MODEL QUESTION PAPER : 2020-21

MATHEMATICS

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections**- I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections**- III, IV and V.**
2. Section **III** comprises of 10 questions of **2 marks** each**.**
3. Section **IV** comprises of 7 questions of **3 marks** each**.**
4. Section **V** comprises of 3 questions of **5 marks** each**.**
5. Internal choice is provided in **3** questions of Section–III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Part – A

Section I

All questions are compulsory. In case of internal choices attempt any one.

1. State the reason for the relation R in the set {1, 2, 3} given by R {(1, 2), (2, 1)} not to be transitive. **[1]**

OR

If A = {1, 2, 3}, B = {4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} is a function from A to B; state whether f is one-one or not. **[1]**

2. How many equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) are there in all ? **[1]**

3. If the set A contains 5 elements and the set B contains 6 elements; then find the number of one-one and onto mappings from A to B ? **[1]**

OR

Let A = {a, b, c} and the relation R be defined on A as follows :

R = {(a, a), (b, c), (a, b)}

Then; write minimum number of ordered pairs to be added in R to make R reflexive and transitive. **[1]**

4. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. If $n = p$, write the order of the matrix $(7X - 5Z)$? **[1]**

5. Let A be a non-singular square matrix of order 3×3. Then; find the value of $|\text{adj } A|$. **[1]**

OR

Simplify :

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta \\ \cos\theta & \sin\theta\end{bmatrix}$$ **[1]**

6. If A_{ij} is the co-factor of the element a_{ij} of the determinant $\begin{vmatrix}2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7\end{vmatrix}$;

then write the value of $a_{32}.A_{32}$? **[1]**

7. Evaluate : $\int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ **[1]**

OR

Evaluate : $\int \frac{x+3}{(x+4)^2} e^x \, dx$ **[1]**

8. Calculate the area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. **[1]**

9. Find the degree of the differential equation : $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ **[1]**

OR

Calculate the number of arbitrary constants in the particular solution of a differential equation of third order. **[1]**

10. Write a unit vector in the direction of the sum of vectors : $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$. **[1]**

11. If the vectors from origin to the points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$ respectively, then find the area of triangle OAB. **[1]**

12. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$. **[1]**

13. Find the direction cosines of the vector $(2\hat{i} + 2\hat{j} - \hat{k})$. **[1]**

14. Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$. **[1]**

15. One bag contains 3 red and 5 black balls. Another bag contains 6 red and 4 black balls. A ball is transferred from first bag to the second bag and then a ball is drawn from the second bag. Find the probability that the ball drawn is red. **[1]**

16. The probability that atleast one of the two events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.3; evaluate $P(\bar{A}) + P(\bar{B})$. **[1]**

Section II

Both the Case study based questions are compulsory. Attempt any 4 sub parts from each question 17 and 18. Each question carries 1 mark

17. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late.

Based on the above information, answer the following :

(i) The conditional probability that the doctor is late when he arrives; given that he comes by scooter is : **[1]**

a) 0.195 b) 0.083

c) 0.33 d) 0.25

(ii) The patient calls the doctor for confirmation of his arrival. If he finds out that the doctor is late, find the probability that he hasn't come by train ? **[1]**

a) 0.44 b) 0.05

c) 0.5 d) 0.37

(iii) The probability that the doctor came by bus and is late is : **[1]**

a) 0.0083 b) 0.075

c) 0 d) 0.066

(iv) Let E be the event of doctor arriving late while visiting the patient and T_1, T_2, T_3, T_4 be the events that he came by train, bus, scooter and by other means of transport; respectively. Then, the value of $\sum_{i=1}^{4} P(T_i / E)$ is : **[1]**

a) 0.20 b) 0.45

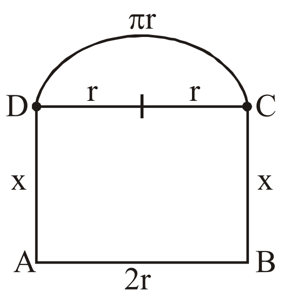
c) 1 d) 0

(v) The total probability of doctor coming late while travelling is : **[1]**

a) 0.15 b) 0.20

c) 0.25 d) 0.60

18. A window is in the form of a rectangle surmounted by a semi-circular opening; as shown below. The total perimeter of the window is 10 m.



Based on the above information, answer the following :

(i) If '2r' and 'x' represent the length and breadth of the rectangular region; then the relation between the variables is : **[1]**

a) $2x + r(\pi + 2) = 10$ b) $x + r(\pi + 2) = 10$

c) $2x + 2r(\pi + 2) = 10$ d) $x + 2r(\pi + 2) = 10$

(ii) The area of the window 'A' expressed as a function of 'r' is : **[1]**

a) $A = 10r - \pi r^2/2$ b) $A = 10r - 2r^2$

c) $A = 10r - \pi r^2/2 - 2r^2$ d) $A = 10r + \pi r^2/2 + 2r^2$

(iii) The maximum or minimum area will be obtained when the value of 'r' is given as : **[1]**

a) $r = \dfrac{10}{5+\pi}$ b) $r = \dfrac{10}{3+\pi}$

c) $r = \dfrac{10}{2+\pi}$ d) $r = \dfrac{10}{4+\pi}$

(iv) The dimensions of the window for maximum area will be : **[1]**

a) $2r = \dfrac{10}{\pi+4}, x = \dfrac{20}{\pi+4}$ b) $2r = \dfrac{20}{\pi+4}, x = \dfrac{10}{\pi+4}$

c) $2r = \dfrac{30}{\pi+4}, x = \dfrac{20}{\pi+4}$ d) $2r = \dfrac{20}{\pi+4}, x = \dfrac{30}{\pi+4}$

(v) The maximum area of the window is given as :

a) $30/4+\pi$ b) $60/4+\pi$ **[1]**

c) $50/4+\pi$ d) $40/4+\pi$

Part – B
Section III

19. Express $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ in the simplest form. **[2]**

20. If there are two values of 'a' which makes determinant;

$$\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86;$$

then what will be the sum of these numbers ? **[2]**

OR

Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$ **[2]**

21. Find the value of k, so that f(x) defined below is continuous at x = 0;

$$f(x) = \begin{cases} \left(\dfrac{1-\cos 4x}{8x^2}\right), & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$$ **[2]**

22. Find the point on the curve $y = x^3 - 11x + 5$ at which the equation of tangent is $y = x - 11$. **[2]**

23. Evaluate : $\int \frac{\sin x}{3+4\cos^2 x} dx$ **[2]**

OR

Evaluate $\int_2^5 \left[|x-2|+|x-3|+|x-5|\right] dx$ **[2]**

24. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$; find the value of 'a'. **[2]**

25. Find the particular solution of the differential equation :

$(1-y^2)(1+\log x)dx + 2xy\ dy = 0$, given that $y = 0$ when $x = 1$. **[2]**

26. If $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = 2\hat{j} - \hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a}+\vec{b})$ and $(\vec{b}+\vec{c})$. **[2]**

27. Find the equation of a plane which is at a distance $3\sqrt{3}$ units from origin and the normal to which is equally inclined to the co-ordinate axes. **[2]**

28. If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$; then find :

(i) $P(A'/B)$ (ii) $P(A'/B')$ **[2]**

OR

The probability distribution of a random variable X is given below : **[2]**

X	0	1	2	3
P(X)	k	k / 2	k / 4	k / 8

(i) Determine the value of k.

(ii) Find $P(X \le 2) + P(X > 2)$.

Section IV

All questions are compulsory. In case of internal choices attempt any one.

29. Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \le b^3\}$ is neither reflexive, nor symmetric nor transitive. **[3]**

30. Find $\frac{dy}{dx}$, if : $y = (\cos x)^x + (\sin x)^{1/x}$. **[3]**

31. Show that the function $f(x) = |x - 3|$; $x \in R$, is not differentiable at $x = 3$. **[3]**

OR

If $x = \cos t\,(3 - 2\cos^2 t)$ and $y = \sin t\,(3 - 2\sin^2 t)$, find the value of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$. **[3]**

32. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is : **[3]**

(a) strictly increasing (b) strictly decreasing

33. Evaluate : $\int \frac{2x^2+1}{x^2(x^2+4)} dx$ **[3]**

34. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle $x^2 + y^2 = 32$. **[3]**

OR

Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$. **[3]**

35. Solve the following differential equation : **[3]**

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}, |x| \neq 1$$

Section V

All questions are compulsory. In case of internal choices attempt any one.

36. Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence; solve the system of equations : **[5]**

$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$

OR

Determine the product :

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

and use it to solve the system of equations. **[5]**

$x - y + z = 4$,
$x - 2y - 2z = 9$,
$2x + y + 3z = 1$

37. Find the equation of the plane which contains the line the intersection of the planes $\vec{r}.(\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r}.(-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$ and whose intercept on x-axis is equal to that on y-axis. **[5]**

OR

Find the coordinates of the point where the line through the points (3, –4, –5) and (2, –3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, –3) and (0, 4, 3). **[5]**

38. Solve the following problem graphically : **[5]**

Minimise and Maximise $Z = 3x + 9y$
subject to constraints :

$x + 3y \leq 60$; $x + y \geq 10$; $x \leq y$; $x \geq 0$; $y \geq 0$.

OR

Find graphically, the maximum value of $Z = 2x + 5y$, subject to constraints given below : **[5]**

$2x + 4y \leq 8$; $3x + y \leq 6$; $x + y \leq 4$; $x \geq 0$; $y \geq 0$.

MATHEMATICS (SOLUTIONS)

1. Here;

$(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$

$\therefore$ R is not transitive

OR

$f = \{(1, 4), (2, 5), (3, 6)\}$

$\therefore f(1) = 4, f(2) = 5, f(3) = 6$

Since different elements have different images; so f is one-one function.

2. Two equivalence relations are there in all;

$\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ and $\{(1, 1), (2, 2), (3, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3,2)\}$

3. Required number of mappings = 0

[Since both the sets have different number of elements]

OR

Given relation, $R = \{(a, a), (b, c), (a, b)\}$

So, required minimum number of ordered pairs to be added are (b, b), (c, c) and (a, c).

4. Order of matrix '7X' is '$2 \times n$' and order of matrix '5Z' is '$2 \times p$' i.e. '$2 \times n$' [$\because$ n = p]

Thus, matrix (7X – 5Z) has order '$2 \times n$'.

5. We know that;

$|\text{adj } A| = |A|^{n-1}$; where 'n' represents the order of matrix.

Hence; $|\text{adj } A| = |A|^{3-1} = |A|^2$

OR

$$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta \\ \cos\theta & \sin\theta\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

6. $a_{32}.A_{32}$

$$= -5\begin{vmatrix}2 & 5 \\ 6 & 4\end{vmatrix}$$

$= -5\ (8 - 30)$

$= 110$

7. $$\int_{-\pi/4}^{\pi/4}\frac{dx}{1+\cos 2x} = \int_{-\pi/4}^{\pi./4}\frac{dx}{2\cos^2 x}$$

$$= \frac{1}{2}\int_{-\pi/4}^{\pi/4}\sec^2 x\ dx = \frac{1}{2}\times 2\int_{0}^{\pi/4}\sec^2 x\ dx$$

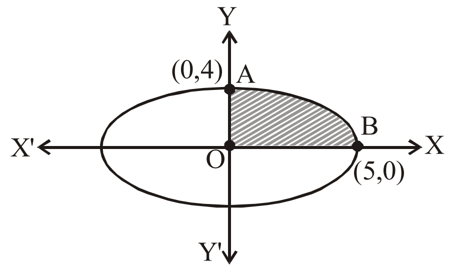
$$= \left[\tan x\right]_0^{\pi/4} = 1$$

OR

$$\int \frac{x+3}{(x+4)^2} e^x dx = \int e^x \left[\frac{(x+4)-1}{(x+4)^2}\right] dx$$

$$= \int e^x \left[\frac{1}{(x+4)} + \left\{\frac{-1}{(x+4)^2}\right\}\right]$$

$$= e^x \left[\frac{1}{x+4}\right] + C$$

8. 

Required Area = $4 \times \int_0^5 \frac{4}{5}\sqrt{5^2 - x^2}\, dx$

$$= \frac{16}{5}\left[\frac{x}{2}\sqrt{5^2 - x^2} + \frac{5^2}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$= \frac{16}{5}\left[0 + \frac{5^2}{2} \times \frac{\pi}{2} - 0 - 0\right]$$

$= 20\,\pi$ sq. units

9. $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$

$\Rightarrow \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$ {on squaring}

Hence; degree = 2

OR

Number of arbitrary constants = 0

10. Let $\vec{r} = \vec{a} + \vec{b} = (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k}) = \hat{i} + 5\hat{k}$

$\Rightarrow \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\hat{i} + 5\hat{k}}{\sqrt{26}} = \frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{j}$

11. Area of Δ OAB

$$= \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$$

$$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|-9\hat{i} + 2\hat{j} + 12\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{229} \text{ sq. units}$$

12. $(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = \vec{0} \cdot \vec{0}$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow 3 + 2\ (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

or $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -3/2$

13. Direction cosines of $(2\hat{i} + 2\hat{j} - \hat{k})$ are :

$$\frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$$

$$= \frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$$

14. The required vector equation of line is given as :

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\ (\hat{i} + 2\hat{j} - 5\hat{k})$

15. P [Red transferred and red drawn or black transferred and red drawn]

$$= \left(\frac{3}{8} \times \frac{7}{11}\right) + \left(\frac{5}{8} \times \frac{6}{11}\right)$$

$$= \frac{51}{88}$$

16. $P(A \cup B) = 0.6,\ P(A \cap B) = 0.3$

$\because\ P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) + P(A \cap B) = [1 - P(\overline{A})] + [1 - P(\overline{B})]$

or $P(\overline{A}) + P(\overline{B}) = 2 - (0.6 + 0.3)$

$= 1.1$

17. Let E be the event that the doctor visits the patient late and let T_1, T_2, T_3, T_4 be the events that the doctor comes by train, bus, scooter and other means of transport respectively.

Then; $P(T_1) = \frac{3}{10},\ P(T_2) = \frac{1}{5},\ P(T_3) = \frac{1}{10},\ P(T_4) = \frac{2}{5}$ (given)

and $P(E/T_1) = \frac{1}{4},\ P(E/T_2) = \frac{1}{3},\ P(E/T_3) = \frac{1}{12},\ P(E/T_4) = 0$ (given)

Now; $P(T_1).P(E/T_1) + P(T_2).P(E/T_2) + P(T_3).P(E/T_3) + P(T_4).P(E/T_4)$

$$= \left(\frac{3}{10}\times\frac{1}{4}\right) + \left(\frac{1}{5}\times\frac{1}{3}\right) + \left(\frac{1}{10}\times\frac{1}{12}\right) + \left(\frac{2}{5}\times 0\right)$$

$= 18/120$ ...(1)

$$\Rightarrow \quad P(T_1/E) = \frac{P(T_1).P(E/T_1)}{P(T_1).P(E/T_1) + P(T_2).P(E/T_2) + P(T_3).P(E/T_3) + P(T_4).P(E/T_4)}$$

$$= \frac{\frac{3}{10}\times\frac{1}{4}}{\frac{18}{120}} = \frac{3}{40}\times\frac{120}{18} = \frac{1}{2} \quad \text{(from (1))} \qquad ...(2)$$

Similarly; $P(T_2/E) = \dfrac{P(T_2).P(E/T_2)}{18/120}$ (from (1))

$$= \frac{\frac{1}{5}\times\frac{1}{3}}{\frac{18}{120}} = \frac{1}{15}\times\frac{120}{18} = \frac{4}{9} \qquad ...(3)$$

$P(T_3/E) = \dfrac{P(T_3).P(E/T_3)}{18/120}$ (from (1))

$$= \frac{\frac{1}{10}\times\frac{1}{12}}{\frac{18}{120}} = \frac{1}{120}\times\frac{120}{18} = \frac{1}{18} \qquad ...(4)$$

and $P(T_4/E) = \dfrac{P(T_4).P(E/T_4)}{18/120}$ (from (1))

$= 0$ ...(5)

(i) (b) Required probability

$= P(E/T_3) = 1/12 = 0.083$

(ii) (c) Required probability

$= 1 - P(T_1/E) = 1 - 1/2 = 1/2 = 0.5$

(iii) (d) Required probability

$= P(T_2).P(E/T_2)$

$= \frac{1}{5} \times \frac{1}{3} = \frac{1}{15} = 0.066$

(iv) (c) $\sum_{i=1}^{4} P(T_i / E)$

$= P(T_1/E) + P(T_2/E) + P(T_3/E) + P(T_4/E)$

$= \frac{1}{2} + \frac{4}{9} + \frac{1}{18} + 0$ (from eqn's (2), (3), (4), (5))

$= \frac{9+8+1}{18} = \frac{18}{18} = 1$

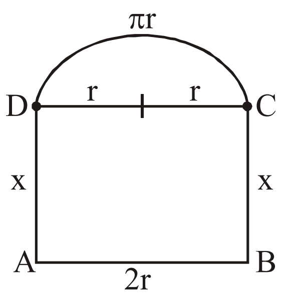
(v) (a) Required probability

$= P(T_1).P(E/T_1) + P(T_2).P(E/T_2) + P(T_3).P(E/T_3) + P(T_4).P(E/T_4)$

$= \frac{18}{120}$ (from (1))

$= 0.15$

18.



Let radius of semi-circle = r

⇒ One side of rectangle = 2r = length,

and other side of rectangle = x = breadth

Let P = Perimeter of the window

⇒ P = 10m (given)

Now; $2x + 2r + \frac{1}{2}(2\pi r) = 10$

⇒ $2x = 10 - r(\pi + 2)$ ...(1)

or $2x + r(\pi + 2) = 10$ ...(2)

Let A be the area of the figure; then :

A = Area of semi-circle + Area of rectangle

$$= \frac{1}{2}\pi r^2 + 2rx$$

$$= \frac{1}{2}\pi r^2 + r\,[10 - r\,(\pi + 2)] \quad \text{(from (1))}$$

$$= \frac{\pi r^2}{2} + 10r - r^2\pi - 2r^2$$

$$\Rightarrow A = 10r - \frac{\pi r^2}{2} - 2r^2 \qquad ...(3)$$

On differentiating twice w.r.t. 'r', we get :

$$\frac{dA}{dr} = 10 - \pi r - 4r$$

and $\frac{d^2A}{dr^2} = -\pi - 4$

For maxima or minima; $\frac{dA}{dr} = 0$

$\Rightarrow \quad 10 - \pi r - 4r = 0$

or $\quad r = \frac{10}{4+\pi} \qquad ...(4)$

Now, $\frac{d^2A}{dr^2} = -(\pi + 4) < 0$

Hence; A has local maximum when $r = \frac{10}{4+\pi}$

$\therefore \quad$ Radius of semi-circle $= \frac{10}{4+\pi}$

$\Rightarrow \quad$ Length of rectangle $= 2r = \frac{20}{4+\pi} \qquad ...(5)$

and Breadth of rectangle = x

$$= \frac{1}{2}\left[10 - r(\pi+2)\right] = \frac{1}{2}\left[10 - \frac{10}{(4+\pi)}(\pi+2)\right] = \frac{10\pi + 40 - 10\pi - 20}{2(\pi+4)}$$

$$\Rightarrow \quad x = \frac{20}{2(\pi+4)} = \frac{10}{\pi+4} \qquad ...(6)$$

And the maximum area of the window is given as :

$$A = 10r - \frac{\pi r^2}{2} - 2r^2 \qquad \text{(from (3))}$$

$$= 10 \times \left[\frac{10}{4+\pi}\right] - \frac{\pi}{2} \times \left[\frac{10}{4+\pi}\right]^2 - 2 \times \left[\frac{10}{4+\pi}\right]^2$$

$$= \frac{100}{4+\pi} - \frac{50\pi}{(4+\pi)^2} - \frac{200}{(4+\pi)^2}$$

$$= \frac{100\pi + 400 - 50\pi - 200}{(4+\pi)^2}$$

$$= \frac{50\pi + 200}{(4+\pi)^2} = \frac{50(\pi+4)}{(\pi+4)^2}$$

$$\Rightarrow \quad A = \frac{50}{4+\pi} m^2 \qquad \text{...(7)}$$

(i) (a) The required relation is :

$2x + r(\pi + 2) = 10$ (from (2))

(ii) (c) Required area is :

$$A = 10r - \frac{\pi r^2}{2} - 2r^2 \qquad \text{(from (3))}$$

(iii) (d) Required value of r is :

$$r = \frac{10}{4+\pi} \qquad \text{(from (4))}$$

(iv) (b) Required dimensions of the window are :

$$2r = \frac{20}{\pi+4}; \; x = \frac{10}{\pi+4} \qquad \text{(from (5), (6))}$$

(v) (c) Required maximum area of window $= \dfrac{50}{4+\pi}$ (from (7))

19. $\cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$$

$$= \cot^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]$$

$$= \cot^{-1}\left[\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right] = \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2}$$

20. $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

$\Rightarrow$ $1(2a^2 + 4) - 2(-4a - 20) + 0 = 86$

or $2a^2 + 8a + 44 = 86$

$\Rightarrow a^2 + 4a - 21 = 0$

or $(a + 7)(a - 3) = 0$

$\Rightarrow a = -7$ and 3

Hence; Sum $= -7 + 3 = -4$

OR

$|A| = 2 \neq 0$

$\Rightarrow \quad A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

Now ;

LHS $= 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

and R.H.S. $= 9I - A = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

Hence; proved

21. $\lim_{x\to 0}\left(\frac{1-\cos 4x}{8x^2}\right) = \lim_{x\to 0}\left(\frac{2\sin^2 2x}{8x^2}\right)$

$= \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2 = 1$

$\because \lim_{x\to 0} f(x) = f(0)$ [as f(x) is continuous at x = 0]

$\Rightarrow k = 1$

22. $y = x^3 - 11x + 5$ ...(1)

$\Rightarrow \frac{dy}{dx} = 3x^2 - 11 =$ slope of tangent

Now, equation of tangent is $y = x - 11$

$\Rightarrow 3x^2 - 11 = 1$

or $x^2 = 4 \Rightarrow x = \pm 2$

From (1); $y = -9$ (at $x = 2$)

or $y = 19$ (at $x = -2$)

But $(-2, 19)$ does not satisfy the equation of tangent.

$\Rightarrow$ Required point is $(2, -9)$

23. Let $I = \int \frac{\sin x}{3+4\cos^2 x} dx$

Put $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\therefore\ I = -\int \frac{dt}{3+4t^2}$$

$$= \frac{-1}{4}\int \frac{dt}{\left(\frac{\sqrt{3}}{2}\right)^2 + t^2}$$

$$= \frac{-1}{4} \cdot \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2t}{\sqrt{3}}\right) + C$$

$$= \frac{-1}{2\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + C$$

OR

$$\int_2^5 \left[|x-2| + |x-3| + |x-5|\right] dx$$

$$= \int_2^3 [(x-2)-(x-3)-(x-5)]\, dx + \int_3^5 [(x-2)+(x-3)-(x-5)]\, dx$$

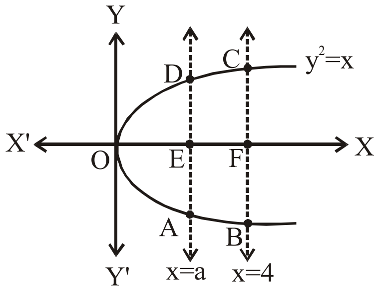
$$= \int_2^3 (-x+6)\, dx + \int_3^5 x\ dx$$

$$= \left[\frac{-x^2}{2} + 6x\right]_2^3 + \left[\frac{x^2}{2}\right]_3^5$$

$$= \left[\left(\frac{-9}{2} + 18\right) - (-2+12)\right] + \frac{1}{2}(25-9)$$

$$= \frac{-9}{2} + 8 + 8 = \frac{23}{2}$$

24.



As per the given conditions;

Ar(OAD) = Ar(ABCD)

$\Rightarrow$ Ar(OED) = Ar(EFCD)

$$\Rightarrow \int_0^a \sqrt{x}\, dx = \int_a^4 \sqrt{x}\, dx$$

$$\text{or } \left[\frac{x^{3/2}}{3/2}\right]_0^a = \left[\frac{x^{3/2}}{3/2}\right]_a^4$$

$\Rightarrow a^{3/2} = 4^{3/2} - a^{3/2}$

or $2a^{3/2} = 8 \Rightarrow a = (4)^{2/3}$

25. $(1-y^2)(1+\log x)dx + 2xy\, dy = 0$

$$\Rightarrow \frac{1+\log x}{x}dx = \frac{-2y}{1-y^2}dy$$

On integrating both sides, we get

$$\frac{(1+\log x)^2}{2} = \log|1-y^2| + C$$

When x = 1, y = 0

$$\Rightarrow \frac{(1+\log 1)^2}{2} = \log(1) + C \Rightarrow C = \frac{1}{2}$$

hence; $(1 + \log x)^2 = 2\log|1 - y^2| + 1$ is the required solution.

26. $(\vec{a}+\vec{b}) = (2\hat{i}-3\hat{j}+\hat{k}) + (-\hat{i}+\hat{k})$

$= \hat{i}-3\hat{j}+2\hat{k}$

and $(\vec{b}+\vec{c}) = (-\hat{i}+\hat{k}) + (2\hat{j}-\hat{k})$

$= -\hat{i}+2\hat{j}$

$$\Rightarrow (\vec{a}+\vec{b})\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -4\hat{i}-2\hat{j}-\hat{k}$$

$\therefore$ Required area of parallelogram

$= \frac{1}{2}\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right| = \frac{1}{2}\sqrt{(-4)^2+(-2)^2+(-1)^2}$

$= \frac{\sqrt{21}}{2}$ sq. units

27. We have; $\cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$

($\because$ normal to plane is equally inclined to axes)

Hence; $\vec{N} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

$\Rightarrow$ Equation of plane is :

$\vec{r}.\hat{N} = 3\sqrt{3}$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).\frac{\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right)}{1} = 3\sqrt{3}$

or $\frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = 3\sqrt{3}$

$\Rightarrow$ Required equation of plane is :

$x + y + z = 9$

28. (i) $P(A'/B) = \frac{P(A'\cap B)}{P(B)}$

$= \frac{P(B) - P(A\cap B)}{P(B)} = \frac{\frac{1}{3}-\frac{1}{4}}{\frac{1}{3}} = \frac{1}{4}$

(ii) $P(A'/B') = \frac{P(A'\cap B')}{P(B')}$

$= \frac{1-P(A\cup B)}{P(B')} = \frac{1-[P(A)+P(B)-P(A\cap B)]}{P(B')}$

$= \frac{1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right]}{1-\frac{1}{3}} = \frac{1-\frac{14}{24}}{\frac{2}{3}} = \frac{5}{8}$

OR

(i) $k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1$

$\Rightarrow 8k + 4k + 2k + k = (1 \times 8)$

or $k = \frac{8}{15}$

(ii) $P(X \leq 2) + P(X > 2)$

$= \left(k + \frac{k}{2} + \frac{k}{4}\right) + \left(\frac{k}{8}\right)$

$= \frac{7k}{4} + \frac{k}{8}$

$= \frac{14k + k}{8} = \frac{15k}{8}$

$= \frac{15}{8} \times \frac{8}{15} = 1$

29. (i) Reflexive :

We observe that $\frac{1}{2} \leq \left(\frac{1}{2}\right)^3$ is not true.

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right) \notin S$. So, S is not reflexive.

(ii) Symmetric :

We observe that $1 \leq 3^3$ but $3 \not\leq 1^3$

i.e. $(1, 3) \in S$ but $(3, 1) \notin S$

So, S is not symmetric

(iii) Transitive :

We observe that $10 \leq 3^3$ and $3 \leq 2^3$ but $10 \not\leq 2^3$

i.e. $(10, 3) \in S$ and $(3, 2) \in S$ but $(10, 2) \notin S$

So, S is not transitive

$\therefore$ S is neither reflexive, nor symmetric, nor transitive.

30. We have ; $y = (\cos x)^x + (\sin x)^{1/x}$

$\Rightarrow \quad y = u + v$ ...(1)

Now; $u = (\cos x)^x \Rightarrow \log u = x.\log \cos x$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = x.\frac{1}{\cos x}(-\sin x) + \log \cos x\ .1$

$\Rightarrow \frac{du}{dx} = (\cos x)^x[-x \tan x + \log \cos x]$ ...(2)

and $v = (\sin x)^{1/x} \Rightarrow \log v = 1/x.\log \sin x$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{1}{x}.\frac{1}{\sin x}(\cos x) + \log \sin x\left(\frac{-1}{x^2}\right)$

$\therefore \dfrac{dv}{dx} = (\sin x)^{1/x}\left[\dfrac{\cot x}{x} - \dfrac{\log \sin x}{x^2}\right]$ ...(3)

From eqn's (1), (2) and (3); we have

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$= (\cos x)^x [\log \cos x - x \tan x] + (\sin x)^{1/x}\left[\frac{\cot x}{x} - \frac{\log \sin x}{x^2}\right]$$

31. We have;

$$f(x) = |x - 3| = \begin{cases} x-3, & x \geq 3 \\ -(x-3), & x < 3 \end{cases}$$

and $f(3) = |3 - 3| = 0$

Now; $L\, f'(3) = \lim\limits_{h \to 0} \dfrac{f(3-h) - f(3)}{(-h)}$

$$= \lim_{h \to 0} \frac{[-(3-h-3)] - 0}{(-h)}$$

$$= \lim_{h \to 0} \frac{h}{(-h)} = -1$$

and $R\, f'(3) = \lim\limits_{h \to 0} \dfrac{f(3+h) - f(3)}{h}$

$$= \lim_{h \to 0} \frac{(3+h-3) - 0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h} = 1$$

$\because\ L\, f'(3) \neq R\, f'(3)$

$\Rightarrow$ f(x) is not differentiable at x = 3

OR

Here;

$x = \cos t\,(3 - 2\cos^2 t)$ and $y = \sin t\,(3 - 2\sin^2 t)$

$$\Rightarrow \frac{dx}{dt} = -\sin t(3 - 2\cos^2 t) + \cos t(2 \times 2 \cos t \sin t)$$

$\therefore \dfrac{dx}{dt} = -3 \sin t + 6 \cos^2 t \sin t$ ...(1)

and $\dfrac{dy}{dt} = \cos t\,(3 - 2\sin^2 t) + \sin t(-2.\ 2 \sin t \cos t)$

$\therefore \dfrac{dy}{dt} = 3\cos t - 6\sin^2 t\cos t$ ...(2)

Hence,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{3\cos t[1-2\sin^2 t]}{3\sin t[-1+2\cos^2 t]} = \frac{3\cos t}{3\sin t}\times\frac{\cos 2t}{\cos 2t}$$

$$\Rightarrow \frac{dy}{dx} = \cot t \Rightarrow \frac{dy}{dx}\left(\text{at } t = \frac{\pi}{4}\right)$$

$$= \cot\frac{\pi}{4} = 1$$

32. We have; $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$\Rightarrow f'(x) = 12x^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

$\therefore f'(x) = 12x(x+1)(x-2)$

Now; $f'(x) = 0$

$\Rightarrow 12x(x+1)(x-2) = 0$ or $x = -1, 0, 2$

Interval	Sign of f '(x)	Nature of function
$(-\infty, -1)$	$(-)(-)(-) < 0$	S. D.
$(-1, 0)$	$(-)(+)(-) > 0$	S. I.
$(0, 2)$	$(+)(+)(-) < 0$	S. D.
$(2, \infty)$	$(+)(+)(+) > 0$	S. I.

(a) f(x) is strictly increasing in $(-1, 0) \cup (2, \infty)$

(b) f(x) is strictly decreasing in $(-\infty, -1) \cup (0, 2)$

33. Let $I = \displaystyle\int \frac{2x^2+1}{x^2(x^2+4)}dx$

If $x^2 = y \Rightarrow \dfrac{2x^2+1}{x^2(x^2+4)} = \dfrac{2y+1}{y(y+4)}$

Then; $\dfrac{2y+1}{y(y+4)} = \dfrac{A}{y} + \dfrac{B}{(y+4)}$

$\therefore A = \dfrac{1}{4}$ and $B = \dfrac{7}{4}$

[$\because 2y + 1 = A(y+4) + By$ and putting $y = 0, -4$]

$$\Rightarrow \frac{2y+1}{y(y+4)} = \frac{1}{4}\times\frac{1}{y} + \frac{7}{4}.\frac{1}{y+4}$$

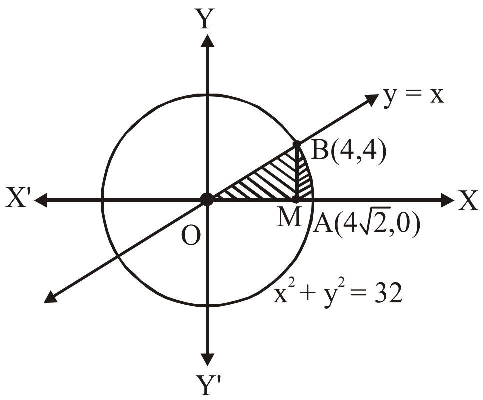
$$\therefore \frac{2x^2+1}{x^2(x^2+4)} = \frac{1}{4}\times\frac{1}{x^2} + \frac{7}{4}\times\frac{1}{x^2+4}$$

$$\Rightarrow \int \frac{2x^2+1}{x^2(x^2+4)}dx = \frac{1}{4}\int x^{-2}dx + \frac{7}{4}\int \frac{1}{x^2+2^2}dx$$

$$= \frac{1}{4}\times\frac{x^{-1}}{(-1)} + \frac{7}{4}\times\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \frac{-1}{4x} + \frac{7}{8}\tan^{-1}\left(\frac{x}{2}\right) + C$$

34. The given equations are : $y = x$ and $x^2 + y^2 = 32$



For intersection point of $y = x$ and $x^2 + y^2 = 32$;

$x^2 + x^2 = 32 \Rightarrow 2x^2 = 32$

or $x^2 = 16 \Rightarrow x = 4$; $y = x = 4$

Hence; Required Area = ar(region OBMO) + ar (region BMAB)

$$= \int_0^4 y\,dx + \int_4^{4\sqrt{2}} y\,dx$$

$$= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32-x^2}\,dx$$

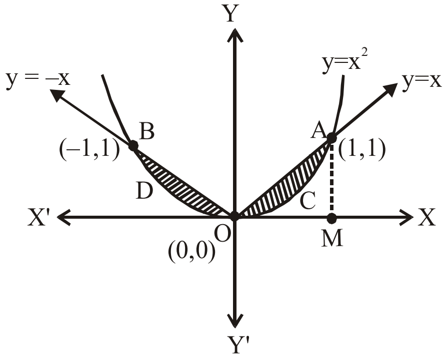
$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\frac{x}{2}\sqrt{32-x^2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$$

$$= \left(\frac{1}{2}\times 4^2\right) + \left[\left(\frac{4\sqrt{2}}{2}\times 0 + \frac{1}{2}\times 32\times\frac{\pi}{2}\right) - \left(\frac{4}{2}\sqrt{32-16} + \frac{1}{2}\times 32\times\frac{\pi}{4}\right)\right]$$

$= 8 + (8\pi - (8 + 4\pi))$

$= 4\pi$ sq. units

OR



The given equations are :

$y = x^2$ and $y = |x|$

For intersection points ; $x^2 = x$

$\Rightarrow x^2 - x = 0$ or $x(x-1) = 0$

$\Rightarrow x = 0,1$ or $y = 0,1$

Hence; required area

$$= 2 \times \int_0^1 (y_2 - y_1)dx$$

$$= 2 \times \int_0^1 (x - x^2)dx$$

$$= 2 \times \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2 \times \left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 2 \times \frac{1}{6} = \frac{1}{3} \text{sq. units}$$

35. We have; $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2 - 1}, |x| \neq 1$

$$\Rightarrow \frac{dy}{dx} + \frac{2xy}{x^2 - 1} = \frac{2}{(x^2 - 1)^2}$$

Comparing it with the linear differential equation of the form :

$\frac{dy}{dx} + Py = Q;$ where

$P = \frac{2x}{x^2 - 1}$ and $Q = \frac{2}{(x^2 - 1)^2}$

$\therefore$ Integrating factor

$= e^{\int P dx} = e^{\int \frac{2x}{x^2-1} dx}$

$= e^{\log_e (x^2-1)} = x^2 - 1$

Hence, solution of differential equation is :

$$y \times (x^2 - 1) = \int \frac{2}{(x^2-1)^2} \times (x^2 - 1)\, dx + C$$

$$\Rightarrow y(x^2 - 1) = 2\int \frac{dx}{x^2 - 1} + C$$

$$\text{or } y(x^2 - 1) = 2 \times \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C$$

$$\Rightarrow y(x^2 - 1) = \log \left| \frac{x-1}{x+1} \right| + C$$

36. Here; $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix}$$

$= 1(-12+6) - 2(-8-6) - 3(-6-9)$

$= 67 \neq 0$

$\therefore A^{-1}$ exists

Now; $A_{11} = -6$, $A_{12} = 14$, $A_{13} = -15$

$A_{21} = 17$, $A_{22} = 5$, $A_{23} = 9$

$A_{31} = 13$, $A_{32} = -8$, $A_{33} = -1$

$$\therefore \text{adj } A = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

$$\text{Hence; } A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$

The given system of equations is :

$x + 2y - 3z = -4$
$2x + 3y + 2z = 2$
$3x - 3y - 4z = 11$

These system of equations can be written as :

AX = B; where

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$\because$ A^{-1} exists; so system of equations has a unique solution given by $X = A^{-1}. B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}.\begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$\Rightarrow$ x = 3, y = –2, z = 1

OR

We have ;

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}.\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 8I$$

$$\Rightarrow \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}.\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = I$$

$$\text{or} \quad \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}^{-1} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

Now, The given system of equations is :

$x–y+z = 4$,

$x–2y–2z = 9$,

and $2x+y+3z = 1$

These can be written as AX = B where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$\because$ A^{-1} exists $\Rightarrow X = A^{-1}.B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$\Rightarrow x = 3, y = -2, z = -1$

37. Required plane is given as $P_1 + \lambda P_2 = 0$

$$\Rightarrow \quad \left[\vec{r}.(\hat{i} - 2\hat{j} + 3\hat{k}) - 4\right] + \lambda\left[\vec{r}.(-2\hat{i} + \hat{j} + \hat{k}) + 5\right] = 0$$

$$\Rightarrow \quad \vec{r}.\left[(1-2\lambda)\hat{i} + (-2+\lambda)\hat{j} + (3+\lambda)\hat{k}\right] = 4 - 5\lambda \qquad ....(1)$$

Now: x-axis intercept = y-axis intercept (given)

$$\Rightarrow \quad \frac{4-5\lambda}{1-2\lambda} = \frac{4-5\lambda}{-2+\lambda}$$

$\therefore$ $1 - 2\lambda = -2 + \lambda$

$\Rightarrow$ $3\lambda = 3$

or $\lambda = 1$, putting this value of λ in equation (1) ; we get

$$\Rightarrow \quad \vec{r}.(-\hat{i} - \hat{j} + 4\hat{k}) = -1$$

$$\vec{r}.(-\hat{i} - \hat{j} + 4\hat{k}) + 1 = 0$$

OR

Equation of line passing through two given points (3, –4, –5) and (2, –3, 1) is

$$\frac{x-3}{2-3} = \frac{y-(-4)}{-3-(-4)} = \frac{z-(-5)}{1-(-5)}$$

$$\Rightarrow \quad \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \qquad ....(1)$$

Now, equation of the plane passing through the points (1, 2, 3); (4, 2, –3) and (0, 4, 3) is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 4-1 & 2-2 & -3-3 \\ 0-1 & 4-2 & 3-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0$$

$(x - 1)(0 + 12) - (y - 2)(0 - 6) + (z - 3)(6) = 0$

$\Rightarrow \quad 12x + 6y + 6z - 42 = 0$

$\Rightarrow \quad 2x + y + z - 7 = 0 \qquad$ ....(2)

From (1), we get

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda \text{ (say)}$$

$\Rightarrow$ A point on the line is given by

$x = -\lambda + 3;\ y = \lambda - 4;\ z = 6\lambda - 5$

This must satisfy the equation of the plane

$2(-\lambda + 3) + 1(\lambda - 4) + 6\lambda - 5 - 7 = 0 \qquad$ (from (2))

$\Rightarrow \quad 5\lambda - 10 = 0$

$\Rightarrow \quad \lambda = 2$

Hence, req. point is (1, –2, 7)

38. The given L.P.P is :

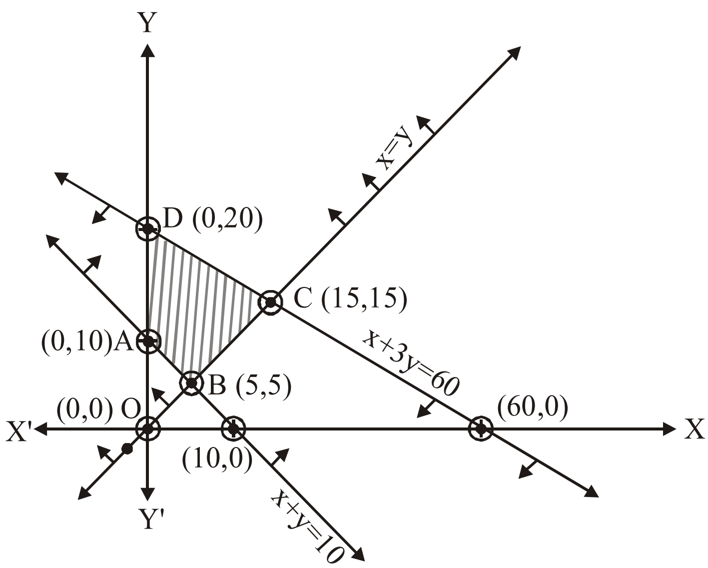
Minimise and Maximise

$Z = 3x + 9y$

subject to the constraints :

$x + 3y \le 60; \quad x + y \ge 10; \quad x \le y; \quad x \ge 0; \quad y \ge 0$

First of all, let us graph the given inequalities and find out the feasible region of the given L.P.P.



As shown in the figure, the given feasible region is bounded, shown by ABCD. And the corner points along–with the corresponding values of Z are shown in the table below :

Corner-points	Corresponding value of Z $Z = 3x + 9y$
A (0,10)	90
B (5,5)	60 (Minimum)
C (15,15)	180 (Maximum)
D (0,20)	180 (Maximum)

We can see that the problem has multiple optimal solutions at the corner points C and D i.e. both points produce same maximum value 180. Hence, it can be concluded that every point on the line-segment CD also gives the same maximum value. Hence, the minimum value of Z is 60 at the point B(5, 5) of the feasible region and the maximum value occurs at the two corner-points C(15, 15) and D(0, 20) and it is 180 in each case.

OR

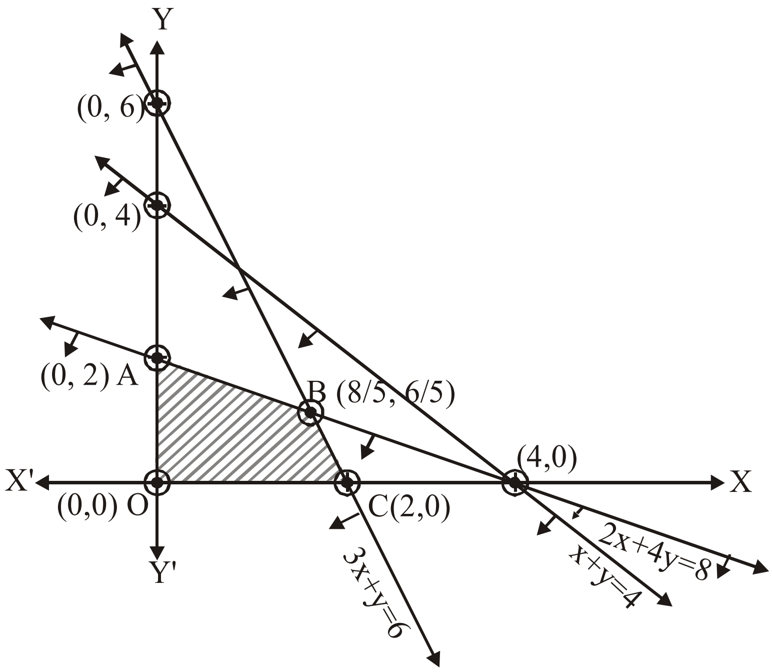
The given L.P.P. is :

Maximise $Z = 2x + 5y$

subject to the constraints

$2x + 4y \le 8,\ \ 3x + y \le 6,\ \ x + y \le 4,\ \ x \ge 0,\ \ y \ge 0$

Firstly, let us represent the given inequalities graphically and find out the feasible region of the given L.P.P.



As shown in the figure; the given feasible region is bounded, shown by OABC. And the corner-points alongwith the corresponding values of Z are shown in the table below :

Corner-points	Corresponding value of Z Z = 2x + 5y
O (0,0)	0
A (0,2)	10 (Maximum)
B (8/5,6/5)	9.2
C (2,0)	4

Hence, the maximum value of Z is 10 at the point A (0, 2) of the feasible region.