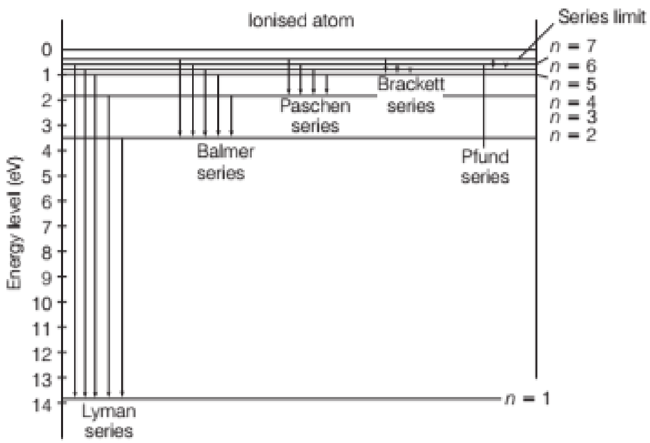
$$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right),\text{ where } n=4,5,6,\ldots$$

(iv) **For Brackett series**

$$\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right),\text{ where } n=5,6,7,\ldots$$

(v) **For Pfund series**

$$\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n^2}\right),\text{ where } n=6,7,8,\ldots$$



Line spectra of the H-atom

> ***Balmer Series In Emission Spectrum Of Hydrogen***
>
> In Balmer series, the line with the longest wavelength (656.3 nm) is red and is called H_α. Next line with wavelength 486.1 nm is blue-green and is called H_β; the third line with 434.1 nm is violet and is called H_γ and so on. As the wavelength decreases, the lines are weaker in intensity and appear closer together.

Explanation

The different series of hydrogen spectrum can be explained by Bohr's theory. According to Bohr's theory, if the ionised state of hydrogen atom be taken as zero energy level, then the energies of the different energy levels of the atom can be expressed by the following formula.

$$E_n = -\frac{Rhc}{n^2}, \text{ where } n = 1, 2, 3, \ldots$$

where, R is Rydberg constant and h is Planck constant. The integer n is called principal quantum number.

When the atom gets energy from outside, its electron goes from the lowest energy level to some higher energy level. But it returns from there, within 10^{-8} s, to the lowest energy level directly or through other lower energy levels. While returning back, the atom emits photons.

EXAMPLE |5| The energy of the electron in the ground state of hydrogen is − 13.6 eV. Calculate the energy of the photon that would be emitted, if electron was to make a transition corresponding to the emission of the first line of the Lyman series of the H-atom.

Sol. Here, energy of e^- in ground state of H-atom = −13.6 eV

i.e. $E_1 = -13.6$ eV

For $n = 2,\ E_2 = -3.4 \text{ eV} \quad \left[\because E_n = -\frac{13.6}{n^2} \text{ eV}\right]$

The energy of photon corresponding to the first line is given by $E = E_2 - E_1$

$\therefore \quad E = [-3.4 - (-13.6)] \text{ eV} = 10.2 \text{ eV}$

EXAMPLE |6| In H-atom, a transition takes place from $n = 3$ to $n = 2$ orbit. Calculate the wavelength of the emitted photon, will the photon be visible? To which spectral series will this photon belong?

(Take, $R = 1.097 \times 10^7 \text{ m}^{-1}$)

Sol. The wavelength of the emitted photon is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

When the transition takes place from $n = 3$ to $n = 2$, then

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\therefore \quad \lambda = \frac{36}{1.097 \times 10^7 \times 5} = 6.563 \times 10^{-7} \text{ m} = 6563 \text{ Å}$$

Since, λ falls in the visible (red) part of the spectrum, hence the photon will be visible. This photon is the first member of the Balmer series.

de-Broglie's Comment on Bohr's Second Postulate

According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving electron.

For an electron revolving in nth circular orbit of radius r_n, total distance covered = circumference of the orbit = $2\pi r_n$.

$\therefore$ For the permissible orbit, $2\pi r_n = n\lambda$

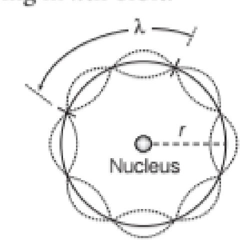
According to de-Broglie wavelength, $\lambda = \frac{h}{mv_n}$

where, v_n is speed of electron revolving in nth orbit.

$$\therefore \quad 2\pi r_n = \frac{nh}{mv_n}$$

$$\text{or} \quad mv_n r_n = \frac{nh}{2\pi} = n(h/2\pi)$$

i.e. angular momentum of electron revolving in nth orbit must be an integral multiple of $h/2\pi$, which is the quantum condition proposed by Bohr in his second postulate.



A standing wave is shown on a circular orbit

EXAMPLE |7| When an electron in hydrogen atom jumps from the third excited state to the ground state, how would the de-Broglie wavelength associated with the electron change? Justify your answer. **Delhi 2015**

Sol. We know that, $\lambda = \frac{h}{p} = \frac{h}{mv}$ or $mv = \frac{h}{\lambda}$

or $mvr = \frac{hr}{\lambda} = \frac{nh}{2\pi}$ or $\lambda = \frac{2\pi}{nh} \times hr = \frac{2\pi r}{n}$

As, $r \propto n^2 \Rightarrow \lambda \propto \frac{1}{n}(n^2) = n$

Thus, we can say that, $\frac{\lambda_3}{\lambda_1} = \frac{4}{1}$ or $\lambda_1 = \frac{\lambda_3}{4}$

Thus, wavelength decreases 4 times as an electron jumps from third excited state to the ground state.

Limitations of Bohr's Model

The limitations of Bohr's model are as follows

(i) This model is applicable only to a simple atom like hydrogen having $Z = 1$. This theory fails, if $Z > 1$.

(ii) It does not explain the fine structure of spectral lines in H-atom.

(iii) This model does not explain why orbits of electrons are taken as circular whereas elliptical orbits are also possible.

Orbital Picture of Electron in an Atom

With the development of quantum mechanics, we have a better understanding of structure of atom. The Schrodinger wave equation gives information about the probability of finding an electron in various regions around the nucleus, which is known as orbital. This function only depends on the coordinates of the electron.

CHAPTER PRACTICE (SOLVED)

OBJECTIVE Type Questions

1. For scattering of α-particles, Rutherford's suggested that

(a) mass of atom and its positive charge were concentrated at centre of atom
(b) only mass of atom is concentrated at centre of atom
(c) only positive charge of atom is concentrated at centre of atom
(d) mass of atom is uniformly distributed throughout its volume

2. In the α-particle scattering experiment, the shape of the trajectory of the scattered α-particles depend upon **[CBSE 2020]**

(a) only on impact parameter
(b) only on the source of α-particles
(c) Both impact parameter and source of α-particles
(d) impact parameter and the screen material of the detector

3. The angular momentum of an electron in hydrogen atom in ground state is

(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$ (c) $\frac{2\pi}{h}$ (d) $\frac{\pi}{h}$

4. If the orbital radius of the electron in a hydrogen atom is 4.7×10^{-11} m. Compute the kinetic energy of the electron in hydrogen atom.

(a) 15.3 eV (b) – 15.3 eV (c) 13.6 eV (d) –13.6 eV

5. A set of atoms in an excited state decays **NCERT Exemplar**

(a) in general to any of the states with lower energy
(b) into a lower state only when excited by an external electric field
(c) all together simultaneously into a lower state
(d) to emit photons only when they collide

6. In Pfund series, ratio of maximum to minimum wavelength of emitted spectral lines is

(a) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{4}{3}$ (b) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{9}{5}$
(c) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{16}{7}$ (d) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{36}{11}$

7. Paschen series of atomic spectrum of hydrogen gas lies in **CBSE All India 2020**

(a) infrared region
(b) ultraviolet region
(c) visible region
(d) partly in ultraviolet and visible region

Direction (Q. Nos. 8-12) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

8. **Assertion** Large angle of scattering of alpha particles led to the discovery of atomic nucleus.

Reason Entire positive charge of atom is concentrated in the central core.

9. **Assertion** Atom as a whole is electrically neutral.

Reason Atom contains equal amount of positive and negative charges.

10. **Assertion** The total energy of an electron revolving in any stationary orbit is negative.

Reason Energy can have positive or negative values.

11. **Assertion** Atoms of each element are stable and emit characteristic spectrum.

Reason The spectrum provides useful information about the atomic structure.

12. **Assertion** Bohr's postulate states that the stationary orbits are those for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$.

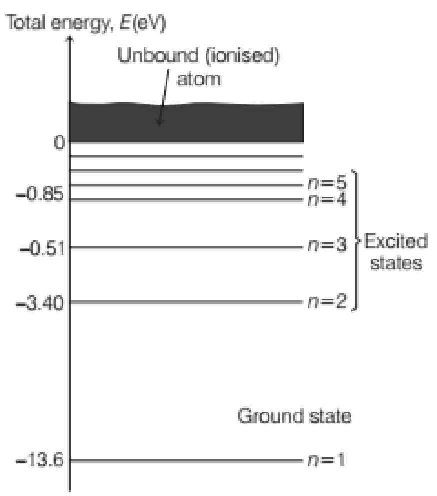
Reason Linear momentum of the electron in the atom is quantised.

Directions (Q.Nos. 13-14) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

13. **Excited State of Atom**

At room temperature, most of the H-atoms are in ground state. When an atom receives some energy (i.e. by electron collisions), the atom may acquire sufficient energy to raise electron to higher energy state. In this condition, the atom is said to be in excited state.

From the excited state, the electron can fall back to a state of lower energy, emitting a photon equal to the energy difference of the orbit.



In a mixture of H–He^+ gas (He^+ is single ionized He atom), H-atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions).

(i) The quantum number n of the state finally populated in He^+ ions is

(a) 2 (b) 3
(c) 4 (d) 5

(ii) The wavelength of light emitted in the visible region by He^+ ions after collisions with H-atoms is

(a) 6.5×10^{-7} m (b) 5.6×10^{-7} m
(c) 4.8×10^{-7} m (d) 4.0×10^{-7} m

(iii) The ratio of kinetic energy of the electrons for the H-atom to that of He^+ ion for $n = 2$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

(iv) The radius of the ground state orbit of H-atom is

(a) $\frac{\varepsilon_0}{h\pi me^2}$ (b) $\frac{h^2\varepsilon_0}{\pi me^2}$ (c) $\frac{\pi me^2}{h}$ (d) $\frac{2\pi h\varepsilon_0}{me^2}$

(v) Angular momentum of an electron in H-atom in first excited state is

(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$ (c) $\frac{2\pi}{h}$ (d) $\frac{\pi}{h}$

14. **α-Particle Scattering Experiment**

In this experiment, H. Geiger and E. Marsden took radioactive source ($^{214}_{83}Bi$) for α-particles. A collimated beam of α-particles of energy 5.5 MeV was allowed to fall on 2.1×10^{-7} m thick gold foil. The α-particles were observed through a rotatable detector consisting of a zinc sulphide screen & microscope and it was found that α-particles got scattered. These scattered α-particles produced scintillations on the zinc sulphide screen.

Observations of this experiment are as follows

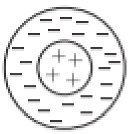
I. Many of the α-particles pass through the foil without deflection.

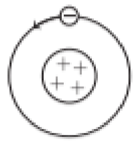
II. Only about 0.14% of the incident α-particles scattered by more than 1°.

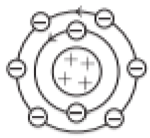
III. Only about one α-particle in every 8000 α-particles deflected by more than 90°.

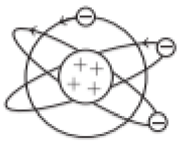
Based on these observation, they were able to proposed a nuclear model of atom, are called planetary model, in which entire positive charge and most of the mass of atom is concentrated in a small volume called the nucleus with electron revolving around the nucleus as planets revolve around the sun.

(i) Rutherford's atomic model can be visualised as

(a) 

(b) 

(c) 

(d) 

(ii) Gold foil used in Geiger-Marsden experiment is about 10^{-8} m thick. This ensures

(a) gold foil's gravitational pull is small or possible

(b) gold foil is deflected when α-particle stream is not incident centrally over it

(c) gold foil provides no resistance to passage of α-particles

(d) most α-particle will not suffer more than 1° scattering during passage through gold foil

(iii) In Geiger-Marsden experiment, detection of α-particles scattered at a particular angle is done by

(a) counting flashes produced by α-particles on a ZnS coated screen

(b) counting spots produced on a photographic film

(c) using a galvanometer detector

(d) using a Geiger-counter

(iv) Atoms consist of a positively charged nucleus is obviously from the following observation of Geiger-Marsden experiment

(a) most of α-particles pass straight through the gold foil

(b) many of α-particles are scattered through the acute angles

(c) very large number of α-particles are deflected by large angles

(d) None of the above

(v) The fact that only a small fraction of the number of incident particles rebound back in Rutherford scattering indicates that

(a) number of α-particles undergoing head-on-collision is small

(b) mass of the atom is concentrated in a small volume

(c) mass of the atom is concentrated in a large volume

(d) Both (a) and (b)

VERY SHORT ANSWER Type Questions

15. Why is the classical (Rutherford) model for an atom of electron orbiting around the nucleus not able to explain the atomic structure? **Delhi 2012**

16. What is the ratio of radii of the orbits corresponding to first excited state and ground state in a H-atom? **Delhi 2010**

17. Consider two different H-atoms. The electron in each atom is in an excited state.

Is it possible for the electrons to have different energies, but the same orbital angular momentum according to the Bohr's model? **NCERT Exemplar**

18. What is the value of angular momentum of electron in the second orbit of Bohr's model of hydrogen atom? **CBSE SQP (Term-I)**

19. When H_α/-line of the Balmer series in the emission spectrum of H-atom is obtained? **Delhi 2013C**

20. Imagine removing one electron from He^4 and He^3. Their energy levels, as worked out on the basis of Bohr's model will be very close. Explain, why? **NCERT Exemplar**

> **Hints:** Niels Bohr proposed a model for hydrogenic (single electron) atoms in order to explain the stability of atoms.

SHORT ANSWER Type Questions

21. Define the distance of closest approach. An α-particle of kinetic energy K is bombarded on a thin gold foil. The distance of the closest approach is r. What will be the distance of closest approach for an α-particle of double the kinetic energy? **All India 2016**

22. An α-particle moving with initial kinetic energy K towards a nucleus of atomic number Z approaches a distance d at which it reverses its direction. Obtain the expression for the distance of closest approach d in terms of the kinetic energy of α-particle K. **Compt. 2016**

23. Using Rutherford's model of the atom, derive the expression for the total energy of the electron in H-atom. What is the significance of total negative energy possessed by the electron? **All India 2014**

24. Explain in brief, why Rutherford's model cannot account for the stability of an atom? **Delhi 2010**

25. Write shortcomings of Rutherford atomic model. Explain, how these were overcome by the postulates of Bohr's atomic model. **CBSE 2020**

26. State Bohr's postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition. **Foreign 2016**

27. Use Bohr's model of hydrogen atom to obtain the relationship between the angular momentum and the magnetic moment of the revolving electron. **CBSE 2020**

28. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. **All India 2015**

29. Using Bohr's postulates of the atomic model, derive the expression for radius of nth electron orbit. Hence, obtain the expression for Bohr's radius. **All India 2014, Delhi 2010**

30. How is the stability of hydrogen atom in Bohr model explained by de-Broglie's hypothesis? **CBSE 2019**

31. Would the Bohr's formula for the H-atom remains unchanged, if proton had a charge $(+4/3)\,e$, and electron had a charge $(-3/4)e$, where, $e = 1.6 \times 10^{-19}$ C. Give reasons for your answer. **NCERT Exemplar**

32. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but same orbital angular momentum according to the Bohr model? Justify your answer. **CBSE SQP (Term-II)**

33. Positronium is just like a H-atom with the proton replaced by the positively charged anti-particle of the electron (called the positron which is as massive as the electron). What would be the ground state energy of positronium? **NCERT Exemplar**

34. How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n?

35. State Bohr's quantisation condition of angular momentum. Calculate the shortest wavelength of the Brackett series and state to which part of the electromagnetic spectrum does it belong. **CBSE 2019**

36. Calculate the orbital period of the electron in the first excited state of hydrogen atom. **CBSE 2019**

LONG ANSWER Type I Questions

37. Draw a plot of α-particle scattering by a thin foil of gold to show the variation of the number of the scattered particles with scattering angle. Describe briefly how the large angle scattering explains the existence of the nucleus inside the atom. Explain with the help of impact parameter picture, how Rutherford scattering serves a powerful way to determine an upper limit on the size of the nucleus. **CBSE All India 2019**

38. Using the relevant Bohr's postulates, derive the expression for the
    (i) velocity of the electron in the nth orbit
    (ii) radius of the nth orbit of the electron in H-atom. **Delhi 2010**

39. Using the postulates of Bohr's model of H-atom, obtain an expression for the frequency of radiation emitted when the atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number $n_f (n_f < n_i)$. **Foreign 2011**

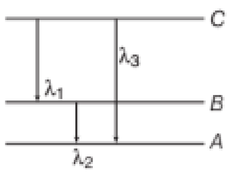
40. Using Bohr's postulates for H-atom, show that the total energy (E) of the electron in the stationary states can be expressed as the sum of kinetic energy (K) and potential energy (U), where $K = -2U$. Hence, deduce the expression for the total energy in the nth energy level of hydrogen atom. **Foreign 2012**

41. (i) Using Bohr's second postulate of quantisation of orbital angular momentum, show that the circumference of the electron in the nth orbital state in H-atom is n times the de-Broglie wavelength associated with it.
    (ii) The electron in H-atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state? **Delhi 2012**

42. Using Bohr's postulates, obtain an expression for the total energy of the electron in the stationary states of the H-atom. Hence, draw the energy level diagram showing how the line spectral corresponding to Balmer series occur due to transition between energy levels. **Delhi 2013**

43. (i) State Bohr's quantisation condition for defining stationary orbits. How does de-Broglie's hypothesis explain the stationary orbits?

(ii) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown below.



Delhi 2016

44. Assume that there is no repulsive force between the electrons in an atom but the force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances, calculate the ground state energy of a He-atom. **NCERT Exemplar**

45. (a) State Bohr's postulate to define stable orbits in hydrogen atom. How does de-Broglie's hypothesis explain the stability of these orbits?

(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. **CBSE 2018**

LONG ANSWER Type II Questions

46. Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level $(n-1)$. For large n, show that this frequency equals to the classical frequency of revolution of the electron in the orbit. **NCERT**

47. (a) State the postulates of Bohr's model of hydrogen atom and derive the expression for Bohr radius.

(b) Find the ratio of the longest and the shortest wavelengths amongst the spectral lines of Balmer series in the spectrum of hydrogen atom. **CBSE 2020**

48. Using Bohr's postulates, derive an expression for the frequency of radiation emitted when electron in H-atom undergoes transition from higher energy state quantum number (n_i) to the lower energy state (n_f).
When electron in H-atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$. Identify the spectral series to which the emission lines belong.

NUMERICAL PROBLEMS

49. Calculate the de-Broglie wavelength associated with the electron in the second excited state of hydrogen atom. The ground state energy of the hydrogen atom is 13.6 eV. **CBSE 2020**

50. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted. **All India 2017, 16**

51. The number of α-particles scattered at 90° is 50 per minute. What will be the number of α-particles, when it is scattered at an angle of 120°?

52. The ground state energy of H-atom is -13.6 eV. What are the kinetic and potential energies of electron in this state? **NCERT, All India 2014 C, All India 2010**

53. Find the ratio of energies of photons produced due to transition of an electron of H-atom from its

(i) second permitted energy level to the first level and

(ii) the highest permitted energy level to the first permitted level. **All India 2010**

54. The gravitational attraction between electron and proton in a H-atom is weaker than the Coulombic attraction by a factor of about 10^{-40}. Estimate the radius of the first Bohr orbit of a H-atom, if the electron and proton were bound by gravitational attraction. **NCERT**

55. In accordance with the Bohr's model, find the quantum number that characterises in the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of the earth = 6×10^{24} kg) **NCERT**

56. The radius of the innermost electron orbit of a H-atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits? **NCERT**

57. In Bohr's model of H-atom, the radius of the first electron orbit is 0.53 Å. What will be the radius of the third orbit and the first orbit of singly ionised helium atom?

58. In the ground state of H-atom, its Bohr radius is given as 5.3×10^{-11} m. The atom is excited such that the radius becomes 21.2×10^{-11} m. Find (i) the value of the principal quantum number and (ii) the total energy of the atom in this excited state. **Delhi 2013C**

59. If the average life time of an excited state of hydrogen is of the order of 10^{-8} s. Estimate how many orbits an electron makes when it is in the state $n = 2$ and before it suffers a transition to state $n = 1$ (Bohr radius, $a_0 = 5.3\times10^{11}$ m)?

60. A H-atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon. **NCERT**

61. The short wavelength limit for the Lyman series of the hydrogen spectrum is 913.4 Å. Calculate the short wavelength limit for Balmer series of the hydrogen spectrum **Delhi 2016**

62. The ground state energy of hydrogen atom is −13.6 eV. If an electron makes a transition from an energy level – 1.51 eV to – 3.4 eV, then calculate the wavelength of the spectral line emitted and name the series of hydrogen spectrum to which it belongs. **Delhi 2016**

63. A photon emitted during the de-excitation of electron from a state n to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2eV, in a photocell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state n. **CBSE 2019**

64. A hydrogen atom in the ground state is excited by an electron beam of 12.5 eV energy. Find out the maximum number of lines emitted by the atom from its excited state. **CBSE 2019**

65. (a) Draw the energy level diagram for the line spectra representing Lyman series and Balmer series in the spectrum of hydrogen atom.

(b) Using the Rydberg formula for the spectrum of hydrogen atom, calculate the largest and shortest wavelengths of the emission lines of the Balmer series in the spectrum of hydrogen atom. **CBSE 2019**

(Use the value of Rydberg constant, $R = 1.1\times10^{7}\ m^{-1}$).

66. Calculate the de-Broglie wavelength associated with the electron revolving in the first excited state of hydrogen atom. The ground state energy of the hydrogen atom is – 13.6 eV. **CBSE 2020**

67. An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond? **Foreign 2016**

68. What is the minimum energy that must be given to a H-atom in ground state so that it can emit an H_γ-line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon? **NCERT Exemplar**

69. Find the quantum number n corresponding to the excited state of He^{+} ion, if on transition to the ground state that ion emits two photons in succession with wavelength 1026.7Å and 304 Å. (Take, $R = 1.097\times10^{7}$ per m)

70. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared, visible, ultraviolet) of hydrogen spectrum does this wavelength lie? **Delhi 2015, All India 2016**

71. Find the ratio between the wavelengths of the 'most energetic' spectral lines in the Balmer and Paschen series of the hydrogen spectrum. **Compt. 2016**

72. What is the shortest wavelength present in the Paschen series of spectral lines? **NCERT**

73. (i) In H-atom, an electron undergoes transition from second excited state to the first excited state and then to the ground state. Identify the spectral series to which these transitions belong.

(ii) Find out the ratio of the wavelengths of the emitted radiations in the two cases. **Delhi 2012**

74. Find out the wavelength of the electron orbiting in the ground state of hydrogen atom. **Dehli 2016**

75. Find the wavelength of the electron orbiting in the first excited state in hydrogen atom. **All India 2016**

75. Use de-Broglie's hypothesis to write the relation for the nth radius of Bohr orbit in terms of Bohr's quantisation condition of orbital angular momentum. **Foreign 2016**

77. The ground state energy of a H-atom is – 13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to – 1.51 eV, then calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong? **All India 2012**

78. The total energy of an electron in the first excited state of the H-atom is about – 3.4 eV.
(i) What is the kinetic energy of the electron in this state?
(ii) What is the potential energy of the electron in this state?
(iii) Which of the answers above would change, if the choice of the zero of potential energy is changed? **NCERT**

79. Obtain the first Bohr radius and the ground state energy of a muonic H-atom (i.e. an atom in which a negatively charged muon $(\bar{\mu})$ of mass about 207 m_e (orbit around a proton). **NCERT**

80. State any two postulates of Bohr's theory of H-atom. What is the maximum possible number of spectral lines when the H-atom is in its second excited state? Justify your answer. Calculate the ratio of the maximum and minimum wavelengths of the radiations emitted in this process. **All India 2010]**

81. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the H-atoms would be excited? Calculate the wavelengths of the first member of Lyman and first member of Balmer series. **All India 2014]**

82. (i) Using the Bohr's model, calculate the speed of the electron in a H-atom in the $n = 1$, 2 and 3 levels.
(ii) Calculate the orbital period in each of these levels. **NCERT**

HINTS AND SOLUTIONS

1. (a) In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away.

2. (a) In α-particle scattering experiment, the shape of the trajectory depends on the impact parameter only.

3. (b) From the formula of angular momentum and Bohr's assumption, $mvr = n(h/2\pi)$.
Here, $n = 1 \Rightarrow \quad mvr = h/2\pi$

4. (a) $K = \dfrac{e^2}{8\pi\varepsilon_0 r} = \dfrac{(9\times10^9\ \text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(2)(4.7\times10^{-11}\ \text{m})}$
$= 2.45\times10^{-18}$ J
$= 15.3$ eV

5. (a) A set of atoms in an excited state decays in general to any of the states with lower energy.

6. (d) In Pfund series,
$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right);\ n = 6, 7, \cdots$$
Maximum wavelength is given by
In transition $6 \rightarrow 5$
$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{5^2} - \frac{1}{6^2}\right)$$
Minimum wavelength is given by
In transition $\infty \rightarrow 5$
$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{5^2} - \frac{1}{\infty}\right)$$
So, ratio is $\dfrac{36}{11}$.

7. (a) Paschen series of hydrogen gas lies in infrared region.

8. (a) α-particle is positively charged, so is the nucleus, so the large angle of scattering of α-particle shows that the nucleus is positively charged and concentrated in the central core.

9. (a)

10. (b)

11. (b)

12. (c)

13. (i) (c) $E_n = \dfrac{-13.6}{n^2}(Z^2)$
In first excited state, $E_{H_2} = 3.4$ eV

and $E_{He} = -13.6$ eV

So, H_2 atom gives excitation energy

(13.6 − 3.4 = 10.2 eV) to helium atom.

Now energy of He ion = − 13.6 + 10.2 = − 3.4 eV

$$\text{Again, } E = \frac{-13.6}{n^2} \times Z^2$$

$$\Rightarrow -3.4 = \frac{-13.6}{n^2} \times (2)^2 \Rightarrow n = 4$$

(ii) (c) $\frac{1}{\lambda} = \frac{13.6 Z^2}{hc}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Here $n_1 = 3$ and $n_2 = 4 \Rightarrow \lambda = 4.8 \times 10^{-7}$ m

(iii) (a) Kinetic energy, $K \propto \frac{Z^2}{n^2}$

$$\frac{K_{H_2}}{K_{He}} = \left(\frac{Z_{H_2}}{Z_{He}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

(iv) (b) Radius of the permitted orbit is $r = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2}$

For hydrogen atom in ground state, i.e.,

$$n = 1, Z = 1 \Rightarrow r = \frac{h^2 \varepsilon_0}{\pi m e^2}$$

(v) (a) Angular momentum for hydrogen atom,

$$L = \frac{nh}{2\pi}$$

For first excited state, $n = 2$

$$\Rightarrow \quad L = \frac{h}{\pi}$$

14. (i) (d) Rutherford's atom had a positively charged centre and electrons were revolving outside it. It is also called the planetary model of the atom as in option (d).

(ii) (d) As the gold foil is very thin, it can be assumed that α-particles will suffer not more than one scattering during their passage through it. Therefore, computation of the trajectory of an α-particle scattered by a single nucleus is enough.

(iii) (a) The scattered α-particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope. The scattered α-particles on striking the screen produced brief light flashes or scintillations. These flashes may be viewed through a microscope and the distribution of the number of scattered particles may be studied as a function of angle of scattering.

(iv) (a) In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away. It is obvious from the observation of Geiger Marsden experiment that most of the α-particles pass straight through the gold foil.

(v) (d) In case of head-on-collision, the impact parameter is minimum and the α-particle rebounds back. So, the fact that only a small fraction of the number of incident particles rebound back indicates that the number of α-particles undergoing head-on collision is small.

This in turn implies that the mass of the atom is concentrated in a small volume.

Hence, options (a) and (b) are correct.

15. (i) Rutherford's model did not explain the stability of nucleus.

(ii) It does not explain the line spectrum of hydrogen atom.

16. For first excited state, $n = 2$

Ground state occurs for $n = 1$

Since, $r_n = r_0 n^2$ and $r_n \propto n^2$

$$\Rightarrow \quad \frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{2}{1}\right)^2$$

So, $r_2 : r_1 = 4 : 1$, where r_2 and r_1 are radii corresponding to first excited state and ground state of the atom.

17. No, it is not possible for the electron to have different energies because according to Bohr's model,

$$E_n = \frac{-13.6}{n^2}$$

The electrons which have different energies, have different values of n.

Angular momentum, $mvr = \frac{nh}{2\pi}$, so as n changes angular momentum changes.

18. The angular momentum of electron revolving in nth orbit of hydrogen atom is

$$L = mvr = \frac{nh}{2\pi}$$

For second orbit, $n = 2$

$$\therefore \quad L = \frac{2h}{2\pi} = \frac{h}{\pi}$$

19. H_α-line of the Balmer series in the emission spectrum of H-atom is obtained in visible region.

20. On removing one electron from He^4 and He^3, the energy levels, as worked out on the basis of Bohr's model will be very close as both the nuclei are very heavy as compared to electron mass.

Also, after removing one electron, He^4 and He^3 atoms contain one electron and are hydrogen like atoms.

21. Refer to text on pages 473 and 474.

The distance of closest approach is given by

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2e \times Ze}{r} = K \qquad \text{...(i)}$$

i.e. $$r \propto \frac{1}{K}$$

Let r_0 be the new distance of closest approach for a twice energetic α-particle.

$$\frac{r_0}{r} = \frac{K}{2K} = \frac{1}{2} \Rightarrow r_0 = \frac{r}{2}$$

22. Refer to text on pages 473 and 474.

23. Refer to text on pages 474 and 475.

24. Refer to text on page 475.

25. Refer to text on pages 474, 475 and 476.

26. An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.

Frequency condition $h\nu = E_i - E_f$

where, ν is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum numbers n_i and n_f respectively (where $n_i > n_f$).

27. Refer to text on page 475.

28. Refer to text on page 476.

29. Refer to text on page 476.

30. Refer to text on page 479. (de-Broglie's Comment on Bohr's Second Postulate)

31. According to Bohr's theory, centripetal force required by the electron for its motion around the nucleus = Electric force between the proton and electron.

$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(q_p)(q_e)}{r^2} \qquad \text{[from Coulomb's law]}$$

where, r = atomic radius, q_p = charge of proton = $+e$

q_e = charge of electron = $-e$

$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(e)(-e)}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-e^2}{r^2}$$

Now, given charge on proton, $q_p = +\frac{4}{3}e$

Charge on electron, $q_e = -\frac{3}{4}e$

Putting the new value (keeping after factors unchanged),

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\left(\frac{4}{3}e\right)\left(-\frac{3}{4}e\right)}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{-e^2}{r^2}$$

i.e. Bohr's formula remain unchanged.

32. No; Because according to Bohr's model, energy of electron in nth orbit of H-atom, $E_n = -\frac{13.6}{n^2}$

Hence, electrons having different energies belong to different energy levels, i.e. different values of n.

Therefore, their angular momentum will be different due to different values of n as

Angular momentum, $L = mvr = \frac{nh}{2\pi}$

33. The reduced mass m of two particle system of masses m_1 and m_2 is given by $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$.

The total energy of the electron in the stationary states of the hydrogen atom is given by $E_n = -\frac{me^4}{8n^2\varepsilon_0^2h^2}$, where signs are as usual but m is the reduced mass of electron and proton. Also, the total energy of the electron in the ground state of the hydrogen atom is -13.6 eV. For H-atom, reduced mass is m_e whereas the positronium, the reduced mass is $m = \frac{m_e}{2}$. Hence, the total energy of the electron in the ground state of the positronium atom is $\frac{-13.6 \text{ eV}}{2} = -6.8$ eV.

34. From the nth state, the atom may go to $(n-1)$ th state, ... , 2nd state or 1st state. So, there are $(n-1)$ possible transitions starting from the nth state.

The atoms reaching $(n-1)$th state may make $n-2$ different transitions. Similarly, for other lower states, the total number of possible transitions is

$$(n-1)+(n-2)+(n-3)+\cdots+2+1 = \frac{n(n-1)}{2}$$

35. Refer to text on pages 507 and 508.
(Bohr's Model of Hydrogen Atom)

For Brackett-series, $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, where

$n = 5, 6, 7, \ldots$

For shortest wavelength, $n = 5$

$$\Rightarrow \frac{1}{\lambda} = 1.097\times 10^7\left(\frac{1}{16} - \frac{1}{25}\right)$$

$$= 1.097\times 10^7 \times \frac{9}{16\times 25} = 0.0246\times 10^7$$

$$\Rightarrow \lambda = 40.514\times 10^{-7} \simeq 4051 \text{ nm}$$

It lies in infrared region of electromagnetic spectrum.

36. The velocity of electron,

$$v = \frac{1}{n}\frac{Ze^2}{2h\varepsilon_0}$$

Here, $Z = 1$, $e = 1.6\times 10^{-19}$ C,

$\varepsilon_0 = 8.85\times 10^{-12}$ NC^{2}m^{-2},

$h = 6.62\times 10^{-34}$ J-s and $n = 2$

(in 1st excited state)

$$\Rightarrow v_2 = \frac{1\times (1.6\times 10^{-19})^2}{2\times 2\times (6.62\times 10^{-34})\times(8.85\times 10^{-12})}$$

$$= 1.09\times 10^6 \text{ m/s}$$

Radius of orbit, $r_2 = \frac{n^2h^2\varepsilon_0}{\pi me^2}$

Here, $m = 9.1\times 10^{-31}$ kg

$$\Rightarrow r_2 = \frac{(2)^2\times(6.62\times 10^{-34})^2\times(8.85\times 10^{-12})}{3.14\times(9.1\times 10^{-31})\times(1.6\times 10^{-19})^2}$$

$$= 2.12\times 10^{-10} \text{ m}$$

Time period or orbital period,

$$T = \frac{2\pi r_2}{v_2}$$

$$= \frac{2 \times 3.14 \times 2.12 \times 10^{-10}}{1.09 \times 10^{6}}$$

$$= 1.22 \times 10^{-15} \text{ s}$$

37. Refer to text on page 473. (Observations)

Refer to text on page 474. (Impact parameter)

38. Refer to text on page 476.

39. Refer to text on page 477.

40. Refer to text on page 477.

41. (i) Bohr's second postulate states that the electron revolves around the nucleus in certain privileged orbit which satisfy certain quantum condition that angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant.

i.e. $L = mvr = \frac{nh}{2\pi}$

where, m = mass of electron, v = velocity of electron and r = radius of orbit of electron.

$$\Rightarrow \quad 2\pi r = n\left(\frac{h}{mv}\right)$$

$\therefore$ Circumference of electron in n th orbit

$= n \times$ de-Broglie wavelength associated with electron. $\left[\because \lambda = \frac{h}{mv}\right]$

(ii) Given, the electron in H-atom is initially in third excited state.

$\therefore \quad n = 4$

And the total number of spectral lines of an atom that can exist is given by the relation

$$= \frac{n(n-1)}{2}$$

Here, $n = 4$

So, number of spectral lines

$$= \frac{4(4-1)}{2} = \frac{4 \times 3}{2} = 6$$

Hence, when a H-atom moves from third excited state to ground state, it emits six spectral lines.

42. Refer to text on page 477.

In H-atom when an electron jumps from the orbit n_i to orbit n_f, the wavelength of the emitted radiation is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

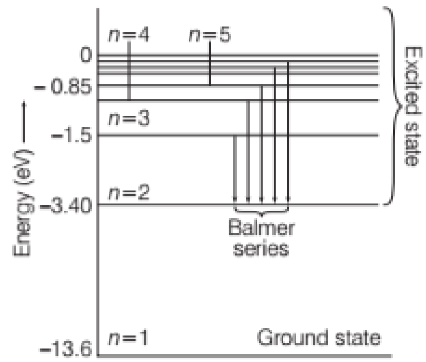
where, R = Rydberg constant $= 1.097 \times 10^{7}\ \text{m}^{-1}$

For Balmer series, $n_f = 2$ and $n_i = 3, 4, 5, \dots$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

where, $n_i = 3, 4, 5, \dots$

These spectral lines lie in the visible region.



43. (i) According to Bohr's principle, electrons revolve in a stationary orbit of which energy and momentum are fixed. The momentum of electrons in the fixed orbit is given by $\frac{nh}{2\pi}$ (where, n = number of orbits).

According to de-Broglie's hypothesis, the electron is associated with wave character. Hence, a circular orbit can be taken to be a stationary energy state only if it contains an integral number of de-Broglie wavelengths, i.e. $2\pi r = n\lambda$

(ii) According to question,

$$E_B - E_C = \frac{hc}{\lambda_1} \quad \text{...(i)}$$

$$E_A - E_B = \frac{hc}{\lambda_2} \quad \text{... (ii)}$$

$$E_C - E_A = \frac{-hc}{\lambda_3} \quad \text{... (iii)}$$

On adding Eqs. (i), (ii) and (iii), we get

$$E_B - E_C + E_A - E_B + E_C - E_A = hc\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_3}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

This is the required expression.

44. $\therefore$ The total energy of the electron in the nth stationary state of hydrogen like atom of atomic number Z is given by

$$E_n = Z^2\left(\frac{-13.6 \text{ eV}}{n^2}\right)$$

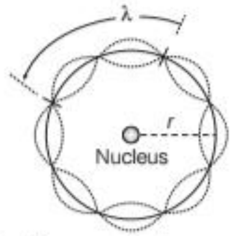
For a He-nucleus with charge $2e$ and electrons of charge $-e$, the energy level in ground state is

$$E_n = Z^2\left(\frac{-13.6 \text{ eV}}{n^2}\right) = 2^2\left(\frac{-13.6 \text{ eV}}{1^2}\right)$$

$$= -54.4 \text{ eV}$$

Thus, the ground state will have two electrons each of energy E and the total ground state energy would be $-(4 \times 13.6)$ eV $= -54.4$ eV

45. (a) Bohr's second postulate defines the stable orbits. This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$, where h is the Planck's constant $(= 6.63 \times 10^{-34}$ J-s).



A standing wave is shown on a circular orbit

According to de-Broglie wavelength of moving electron $\lambda = \dfrac{h}{mv_n}$

where, v_n is speed of electron revolving in nth orbit.

As, $2\pi r_n = n\lambda$ [from figure]

$\therefore \quad 2\pi r_n = \dfrac{nh}{mv_n}$

or $\quad mv_n r_n = \dfrac{nh}{2\pi} = n(h/2\pi)$

i.e. Angular momentum of electron revolving in nth orbit must be an integral multiple of $h/2\pi$, which is the quantum condition proposed by Bohr in his second postulate.

(b) We know that, energy of electron in nth orbit is

$$E_n = -\frac{13.6}{n^2}\text{ eV}$$

For $n = 1$,

$$E_1 = -13.6\text{ eV}$$

Similarly, for $n = 4$,

$$E_4 = -\frac{13.6}{(4)^2}\text{ eV}$$

$\therefore$ Energy difference, $\Delta E = E_4 - E_1$

$$= \left[-\frac{13.6}{16} - (-13.6)\right]\text{ eV} \quad \text{... (i)}$$

Also, energy of photon is

$$\Delta E = h\nu$$

$$\Rightarrow \quad \nu = \frac{\Delta E}{h} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$\nu = \left(-\frac{13.6}{16} + 13.6\right) \times \frac{1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$\therefore \quad \nu = 3.1 \times 10^{15}\text{ Hz}$$

46. Let ν be the frequency when a hydrogen atom jumps from level n to $(n-1)$.

i.e. $n_1 = (n-1)$

$n_2 = n$

Energy, $E = h\nu = E_2 - E_1$

$$\Rightarrow \quad \nu = \frac{1}{2} \cdot \frac{mc^2\alpha^2}{h} \times \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{mc^2\alpha^2}{2h}\left[\frac{n^2 - (n-1)^2}{n^2(n-1)^2}\right]$$

$$= \frac{mc^2\alpha^2[(n+n-1)(n-n+1)]}{2hn^2(n-1)^2}$$

$$= \frac{mc^2\alpha^2(2n-1)}{2hn^2(n-1)^2}$$

For large values of n, $(2n-1 = 2n)$, $(n-1 = n)$, we have

$$\nu = \frac{mc^2\alpha^2\, 2n}{2hn^2 n^2} = \frac{mc^2\alpha^2}{hn^3} \qquad \left[\because \alpha = \frac{2\pi Ke^2}{cn}\right]$$

$$= \frac{mc^2}{hn^3} \cdot \frac{4\pi^2 K^2 e^4}{c^2 n^2}$$

$$= \frac{4\pi^2 K^2 m e^4}{hn^5} \quad \text{...(i)}$$

In Bohr's atomic model, velocity of nth orbit, $v = \dfrac{hn}{2\pi mr}$

and radius, $r = \dfrac{n^2h^2}{4\pi^2 mKe^2}$

Thus, frequency of oscillation

$$\nu = \frac{v}{2\pi r} = \frac{nh}{2\pi mr}\left(\frac{4\pi^2 mKe^2}{2\pi n^2 h^2}\right)$$

$$= \frac{Ke^2}{nhr} = \frac{Ke^2}{nh}\left(\frac{4\pi^2\, mKe^2}{n^2h^2}\right) = \frac{4\pi^2 mK^2e^4}{n^3h^3}$$

It is same as Eq. (i).

So, we can say that for large values of n, the classical frequency of revolution of electron in nth orbit is same as the frequency of radiation emitted when hydrogen atom de-excites from level n to level $(n-1)$.

47. (a) Refer to text on pages 475 and 476.

(b) The wavelength of the lines in Balmer series is expressed by the formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

For shortest wavelength of the spectral line emitted in Balmer series is given by

$$\frac{1}{\lambda_s} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) \qquad (\because n = \infty)$$

$$= \frac{10^7}{4} \qquad [\because R = 10^7]$$

$$\Rightarrow \quad \lambda_s = \frac{4}{10^7} = 4 \times 10^{-7}\text{ m} = 4000\text{ Å}$$

For longest wavelength in Balmer series,

$$\frac{1}{\lambda_l} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \qquad (\because n = 3)$$

$$\Rightarrow \quad \frac{1}{\lambda_l} = \frac{5 \times 10^7}{36}$$

or $\quad \lambda_l = 7.2 \times 10^{-7}\text{ m} = 7200\text{ Å}$

$$\therefore \quad \frac{\lambda_l}{\lambda_s} = \frac{7200}{4000} = \frac{9}{5}$$

48. Refer to text on pages 477 and 478.

As, $\nu = Rc \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Here, higher state is $n_i = 4$ and lower state is

$n_f = 3, 2, 1$

For the transition,

$n_i = 4$ to $n_f = 3 : \longrightarrow$ Paschen series

$n_i = 4$ to $n_f = 2 : \longrightarrow$ Balmer series

$n_i = 4$ to $n_f = 1 : \longrightarrow$ Lyman series

49. Energy in second excited state,

$$E_2 = -\frac{13.6}{(3)^2}\text{ eV}$$

$$= -\frac{13.6}{9} = -1.51\text{ eV}$$

Energy in ground state,

$$E_0 = -13.6\text{ eV}$$

$$\Delta E = E_2 - E_0$$

$$= 1.51 - (-13.6)$$

$$= -1.51 + 13.6$$

$$= 12.09\text{ eV}$$

Wavelength, $\lambda = \frac{12375}{\Delta E}\text{ Å} = \frac{12375}{12.09}\text{ Å} = 1023\text{ Å}$

50. Energy of electron beam,

$$E = 12.5\text{ eV}$$

$$= 12.5 \times 1.6 \times 10^{-19}\text{ J}$$

Planck constant, $h = 6.63 \times 10^{-34}$ J-s

Velocity of light, $c = 3 \times 10^8$ m/s

Using the relation, $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{12.5 \times 1.6 \times 10^{-19}}$$

$$= 0.993 \times 10^{-7}\text{ m}$$

$$= 993 \times 10^{-10}\text{ m}$$

$$= 993\text{ Å}$$

So, wavelength falls in the range of Lyman series from 912 Å to 1216 Å.

51. Approx. 22 per minute, refer to Example 1 on page 473.

52. Refer to Example 4 part (ii) on page 478.

53. (i) 10.2 eV; refer to Example 5 on page 479.

(ii) The highest permitted energy level to the first permitted level,

$$\Delta E = E_\infty - E_1 = 0 - (-13.6) = 13.6\text{ eV}$$

Ratio of energies of photon

$$= \frac{10.2}{13.6} = \frac{3}{4} = 3 : 4$$

54. As, we know that the radius of first Bohr orbit of

H-atom is $\quad r_0 = \frac{4\pi\varepsilon_0\left(\frac{h}{2\pi}\right)^2}{m_e e^2}$

Let us consider that the atom is bound by the gravitational force $= \frac{Gm_p m_e}{r^2}$

We replace $\frac{e^2}{4\pi\varepsilon_0}$ by $Gm_p m_e$. In that case, radius of first orbit (Bohr) of H-atom would be

$$r_0 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p \cdot m_e^2}$$

By substituting the standard values, we get

$$r_0 = \frac{\left(\frac{6.63 \times 10^{-34}}{2 \times 3.14}\right)^2}{6.6 \times 10^{-11} \times 1.67 \times 10^{-27} \times (9.1 \times 10^{-31})^2}$$

$$= 1.2 \times 10^{29}\text{ m}$$

55. Given, radius of the orbit of the earth around the sun,

$$r = 1.5 \times 10^{11}\text{ m}$$

Orbital speed of the earth, $v = 3 \times 10^4$ m/s

Mass of the earth, $m = 6 \times 10^{24}$ kg

According to Bohr's model, angular momentum is quantised and given as, $mvr = \frac{nh}{2p}$

where, h = Planck constant $= 6.63 \times 10^{-34}$ J-s

n = quantum number.

$$\therefore \quad n = \frac{mvr\, 2\pi}{h} = \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11}}{6.63 \times 10^{-34}}$$

$$= 25.61 \times 10^{73}$$

$$= 2.6 \times 10^{74}$$

Hence, the quantum number that characterises the earth's revolution is 2.6×10^{74}.

56. 2.12×10^{-10} m and 4.47×10^{-10} m; refer to example 4 (i) on page 510.

57. Radius of the nth Bohr orbit, $r = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2}$

Again, $\quad r \propto \frac{1}{Z}$

$$\therefore \quad \frac{r_{He^+}}{r_H} = \frac{Z_H}{Z_{He^+}}$$

For hydrogen, $Z = 1$ and for helium, $Z = 2$

$\therefore \qquad \dfrac{r_{He^+}}{r_H} = \dfrac{1}{2}$

$\Rightarrow \qquad r_{He^+} = \dfrac{1}{2} r_H$

$= \dfrac{0.53}{2} = 0.265 \text{Å}$

For radius of third orbit, i.e. for $n = 3$

$r_3 = (3)^2 \times 0.265 \text{Å}$

$= 9 \times 0.265 \text{Å}$

$= 2.38 \text{Å}$

58. (i) We know that, $r \propto n^2$

$$\frac{r_1}{r_2} = \frac{n_1^2}{n_2^2}$$

$$\Rightarrow \quad \frac{1}{n_2^2} = \frac{5.3 \times 10^{-11}}{21.2 \times 10^{-11}}$$

$$n_2^2 = 4$$

$$n_2 = 2$$

(ii) We know that, $E = \dfrac{-13.6}{n^2} = \dfrac{-13.6}{4}$

$= -3.4$ eV

59. Angular momentum of an electron in nth orbit $= \dfrac{nh}{2\pi}$

By Bohr's hypothesis, we have

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

Time period to complete a revolution in an orbit,

$$T = \frac{2\pi r}{v} = \frac{2\pi r(2\pi mr)}{nh} = \frac{4\pi^2 mr^2}{nh}$$

Since, radius of the orbit is proportional to n^2, hence

$$r \propto n^2$$

$$\Rightarrow \quad r = a_0 n^2$$

$$\therefore \quad T = \frac{4\pi^2 m a_0{}^2 n^4}{nh} = \frac{4\pi^2 m a_0^2 n^3}{h}$$

$\therefore$ Number of orbits completed in 10^{-8} s

$$= \frac{10^{-8}}{T} = \frac{10^{-8} \times h}{4\pi^2 m a_0^2 n^3}$$

$$= \frac{10^{-8} \times 6.6 \times 10^{-34}}{4 \times (3.14)^2 \times 9.1 \times 10^{-31} \times (5.3 \times 10^{-11})^2 \times (2)^3}$$

$$= 8 \times 10^6$$

60. To find the wavelength and frequency of photon, use the relation of energy of electron in hydrogen atom.

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\because \quad E_n = \frac{-13.6}{n^2} \text{eV}$$

$$\Rightarrow \quad E_1 = \frac{-13.6}{1^2} \text{ eV}$$

$$= -13.6 \text{ eV}$$

$$E_4 = \frac{-13.6}{4^2} \text{ eV}$$

$$= -\frac{13.6}{16} \text{eV} = -0.85 \text{ eV}$$

$$\Delta E = E_4 - E_1 = -0.85 - (-13.6)$$

$$= 12.75 \text{ eV}$$

$$\Delta E = 12.75 \text{eV} = 12.75 \times 1.6 \times 10^{-19}$$

$$= 20.4 \times 10^{-19} \text{ J}$$

$$\because \quad \Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{20.4 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 3.1 \times 10^{15} \text{ Hz}$$

Wavelength of photon, $\lambda = \dfrac{c}{\nu} = \dfrac{3 \times 10^8}{3.1 \times 10^{15}}$

$= 9.74 \times 10^{-8}$ m

Thus, the wavelength is 9.74×10^{-8} m and frequency is 3.1×10^{15} Hz.

61. Lyman series, $n = 2, 3, 4...$ to $n = 1$

For short wavelength, $n = \infty$ to $n = 1$

Energy, $\quad E = \dfrac{12375}{\lambda(\text{Å})} = \dfrac{12375}{913.4}$ eV

$= 13.54$ eV

Also, energy of nth orbit, $E = \dfrac{13.54}{n^2}$

So, energy of $n = 1$, energy level $= 13.54$ eV

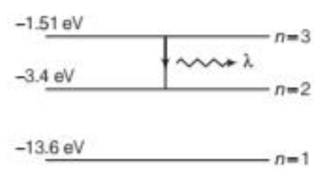
Energy of $n = 2$, energy level

$= \dfrac{13.54}{2^2} = 3.387$ eV

So, short wavelength of Balmer series $= \dfrac{12375}{3.387}$

$= 3653$ Å

62. Energy levels of H-atom are as shown below



Wavelength of spectral line emitted,

$$\lambda = \frac{hc}{\Delta E}$$

Taking, $\quad hc = 1240$ eV-nm,

We have, $\quad \Delta E = -1.51 - (-3.4)$

$= 1.89$ eV

$$\therefore \quad \lambda = \frac{1240}{1.89}$$

$= 656$ nm

This belongs to Balmer spectral series.

63. Here, $\phi = 2\text{ eV}$, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \phi + \text{KE}$$

Also, $\text{KE} = eV_0$

$$n_1 = 2, n_2 = n$$

$$hcR\left(\frac{1}{4} - \frac{1}{n^2}\right) = 2 \times 1.6 \times 10^{-19} + 1.6 \times 10^{-19} \times 0.55$$

$$\Rightarrow 6.62 \times 10^{-34} \times 3 \times 10^8 \times 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$= (3.2 + 0.88) \times 10^{-19}$$

$$\Rightarrow 21.786 \times 10^{-19}\left(\frac{1}{4} - \frac{1}{n^2}\right) = 4.08 \times 10^{-19}$$

$$\frac{1}{4} - \frac{1}{n^2} = 0.187 \Rightarrow n \doteq 4$$

64. The energy absorbed by it is

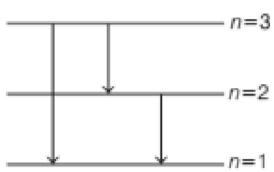
$$-13.6 + 12.5 = -1.1\text{ eV}$$

The energy, $E_n = -\frac{13.6}{n^2}$

$$\Rightarrow \quad n^2 = \frac{-13.6}{-1.1} = 12.36$$

$$\Rightarrow \quad n \doteq 3$$

Thus, number of transitions = 3



65. (a) Refer to diagram on page 511.
(Line Spectrum of the H-atom)

(b) For largest wavelength, $n = \infty$

$$\frac{1}{\lambda_m} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$$

$$\frac{1}{\lambda_m} = \frac{1.1 \times 10^7}{4}$$

$$\Rightarrow \quad \lambda_m = \frac{4}{1.1} \times 10^{-7}$$

$$= 3.636 \times 10^{-7}\text{ m}$$

$$= 3636\text{ Å}$$

For shortest wavelength,

$$n = 3$$

$$\frac{1}{\lambda_s} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 1.1 \times 10^{-7} \times \frac{5}{36}$$

$$\Rightarrow \quad \lambda_s = \frac{36}{5.5} \times 10^{-7}$$

$$= 6.545 \times 10^{-7}\text{ m}$$

$$= 6545\text{ Å}$$

66. Energy stored in first excited state,

$$E_1 = \frac{-13.6}{(2)^2}\text{ eV}$$

$$= \frac{-13.6}{4} = -3.4\text{ eV}$$

Energy in ground state, $E_0 = -13.6\text{ eV}$

$$\Delta E = E_0 - E_1 = -13.6 - (-3.4)$$

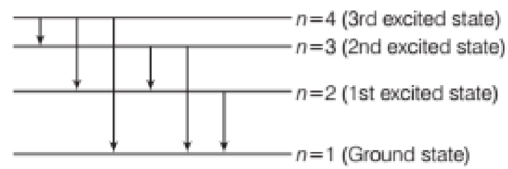
$$= -10.2\text{ eV}$$

$$\text{Wave length }(\lambda) = \frac{12375\text{Å}}{\Delta\varepsilon} = \frac{12375}{10.2}\text{Å}$$

$$= 1213.23\text{ Å}$$

67. Number of spectral lines obtained due to transition of electron from $n = 4$ (3rd excited state) to $n = 1$ (ground state) is

$$N = \frac{(4)(4-1)}{2} = 6$$



These lines correspond to Lyman series.

68. The third line in Balmer series in the spectrum of hydrogen atom is H_γ. H_γ in Balmer series corresponds to transition $n = 5$ to $n = 2$. So, the electron in ground state, i.e. from $n = 1$ must first be placed in state $n = 5$.

Energy required for the transition from $n = 2$ to $n = 5$ is given by

$$= E_1 - E_5 = 13.6 - 0.54 = 13.06\text{ eV}$$

Since, angular momentum is conserved.

Angular momentum corresponding to H_γ photon = Change in angular momentum of electron

$$= L_5 - L_2 = 5\hbar - 2\hbar = 3\hbar = 3 \times 1.06 \times 10^{-34}$$

$$= 3.18 \times 10^{-34}\text{ kg-m}^2/\text{s}$$

69. Given, $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = RZ^2\left(1 - \frac{1}{n^2}\right)$

$$\Rightarrow \quad \frac{1}{n^2} = 1 - \left[\frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2} \times \frac{1}{RZ^2}\right]$$

$$= 1 - \left[\frac{1026.7 + 304}{1026.7 \times 304} \times \frac{1}{4 \times 1.097 \times 10^7}\right]$$

$$\Rightarrow \quad \frac{1}{n^2} = 0.0275$$

$$\Rightarrow \quad n = 6.03$$

Hence, the principal quantum number is 6.

70. Since, we know that for Balmer series,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right), \quad n_2 = 3, 4, 5, \ldots..$$

For shortest wavelength in Balmer series, the spectral series is given by

$$n_1 = 2, n_2 = \infty$$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = R \times \frac{1}{4} \Rightarrow \frac{1}{\lambda} = \frac{R}{4}$$

$$\Rightarrow \quad \lambda = \frac{4}{R} = \frac{4}{1.097 \times 10^7}$$

$$[\because R = 1.097 \times 10^7 \text{ m}^{-1}]$$

$$\Rightarrow \quad \lambda = 3.64 \times 10^{-7} \text{ m}$$

The lines of Balmer series are found in the visible part of the spectrum.

71. For Balmer series, $\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)r$

For highest energy $n \to \infty \Rightarrow \lambda_B = \frac{4}{R}$

$$\Rightarrow \quad \frac{1}{\lambda_B} = \frac{R}{2^2} = \frac{R}{4}$$

For Paschen series, $\frac{1}{\lambda_P} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$

For highest energy,

$$n \to \infty \Rightarrow \lambda_P = \frac{9}{R}$$

$$\Rightarrow \quad \lambda_B : \lambda_P = \frac{4}{R} : \frac{9}{R} \Rightarrow 4 : 9$$

72. Using formula for Paschen series,

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right], \quad n_2 = 4, 5, 6, \ldots$$

For shortest wavelength, $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right] = \frac{R}{9}$$

$$\text{or} \quad \lambda = \frac{9}{R} \simeq 820.4 \text{ nm}$$

73. (i) An electron undergoes transition from second excited state to the first excited state which corresponds to Balmer series and then to the ground state which corresponds to Lyman series.

(ii) The wavelength of the emitted radiations in the two cases.

We know that, $\lambda = \frac{hc}{\Delta E}$

From $n_3 \to n_2$,

$$\lambda_1 = \frac{hc}{E_3 - E_2} = \frac{hc}{(-1.5) - (-3.4)} = \frac{hc}{1.9}$$

From $n_2 \to n_1$,

$$\lambda_2 = \frac{hc}{E_2 - E_1} = \frac{hc}{(-3.4) - (-13.6)} = \frac{hc}{10.20}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{10.20}{1.9} = 5.3$$

74. For an electron revolving in nth orbit of radius r_0, then we have, $\quad n\lambda = 2\pi r_0$

For electron orbiting in ground state, $n = 1$.

$$1\lambda = 2\pi r_0 = 2\pi \times 0.5 \text{Å} \quad [\because r_0 = 0.5 \text{ Å}]$$

$$= \pi \text{Å}$$

$$\text{or} \quad \lambda = 3.14 \text{ Å}$$

75. For electron in first excited state, i.e. $n = 2$.

So, if λ be its wavelength (de-Broglie), then we have

$$n\lambda = 2\pi r_n$$

where, r_n is the radius of second orbit.

$$r_n = 0.5 \times n^2 \text{ (in Å)}$$

$$= 0.5 \times 4 = 2 \text{ Å}$$

$$\therefore \quad 2 \times \lambda = 2 \times \pi \times 2 \text{ Å}$$

$$\Rightarrow \quad \lambda = 2\pi \text{ (Å)} = 6.28 \text{ Å}$$

76. According to Bohr's postulates,

$$mvr = \frac{nh}{2\pi} \quad \ldots\text{(i)}$$

(where, mvr = angular momentum of an electron and n is an integer).

Thus, the centripetal force, $\frac{mv^2}{r}$ (experienced by the electron) is due to the electrostatic attraction, $\frac{kZe^2}{r^2}$

where, Z = Atomic number of the atom.

$$\text{Therefore,} \quad \frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

Substituting the value of v^2 from Eq. (i), we obtain

$$\frac{m}{r} - \frac{n^2h^2}{4\pi^2 m^2 r^2} = \frac{kZe^2}{r^2}$$

$$\Rightarrow \quad r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

The relation for the nth radius of Bohr orbit in terms of Bohr's quantisation condition of orbital angular momentum $= \dfrac{n^2h^2}{4\pi^2 mkZe^2}$

77. 18751 Å; refer to Q. 62 on page 485.

The wavelength belongs to Paschen series of hydrogen spectrum.

78. (i) 3.4 eV and (ii) – 6.8 eV; refer to Example 4 part (ii) on page 478.

(iii) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since, total energy is the sum of kinetic and potential energies, total energy of the system will also change.

79. Muonic hydrogen is the atom in which a negatively charged muon of mass about 207 m_e revolves around a proton.

In Bohr's atomic model, $r \propto \dfrac{1}{m}$

$$\frac{r_{\text{muon}}}{r_{\text{electron}}} = \frac{m_e}{m_\mu} = \frac{m_e}{207\ m_e} = \frac{1}{207} \qquad [\because m_\mu = 207 m_e]$$

Here, r_e is radius of orbit of electron in H-atom = 0.53 Å

$$r_\mu = \frac{r_e}{207} = \frac{0.53 \times 10^{-10}}{207}$$

$$= 2.56 \times 10^{-13}\ \text{m}$$

Again in Bohr's atomic model,

$$E \propto m$$

$$\therefore \quad \frac{E_\mu}{E_e} = \frac{m_\mu}{m_e} = \frac{207\ m_e}{m_e}$$

$$\Rightarrow \quad E_\mu = 207\ E_e$$

For ground state, energy of electron in H-atom,

$$E_e = -\ 13.6\ \text{eV}$$

$$\therefore \quad E_\mu = 207\ (-\ 13.6)$$

$$= -\ 2815.2\ \text{eV}$$

$$= -\ 2.8152\ \text{keV}$$

80. Refer to text on pages 475 and 476.

In second excited state, i.e. $n = 3$, three spectral lines namely Lyman series and Balmer series can be obtained corresponding to transition of electron from $n = 3$ to $n = 1$ and $n = 3$ to $n = 2$, respectively and $n = 2$ to $n = 1$.

For Lyman series (minimum wavelength) $n = 3$ to $n = 1$,

$$\frac{1}{\lambda_{\text{min}}} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 8R/9 \qquad \text{...(i)}$$

For Balmer series (maximum wavelength)

$n = 3$ to $n = 2$,

$$\frac{1}{\lambda_{\text{max}}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{9-4}{36} = \frac{5}{36}R$$

$$\Rightarrow \quad \frac{1}{\lambda_{\text{max}}} = \frac{5R}{36} \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\lambda_{\text{max}}}{\lambda_{\text{min}}} = \frac{\frac{8R}{9}}{\frac{5R}{36}}$$

$$= \frac{8R}{9} \times \frac{36}{5R} = \frac{32}{5}$$

$$\lambda_{\text{max}} : \lambda_{\text{min}} = 32 : 5$$

81. The energies of gaseous hydrogen at room temperature are as given below

$$E_1 = -13.6\ \text{eV},\ E_2 = -\ 3.4\ \text{eV}$$

$$E_3 = -\ 1.51\ \text{eV},\ E_4 = -\ 0.85\ \text{eV}$$

$$E_3 - E_1 = -\ 1.51 - (-13.6) = 12.09\ \text{eV}$$

and $E_4 - E_1 = -\ 0.85 - (-13.6) = 12.75\ \text{eV}$

As, both the values do not match the given value, but it is nearest to $E_4 - E_1$.

$\therefore$ Upto $E_4 - E_1$ energy level, the H-atoms would be excited.

Lyman series, $\dfrac{1}{\lambda} = R\left[\dfrac{1}{1^2} - \dfrac{1}{n^2}\right]$

For first member, $n = 2$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$$

$$= 1.097 \times 10^7 \left[\frac{4-1}{4}\right]$$

$$\Rightarrow \quad \lambda_1 = 1.215 \times 10^{-7}\ \text{m}$$

Balmer series, $\dfrac{1}{\lambda} = R\left[\dfrac{1}{2^2} - \dfrac{1}{n^2}\right]$

For first member, $n = 3$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \lambda_1 = 6.56 \times 10^{-7}\ \text{m}$$

82. (i) Let v_1 be the orbital speed of the electron in a H-atom in the ground state level, $n_1 = 1$. For charge (e) of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi\,\varepsilon_0 \left(\dfrac{h}{2\pi}\right)} = \frac{e^2}{2\varepsilon_0 h}$$

where, e = charge on an electron

$= 1.6 \times 10^{-19}$ C

ε_0 = permittivity of free space

$$= 8.85 \times 10^{-12}\ N^{-1}C^2m^{-2}$$

$$h = \text{Planck constant} = 6.63 \times 10^{-34}\ \text{J-s}$$

$$\therefore \quad v_1 = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.63 \times 10^{-34}}$$

$$= 0.0218 \times 10^8 = 2.18 \times 10^6\ \text{m/s}$$

We know that, $v_n = v_1 / n$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as,

$$v_2 = \frac{v_1}{2} = \frac{2.18 \times 10^6}{2}$$

$$= 1.09 \times 10^6\ \text{m/s}$$

and for level $n_3 = 3$, we can write the relation for the corresponding orbital speed as

$$v_3 = \frac{v_1}{3} = \frac{2.18 \times 10^6}{3} = 7.27 \times 10^5\ \text{m/s}$$

Hence, the speed of the electron in a H-atom in $n = 1$, $n = 2$ and $n = 3$ is 2.18×10^6 m/s, 1.09×10^6 m/s and 7.27×10^5 m/s, respectively.

(ii) Let T_1 be the orbital period of the electron and is given by $T = \frac{2\pi r}{v}$

where, r = radius of the orbit $= \frac{n^2 h^2 \varepsilon_0}{\pi m e^2}$

h = Planck constant $= 6.63 \times 10^{-34}$ J-s

e = charge on an electron $= 1.6 \times 10^{-19}$ C

ε_0 = permittivity of free space

$= 8.85 \times 10^{-12}\ N^{-1}\ C^2 m^{-2}$

m = mass of an electron $= 9.1 \times 10^{-31}$ kg

For $n = 1$,

$$T_1 = \frac{2\pi r_1}{v_1} = \frac{2 \times 3.14 \times 0.53 \times 10^{-10}}{2.18 \times 10^6}$$

$$= 1.52 \times 10^{-16}\text{s}$$

As, $\quad T_n = n^3\ T_1$

Then, $\quad T_2 = (2)^3\ T_1 = 8 \times 1.52 \times 10^{-16}$

$$= 1.22 \times 10^{-15}\text{s}$$

$$T_3 = (3)^3\ T_1 = 27 \times 1.52 \times 10^{-16}$$

$$= 4.10 \times 10^{-15}\text{s}$$

Then, the orbital period in each of these levels is 1.52×10^{-16} s, 1.22×10^{-15} s and 4.10×10^{-15} s, respectively.

SUMMARY

- **α-Particle Scattering Experiment by Rutherford** In this experiment, a collimated beam of α-particles of energy 5.5 MeV was allowed to fall on 2.1×10^{-7}m thick gold foil. The α-particles were observed through a rotatable detector consisting of zinc sulphide screen and microscope and it was found that α-particles got scattered, which produce scintillations on zinc sulphide screen.
- **Rutherford's Model of Atom** According to this model, every atom consists of a central core called the nucleus of an atom and the size of the nucleus is of the order of 10^{-15} m. The atomic nucleus is surrounded by certain number of electrons and they revolve around the nucleus in various circular orbits.
- **Electron Orbits** The total mechanical energy E of electron in a hydrogen atom is $E = \dfrac{-e^2}{8\pi\varepsilon_0 r}$.
- **Drawbacks of Rutherford's Model** It could not explain
  (i) stability of atom.
  (ii) line spectrum.
- **Distance of Closest Approach** At a certain distance r_0 from the nucleus, whole of the kinetic energy of α-particles cannot go further closed to the nucleus.
- **Scattering Angle** Angle by which α-particles get deviated from its original path around the nucleus is called angle of scattering.
- **Impact Parameter** Perpendicular distance of the velocity vector of the α-particles from the central line of the nucleus of an atom is called impact parameter.
- **Bohr's Model of Hydrogen Atom** Bohr's combined classical and early quantum concepts and gave his theory in the form of three postulates.
  First postulate state that an electron could revolve in a certain stable orbits without the emission of radiant energy.
  Second postulate tells that $mvr = \dfrac{nh}{2\pi}$.
  Third postulate tells that $h\nu = E_2 - E_1$.
- **Radii of Bohr's Stationary Orbits**
  $$r = \frac{n^2h^2}{4\pi^2 mke^2}$$
  $$\Rightarrow \quad r \propto n^2$$
- **Velocity of Electrons in Bohr's Stationary Orbits**
  $$v = \frac{2\pi\, Zke^2}{n\,h}$$
- **Frequency of Electrons in Bohr's Stationary Orbits**
  $$\nu = \frac{kZe^2}{nhr} \;\Rightarrow\; \nu \propto \frac{1}{n}$$
- **Total Energy of Electrons in Bohr's Stationary Orbits**
  $$E = \frac{-me^4Z^2}{8n^2\varepsilon_0^2h^2}$$
- **Hydrogen Spectrum** It consists of discrete bright lines a dark background and is known as hydrogen emission spectrum. There is one more type of hydrogen spectrum exists, where we get dark lines on the bright background, it is known as absorption spectrum.
- **Formulae for the Spectral Series of Hydrogen**
  Lyman series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)$, where $n = 2, 3, 4, \ldots$
  Balmer series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$, where $n = 3,4,5, \ldots$
  Paschen series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2} - \dfrac{1}{n^2}\right)$, where $n = 4,5,6, \ldots$
  Brackett series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{4^2} - \dfrac{1}{n^2}\right)$ where $n = 5, 6, 7, \ldots$
  Pfund series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{5^2} - \dfrac{1}{n^2}\right)$, where $n = 6, 7, 8, \ldots$
- **de-Broglie's Comment on Bohr's second Postulate** According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving electron.

CHAPTER PRACTICE (UNSOLVED)

OBJECTIVE Type Questions

1. Atoms consist of a positively charged nucleus is obvious from the following observation of Geiger-Marsden experiment
   (a) most of α-particles do not pass straight through the gold foil
   (b) many of α-particles are scattered through the acute angles
   (c) very large number of α-particles are deflected by large angles
   (d) None of the above

2. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because **NCERT Exemplar**
   (a) of the electrons not being subject to a central force
   (b) of the electrons colliding with each other
   (c) of screening effects
   (d) the force between the nucleus and an electron will no longer be given by Coulomb's law

3. Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about **NCERT Exemplar**
   (a) 53 pm (b) 27 pm
   (c) 18 pm (d) 13 pm

4. In Bohr's atomic model, in going to a higher level (PE = potential energy, TE = total energy)
   (a) PE decreases, TE increases
   (b) PE increases, TE increases
   (c) PE decreases, TE decreases
   (d) PE increases, TE decreases

5. The kinetic energy in ground state of hydrogen atom is – 13.6 eV. What will be the potential energy of electron in this state?
   (a) – 27.2 eV (b) + 27.2 eV
   (c) – 13.6 eV (d) 0 eV

6. Balmer formula is valid for
   (a) hydrogen
   (b) singly ionised helium
   (c) doubly ionised lithium
   (d) All of the above

7. In hydrogen spectrum, H_α lines lies in
   (a) Lyman series
   (b) Balmer series
   (c) Paschen series
   (d) Brackett or Pfund in one of them

8. The number of spectral lines produced due to transition among three energy levels will be
   (a) 10 (b) 8 (c) 6 (d) 3

9. The de-Broglie wavelength of an electron in first Bohr's orbit is
   (a) equal to $\frac{1}{4}$ of circumference of orbit
   (b) equal to $\frac{1}{2}$ of circumference of orbit
   (c) equal to twice of circumference of orbit
   (d) equal to the circumference of orbit

VERY SHORT ANSWER Type Questions

10. What is the impact parameter for scattering of α-particle by 180°?

11. What is the ratio of mass of an α-particle to that of an electron?

12. Calculate the radius of the first orbit of H-atom. Show that the velocity of electron in the first orbit is 1/137 times the velocity of light.

13. Name the spectral series of H-atom lying in the infrared region.

SHORT ANSWER Type Questions

14. Explain, why scattering of α-particles in Rutherford's experiment is not affected by the mass of the nucleons?

15. Derive the expression for the wavelength of the H-atom for different spectral series.

16. Derive the Bohr's quantisation condition for angular momentum of the orbiting of electron in hydrogen atom, using de-Broglie's hypothesis.
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LONG ANSWER Type I Questions

17. Explain the significance of negative energy of an electron in an orbit. What is the energy possessed by an electron for $n = \infty$?

18. Draw energy levels for the hydrogen atom.

19. What does an emperical formula mean? Hence, explain that how Balmer proposed this formula?

20. Using Bohr's postulates, derive the expression for the total energy of the electron revolving in n th orbit of hydrogen atom. Find the wavelength of H_α line, given the value of Rydberg constant, $R = 1.1 \times 10^7 \text{ m}^{-1}$.

21. What is the difference between emission spectra and absorption spectra?

22. Explain the origin of spectral lines of hydrogen using Bohr's theory.

23. How did de-Broglie's equation lead to the quantisation condition laid down by Bohr?

LONG ANSWER Type II Questions

24. (i) State the basic assumption of the Rutherford model of an atom. Explain in brief, why this model cannot account for the stability of an atom?

(ii) Using Bohr's postulates, derive the expression for radius of electron in nth orbit of electron in hydrogen atom.

25. (i) Using postulates of Bohr's theory of hydrogen atom, show that
(a) the radii of orbits increase as n^2 and
(b) the total energy of the electron increases as $1/n^2$, where n is the principal quantum number of the atom.

(ii) Calculate the wavelength of H_α-line in Balmer series of hydrogen atom.
Given, Rydberg constant, $R = 1.097 \times 10^7 \text{ m}^{-1}$.

ANSWERS

1. (b) 2. (a) 3. (c) 4. (b) 5. (b)
6. (d) 7. (b) 8. (d) 9. (d)

10. We know that, impact parameter $(b) = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2 \cot\phi/2}{K}$

Here, $\phi = 180°$

$\therefore \quad b = 0 \quad [\because \cot 180° = 0]$

So, impact parameter becomes zero when scattering of α-particles occurs at 180°.

11. We have, mass of α-particle

$$({}_2He^4) = 4 \times 1.67 \times 10^{-27} \text{ kg}$$

Mass of electron $= 9.1 \times 10^{-31}$ kg

$$\therefore \quad \frac{m_\alpha}{m_e} = \frac{4 \times 1.67 \times 10^{-27}}{9.1 \times 10^{-31}} = 7341$$

Hence, α-particle is 7341 times heavier than electron.

12. We have, radius of nth orbit, $r_n = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2}$ ...(i)

For $n = 1$,

$$r_1 = \frac{h^2\varepsilon_0}{\pi m e^2} = \frac{(6.63 \times 10^{-34})^2 \times (8.85 \times 10^{-12})}{(3.14)(9.1 \times 10^{-31})(1.6 \times 10^{-19})^2} = 0.53 \text{ Å}$$

By Bohr's postulate, we can also write as

$$v = \frac{nh}{2\pi m r}$$

...(ii)

Putting Eq. (ii) in Eq. (i), we get

$$v = \frac{Ze^2}{2h\varepsilon_0} \cdot \frac{1}{n}$$

For $n = 1$, we get $v_1 = \frac{e^2}{2h\varepsilon_0}$ or we can write it as $= \frac{c}{137}$

13. Brackett and Pfund series of H-atom lying in the infrared region.

14. The electrostatic force of attraction between α-particles and nucleus is $= 10^{36}$ stronger than the gravitational force, i.e.

$$\frac{F_g}{F_e} = \frac{\frac{Gm_\alpha M_{\text{nucleus}}}{r^2}}{\frac{K q_\alpha q_{\text{nucleus}}}{r^2}} = 10^{-36}$$

Hence, scattering of α-particles is not affected by the mass of nucleus significantly.

15. Refer to text on page 478.

16. Refer to text on page 479.

17. Refer to text on page 477.

18. Refer to text on page 477.

19. An empirical formula is based solely on observation and experiments but not necessarily supported by theory.

The spacing between the spectral lines within certain sets of the hydrogen spectrum decreases in a regular way. As the wavelength decreases, teh lines appear closes together and are weaker in intensity. Balmer found a simple empirical formula for the observed comelengths.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

20. Refer to text on pages 477 and 478.

21. Refer to text on page 478.

22. Refer to text on page 478.

23. Refer to text on page 479.

24. (i) Refer to the text on pages 473 and 475.
(ii) Refer to the text on page 476.

25. (i) (a) Refer to the text on page 476.
(b) Refer to the text on page 477.

(ii) We know that, $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For H_α-line in Balmer series.

$$n_1 = 2, n_2 = 3$$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 1.097 \times 10^7\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$= 1.097 \times 10^7\left[\frac{9-4}{36}\right]$$

$$\frac{1}{\lambda} = 1.097 \times 10^7\left[\frac{5}{36}\right]$$

$$\text{or} \quad \lambda = \frac{36}{5 \times 1.097} \times 10^{-7}$$

$$\lambda = 6.56 \times 10^{-7}\ \text{m}$$